# Example 12 Worksheet: Fast Modular Exponentiation

**Problem:** Use Algorithm 5 to find $3^{544}$ (mod 645).

## Step-by-Step Solution

Algorithm 5 works by:

1. Writing the exponent (544) in binary.

2. Iterating through the bits, squaring at each step, and multiplying only when the current bit is 1.

3. Reducing by the modulus after every multiplication.

Binary form of 544: $(1000100000)_2$.
Steps (values of $i$, $x$, and *power*):

- $i = 0$: $a_0 = 0$, $x = 1$, *power* $= 3^2$ (mod 645) $= 9$.

- $i = 1$: $a_1 = 0$, $x = 1$, *power* $= 9^2$ (mod 645) $= 81$.

- $i = 2$: $a_2 = 0$, $x = 1$, *power* $= 81^2$ (mod 645) $= 111$.

- $i = 3$: $a_3 = 0$, $x = 1$, *power* $= 111^2$ (mod 645) $= 66$.

- $i = 4$: $a_4 = 0$, $x = 1$, *power* $= 66^2$ (mod 645) $= 486$.

- $i = 5$: $a_5 = 1$, $x = (1 \cdot 486)$ (mod 645) $= 486$, *power* $= 486^2$ (mod 645) $= 126$.

- $i = 6$: $a_6 = 0$, $x = 486$, *power* $= 126^2$ (mod 645) $= 396$.

- $i = 7$: $a_7 = 0$, $x = 486$, *power* $= 396^2$ (mod 645) $= 81$.

- $i = 8$: $a_8 = 0$, $x = 486$, *power* $= 81^2$ (mod 645) $= 111$.

- $i = 9$: $a_9 = 1$, $x = (486 \cdot 111)$ (mod 645) $= 36$, *power* $= 111^2$ (mod 645) $= 36$.

Final answer: $3^{544}$ (mod 645) $= \boxed{36}$.

## Practice Problems

1. Easier: Compute $2^{13}$ (mod 19) using Algorithm 5.

2. Medium: Compute $7^{45}$ (mod 50) using Algorithm 5.

3. Harder: Compute $11^{117}$ (mod 221) using Algorithm 5.