Solutions: Decimal Expansions from Binary Numbers

Part A: Worked Example (from text)

Problem: Find the decimal expansion of the binary integer $(0101011111)_2$.

Solution:

$$(0101011111)_2 = 0 \cdot 2^9 + 1 \cdot 2^8 + 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$= 256 + 64 + 16 + 8 + 4 + 2 + 1$$

$$= 351$$

So, $\mathbf{(0101011111)_2 = (351)_{10}}$.

—

Part B: Easier Practice

Problem: Convert $(1011)_2$ to decimal.

Solution:

$$(1011)_2 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 8 + 0 + 2 + 1 = 11$$

So, $\mathbf{(1011)_2 = (11)_{10}}$.

—

Part C: Harder Practice

Problem: Convert $(11011010101)_2$ to decimal.

Solution:

$$(11011010101)_2 = 1 \cdot 2^{10} + 1 \cdot 2^9 + 0 \cdot 2^8 + 1 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$= 1024 + 512 + 0 + 128 + 64 + 0 + 16 + 0 + 4 + 0 + 1$$

$$= 1749$$

So, $\mathbf{(11011010101)_2 = (1749)_{10}}$.

—

Part D: Reflection

Binary expansions are sums of powers of 2, just like decimal expansions are sums of powers of 10. The only difference is the base. Each base-b expansion expresses a number as digits multiplied by powers of b.

Solutions: Decimal Expansion from Octal (Example 2)

Part A — Worked Example

Problem. $(7016)_8$.

Solution (place value).

$$(7016)_8 = 7 \cdot 8^3 + 0 \cdot 8^2 + 1 \cdot 8^1 + 6 \cdot 8^0 = 7 \cdot 512 + 0 \cdot 64 + 1 \cdot 8 + 6 \cdot 1$$
$$= 3584 + 0 + 8 + 6 = \mathbf{3598}.$$

Alternative (Horner).

$$(((7) \cdot 8 + 0) \cdot 8 + 1) \cdot 8 + 6 = 3598.$$

Therefore, $\boxed{(7016)_8 = (3598)_{10}}$.

Part B — Easier Practice

Problem. $(52)_8$.

Solution.

$$(52)_8 = 5 \cdot 8^1 + 2 \cdot 8^0 = 5 \cdot 8 + 2 \cdot 1 = 40 + 2 = \boxed{42}.$$

Horner check: $(5) \cdot 8 + 2 = 42$.

Part C — Harder Practice

Problem. $(574321)_8$.

Solution.

$$(574321)_8 = 5 \cdot 8^5 + 7 \cdot 8^4 + 4 \cdot 8^3 + 3 \cdot 8^2 + 2 \cdot 8^1 + 1 \cdot 8^0.$$
$$8^5 = 32768,\ 8^4 = 4096,\ 8^3 = 512,\ 8^2 = 64,\ 8^1 = 8,\ 8^0 = 1.$$
$$= 5 \cdot 32768 + 7 \cdot 4096 + 4 \cdot 512 + 3 \cdot 64 + 2 \cdot 8 + 1 \cdot 1$$
$$= 163{,}840 + 28{,}672 + 2{,}048 + 192 + 16 + 1 = \boxed{194{,}769}.$$

Horner check:

$$(((((5) \cdot 8 + 7) \cdot 8 + 4) \cdot 8 + 3) \cdot 8 + 2) \cdot 8 + 1) = 194{,}769.$$

Teaching Notes

- Emphasize base-b place value: $\sum d_i b^i$ mirrors decimal exactly.
- Encourage Horner's method for speed and fewer big intermediate sums.
- Common pitfalls: mis-ordering powers, forgetting $8^0 = 1$, and dropping a digit.

Solutions: Decimal Expansion from Hexadecimal (Example 3)

Part A — Worked Example

$$(FACE)_{16} = 15 \cdot 16^3 + 10 \cdot 16^2 + 12 \cdot 16^1 + 14 \cdot 16^0$$
$$= 61{,}440 + 2{,}560 + 192 + 14 = \boxed{64{,}206}.$$

—

Part B — Easier Practice

Problem: $(2B)_{16}$.

$$2 \cdot 16^1 + 11 \cdot 16^0 = 32 + 11 = \boxed{43}.$$

—

Part C — Easier Practice

Problem: $(7F)_{16}$.

$$7 \cdot 16^1 + 15 \cdot 16^0 = 112 + 15 = \boxed{127}.$$

—

Part D — Harder Practice

Problem: $(BEEF)_{16}$. Digits: $B = 11,\ E = 14,\ F = 15$.

$$(BEEF)_{16} = 11 \cdot 16^3 + 14 \cdot 16^2 + 14 \cdot 16^1 + 15 \cdot 16^0.$$

$$= 11 \cdot 4096 + 14 \cdot 256 + 14 \cdot 16 + 15 \cdot 1.$$

$$= 45{,}056 + 3{,}584 + 224 + 15 = \boxed{48{,}879}.$$

—

Teaching Notes

- Emphasize hex digits A–F map to 10–15.
- Point out binary shortcut: each hex digit corresponds to a 4-bit binary block.
- Common pitfalls: forgetting to expand digits above 9, or miscomputing $16^2 = 256$.

Solutions: Octal Expansion (Example 4)

Part A — Worked Example

Already shown in worksheet:

$$12345 \div 8 \Rightarrow 1543\,r1$$

$$1543 \div 8 \Rightarrow 192\,r7$$

$$192 \div 8 \Rightarrow 24\,r0$$

$$24 \div 8 \Rightarrow 3\,r0$$

$$3 \div 8 \Rightarrow 0\,r3$$

Digits: $(30071)_8$.

—

Part B — Easier Practice Solutions

1. $(25)_{10}$:

$$25 = 8 \cdot 3 + 1 \quad \Rightarrow r1$$

$$3 = 8 \cdot 0 + 3 \quad \Rightarrow r3$$

Answer: $(25)_{10} = (31)_8$.

2. $(64)_{10}$:

$$64 = 8 \cdot 8 + 0$$

$$8 = 8 \cdot 1 + 0$$

$$1 = 8 \cdot 0 + 1$$

Digits: $(100)_8$.

3. $(255)_{10}$:

$$255 = 8 \cdot 31 + 7$$

$$31 = 8 \cdot 3 + 7$$

$$3 = 8 \cdot 0 + 3$$

Digits: $(377)_8$.

—

Part C — Harder Challenge Solution

Convert $(54321)_{10}$:

$$54321 \div 8 = 6790\,r1$$
$$6790 \div 8 = 848\,r6$$
$$848 \div 8 = 106\,r0$$
$$106 \div 8 = 13\,r2$$
$$13 \div 8 = 1\,r5$$
$$1 \div 8 = 0\,r1$$

Digits (bottom-to-top): $1, 5, 2, 0, 6, 1$.

$$(54321)_{10} = (152061)_8.$$

—

Teaching Notes

- Emphasize writing quotients and remainders in columns.
- Students often forget to read remainders bottom-to-top.
- Always check by converting octal back to decimal.

Solutions: Hexadecimal Expansion (Example 5)

Part A — Worked Example

Shown step-by-step:

$$177130 \div 16 = 11070\, r10 \quad (A)$$
$$11070 \div 16 = 691\, r14 \quad (E)$$
$$691 \div 16 = 43\, r3$$
$$43 \div 16 = 2\, r11 \quad (B)$$
$$2 \div 16 = 0\, r2$$

Digits: $(2B3EA)_{16}$.
—

Part B — Easier Practice Solutions

1. $(255)_{10}$:

$$255 \div 16 = 15\, r15 \quad (F)$$
$$15 \div 16 = 0\, r15 \quad (F)$$

Answer: $(255)_{10} = (FF)_{16}$.

2. $(4095)_{10}$:

$$4095 \div 16 = 255\, r15 \quad (F)$$
$$255 \div 16 = 15\, r15 \quad (F)$$
$$15 \div 16 = 0\, r15 \quad (F)$$

Answer: $(4095)_{10} = (FFF)_{16}$.
—

Part C — Harder Challenge Solution

$$1048575 = 2^{20} - 1.$$

Successive division by 16 gives all remainders equal to 15:

$$1048575 \div 16 = 65535\, r15 \quad (F)$$
$$65535 \div 16 = 4095\, r15 \quad (F)$$
$$4095 \div 16 = 255\, r15 \quad (F)$$
$$255 \div 16 = 15\, r15 \quad (F)$$

$$15 \div 16 = 0\,r15 \quad (F)$$

Answer:

$$(1048575)_{10} = (FFFFF)_{16}.$$

—

Teaching Notes

- Highlight that A–F are digits 10–15.
- Easier problems illustrate short expansions (FF, FFF).
- The challenge problem emphasizes recognizing special forms like $2^n - 1$.

Solutions: Binary Expansion (Example 6)

Part A — Worked Example

$$
\begin{aligned}
241 \div 2 &= 120\, r1 \\
120 \div 2 &= 60\, r0 \\
60 \div 2 &= 30\, r0 \\
30 \div 2 &= 15\, r0 \\
15 \div 2 &= 7\, r1 \\
7 \div 2 &= 3\, r1 \\
3 \div 2 &= 1\, r1 \\
1 \div 2 &= 0\, r1
\end{aligned}
$$

Reading bottom-to-top: $(11110001)_2$.

—

Part B — Easier Practice Solutions

1. $(13)_{10}$:

$$13 \div 2 = 6\, r1,\ 6 \div 2 = 3\, r0,\ 3 \div 2 = 1\, r1,\ 1 \div 2 = 0\, r1.$$

Answer: $(13)_{10} = (1101)_2$.

2. $(100)_{10}$:

$$100 \div 2 = 50\, r0,\ 50 \div 2 = 25\, r0,\ 25 \div 2 = 12\, r1,\ 12 \div 2 = 6\, r0,\ 6 \div 2 = 3\, r0,\ 3 \div 2 = 1\, r1,\ 1 \div 2 = 0\, r1.$$

Answer: $(100)_{10} = (1100100)_2$.

—

Part C — Harder Challenge Solution

$(1023)_{10}$. Note: $1023 = 2^{10} - 1$.

This means the binary expansion will be ten 1's in a row.

$$(1023)_{10} = (1111111111)_2.$$

—

Teaching Notes

- Reinforce the bottom-to-top reading of remainders.
- Use powers of 2 to recognize special forms (like $2^n - 1$).
- Encourage students to double-check by recomputing in decimal.

Solutions: Conversions Between Binary, Octal, and Hexadecimal (Example 7)

Part A — Worked Example

1. $(11\ 1110\ 1011\ 1100)_2 \rightarrow$ Octal:

$$011\ 111\ 010\ 111\ 100 \quad \Rightarrow \quad 3, 7, 2, 7, 4$$

Answer: $(37274)_8$.

2. $(11\ 1110\ 1011\ 1100)_2 \rightarrow$ Hexadecimal:

$$0011\ 1110\ 1011\ 1100 \quad \Rightarrow \quad 3, E, B, C$$

Answer: $(3EBC)_{16}$.

3. $(765)_8 \rightarrow$ Binary:

$$7 = 111,\ 6 = 110,\ 5 = 101$$

Answer: $(111110101)_2$.

4. $(\mathrm{A8D})_{16} \rightarrow$ Binary:

$$A = 1010,\ 8 = 1000,\ D = 1101$$

Answer: $(101010001101)_2$.

—

Part B — Easier Problems

1. $(101101)_2$ to octal: Group into 3's: $101, 101 = 5, 5$. Answer: $(55)_8$.

2. $(47)_8$ to binary:

$$4 = 100,\ 7 = 111$$

Answer: $(100111)_2$.

—

Part C — Harder Challenge

$(111011110101)_2$.

- Octal grouping (3's): $111, 011, 110, 101 = 7, 3, 6, 5$. Answer: $(7365)_8$.
- Hex grouping (4's): $1110, 1111, 0101 = E, F, 5$. Answer: $(EF5)_{16}$.

—

Teaching Notes

- Remind students: grouping into 3's for octal, 4's for hex is the fastest way.
- Encourage them to pad with leading zeros if needed.
- Cross-check: convert both octal and hex back to binary to confirm.

Example 8: Binary Addition (Teacher's Guide)

We are adding:

$$a = (1110)_2, \quad b = (1011)_2$$

Step-by-Step Solution

$$\begin{array}{cccccc}
 & 1 & 1 & 1 & & \\
 & 1 & 1 & 1 & 0 & \\
+ & 1 & 0 & 1 & 1 & \\
\hline
 & 1 & 1 & 0 & 0 & 1
\end{array}$$

Hence:

$$(1110)_2 + (1011)_2 = (11001)_2$$

—

Practice Problem Solutions

1. $(101)_2 + (11)_2$:

$$101_2 = 5_{10}, \quad 11_2 = 3_{10}, \quad 5 + 3 = 8$$

So, $(101)_2 + (11)_2 = (1000)_2$.

2. $(111)_2 + (1)_2$:

$$111_2 = 7_{10}, \quad 1_2 = 1_{10}, \quad 7 + 1 = 8$$

So, $(111)_2 + (1)_2 = (1000)_2$.

3. $(11011)_2 + (11101)_2$:

$$11011_2 = 27_{10}, \quad 11101_2 = 29_{10}, \quad 27 + 29 = 56$$

So, $(11011)_2 + (11101)_2 = (111000)_2$.

Section 4.2 — Example 9 Teacher Manual
Counting Bit Additions in Algorithm 2

Detailed explanation (matching Example 9)

At each bit position j Algorithm 2 computes

$$d = \left\lfloor \frac{a_j + b_j + c}{2} \right\rfloor, \qquad s_j = a_j + b_j + c - 2d, \qquad c \leftarrow d.$$

Implementing the three-bit sum $a_j + b_j + c$ uses at most two one-bit additions:

$$t_1 = (a_j + b_j) \quad \text{and} \quad t_2 = t_1 + c.$$

If $c = 0$ and $a_j + b_j < 2$, the second add is effectively a no-op, so the total count is *strictly less than* $2n$. In any case, the bound $\leq 2n$ yields linear time $O(n)$.

Worked examples

A. Two-bit example: $(11)_2 + (01)_2$

j	a_j	b_j	c (in)	s_j	c (out)
0	1	1	0	0	1
1	1	0	1	0	1

$s_2 = 1$

Result: $(100)_2$. Bit additions used ≤ 4.

B. Four-bit example: $(1011)_2 + (0110)_2$

j	a_j	b_j	c (in)	s_j	c (out)
0	1	0	0	1	0
1	1	1	0	0	1
2	0	1	1	0	1
3	1	0	1	0	1

$s_4 = 1$

Result: $(10001)_2$. Bit additions used ≤ 8.

Solutions to student practice

1) $(0101)_2 + (0011)_2$

j	a_j	b_j	c (in)	s_j	c (out)
0	1	1	0	0	1
1	0	1	1	0	1
2	1	0	1	0	1
3	0	0	1	1	0

$s_4 = 0$

Answer: $(10000)_2$.

2) $(1001)_2 + (0001)_2$

j	a_j	b_j	c (in)	s_j	c (out)
0	1	1	0	0	1
1	0	0	1	1	0
2	0	0	0	0	0
3	1	0	0	1	0

$s_4 = 0$

Answer: $(1010)_2$.

3) $(11101101)_2 + (10111011)_2$

Work right-to-left; lots of carries chain through:

j	a_j	b_j	c (in)	s_j	c (out)
0	1	1	0	0	1
1	0	1	1	0	1
2	1	1	1	1	1
3	1	0	1	0	1
4	0	1	1	0	1
5	1	1	1	1	1
6	1	0	1	0	1
7	1	1	1	1	1

$s_8 = 1$

Answer: $(1\,011010000)_2 = (1011010000)_2$.

Bit-addition count bound. In all three problems, the number of one-bit additions is $< 2n$ and $\leq 2n$, so linear in the input length.

Ch 4.2 — Example 10 Teacher Solutions
Binary Multiplication via Algorithm 3

Instructor Key

Reference: Algorithm 3 (Multiplication)

For $a = (a_{n-1} \ldots a_0)_2$ and $b = (b_{n-1} \ldots b_0)_2$,

$$c_j = \begin{cases} a \text{ shifted left } j, & b_j = 1, \\ 0, & b_j = 0, \end{cases} \qquad p = \sum_{j=0}^{n-1} c_j,$$

where the sum is carried out with Algorithm 2.

Worked Example (text): $(110)_2 \times (101)_2$

Multiplier bits: $b_0 = 1$, $b_1 = 0$, $b_2 = 1$.

$$c_0 = (110)_2, \qquad c_1 = (000)_2, \qquad c_2 = (11000)_2.$$

Add (Algorithm 2):

$$\begin{array}{r} 11000 \\ 00000 \\ +00110 \\ \hline 11110 \end{array} \quad \Rightarrow \quad (11110)_2.$$

Decimal check: $6 \cdot 5 = 30$, $(11110)_2 = 30$.

Solutions to Student Practice

A. Easier

1) $(101)_2 \times (11)_2$.

$a = (101)_2$, $b = (011)_2$ (writing as three bits helps). Bits: $b_0 = 1$, $b_1 = 1$, $b_2 = 0$.

$$c_0 = (101)_2, \quad c_1 = (1010)_2, \quad c_2 = (0000)_2.$$

Add:

$$\begin{array}{r} 01010 \\ 00101 \\ \hline 01111 \end{array} \quad \Rightarrow \quad (1111)_2.$$

Check: $5 \times 3 = 15$, $(1111)_2 = 15$.

B. Harder

2) $(10011)_2 \times (1011)_2$.

$a = (10011)_2$, $b = (1011)_2$ has $b_0 = 1, b_1 = 1, b_2 = 0, b_3 = 1$.

$$\begin{aligned} c_0 &= (10011)_2, \\ c_1 &= (100110)_2, \\ c_2 &= (000000)_2, \\ c_3 &= (10011000)_2. \end{aligned}$$

Add in two stages (Algorithm 2).

Stage 1: $c_0 + c_1$:

$$\begin{array}{r} 100110 \\ +10011 \\ \hline 111001 \end{array} \quad \Rightarrow \quad (111001)_2.$$

Stage 2: add c_3 (align lengths):

$$\begin{array}{r} 10011000 \\ 00111001 \\ \hline 11010001 \end{array} \quad \Rightarrow \quad (11010001)_2.$$

Answer: $(11010001)_2$.

Decimal check: $(10011)_2 = 19$, $(1011)_2 = 11$, $19 \cdot 11 = 209$; $(11010001)_2 = 209$.

3) $(111010)_2 \times (10111)_2$.

$a = (111010)_2$, $b = (10111)_2$ with bits $b_0 = 1, b_1 = 1, b_2 = 1, b_3 = 0, b_4 = 1$.

$$\begin{aligned} c_0 &= (111010)_2, \\ c_1 &= (1110100)_2, \\ c_2 &= (11101000)_2, \\ c_3 &= (000000000)_2, \\ c_4 &= (1110100000)_2. \end{aligned}$$

Add progressively (Algorithm 2).

(i) $c_0 + c_1$:

$$\begin{array}{r} 1110100 \\ 0111010 \\ \hline 10111110 \end{array} \Rightarrow (10111110)_2.$$

(ii) add c_2:

$$\begin{array}{r} 11101000 \\ 010111110 \\ \hline 1010011110 \end{array} \Rightarrow (1010011110)_2.$$

(iii) add c_4 (align):

$$\begin{array}{r} 1110100000 \\ 001010011110 \\ \hline 0100011011110 \end{array} \Rightarrow (100011011110)_2.$$

Answer: $(100011011110)_2$.

Decimal check: $(111010)_2 = 58$, $(10111)_2 = 23$, $58 \cdot 23 = 1334$; $(100011011110)_2 = 1024+256+32+16+8-?$ (Compute): $1024+256+32+16+8+4+2 = 1342?$ — re-check.

Tidy recomputation (columns):

$(100011011110)_2 = 2^{11}+2^7+2^6+2^4+2^3+2^2+2^1 = 2048+128+64+16+8+4+2 = 2{,}270$ (too large).

So we misaligned in (ii). Let's recompute carefully with consistent widths.

Clean column addition Write all partials to 11 columns (max length of c_4 is 10 + safety):

$$\begin{array}{rl} 000111010 & (c_0) \\ 001110100 & (c_1) \\ 011101000 & (c_2) \\ 000000000 & (c_3) \\ 1110100000 & (c_4) \\ \hline \end{array}$$

Add top to bottom (carry shown conceptually):

$$\begin{array}{r} 1110100000 \\ +001110100 \\ +000111010 \\ +011101000 \\ \hline 10000111110 \end{array} \Rightarrow (10000111110)_2.$$

Now check in decimal: $(10000111110)_2 = 2^{10}+2^4+2^3+2^2+2^1 = 1024+16+8+4+2 = 1054$ — still not $58 \cdot 23$.

Better path: verify with decimal first: $58 \times 23 = 1334$. Binary of 1334:

$$1334 = 1024 + 256 + 32 + 16 + 4 + 2 \Rightarrow (10100110110)_2.$$

Let's recompute the partials *precisely*:

$a = (111010)_2$:

$$a = 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 = 58.$$

$b = (10111)_2$ with bits $b_4 = 1, b_3 = 0, b_2 = 1, b_1 = 1, b_0 = 1$.

$$\begin{aligned} c_0 &= a \cdot 2^0 = (111010)_2, \\ c_1 &= a \cdot 2^1 = (1110100)_2, \\ c_2 &= a \cdot 2^2 = (11101000)_2, \\ c_3 &= 0, \\ c_4 &= a \cdot 2^4 = (1110100000)_2. \end{aligned}$$

Now align on the right and add:

$$
\begin{array}{r}
1110100000 \\
0011101000 \\
0001110100 \\
0000000000 \\
\underline{0000111010} \\
\hline
10100110110
\end{array}
$$

Result: $(10100110110)_2$. Decimal check: $1024 + 256 + 32 + 16 + 4 + 2 = 1334$. Correct.

Answer. $(111010)_2 \times (10111)_2 = (10100110110)_2$.

Operation counts (optional talking points)

If b has k ones among n bits, Algorithm 3 forms k nonzero partial products and performs up to $k-1$ multiword additions (Algorithm 2). Each addition is $O(n)$, so multiplication is $O(kn) \subseteq O(n^2)$ in the worst case ($k \approx n$).

Example 12 Teacher's Solutions: Fast Modular Exponentiation

Worked Example

We already computed:

$$3^{544} \pmod{645} = 36$$

with full details shown in the worksheet.

—

Practice Problem Solutions

1. Easier: $2^{13} \pmod{19}$

   Binary expansion of $13 = (1101)_2$. Steps:

   $$2^1 \equiv 2,\ 2^2 \equiv 4,\ 2^4 \equiv 16,\ 2^8 \equiv 9 \pmod{19}.$$

   Multiply relevant powers: $2^8 \cdot 2^4 \cdot 2^1 \equiv 9 \cdot 16 \cdot 2 \equiv 288 \equiv 3 \pmod{19}$.

   Answer: $\boxed{3}$.

   —

2. Medium: $7^{45} \pmod{50}$

   Binary expansion of $45 = (101101)_2$. Steps:

   $$7^1 \equiv 7,\ 7^2 \equiv -1 \equiv 49 \pmod{50}.$$

   Notice $7^2 \equiv -1$. Then $7^{44} = (7^2)^{22} \equiv (-1)^{22} \equiv 1 \pmod{50}$. Multiply one more factor of 7: $7^{45} \equiv 7 \pmod{50}$.

   Answer: $\boxed{7}$.

   —

3. Harder: $11^{117} \pmod{221}$

   Note: $221 = 13 \cdot 17$. Apply the Chinese Remainder Theorem.

   Mod 13: $\varphi(13) = 12$. Reduce $117 \equiv 9 \pmod{12}$. So $11^{117} \equiv 11^9 \pmod{13}$. Compute: $11 \equiv -2 \pmod{13}$, so $(-2)^9 \equiv -512 \equiv 11 \pmod{13}$.

   Mod 17: $\varphi(17) = 16$. Reduce $117 \equiv 5 \pmod{16}$. So $11^{117} \equiv 11^5 \pmod{17}$. Compute: $11^2 = 121 \equiv 2 \pmod{17}$, $11^4 \equiv 2^2 = 4$, so $11^5 \equiv 11 \cdot 4 = 44 \equiv 10 \pmod{17}$.

   Solve CRT system:

   $$x \equiv 11 \pmod{13}, \quad x \equiv 10 \pmod{17}.$$

   Answer: $\boxed{142}$.