Solutions: Binary Expansion (Example 6)

Part A — Worked Example

$$\begin{aligned}
241 \div 2 &= 120\, r1 \\
120 \div 2 &= 60\, r0 \\
60 \div 2 &= 30\, r0 \\
30 \div 2 &= 15\, r0 \\
15 \div 2 &= 7\, r1 \\
7 \div 2 &= 3\, r1 \\
3 \div 2 &= 1\, r1 \\
1 \div 2 &= 0\, r1
\end{aligned}$$

Reading bottom-to-top: $(11110001)_2$.

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Part B — Easier Practice Solutions

1. $(13)_{10}$:

$$13 \div 2 = 6\, r1,\ 6 \div 2 = 3\, r0,\ 3 \div 2 = 1\, r1,\ 1 \div 2 = 0\, r1.$$

Answer: $(13)_{10} = (1101)_2$.

2. $(100)_{10}$:

$$100 \div 2 = 50\, r0,\ 50 \div 2 = 25\, r0,\ 25 \div 2 = 12\, r1,\ 12 \div 2 = 6\, r0,\ 6 \div 2 = 3\, r0,\ 3 \div 2 = 1\, r1,\ 1 \div 2 = 0\, r1.$$

Answer: $(100)_{10} = (1100100)_2$.

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Part C — Harder Challenge Solution

$(1023)_{10}$. Note: $1023 = 2^{10} - 1$.

This means the binary expansion will be ten 1's in a row.

$$(1023)_{10} = (1111111111)_2.$$

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Teaching Notes

- Reinforce the bottom-to-top reading of remainders.
- Use powers of 2 to recognize special forms (like $2^n - 1$).
- Encourage students to double-check by recomputing in decimal.