SECI1013 DISCRETE STRUCTURE

SEMESTER 1 (2023/2024)

ASSIGNMENT 2

(CHAPTER 2: RELATIONS AND FUNCTIONS)

NAME: NAJMA SHAKIRAH BINTI SHAHRULZAMAN

MATRIC NO.: A23CS0140

LECTURER'S NAME: DR NOR HAIZAN BINTI MOHAMED RADZI

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Q1 Relation

1. $A = \{2, 3, 4, 5, 6, 7, 8\}$

xRy if $x - y = 3n$ (if difference of x and y is divisible by 3)

$R = \{(2,2), (2,5), (2,8), (3,3), (3,6), (4,4)$
$(4,7), (5,2), (5,5), (5,8), (6,3), (6,6)$
$(7,4), (7,7), (8,2), (8,5), (8,8)\}$

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad M_R^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Reflexive - main diagonal is 1

Symmetric - transpose matrix $M_R^T = M_R$

Transitive - Boolean product of matrix is equal to M_R

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

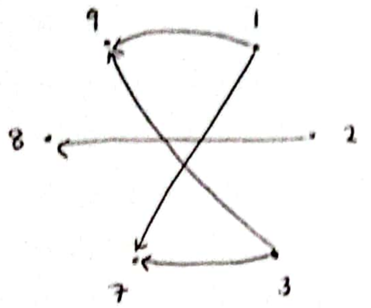
2. $A = \{1, 2, 3\}$, $B = \{9, 8, 7\}$

a) $A \times B = \{(1,9), (1,8), (1,7), (2,9), (2,8), (2,7)$
$(3,9), (3,8), (3,7)\}$

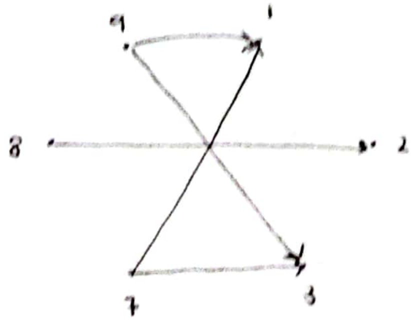
(a,b) if and only if $a + b$ is an <u>even</u> number

$R = \{(1,9), (1,7), (2,8), (3,9), (3,7)\}$
$R^{-1} = \{(9,1), (7,1), (8,2), (9,3), (7,3)\}$

b) R



R^{-1}



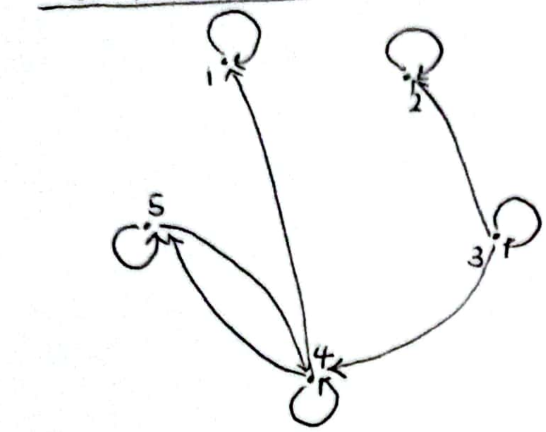
c) R-1 is the inverse relation of R

if $(x, y) \in R$, then $(y, x) \in R^{-1}$

The arrow of digraph is reversed.

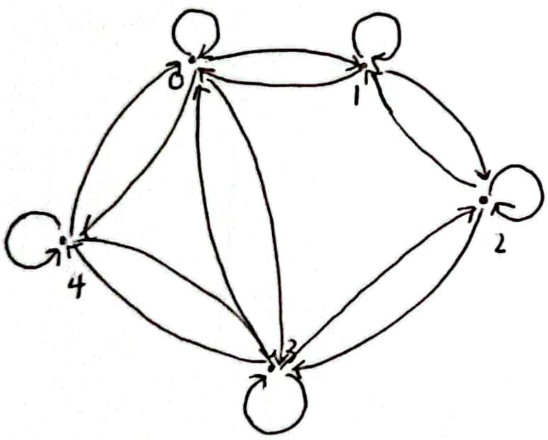
3. R = {(1,1),(2,2),(3,2),(3,3),(3,4),(4,1)
(4,4),(4,5),(5,4),(5,5)}

Digraph of R



	1	2	3	4	5
In-degree	2	2	1	3	2
Out-degree	1	1	3	3	2

4. Digraph of R



R is reflexive relation because the digraph has a loop at every vertex. → (0,0), (1,1),(2,2), (3,3) (4,4)

R is symmetric relation because whenever there is a directed edge from v to w, there is also a directed edge from w to v. → (0,1) (1,0) (3,0) (0,3) (1,2) (2,1) (2,3) (3,2) (3,4) (4,3) (4,0) (0,4)

R is transitive relation because there is a directed edge from a to b, b to c and c to a → (0,1) (1,2) (2,3) (3,4) (4,0) (0,4) (4,3) (3,2) (2,1) (1,0)

5. $A = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$

$3x - y = 0$

$3x = y$

$x = \frac{y}{3}$ (3 divides y)

$R = \{(1,3),(2,6),(3,9),(4,12)\}$

a) R is irreflexive relation because for every $x \in A$, $(x,x) \notin R$

b) R is asymmetric relation because for all $a,b \in A$, if $(a,b) \in R$ the $(b,a) \notin R$

c) R is not transitive because $(1,3),(3,9)$ exists but $(9,1)$ does not exist

6. a)

$$\begin{array}{ccccc} R & & S & & RS \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \otimes & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} & = & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

b)

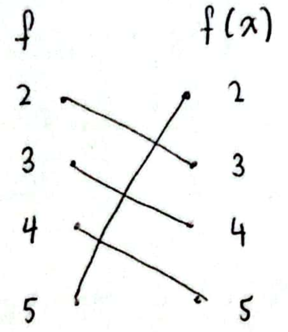
$$\begin{array}{ccccc} S & & R & & SR \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} & \otimes & \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} & = & \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

7.

All functions are relations, but not all relations are functions.

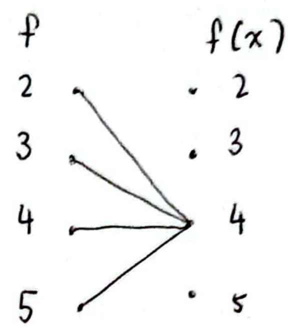
A function f is a relation from x to y.
If any value of x is repeated, then it is not a relation.

8. i)



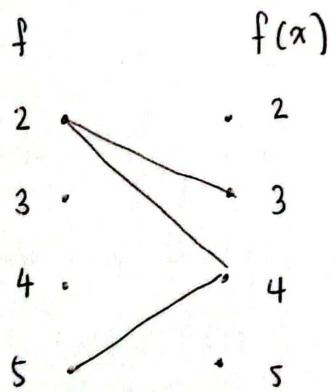
function - all values of x has one value of y

ii)



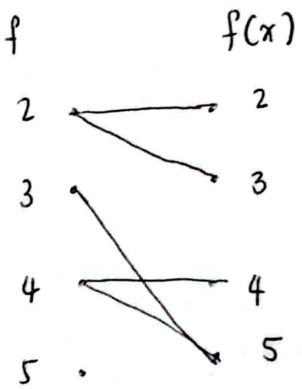
function - all values of x has one value of y

iii)



not a function - not all values of x has y value and an x value has multiple y values

iv)



not a function - not all values of x has value of y and an x value has multiple value of y

9. domain, $x = \{1, 2, 3, 4, 5\}$

range, $y = \{6, 7, 8, 9, 10\}$

$R = \{(1,6), (2,7), (3,8), (4,9), (5,10)\}$

10. v) $f: R \to R$, $f(x) = 1-2x$

$f(x) = 1-2x$

$1-2x_1 = 1-2x_2$

$-2x_1 = -2x_2$

$x_1 = x_2$

$f(x)$ is one-to-one

$f(x) = 1-2x$

$y = 1-2x$

$2x = 1-y$

$x = \frac{1-y}{2} \quad \forall\, y \in R$

$f(x)$ is onto

$\therefore f(x) = 1-2x$ is bijective

vi) $f(x) = 5x^2 - 1$

$5x_1^2 - 1 = 5x_2^2 - 1$

$5x_1^2 = 5x_2^2$

$x_1^2 = x_2^2$

$x_1 = x_2$

$f(x)$ is one-to-one

$f(x) = 5x^2 - 1$

$y = 5x^2 - 1$

$5x^2 = y+1$

$x^2 = \frac{y+1}{5}$

$x = \sqrt{\frac{y+1}{5}} \quad y > 0$

$f(x)$ is not onto

$\therefore f(x) = 5x^2 - 1$ is not bijective

CamScanner

vii) $f(x) = x^4$

$x_1^4 = x_2^4$

$x_1 = x_2$

$f(x)$ is one-to-one

$f(x) = x^4$

$y = x^4$

$x = \pm\sqrt[4]{y} \quad (y > 0)$

$f(x)$ is not onto

$\therefore f(x)$ is not bijective

viii) $f(x) = \left(\frac{x-2}{x-3}\right)$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$

$-3x_1 - 2x_2 = 3x_2 - 2x_1$

$3x_1 - 2x_1 = 3x_2 - 2x_2$

$x_1 = x_2$

$f(x)$ is one-to-one

$y = \frac{x-2}{x-3}$

$(x-3)y = x - 2$

$xy - 3y = x - 2$

$xy - x = -2 + 3y$

$x(y-1) = -2 + 3y$

$x = \frac{-2+3y}{y-1}$

$y \neq 1$

$f(x)$ is not onto

$\therefore f(x)$ is not bijective

CS CamScanner

11. ix) $f(x) = 3x - 1$; $g(x) = x^2 - 1$

$$f[g(x)] = f[x^2 - 1]$$
$$= 3(x^2 - 1) - 1$$
$$= 3x^2 - 3 - 1$$
$$= 3x^2 - 4$$

x) $f(x) = x^2$; $g(x) = 5x - 6$

$$f[g(x)] = f[5x - 6]$$
$$= (5x - 6)^2$$
$$= 25x^2 - 60x + 36$$

xi) $f(x) = x - 1$; $g(x) = x^3 + 1$

$$f[g(x)] = f[x^3 + 1]$$
$$= (x^3 + 1) - 1$$
$$= x^3$$

Q3 Recurrence Relation

12. xii) $a_n = 6a_{n-1} - 9a_{n-2}$; initial condition $a_0 = 1$ and $a_1 = 6$

$a_2 = 6a_1 - 9a_0$
$= 6(6) - 9(1)$
$= 27$

xiii) $a_n = 6a_{n-1} - 11_{n-2} + 6a_{n-3}$
initial cond. $a_0 = 2, a_1 = 5, a_2 = 15$

$a_3 = 6a_2 - 11a_1 + 6a_0$
$= 6(15) - 11(5) + 6(2)$
$= 47$

xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$
initial cond. $a_0 = 1, a_1 = -2, a_2 = -1$

$a_3 = -3a_2 - 3a_1 + a_0$
$= -3(-1) - 3(-2) + 1$
$= 10$

13.

i) $a_1 = k$

$a_2 = 5a_1 - 3$

$= 5k - 3$

$a_3 = 5a_2 - 3$

$= 5(5k-3) - 3$

$= 25k - 15 - 3$

$= 2k - 18$

$a_4 = 5a_3 - 3$

$= 5(25k - 18) - 3$

$= 125k - 90 - 3$

$= 125k - 93$

ii) $a_4 = 125(7) - 93$

$= 782$