COMPUTER ENGINEERING

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ADVANCED DIGITAL SIGNAL PROCESSING
Chapter 6: MIMO Encoding/Decoding

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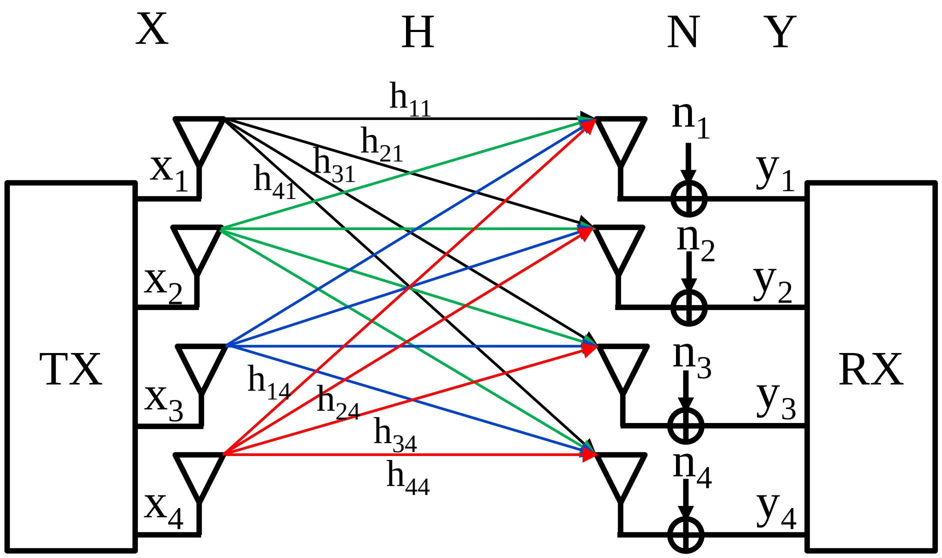
Content

- Introduction
- Spatial Multiplexing
  - ZF (Zero-Forcing)
  - MMSE (Minimum Mean Square Error)
  - MLD (Maximum Likelihood Detection)
  - K-Best
- Spatial Diversity
  - STBC

Introduction

MIMO System Model

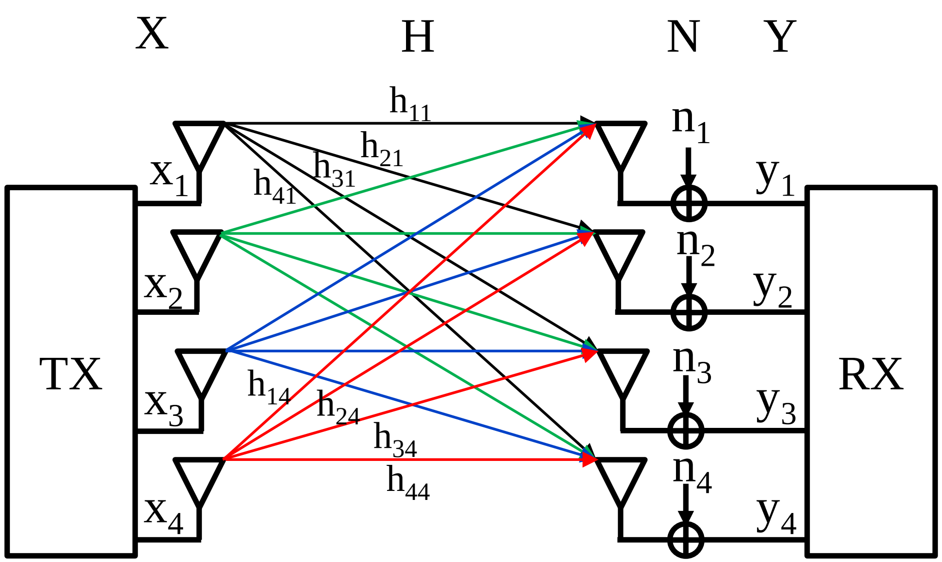


$$Y = HX + N$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix}$$

Introduction

MIMO System Model



$$Y = HX + N$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} h_{11} \times x_1 + h_{12} \times x_2 + h_{13} \times x_3 + h_{14} \times x_4 + n_1 \\ h_{21} \times x_1 + h_{22} \times x_2 + h_{23} \times x_3 + h_{24} \times x_4 + n_2 \\ h_{31} \times x_1 + h_{32} \times x_2 + h_{33} \times x_3 + h_{34} \times x_4 + n_3 \\ h_{41} \times x_1 + h_{42} \times x_2 + h_{43} \times x_3 + h_{44} \times x_4 + n_4 \end{bmatrix}$$

Introduction

Spatial Gain

Diversity – Multiplexing Tradeoff

Bad Data Rate
Good BER

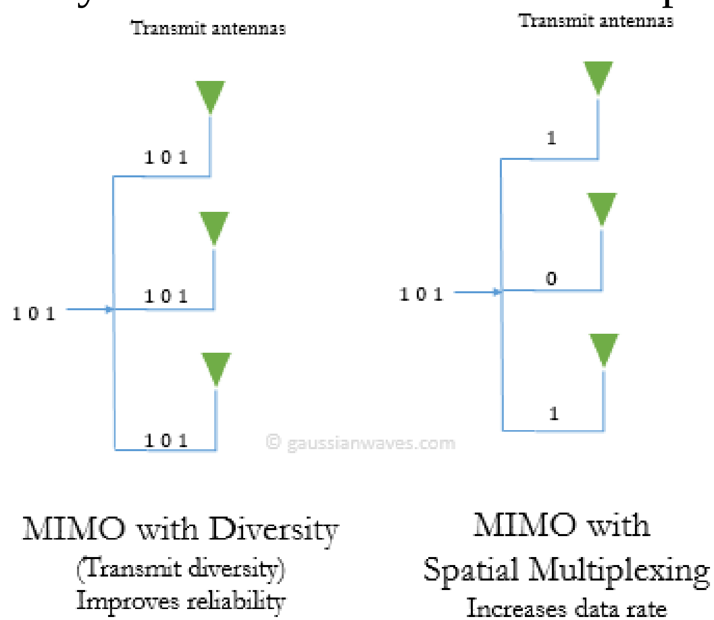
Good Data Rate
Bad BER

Spatial Diversity Gain

STBC

Spatial Multiplexing Gain

ZF
MMSE
MLD
K-Best

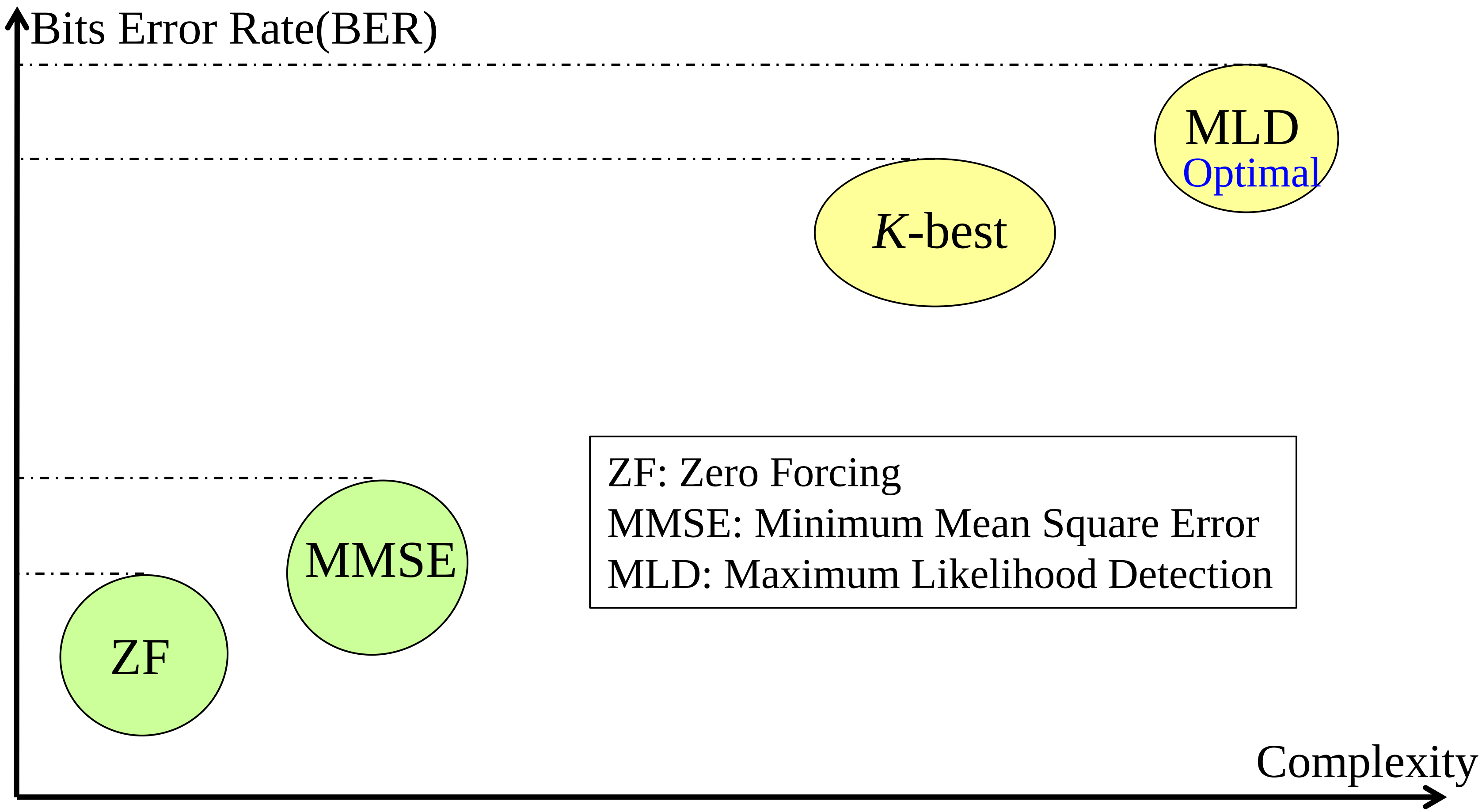


Spatial Multiplexing

- Space Division Multiplexing Access (SDMA)
  - Independent transmitted signal X = $(x_1 \;\; x_2 \;\; x_3 \;\; x_4)^T$ at every time slot on different Tx antennas to increase the data rate
  - Recovering the transmitted signal X = $(x_1 \;\; x_2 \;\; x_3 \;\; x_4)^T$ at the receiver (RX)
  - Linear MIMO decoding: Zero Forcing (ZF) and Minimum Mean Square Error (MMSE)
  - Non-linear MIMO decoding: MLD, K-Best

Introduction

- Complexity Vs BER



Zero Forcing

- Received signal Y:

$$Y = HX + N$$

- Multiplying both sides to ZF decoding matrix (W_{ZF})

$$W_{ZF}Y = W_{ZF}HX + W_{ZF}N$$

- The interference can be nullified by the following matrix

$$W_{ZF} = H^{-1} \quad \text{if H is square matrix}$$
$$W_{ZF} = (H^H H)^{-1} H^H \quad \text{if H is not square matrix}$$

Zero Forcing

- H is square matrix

$$
\begin{aligned}
W_{ZF}Y &= W_{ZF}HX + W_{ZF}N \\
\Rightarrow H^{-1}Y &= H^{-1}HX + H^{-1}N \\
\Rightarrow X \quad &= H^{-1}Y - \boxed{H^{-1}N}
\end{aligned}
$$

N_{ZF}: Unknown

- H is non-square matrix

$$
\begin{aligned}
W_{ZF}Y &= W_{ZF}HX + W_{ZF}N \\
\Rightarrow (H^H H)^{-1} H^H Y &= (H^H H)^{-1} H^H HX + (H^H H)^{-1} H^H N \\
\Rightarrow X \quad &= (H^H H)^{-1} H^H Y - \boxed{(H^H H)^{-1} H^H N}
\end{aligned}
$$

N_{ZF}: Unknown

- The error performance is directly connected to the power of noise N_{ZF}

MMSE

- Received signal Y:

$$Y = HX + N$$

- Multiplying both sides to MMSE decoding weight matrix (W_{MMSE})

$$W_{MMSE}Y = W_{MMSE}HX + W_{MMSE}N$$

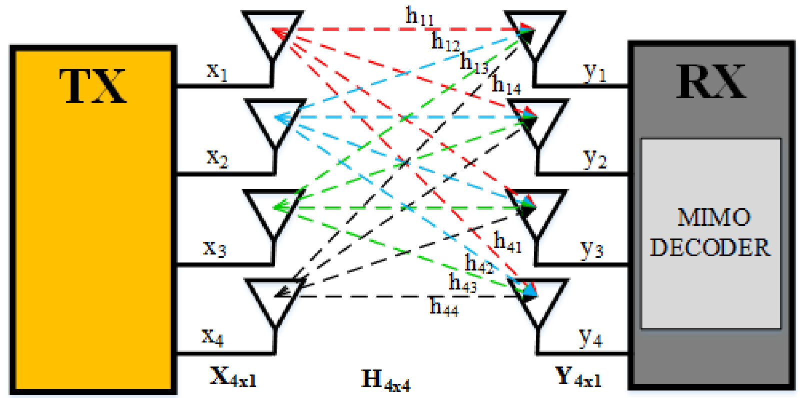
- The interference can be nullified by the following weight matrix

$$W_{MMSE} = \left(H^H H + \rho^{-1} I_{N_t}\right)^{-1} H^H$$

N_t: $Number\ transmitted\ antennas$
ρ : SNR
I_{N_t}: $Identity\ matrix$

Maximum Likelihood Detection (MLD)

■ MLD algorithm



X: transmit signal vector
H: Channel matrix
Y: Received signal vector
N: Noise vector
Q : Orthogonal matrix
Q^H: Hermitian transpose matrix
R : Upper Triangular matrix

$$Y = H \times X + N \qquad (1)$$

$$H = Q \times R \qquad (3)$$

$$Z = Q^H \times Y \qquad (4)$$

$$Z = R \times X + N' \qquad (5), with\ N' = Q^H \times N$$

$$\hat{X} = \underbrace{argmin}_{X} \|Z - R \times X\| \qquad (6)$$

Maximum Likelihood Detection (MLD)

- Full MLD algorithm

$$\{\hat{x_1}, \hat{x_2}, \hat{x_3}, \hat{x_4}\} = \arg\min_{x_1,x_2,x_3,x_4} \left\| \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} - \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ 0 & r_{22} & r_{23} & r_{24} \\ 0 & 0 & r_{33} & r_{34} \\ 0 & 0 & 0 & r_{44} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right\|^2 \quad (7)$$

$$D_4 = |z_4 - r_{44} \times x_4|^2 \quad (8)$$

$$D_3 = |z_3 - r_{33} \times x_3 - r_{34} \times x_4|^2 \quad (9)$$

$$D_2 = |z_2 - r_{22} \times x_2 - r_{23} \times x_3 - r_{24} \times x_4|^2 \quad (10)$$

$$D_1 = |z_1 - r_{11} \times x_1 - r_{12} \times x_2 - r_{13} \times x_3 - r_{14} \times x_4|^2 \quad (11)$$

$$\{\hat{x_1}, \hat{x_2}, \hat{x_3}, \hat{x_4}\} = \arg\min_{x_1,x_2,x_3,x_4} (D_4 + D_3 + D_2 + D_1) \quad (12)$$

Maximum Likelihood Detection (MLD)

- K-Best algorithm

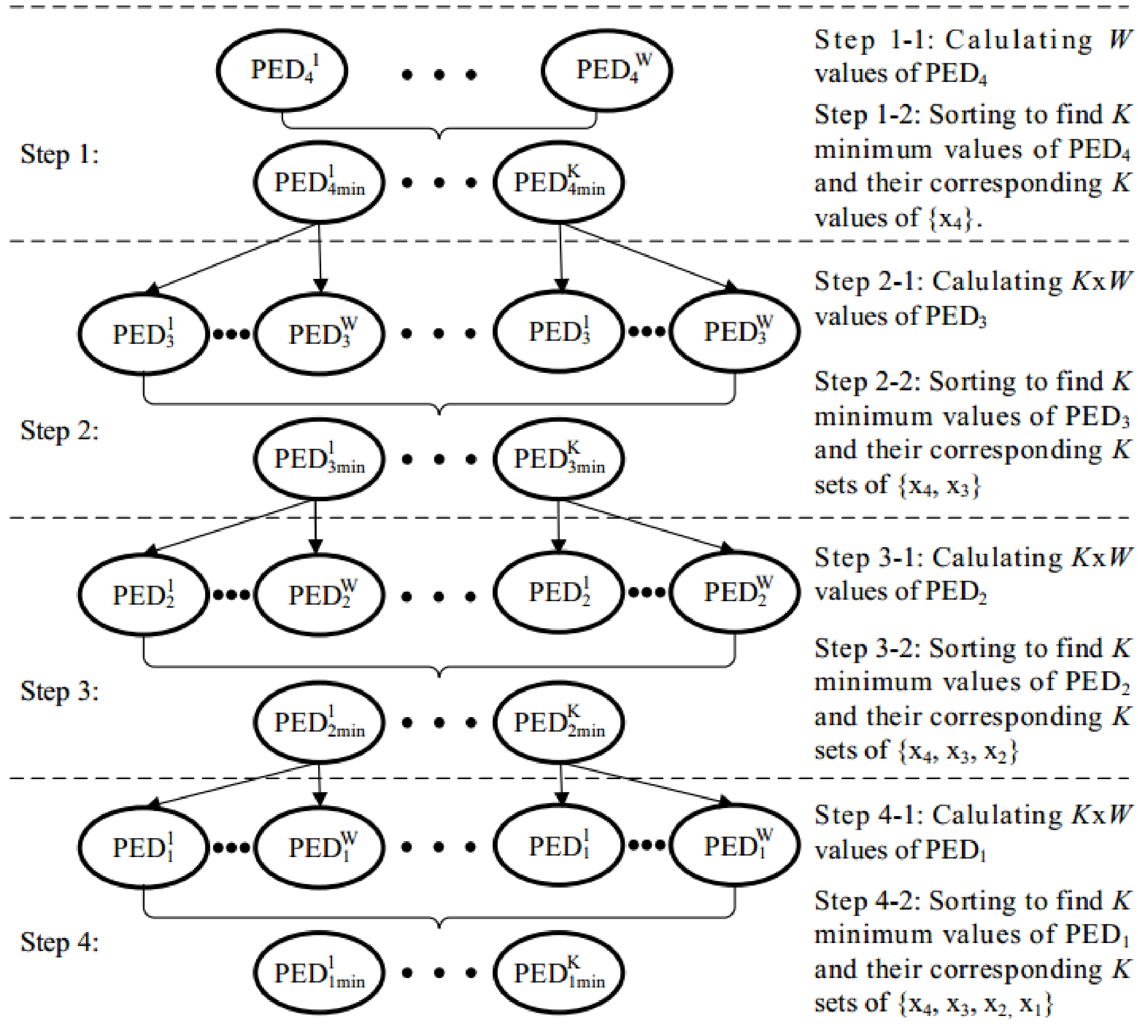
$$PED_4 = D_4 \quad (13)$$

$$PED_3 = PED_4 + D_3 \quad (14)$$

$$PED_2 = PED_3 + D_2 \quad (15)$$

$$PED_1 = PED_2 + D_1 \quad (16)$$

PED: Partial Euclidean Distance
W: Modulation Constellation Points



Spatial Diversity

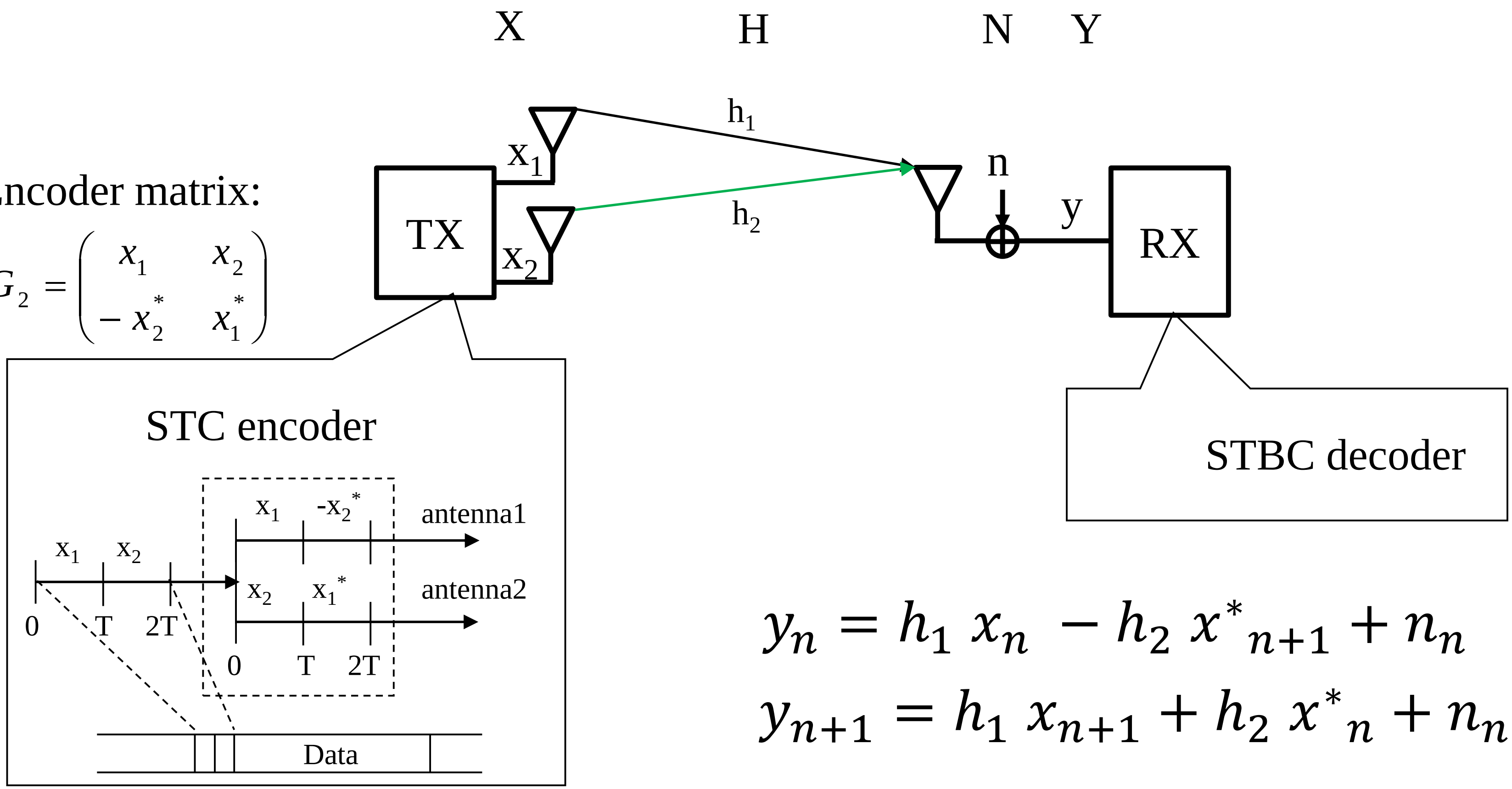
- Space-time block coding: utilizes multiple transmit antennas to create spatial diversity.
  - This allows a system to have better performance in a fading environment.
- Benefits:
  - Good performance with minimal decoding complexity.
  - Can achieve maximum diversity gain equivalent to space-time trellis codes.
  - Receivers that use only linear processing

Spatial Diversity

Alamouti coding

Encoder matrix:

$$G_2 = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}$$



$$y_n = h_1\, x_n - h_2\, {x^*}_{n+1} + n_n$$

$$y_{n+1} = h_1\, x_{n+1} + h_2\, {x^*}_n + n_{n+1}$$

Spatial Diversity

$\mathbf{s} = a+ib$

$\Downarrow$

$\mathbf{s}^* = a-ib$

(complex conjugate)

Alamouti decoding

$$y_n = h_1 x_n - h_2 x^*_{n+1} + n_n$$
$$y_{n+1} = h_1 x_{n+1} + h_2 x^*_n + n_{n+1}$$

Complex Conjugate

$$y_n = h_1 x_n - h_2 x^*_{n+1} + n_n$$
$$y^*_{n+1} = h^*_1 x^*_{n+1} + h^*_2 x_n + x^*_{n+1}$$

$$\Longleftrightarrow \quad y_n = h_1 x_n - h_2 x^*_{n+1} + n_n$$
$$y^*_{n+1} = h^*_2 x_n + h^*_1 x^*_{n+1} + x^*_{n+1}$$

$$\Longleftrightarrow \quad \begin{bmatrix} y_n \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} h_1 & -h_2 \\ h^*_2 & h^*_1 \end{bmatrix} \begin{bmatrix} x_n \\ x^*_{n+1} \end{bmatrix} + \begin{bmatrix} n_n \\ n^*_{n+1} \end{bmatrix}$$

Apply ZF, MMSE, MLD to decode x_n and x_{n+1}

Alamouti's code

- High order STBCs

Transmit Antenna	Rate	OSTBC Codeword Matrix	
2	1	$\boxed{\begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix}}$	**Alamouti**
3	1/2	$\begin{pmatrix} s_1 & s_2 & 0 \\ -s_2^* & s_1^* & 0 \\ 0 & 0 & s_1 \\ 0 & 0 & -s_2^* \end{pmatrix}$	
3	3/4	$\begin{pmatrix} s_1 & s_2 & s_3 \\ -s_2^* & s_1^* & 0 \\ s_3^* & 0 & -s_1^* \\ 0 & s_3^* & -s_2^* \end{pmatrix}$	
4	1/2	$\boxed{\begin{pmatrix} s_1 & s_2 & 0 & 0 \\ -s_2^* & s_1^* & 0 & 0 \\ 0 & 0 & s_1 & s_2 \\ 0 & 0 & -s_2^* & s_1^* \end{pmatrix}}$	**Inplement in Mathlab**
4	3/4	$\begin{pmatrix} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & s_3 \\ s_3^* & 0 & -s_1^* & s_2 \\ 0 & s_3^* & -s_2^* & -s_1 \end{pmatrix}$	

Summary

- MIMO Encode – Decoding
  - Spatial Multiplexing: Improve data rate
    - ZF
    - MMSE
    - MLD
    - B-Best
  - Spatial Diversity: Improve BER performance
    - STBC

END