

Worksheet: Arithmetic Modulo m

In this worksheet, we explore arithmetic modulo m . These are operations done with remainders when dividing by m . We'll practice using definitions, examples, and exercises to build understanding.

Key Definitions

Addition Modulo m : $a \oplus b = (a + b) \bmod m$

Multiplication Modulo m : $a \otimes b = (a \times b) \bmod m$

Example 8 (from the text):

Compute $7 \oplus 11$ and $7 \otimes 11$ in \mathbb{Z}_{12} .

Solution steps are provided in the instructor manual. Use this space to show your own work.

Practice Example (Easier):

Compute $3 \oplus 7$ in \mathbb{Z}_8 and $3 \otimes 7$ in \mathbb{Z}_8 .

Practice Example (Harder):

Compute $(12 \otimes 15) \oplus 9$ in \mathbb{Z}_{17} .

Exploring Properties with Real Numbers

1. Closure: Show examples with integers (e.g., $2+3$, 4×7).
2. Associativity: Work with numbers like $(2+3)+4$ and $2+(3+4)$.
3. Commutativity: Try $5+7$ vs $7+5$ and 3×8 vs 8×3 .
4. Identity: Identify 0 and 1 as additive and multiplicative identities.
5. Additive Inverse: Find the opposite number that sums to zero (e.g., 5 and -5).
6. Distributivity: Check $(2 \times (3+4))$ vs $(2 \times 3 + 2 \times 4)$.

Reflection: How does modular arithmetic compare with ordinary arithmetic? Why do you think these properties are useful when working with modular arithmetic?