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Performance Analysis of ZF and MMSE Receiver Algorithms

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Abstract
The use of multiple input-multiple output (MIMO) system is rapidly becoming the new leading edge technology of wireless communication systems with increase in capacity and spectral efficiency through the use of multiple antennas at both transmitter and receiver. As there are multiple link(antennas)MIMO technology can be employed for diversity gain it means most robustness to fading. MIMO not only gives the diversity gain but also increase the data rate by transmitting several information stream in parallel at same transmit power. The main challenge in the practical realization of MIMO wireless system lies in the efficient implementation of receiver which needs to separate parallel data stream. To achieve this, different linear and non linear algorithms are reported in the literature. This paper discusses the development in the various linear receiver algorithms and their performance.

Key Words - MIMO, Wireless communication, ZF, MMSE, Linear Receiver, Spatial multiplexing.

1. Introduction

The performance improvement resulting from MIMO wireless technology comes at the cost of increasing computational complexity in the receiver. The design of low complexity receiver is therefore one of the key problem in MIMO wireless system design. Basically the computational complexity of a MIMO detection algorithm depends on the symbol constellation size and the number of spatially multiplex data streams, but often also on instantaneous MIMO channels realization and signal to noise ratio. On the other hand the overall decoding effort is typically constrained by the system band width, the latency requirement and the search to keep chip area and power consumption as low as possible.

Before describing the performance and computational complexity of the linear MIMO detection algorithms the system model for a narrow band MIMO link is introduced.

A. Narrowband MIMO system Model

The System model considered has MT transmit and M R receive antennas with M R $\geqslant$ MT denoted as MR x MT. The transmitted symbols are taken independently from a quadrature amplitude modulation (QAM) constellation of P points and the M R dimensional received signal vector, using matrix notation is given by

$$y = Hs + n \quad (1)$$

Where H denotes the MR X MT channel matrix, s = [s1 s2 ………sMT] T is the MT dimensional transmit signal vector, and n stands for the MR –dimensional additive independent identically distributed circularly symmetric complex Gaussian noise vector [1].

B. Spatial Multiplexing

In a MIMO system both transmit and receive antenna combine to give a large diversity order. In which spatial diversity gain can be obtained when multiple antenna are present at either the transmit or the receive side. In a spatial multiplexing system [2,3] the data stream to be multiplexed into MT lower rate stream which are then simultaneously sent from the MT transmit antennas after coding and modulation and all the transmitted streams occupy the same frequency band(i.e. they are co-channel signals). At the receiver side, each receive antenna observes a superposition of the transmitted signals. The receiver then separates them into constituent data streams and remultiplexes them to recover the original data stream. And this separation step at the receiver needs to determine the performance and computational complexity of the receiver. Algorithms to separate the parallel data streams corresponding to the MT transmit antenna can be divided into four categories. Next sections discuss these algorithms, their performance and computational complexity .It includes mainly the two linear receivers and their performance comparison by considering the various parameters.

2. Linear Receiver Algorithms

In linear receivers, the received signal vector y is linearly transformed by a matrix equalizer that basically undoes the effect of the channel H to obtain an estimate of the transmitted symbol vector $\hat{s}$.The matrix equalizer can be computed according to different criteria.

A. Zero Forcing (ZF) Receiver Algorithm

As given in [4, 5] a simple linear receiver is the zero-forcing (ZF) receiver which basically inverts the channel transfer matrix, i.e., assuming that H is invertible an estimate of the MT x1 transmitted data symbol vector s is obtained as

$$\hat{s} = H^{-1} y \tag{2}$$

The ZF receiver hence perfectly separates the co-channel signals si (i= 0, 1,…, MT -1).For ill conditioned H, the ZF receiver performs well in high SNR regime, whereas in the low SNR regime there will be significant noise enhancement. The zero forcing criteria used in the receiver have the disadvantage that the inverse filter may excessively amplify noise at frequencies where folded channel spectrum has high attenuation. The ZF equalizer thus suffers from noise enhancement since it focuses on canceling the effects of the channel response at the expense of enhancing the noise, and is not often used for wireless link, and it also has poor bit error rate performance [5].

B. Minimum Mean Square Error (MMSE) Receiver Algorithm

An alternative linear receiver is a minimum mean square error (MMSE) receiver, which minimizes the overall error due to noise and mutual interference. For the MMSE the estimate of signal vector s is obtained according to

$$\hat{s} = \frac{\rho}{M_T} H^H \left(\sigma_n I_{MR} + \frac{\rho}{M_T} HH^T \right)^{-1} r \tag{3}$$

Where IMR is the mutual Information between the transmitter and receiver.The MMSE receiver is less sensitive to noise at the cost of reduced signal separation quality. In the high SNR case the MMSE receiver converges to the ZF receiver [4, 5].

C. Performance Analysis and Associated Complexity of ZF and MMSE Algorithms

In [7]-[9], the asymptotic performance of linear receiver in MIMO fading channels had been discuss by considering two cases. First one is for fixed number of antennas, the limit of error probability in the high–signal to noise ratio (SNR regime) in terms of the diversity–multiplexing tradeoff (DMT), second is the error probability for fixed SNR in the regime of large (but finite) number of antennas.

In comparison with the above two cases, as per as DMT is concerned, the ZF and MMSE receiver achieve the same (DMT), which is largely suboptimal even in the case where outer coding and decoding is performed across the antennas whereas behavior of the ZF and MMSE receivers at finite rate and non asymptotic SNR, is different. The ZF receiver achieves poor diversity at any finite rate, the MMSE receiver error curve slope flattens out progressively, as the coding rate increases. If the second case is considered i.e. when SNR is fixed and the numbers of antennas become large, the mutual information at the output of ZF and MMSE linear receiver has fluctuations that converge in distribution to a Gaussian random variable whose mean and variance can be characterized in closed form.

Based on the analysis of [7], [9] to achieve a required target spectral efficiency at a given block error rate and SNR operating point, an attractive design option may consist of increasing the no. of antennas (especially at the receiver) and using a low complexity linear receiver.

In [10], the authors consider a MMSE receiver and develop receive antenna selection algorithms to maximize the channel capacity, which again need not be optimal as far as error performance is concerned. In fact, such schemes perform only slightly better than deterministic antenna selection. Therefore, it is essential to model antenna selection problem with the aim of minimizing the error rate of a link, taking receiver processing into consideration this issue is appeared in [11,13]. In [12], authors proposed transmit antenna selection strategy for ZF receivers to moderate the effect of transmit antenna correlation. The suggested algorithm pre-determines the subset of antennas to use, exploiting a priori knowledge of transmit correlation matrix at the transmitter. However, the proposed algorithm fails to exploit the transmit diversity gain that could be leveraged by selecting transmit antennas that exploit the current state of the fading channel. In [12] the authors also presented a transmit antenna selection algorithm for ZF receivers and later in [13] extended it to lattice-reduction-aided (LRA) ZF receivers, which have additional complexity, compared to traditional ZF receivers. The authors also proposed an approximate selection rule for LRA-MMSE receivers based on the minimization of maximum mean square error [Eq (24), 13]. The analysis in [11]-[13] is limited to single user and in [13], [14] antenna selection is considered only at the transmitter. Moreover, no exact analysis is provided for the MMSE receiver, which is known to have much superior performance than the ZF receiver at low and moderate SNR [15].

In [16], the authors consider the case in which the base station uses the linear minimum mean-square-error (MMSE) detector in order to detect the signals from the number of users. The performance objective for this receiver is the normalized MSE and main attempt is to extent the results of [17] for the MIMO transmission model and compute the normalized MSE as a function of the transmit covariance matrices and power allocation of the users and minimize the average MSE under a sum power constraint and under individual power constraints. In addition to this, the authors derive the optimization problem that balances the MSE requirements of users. The individual MSE are functions of the transmit covariance matrices and the achievable individual MSE region has been analyzed and It has been shown that the achievable MSE region is convex for the two user MIMO scenario. In [17] the linear MMSE multiuser receiver for synchronous code-division multiple-access (CDMA) systems was analyzed.

In [18], the linear precoder which minimizes the bit error rate (BER) was derived for block transmission systems in which zero-forcing equalization and threshold detection techniques were used .The authors found the analytic solutions for the optimal precoder by using two standard schemes of eliminating inter-block interference (namely, zero-padded and cyclic-prefix schemes). It was found that the CP transmission scheme with the MBER precoding can be regarded as variation on standard DMT in which MMSE power loading replaces water-filling power loading, and the diagonal power loading matrix is post-multiplied by a DFT matrix and a flexible scheme for dropping channels to ensure the validity of the analytic solution was proposed.

In [19], the authors concluded that, the MMSE receiver is doing the best compromising between noise amplification and noise enhancement by selecting an appropriate linear combination of ZF receivers. However, it does not so in view of error-rate minimization but merely in order to minimize the total interference plus noise power. To closely approach the behavior of the Minimum Error Rate receiver, the residual noise plus interference should be Gaussian distributed. This assumption is acceptable in multiuser detection with a large number of users but not in typical MIMO antenna systems.In fact the implicit Gaussian assumption made in the MMSE receiver is a negative one, since Gaussian noise is generally known to be the worst possible additive noise.

New algorithms for the construction of approximate minimum-error rate linear MIMO receivers have been introduced by [20].While introducing this 2 X 2 MIMO system was initially considered. In the general case of no of transmit and receive the authors extended the method using cascaded MMSE and MER-based receivers. The MER-based receiver was applied on selected pairs of inputs so as to minimize the probability of error. The advantage of applying the receiver on selected pairs of inputs over existing linear MMSE-type receivers is better with respect to ill-conditioned MIMO channel matrix arising in correlated fading circumstances. The advantage of the minimum error rate approach is increased in the case of unequal input power allocation as predicted multiuser detection theory.

In [21], the downlink capacity of interference limited MIMO cellular systems operating in fading channels has been discussed. It has been found that among the various multiuser detection techniques, linear MMSE multiuser detection have been shown to be feasible and effective .The success of linear MMSE processing arises because of its ability to suppress interference and also its ability of producing Gaussian like interference [22].

3. Conclusion

In this paper the performance of the zero forcing and the minimum mean square error linear receiver algorithm has been studied. Basically in linear receivers, the received signal vector is linearly transformed by a matrix equalizer and all the layers are detected jointly. While using the zero forcing approach it has been found that it suffers from noise enhancement since it focuses on cancelling the effects of the channel response at the expense of enhancing the noise, and is not often used for wireless link, and it also has poor bit error rate performance whereas the MMSE receiver is less sensitive to noise at the cost of reduced signal separation quality. Aamong the two linear receiver algorithms i.e. ZF and MMSE, it has been found that MMSE is having better performance than the ZF in the ill condition SNR regime.

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