

Ch 4.2 — Example 10 Worksheet

Binary Multiplication via Algorithm 3

Discrete Structures

Goal

Multiply two positive integers given in binary using **Algorithm 3 (Multiplication of Integers)**:

- For each bit b_j of the multiplier b , form a partial product c_j :

$$c_j = \begin{cases} a \text{ shifted left } j \text{ places,} & \text{if } b_j = 1, \\ (0)_2, & \text{if } b_j = 0. \end{cases}$$

- Add all partial products using **Algorithm 2 (binary addition)**.

Worked Example (from the text)

Find the product of $a = (110)_2$ and $b = (101)_2$.

Step 1 — Identify bits of the multiplier

$b = (101)_2$ has bits (from least significant to most) $b_0 = 1$, $b_1 = 0$, $b_2 = 1$.

Step 2 — Form partial products

Shift $a = (110)_2$ by j places when $b_j = 1$.

$$b_0 = 1 : c_0 = (110)_2 \cdot 2^0 = (110)_2.$$

$$b_1 = 0 : c_1 = (000)_2 \quad (\text{all zeros}).$$

$$b_2 = 1 : c_2 = (110)_2 \cdot 2^2 = (11000)_2.$$

Step 3 — Add the partial products (Algorithm 2)

Pad with initial zeros to align columns and add:

$$\begin{array}{r} 11000 \\ 00000 \\ + 00110 \\ \hline 11110 \end{array} \Rightarrow (11110)_2.$$

Step 4 — Quick decimal check (optional)

$(110)_2 = 6$, $(101)_2 = 5$, and $6 \cdot 5 = 30$, while $(11110)_2 = 16 + 8 + 4 + 2 = 30$. Checks out.

Answer. $(110)_2(101)_2 = (1\ 1110)_2$.

Student Practice

Show clear partial products and use Algorithm 2 for each addition.

A. Easier (warm-up)

1) $(101)_2 \times (11)_2$

B. Harder (challenge)

2) $(10011)_2 \times (1011)_2$

(two 1s close together; more carries)

3) $(111010)_2 \times (10111)_2$

(longer; many partials)

Space for work: