

# Teacher Solutions – Section 5.1.3: Strong Induction

SWOSU Discrete Structures

## **Solution: Every Integer $\geq 1$ is a Product of Primes**

**Base Case:**  $n = 2$  is prime — true.

**Inductive Hypothesis:** Assume that for all integers  $2 \leq n \leq k$ , each can be written as a product of primes.

**Inductive Step:** For  $k + 1$ :

- If  $k + 1$  is prime, we're done.
- If  $k + 1 = ab$  where  $2 \leq a, b \leq k$ , then both  $a$  and  $b$  are products of primes by hypothesis. Therefore,  $k + 1 = ab$  is a product of primes.

Hence, by strong induction, every integer  $n > 1$  can be expressed as a product of primes.

## **Instructor Notes**

- Emphasize the difference between \*regular\* and \*strong\* induction. Students should see that strong induction assumes all previous cases, not just one. - Great demo: Use Jenga blocks as “integers” — show that pulling one requires all below it to be solid. - Encourage playful examples (pizza slices, staircases, Pokémon evolutions).