Homework V

Deadline: 2017-11-15

1. (10 pts) Let $\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 2 \\ 2 & -2 & 2 \\ -1 & -1 & -1 \\ 2 & 0 & 2 \end{pmatrix}$. Factorize $\boldsymbol{A}$ into the following form $\boldsymbol{A} = \boldsymbol{QL}$ step by step, where $\boldsymbol{Q}$ is a 4×3 orthogonal matrix and $\boldsymbol{L}$ is a 3×3 lower triangular matrix. (**Hint:** use an analogue of classical or modified Gram-Schmidt but start from the last column and proceed towards the first column).

2. (15 pts) Write your own codes for Householder QR and conduct random tests to show the accuracy of your codes.

3. (15 pts) Sample 30 equally spaced points $\{x_i\}_{i=1}^{30}$ over $[0, 1]$ (**Hint:** Use `linspace`) and evaluate the function $\cos(10x)$ on those points and get $\{y_i\}_{i=1}^{30}$. Fit a degree 5 polynomial through points $\{(x_i, y_i)\}_{i=1}^{30}$ (**Note:** Use the QR factorization computed from your codes for last problem to solve the least-squares problem). Plot the discretized points and your fitted polynomial on a same figure.