Section 5.1.2 – The Principle of Mathematical Induction

SWOSU Discrete Structures

The Principle of Mathematical Induction

Mathematical induction is like proving that a line of dominoes will all fall over. To show this:

1. You knock over the **first domino** (the **basis step**).
2. You show that **whenever one domino falls**, it knocks over the next (**inductive step**).

Once those two steps are shown, every domino falls, every ladder rung is reached, and every integer gets its proof hug.

Expressed formally:

$$\big(P(1) \wedge \forall k(P(k) \to P(k+1))\big) \to \forall n\, P(n)$$

Try It Yourself

Statement: $1 + 3 + 5 + \cdots + (2n-1) = n^2$

a) **Base Case:** Prove the formula works when $n = 1$.

b) **Inductive Hypothesis:** Assume the formula works for some k.

c) **Inductive Step:** Prove it works for $k + 1$ by using your hypothesis.

d) **Conclusion:** What does this mean for all positive integers n?

Historical Note

The method dates back to the 1500s with Francesco Maurolico — a mathematician who proved things before proofs were cool. He showed, for example, that the sum of the first n odd integers equals n^2 (yep, the problem you just did).

Visual Thinking Challenge

Draw a quick sketch of dominoes labeled 1 through 6. Show which two statements ($P(1)$ and $P(k) \to P(k+1)$) guarantee the entire chain falls.