

Teacher Solutions – Section 5.1.2: The Principle of Mathematical Induction

SWOSU Discrete Structures

Solution: Sum of Odd Integers Equals n^2

Base Case: For $n = 1$, LHS = 1, RHS = $1^2 = 1$. True.

Inductive Hypothesis: Assume for $n = k$,

$$1 + 3 + 5 + \cdots + (2k - 1) = k^2.$$

Inductive Step: Then for $n = k + 1$:

$$1 + 3 + 5 + \cdots + (2k - 1) + (2(k + 1) - 1) = k^2 + (2k + 1) = (k + 1)^2.$$

Therefore, by induction, the statement holds for all positive integers n .

Instructor Notes:

- Reinforce that $P(k)$ is *assumed true only for one integer k* , not all integers.
- Emphasize that this is not circular reasoning.
- The domino and ladder metaphors are helpful mental models—keep them visual.
- Let students explain the process in their own metaphors (stairs, cookies, chain reactions, etc.)