

Assigned: 25 March 2020

Project #7 – Expectation Maximization

EE 511: Spring 2020

Due: Wednesday, 08 April 2020 at 14:00. Late penalty: 15% per day before 10 April at 14:00.

1. Implement a random number generator for a random vector $X = [X_1, X_2, X_3]^T$ having multivariate Gaussian distribution with

$$\mu = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & 3 \\ 1 & 3 & 4 \end{bmatrix}.$$

2. Implement a random number generator for a random variable with the following mixture distribution: $f(x) = 0.4N(-1,1) + 0.6N(1,1)$. Generate a histogram and overlay the theoretical p.d.f. of the random variable.
3. Implement a 2-dimensional random number generator for a Gaussian mixture model (GMM) pdf with 2 subpopulations. Use the expectation maximization (EM) algorithm to estimate the pdf parameters of the 2-D GMM from samples. Compare the quality and speed of your GMM-EM estimates using 300 samples from different GMM distributions (e.g. spherical vs ellipsoidal covariance, close vs well-separated subpopulations, etc.).
4. A geyser is a hot spring characterized by an intermittent discharge of water and steam. Old Faithful is a famous cone geyser in Yellowstone National Park, Wyoming. It has a predictable geothermal discharge and since 2000 it has erupted every 44 to 125 minutes. Refer to the addendum data file that contains waiting times and the durations for 272 eruptions.
 - a. Generate a 2-D scatter plot of the data. Run a k -means clustering routine on the data for $k = 2$. Show the two clusters on a scatterplot.
 - b. Use a GMM-EM algorithm to fit the dataset to a GMM pdf. Draw a contour plot of your final GMM pdf. Overlay the contour plot with a scatterplot of the data set. How can you use the GMM pdf estimates to cluster the data?