Solutions: Decimal Expansions from Binary Numbers

Part A: Worked Example (from text)

Problem: Find the decimal expansion of the binary integer $(0101011111)_2$.

Solution:

$$(0101011111)_2 = 0\cdot 2^9+1\cdot 2^8+0\cdot 2^7+1\cdot 2^6+0\cdot 2^5+1\cdot 2^4+1\cdot 2^3+1\cdot 2^2+1\cdot 2^1+1\cdot 2^0$$

$$= 256 + 64 + 16 + 8 + 4 + 2 + 1$$

$$= 351$$

So, $(\mathbf{0101011111})_{\mathbf{2}} = (\mathbf{351})_{\mathbf{10}}$.

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Part B: Easier Practice

Problem: Convert $(1011)_2$ to decimal.

Solution:

$$(1011)_2 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 8 + 0 + 2 + 1 = 11$$

So, $(\mathbf{1011})_{\mathbf{2}} = (\mathbf{11})_{\mathbf{10}}$.

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Part C: Harder Practice

Problem: Convert $(11011010101)_2$ to decimal.

Solution:

$$(11011010101)_2 = 1\cdot 2^{10}+1\cdot 2^9+0\cdot 2^8+1\cdot 2^7+1\cdot 2^6+0\cdot 2^5+1\cdot 2^4+0\cdot 2^3+1\cdot 2^2+0\cdot 2^1+1\cdot 2^0$$

$$= 1024 + 512 + 0 + 128 + 64 + 0 + 16 + 0 + 4 + 0 + 1$$

$$= 1749$$

So, $(\mathbf{11011010101})_{\mathbf{2}} = (\mathbf{1749})_{\mathbf{10}}$.

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Part D: Reflection

Binary expansions are sums of powers of 2, just like decimal expansions are sums of powers of 10. The only difference is the base. Each base-b expansion expresses a number as digits multiplied by powers of b.