Teacher Solutions: Induction Worksheet

SWOSU Discrete Structures

Solution: The Ladder Proof

Base Case: For $n = 1$, LHS $= 1$, RHS $= \frac{1(1+1)}{2} = 1$. Works.
Inductive Step: Assume for some k,

$$1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}.$$

Then for $k + 1$:

$$1 + 2 + 3 + \cdots + k + (k+1) = \frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2}.$$

Thus, by induction, the statement holds for all positive integers n.

Solution: Divisibility by 3

For $n^3 - n$:
$$n^3 - n = n(n-1)(n+1),$$

which is the product of three consecutive integers — one of them must be divisible by 3. Therefore, $n^3 - n$ is divisible by 3 for all $n \in \mathbb{Z}^+$.

Instructor Note:

Make this lesson interactive — have students build a "ladder of proofs" on the board. Each rung = one n. Then ask who can "climb" to the next.