Chapter 5.1: Mathematical Induction – The Infinite Ladder of Logic

SWOSU Discrete Structures

Introduction: Climbing the Infinite Ladder

Suppose we have an **infinite ladder**. You're standing at the bottom, holding your coffee in one hand and your sense of dread in the other. You know two things:

1. You can reach the first rung. (Victory!)
2. If you can reach any rung k, then you can also reach the next rung $k + 1$.

By pure logic (and maybe caffeine), you can conclude that you'll eventually reach every rung of that ladder. *This, my friend, is the essence of mathematical induction.*

Student Challenge

Prove that for all positive integers n:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Hint: Think of this as a staircase to mathematical greatness. Start small (the base case), then prove you can keep going (the inductive step).

a) **Base Case:** Prove it works for $n = 1$. (You can do this in your sleep.)

b) **Inductive Step:** Assume it works for some k, and show it must work for $k + 1$.

c) **Victory Lap:** Conclude that it works for all n. Then treat yourself to a cookie.

Example: Inductive Thinking in the Wild

Let's prove that $n^3 - n$ is divisible by 3 for all positive integers n.
Try it yourself: test $n = 1, 2, 3$ — see the pattern? Now imagine the induction domino effect at work — if it's true for one, it's true for the next, and so on.

Reflect

Where in life have you ever used an "inductive" idea — believing something is true because it's true in small cases and keeps working? (*Hint:* "My coffee cup always empties itself if I keep drinking" counts.)