Homework IV

Deadline: 2017-11-8

1. (5 pts) Give your own example of a 4×3 orthogonal matrix $\boldsymbol{Q}$ and verify that $\boldsymbol{Q}^T\boldsymbol{Q} = \boldsymbol{I}$ but $\boldsymbol{Q}\boldsymbol{Q}^T \neq \boldsymbol{I}$, where $\boldsymbol{I}$ is an identity matrix. (**Note:** the constructed matrix can have **at most** two zero entries.)

2. (10 pts) Let
$$\boldsymbol{A} = \begin{pmatrix} -1 & -1 & -1 \\ 1 & 3 & 3 \\ -1 & -1 & 5 \\ 1 & 3 & 7 \end{pmatrix}.$$
Compute the QR factorization of $\boldsymbol{A}$ step by step using classical Gram-Schmidt and modified Gram-Schmidt, respectively. Whether the results from those two methods coincide with each other under exact arithmetic?

3. (15 pts) Write your own codes for classical Gram-Schmidt (CGS) and modified Gram-Schmidt (MGS), respectively. Test the codes for CGS and MGS on a 500×500 random matrix and repeat the tests for 30 random problem instances. Report the orthogonality error $\left\|\boldsymbol{Q}^T\boldsymbol{Q} - \boldsymbol{I}\right\|_F$ of all the 30 tests using the `semilogy` in a same figure. What is your observation? (**Hint:** In order to see the difference of the two methods, better not use high precision for your MATLAB. In addition, you may want to generate a random matrix $\boldsymbol{A}$ as follows: `A=randn(500);` `A=A'*A`.)