Solutions: Decimal Expansion from Octal (Example 2)

Part A — Worked Example

Problem. $(7016)_8$.

Solution (place value).

$$(7016)_8 = 7 \cdot 8^3 + 0 \cdot 8^2 + 1 \cdot 8^1 + 6 \cdot 8^0 = 7 \cdot 512 + 0 \cdot 64 + 1 \cdot 8 + 6 \cdot 1$$

= $3584 + 0 + 8 + 6 = 3598$.

Alternative (Horner).

$$(((7) \cdot 8 + 0) \cdot 8 + 1) \cdot 8 + 6 = 3598.$$

Therefore, $(7016)_8 = (3598)_{10}$.

Part B — Easier Practice

Problem. $(52)_8$.

Solution.

$$(52)_8 = 5 \cdot 8^1 + 2 \cdot 8^0 = 5 \cdot 8 + 2 \cdot 1 = 40 + 2 = \boxed{42}.$$

Horner check: $(5) \cdot 8 + 2 = 42$.

Part C — Harder Practice

Problem. $(574321)_8$.

Solution.

$$(574321)_8 = 5 \cdot 8^5 + 7 \cdot 8^4 + 4 \cdot 8^3 + 3 \cdot 8^2 + 2 \cdot 8^1 + 1 \cdot 8^0.$$

$$8^5 = 32768, \ 8^4 = 4096, \ 8^3 = 512, \ 8^2 = 64, \ 8^1 = 8, \ 8^0 = 1.$$

$$= 5 \cdot 32768 + 7 \cdot 4096 + 4 \cdot 512 + 3 \cdot 64 + 2 \cdot 8 + 1 \cdot 1$$

$$= 163,840 + 28,672 + 2,048 + 192 + 16 + 1 = \boxed{194,769}.$$

Horner check:

$$(((((((5) \cdot 8 + 7) \cdot 8 + 4) \cdot 8 + 3) \cdot 8 + 2) \cdot 8 + 1) = 194,769.$$

Teaching Notes

- Emphasize base-b place value: $\sum d_i b^i$ mirrors decimal exactly.
- Encourage Horner's method for speed and fewer big intermediate sums.
- Common pitfalls: mis-ordering powers, forgetting $8^0 = 1$, and dropping a digit.