Calculus Assignment 2

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Exercise 7

When trying to find the limit towards ∞ of a rational function, we first have to divide both sides of the fraction by the highest degree of x. For this function, it is x^3, so the new function we get after this is

$$\frac{1+\frac{2}{x}+\frac{2}{x^3}}{3+\frac{1}{x^2}+\frac{4}{x^3}}$$

Now, we substitute all x's for 0, and we get $\frac{1}{3}$, which means that

$$\lim_{x\to-\infty}\frac{x^3+2x^2+2}{3x^3+x+4}=\frac{1}{3}$$

Exercise 8

The function is : $f(x) = 2x + 3$

$$\frac{2(x+h)+3-2x-3}{h}$$

$$\frac{2x+2h+3-2x-3}{h}$$

$$\frac{2h}{h}$$

Cancel the h on both sides

$$\lim_{h\to 0} = 2$$

Exercise 9

a) In order to find the tangent line of a function at any point, we must first find the derivative This function is a fraction, but because there is a 1 as the numerator, we can re-write it as $(1 + x^{-1})^{-1}$, as $\frac{1}{x} = x^{-1}$

Now, we can use the chain rule to solve this

$$-(1 + x^{-1})^{-2} \cdot -x^{-2}$$

$$-\left(\frac{1}{(1+\frac{1}{x})^2}\right) \cdot -\frac{1}{x^2}$$

$$-\frac{1}{\frac{1}{x^2} + \frac{2}{x} + 1} \cdot -\frac{1}{x^2}$$

$$\frac{1}{x^2(\frac{1}{x^2} + \frac{2}{x} + 1)}$$

$$f'(x) = \frac{1}{x^2 + 2x + 1}$$

Now that we have found the derivative, we can plug in the value of $x = 2$ to find the slope of the tangent line at that point

$$\frac{1}{(2)^2 + 2(2) + 1} = \frac{1}{9}$$

The slope of $f(x) = \frac{1}{1+\frac{1}{x}}$ at $x = 2$ is $\frac{1}{9}$

Now that we have the slope, we can use a formula to find the equation of the tangent line, but first we need a y value for when $x = 2$

$$\frac{1}{1 + \frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

We have a point $2, \frac{2}{3}$, so we can find the line

$$
\begin{aligned}
y &= mx + c \\
\left(\frac{2}{3}\right) &= \left(\frac{1}{9}\right) \cdot (2) + c \\
\frac{2}{3} &= \frac{2}{9} + c \\
c &= \frac{2}{3} - \frac{2}{9} \\
c &= \frac{4}{9}
\end{aligned}
$$

From this, we can get that the equation of the line is $y = \frac{x}{9} + \frac{4}{9}$ or $9y = x + 4$

b) We have already found the equation for the tangent line of this function which is $\frac{1}{x^2+2x+1}$, so to find the limit going to ∞, we see based on the highest degree of the polynomial, what is that limit Since the highest degree is x^2, and $\lim\limits_{x\to\infty} \frac{1}{x^2} = 0$, therefore we can see that

$$\lim_{x\to\infty} \frac{1}{x^2 + 2x + 1} = 0$$

Exercise 10

a) I will use the **Chain rule** for this problem

$$\frac{1}{\cos^2(\cos(x))} \cdot -\sin(x)$$

$$-\frac{\sin(x)}{\cos^2(\cos(x))}$$

$$f'(x) = -\frac{\sin(x)}{\cos^2(\cos(x))}$$

b) I will use the chain rule to find the first derivative of this function

$$g'(x) = -\sin(3x) * 3 = -3\sin(3x)$$

Now, I will find the first 8 derivatives to try and see a pattern

$$\begin{aligned}
g^2(x) &= -3\cos(3x) \cdot 3 = -9\cos(3x) \\
g^3(x) &= 9\sin(3x) \cdot 3 = 27\sin(3x) \\
g^4(x) &= 27\cos(3x) \cdot 3 = 81\cos(3x) \\
g^5(x) &= -81\sin(3x) \cdot 3 = -243\sin(3x) \\
g^6(x) &= -243\cos(3x) \cdot 3 = -729\cos(3x) \\
g^7(x) &= 729\sin(3x) \cdot 3 = 2187\sin(3x) \\
g^8(x) &= 2186\cos(3x) \cdot 3 = 6461\cos(3x)
\end{aligned}$$

Now, we can see that the pattern is that the coefficient is always 3^d, where d is which derivative it is, and if $d \equiv 0 \pmod 4$, than the trigonometric function is $\cos(3x)$, if $d \equiv 1 \pmod 4$, then the triginometric function is $-\sin(3x)$, if $d \equiv 2 \pmod 4$ then the triginometric function is $-\cos(3x)$, and if $d \equiv 3 \pmod 4$, then the trigonometric function is $\sin(3x)$

We need to calculate $2020 \mod 4 \equiv 0$, so therefore

$$g^{2020}(x) = 3^{2020}\cos(3x)$$

c) This will use the chain rules, as $\sqrt{x} = x^{frac12}$

$$\frac{1}{2}\left(\sin(x^2) + \cos(2x)\right)^{-\frac{1}{2}} \cdot \left[\sin(x^2) + \cos(2x)\right]'$$

Differentiate $\sin(x^2)$

$$\cos(x^2) \cdot 2x$$
$$2x\cos(x^2)$$

Differentiate $\cos(2x)$

$$-\sin(2x)\cdot 2$$
$$-2\sin(2x)$$

$$-2\sin(2x)+2x\cos(x^2)=2(x\cos(x^2)-\sin(2x))$$

Now we can put this back into the equation for the whole derivative

$$\frac{1}{2}\cdot -\sqrt{(}\sin(x^2)+\cos(2x))\cdot 2(x\cos(x^2)-\sin(2x))$$
$$\frac{(x\cos(x^2)-\sin(2x))}{\sqrt{(}\sin(x^2)+\cos(2x))}$$

Exercise 12

a) To make the equation easier, I will rewrite it as $(x^2+y^2-1)^3=x^2y^3$

Now, we can use the rules for implicit differentiation to find the equation for the slope

We differentiate both sides

$$\frac{d}{dx}(x^2+y^2-1)^3=\frac{d}{dx}(x^2y^3)$$

We use the chain rule on the left-hand side

$$3(x^2+y^2-1)^2\cdot 2x+2y\frac{dy}{dx}$$
$$3(x^4+y^4+2x^2y^2-x^2-y^2+1)\cdot\frac{dy}{dx}$$

I will use the product rule on the left hand side

$$2x\cdot y^3+x^2\cdot 3y^2\frac{dy}{dx}$$