

Worksheet: The Division Algorithm

The Division Algorithm is a way to describe what happens when we divide one integer by another positive integer.

Definition (The Division Algorithm): Let a be an integer and d a positive integer. Then there exist unique integers q (the quotient) and r (the remainder), such that:

$$a = dq + r, \text{ with } 0 \leq r < d.$$

Here's what the variables mean:

- a : the dividend (the number being divided).
- d : the divisor (the positive number we are dividing by).
- q : the quotient (how many times d fits into a).
- r : the remainder (what's left over after dividing).

Pronunciations:

- Dividend: 'DIV-uh-dend'
- Divisor: 'dih-VY-zor'
- Quotient: 'KWO-shunt'
- Remainder: 'rih-MAYN-der'

Examples:

Example 1: Find the quotient and remainder when 101 is divided by 11.

Work it out: $101 = 11 * q + r$

Try this on your own!

Example 2: Find the quotient and remainder when -11 is divided by 3.

Work it out: $-11 = 3 * q + r$, with $0 \leq r < 3$

Try this on your own!

Your Turn! Practice Problems:

- Find the quotient and remainder when 123 is divided by 7.
- Find the quotient and remainder when -25 is divided by 4.
- Find the quotient and remainder when 250 is divided by 13.

Python and the Modulo Operator

In Python, we can use the % symbol to compute the remainder of a division.

Example:

```
>>> 101 % 11 # remainder when 101 is divided by 11
```

```
2
```

```
>>> -11 % 3 # remainder when -11 is divided by 3
```

```
1
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