Information Retrieval

Exercise Sheet 3

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Exercise 1

Proof $\Theta(\log_2 d_i \forall i$ for galloping-search.

The number of exponential steps is $\log_2 di$, because:
For computing upper border applies $j' \geq j_i$, $j' = 2^{\lceil \log_2 d_{j'} \rceil}$.
$O_{exp}(\log_2 d_i) = \begin{cases} d_i > d_{j'} & or \\ d_i = d_{j_i} \end{cases}$
So the exponential search for $B[j_i]$ is over the distance $d_{j'}$, so it always needs the same number of steps.

The binary search for the complete distance d_i is always overt the area of $d_{j'}$, which is, like the exponential search, in $O_{bin}(\log_2 d_i)$. If $d_i = d_{j'}$ the binary search is not needed.
$\Theta(log_2 d_i) = \begin{cases} O(\log_2 d_i) & \text{full exponential and binary search} \\ \Omega(\log_2 d_i) & \text{only exonential search} \end{cases}$

Exercise 2

- Proof why $\mathcal{O}(\sum_{i=1}^{k} log_2 d_i)$ ist not correct unsing counterexample:
  when the gaps are always one then $d_i = 1$

  $$\Longrightarrow \mathcal{O}(\sum_{i=1}^{k} \log_2 d_i) = \mathcal{O}(\sum_{i=1}^{k} \log_2 1) = 0.$$

  and the time complexity cannot be 0.

- Proof $\mathcal{O}(k. \sum_{i=1}^{k} \log_2 (n/k))$ ist not correct unsing counterexample:
  When $(len(A) = len(B)) \Longrightarrow k = n$

  $$\Longrightarrow \mathcal{O}(k. \sum_{i=1}^{k} \log_2 (n/k)) = \mathcal{O}(k. \sum_{i=1}^{k} \log_2 1) = 0.$$

  and the time complexity cannot be 0.

Exercise 3

$k.\log_2(1+n/k) \leq k+n$
constraint $:= k.\log_2(1+n/k) - k - n = 0$

$$\Longrightarrow L := k.\log_2(1+n/k) + \lambda(k+n)$$
$$= k.\log_2(1+n/k) + \lambda k + \lambda n$$

- $\frac{\partial L}{\partial k} = \log_2(1+n/k) + k\frac{-n}{k^2}.\frac{1}{1+\frac{n}{k}} + \lambda$
  (I)

- $\frac{\partial L}{\partial n} = \frac{k}{k^2}.k\frac{1}{1+\frac{n}{k}} + \lambda = \frac{1}{1+\frac{n}{k}} + \lambda$
  (II)

- $\frac{\partial L}{\partial \lambda} = -k - n$
  (III)

from (I) $\Longrightarrow \lambda = -\log_2(1+n/k) + \frac{n}{k}.\frac{1}{1+\frac{n}{k}}$
from (II) $\Longrightarrow \lambda = \frac{-1}{1+\frac{n}{k}}$

from (I) and (II):
$$-\log_2(1+n/k) + \frac{n}{k}.\frac{1}{1+\frac{n}{k}} = \frac{-1}{1+\frac{n}{k}}$$
$$-\log_2(1+n/k) + \frac{n}{k+n} = \frac{-1}{1+\frac{n}{k}}$$