

Worksheet: Decimal Expansion from Octal (Example 2)

Goal

Convert an octal (base 8) numeral into its decimal (base 10) value using place value.

Part A — Worked Example (detailed)

Problem. What is the decimal expansion of $(7016)_8$?

Idea. In base 8, each position is a power of 8: from right to left $8^0, 8^1, 8^2, 8^3, \dots$. For digits $d_3 d_2 d_1 d_0$ we have:

$$(d_3 d_2 d_1 d_0)_8 = d_3 \cdot 8^3 + d_2 \cdot 8^2 + d_1 \cdot 8^1 + d_0 \cdot 8^0.$$

Step 1: Label digits and place values.

$$(7016)_8 = 7 \cdot 8^3 + 0 \cdot 8^2 + 1 \cdot 8^1 + 6 \cdot 8^0.$$

Step 2: Evaluate powers of 8.

$$8^3 = 512, \quad 8^2 = 64, \quad 8^1 = 8, \quad 8^0 = 1.$$

Step 3: Multiply digits by powers.

$$7 \cdot 512 = 3584, \quad 0 \cdot 64 = 0, \quad 1 \cdot 8 = 8, \quad 6 \cdot 1 = 6.$$

Step 4: Add the contributions.

$$3584 + 0 + 8 + 6 = 3598.$$

Conclusion. $\boxed{(7016)_8 = (3598)_{10}}$

(Optional) Horner's Method (left-to-right accumulate).

$$(((7) \cdot 8 + 0) \cdot 8 + 1) \cdot 8 + 6 = (56 \cdot 8 + 1) \cdot 8 + 6 = (449) \cdot 8 + 6 = 3598.$$

Same answer, fewer big numbers along the way.

Part B — Easier Practice

Convert $(52)_8$ to decimal. Show all steps (place values, multiply, add).

Work:

$$\begin{aligned} (52)_8 &= \underline{\hspace{2cm}} \cdot 8^1 + \underline{\hspace{2cm}} \cdot 8^0 \\ &= \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \end{aligned}$$

Part C — Harder Practice

Convert $(574321)_8$ to decimal. *Hint:* write powers $8^5, 8^4, 8^3, 8^2, 8^1, 8^0$ first.

Work setup:

$$(574321)_8 = 5 \cdot 8^5 + 7 \cdot 8^4 + 4 \cdot 8^3 + 3 \cdot 8^2 + 2 \cdot 8^1 + 1 \cdot 8^0$$

$$8^5 = \underline{\hspace{2cm}}, \quad 8^4 = \underline{\hspace{2cm}}, \quad 8^3 = \underline{\hspace{2cm}}, \quad 8^2 = \underline{\hspace{2cm}}, \quad 8^1 = \underline{\hspace{2cm}}, \quad 8^0 = \underline{\hspace{2cm}}$$

Quick Self-Check

Why does the method above look exactly like the base-10 method, except with 8 instead of 10?