UNIVERSITI TEKNOLOGI MALAYSIA

SECI1013 DISCRETE STRUCTURE

SEMESTER 1 (2023/2024)

ASSIGNMENT 1

(CHAPTER 1: SET THEORY, LOGIC, AND PROOF TECHNIQUES)

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Assignment 1 – Chapter 1

Question 1

a) In order to know the habits of the FC students about social networks we have asked 150 students if they have an active account in some of the most famous social networks: Facebook, Instagram or Twitter. Considering the positive answers, we obtain that 25 people have only Facebook account, 30 have only Instagram account, and 20 have only Twitter account. 15 of them have Facebook and Instagram accounts, but not a Twitter one. Only 5 people have an account in the three social networks. After the experiment, we obtain 65 Facebook users, 55 Instagram users and 50 Twitter users.

i) Draw a Venn diagram to represent to above problem. (2 marks)

ii) How many students do not have an account in any of the three social networks? (2 marks)

iii) How many students have exactly two social networks? (2 marks)

iv) How many students have social media account other than Facebook? (2 marks)

b) Suppose, $A = \{n \in N | n\ odd, 1 < n < 10\}$, where $N = \{natural\ number\}$
$B = \{n \in N | n\ is\ prime, 1 < n < 10\}$, $C = \{n \in N | n\ divisible\ by\ 3, 1 < x < 10\}$

i) Find $|A|, |B|$and $|C|$, (3 marks)

ii) Find the number of proper subsets of A. (3 marks)

iii) Find $C \times B$ (2 marks)

Question 2

a) Verify $\sim(p \vee q) \vee (\sim p \wedge \quad q) \equiv \sim p$, using both truth table and logic property law (6 marks)

b) Write the following statement using *p* and *q* and logical connective

p: I go to the beach
q: it is a sunny summer day
r : it is Sunday

i) I go to the beach whenever it is Sunday and sunny summer day (2 marks)

ii) If it is not either Sunday or sunny summer day then I do not go to the beach. (2 marks)

iii) If I do not go to the beach then it is not either Sunday or sunny summer day. (2 marks)

c) Write the negation of $\forall x(x^2 + 2x - 3 = 0)$ and determine the resulting proposition is TRUE or FALSE with the domain of discourse is integer (5 marks)

d) Express the following statement using predicates, quantifier and logical connective with the domain of discourse consist of all students at your school (6 marks)

i) There is a student at your school who can speak Russian but does not know C++

ii) Every student at your school either can speak Russian or knows C++

iii) No student at your school can speak Russian or knows C++

Question 3

a) Prove the following theorem using indirect proof method. (5 marks)

For all integers, if $a^2 - 3b$ *is even then a is even and b is even*

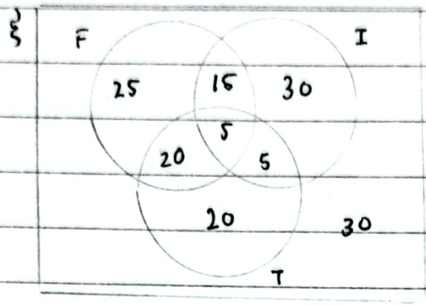
No.:

Assignment 1

Chapter 1 : Set Theory, Logic & Proof Technique

Question 1

a) i)



Let F = facebook users,

I = Instagram users,

T : Twitter users

ii) $U = 150$

$F \cup I \cup T = 25 + 15 + 30 + 20 + 5 + 5 + 20$

$= 120$

$(F \cup I \cup T)' = 150 - 120$

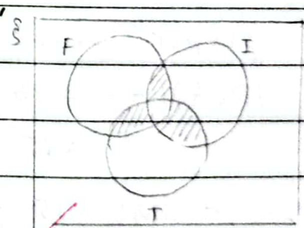
$= 30$

$\therefore$ 30 students does not have any three social networks

iii) Students with exactly two social networks

$((F \cap I) \cup (F \cap T) \cup (T \cap I)) \cap (F \cap I \cap T)'$

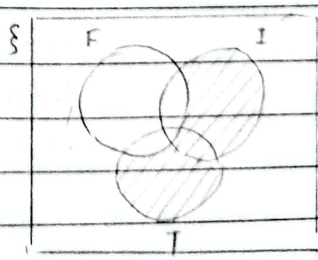
$= 15 + 20 + 5 = 40$ students



iv) students with social networks other than Facebook

$(T \cup I) \cap F'$

$= 30 + 20 + 5 = 55$ students



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No.:

b) $1 < n < 10$ $\quad N = \{\text{natural numbers}\}$

$n = \{2,3,4,5,6,7,8,9\}$

$A = \{n \in N \mid n \text{ is odd}, 1 < n < 10\}$

$A = \{3,5,7,9\}$

$B = \{n \in N \mid n \text{ is prime}, 1 < n < 10\}$

$B = \{2,3,5,7\}$

$C = \{n \in N \mid n \text{ is divisible by } 3, 1 < n < 10\}$

$C = \{3,6,9\}$

i) $|A| = 4$

$|B| = 4$

$|C| = 3$

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ii) subset

$|P(A)| = 2^4$

$= 16$

proper subset

$|P(A)| - 1 = 16 - 1$

$= 15$

$\therefore$ A has 15 proper subsets.

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iii) $C \times B = \{(3,2),(3,3),(3,5),(3,7)$

$(6,2),(6,3),(6,5),(6,7)$

$(9,2),(9,3),(9,5),(9,7)\}$

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No.:

Question 2

a) $\sim(p \vee q) \vee (\sim p \wedge q)$

Truth Table

p	q	$\sim p$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge q$	$\sim(p \vee q) \vee (\sim p \wedge q)$
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T

$\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

$\therefore$ Based on the truth table,

$\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to $\sim p$

Logic Property Law

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$\sim(p \vee q) \vee (\sim p \wedge q)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ (De Morgan's Law)

$= \sim p \wedge (\sim q \vee q)$ (Distributive Law)

$= \sim p \wedge U$ (Complement Law)

$= \sim p$ (Identity Law)

$\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

$\therefore$ Based on logic property law,

$\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to $\sim p$

No.:

b) p : I go to the beach
q : it is a sunny summer day
r : it is Sunday

i) I go to the beach whenever it is Sunday and sunny summer day.

I go to the beach : p
whenever (if and only if) : $\longleftrightarrow$
it is Sunday and sunny summer day : $r \wedge q$

$p \longleftrightarrow (r \wedge q)$ 4

ii) If ..., then : $\longrightarrow$
not either Sunday or sunny summer day : $\sim(r \vee q)$
I do not go to the beach : $\sim p$

$\sim(r \vee q) \longrightarrow \sim p$ 2

iii) If ..., then : $\longrightarrow$
I do not go to the beach : $\sim p$
it is not either Sunday or sunny summer day : $\sim(r \vee q)$

$\sim p \longrightarrow \sim(r \vee q)$ 2

No.:

c)

$\forall x\,(x^2 + 2x - 3 = 0)$

↳ for all values of x, $x^2 + 2x - 3 = 0$ is true

Negation of $\forall x(x^2+2x-3=0)$

$\sim(\forall x\,(x^2 + 2x - 3 = 0))$; $\exists x \sim (x^2 + 2x - 3 = 0)$

$\exists x \sim (x^2 + 2x - 3 = 0)$

↳ for some values of x, $x^2+2x-3=0$ is false

when $x = 2$,

$x^2 + 2x - 3 = (2)^2 + 2(2) - 3$

$= 5$

$x^2 + 2x - 3 \neq 0$,

therefore, $x^2 + 2x - 3 = 0$ is false.

∴ Statement is true because it is possible to find at least one integer number x to make proposition is true.

No.:

d) Let $R(x)$: student who can speak Russian

Let $C(x)$: student who knows C++

i) There is a student at your school who can speak Russian but does not know C++

There is : $\exists x$

can speak Russian : $R(x)$

but (and) : $\wedge$

does not know C++ : $C(x)$

$\exists x\,(R(x) \wedge \sim C(x))$

ii) Every student at your school either can speak Russian or knows C++

Every : $\forall x$

can speak Russian : $R(x)$

either...or : $\vee$

knows C++ : $C(x)$

$\forall x\,(R(x) \vee C(x))$

iii) No student at your school can speak Russion or knows C++

No student : $\sim \exists x$

can speak Russian : $R(x)$

or : $\vee$

knows C++ : $C(x)$

$\sim \exists x\,(R(x) \vee C(x))$

No.:

Question 3

1) If $a^2 - 3b$ is even, then a is even and b is even
(p) (q)

Indirect proof

→ contrapositive of statement has to be true

$p \rightarrow q$ (true) $\sim q \rightarrow \sim p$ (true)

Contrapositive :

If a is odd or b is odd, then $a^2 - 3b$ is odd
$(\sim q)$ $(\sim p)$

Three cases to prove :

$a^2 - 3b$ is odd when a is odd, b is even

$a^2 - 3b$ is odd when a is even, b is odd

$a^2 - 3b$ is odd when a is odd, b is odd

case 1 (a is odd, b is even)

Let $a = 2k + 1$, $b = 2k$

$$a^2 - 3b = (2k+1)^2 - 3(2k)$$
$$= 4k^2 + 4k + 1 - 6k$$
$$= 4k^2 - 2k + 1$$
$$= 2(2k^2 - k) + 1 \quad \text{Let } m = 2k^2 - k$$
$$= 2m + 1 \text{ (odd)}$$

$a^2 - 3b$ is odd when a is odd and b is even.

Therefore, case 1 is proven to be true.

No.:

Case 2 (a is even, b is odd)

Let $a = 2k+2$, $b = 2k+1$

$$\begin{aligned} a^2 - 3b &= (2k+2)^2 - 3(2k+1) \\ &= 4k^2 + 8k + 4 - 6k - 3 \\ &= 4k^2 + 2k + 1 \\ &= 2(2k^2 + k) + 1 \quad \text{Let } m = 2k^2 + k \\ &= 2m + 1 \text{ (odd)} \end{aligned}$$

$a^2 - 3b$ is odd when a is even, b is odd

Therefore, case 2 is proven to be true.

Case 3 (a is odd, b is odd)

Let $a = 2k+1$, $b = 2k+1$

$$\begin{aligned} a^2 - 3b &= (2k+1)^2 - 3(2k+1) \\ &= 4k^2 + 4k + 1 - 6k - 3 \\ &= 4k^2 - 2k - 2 \\ &= 2(2k^2 - k - 1) \quad \text{Let } m = 2k^2 - k - 1 \\ &= 2m \text{ (even)} \end{aligned}$$

$a^2 - 3b$ is even when a is odd, b is odd

Therefore, case 3 is proven to be false

∴ Only case 1 and case 2 are proven to be true but case 3 is false. Therefore, the theorem is not true.

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