Section 5.1.3 – Strong Induction: Proving the Unprovable (Almost)

SWOSU Discrete Structures

When Regular Induction Isn't Enough

Sometimes one domino can't knock down the next — you need a running start! That's where **strong induction** swoops in like a proof superhero.

Formally, if:

$$\big(P(1) \wedge P(2) \wedge \cdots \wedge P(k) \Rightarrow P(k+1)\big)$$

then we can conclude $P(n)$ is true for all $n \geq 1$.

Try It Yourself: Breaking Down Integers

Statement: Every integer $n > 1$ can be written as a product of primes.

a) **Base Case:** Prove it for $n = 2$. (Spoiler: it's prime.)

b) **Inductive Hypothesis:** Assume all integers $2, 3, \ldots, k$ can be written as products of primes.

c) **Inductive Step:** Show that $k + 1$ can also be written as a product of primes.

d) **Reflect:** Why do we need *strong* induction here instead of ordinary induction?

Strong Induction in the Wild

If you can climb any rung below you, not just the last one, you'll always make it higher. This is how mathematicians prove things like:

- Every amount of postage above 12¢ can be made with 4¢ and 5¢ stamps.
- Every integer greater than 1 has a prime factorization.
- Every coffee cup becomes a donut if you stretch it gently enough (okay, that's topology, but still fun).