Homework III

Deadline: 2017-11-1

1. (5 pts) Show that the matrix $\boldsymbol{A}^T\boldsymbol{A} + \alpha\boldsymbol{I}$ is positive definite, where $\boldsymbol{A} \in \mathbb{R}^{m\times n}$, $\alpha > 0$, and $\boldsymbol{I}$ is an n by n identity matrix.

2. (10 pts) Reduce

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 8 & 7 & 9 & 5 \\ 6 & 7 & 9 & 8 \end{pmatrix}$$

   into an upper triangular matrix step by step using Gaussian elimination with partial pivoting.

3. (10 pts) Find the Cholesky decomposition of

$$\boldsymbol{A} = \begin{pmatrix} 3 & -1 & -3 & 1 \\ -1 & 7 & -3 & 7 \\ -3 & -3 & 10 & -4 \\ 1 & 7 & -4 & 9 \end{pmatrix}$$

   step by step.

4. (15 pts) Write your own for the Cholesky decomposition of a positive definite matrix $\boldsymbol{A} \in \mathbb{R}^{n\times}$, test the correctness of your codes for **at least** three different n by computing the error between $\boldsymbol{A}$ and $\boldsymbol{L}\boldsymbol{L}^T$, where $\boldsymbol{L}$ is the lower triangular matrix obtained from your codes. (**Hint:** You can generate a positive definite matrix as follows: `A = randn(n); A = A'*A.`)