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# Assignment (LA)

1)

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$
$R_4 \rightarrow R_4 - 6R_1$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$ Swap $R_2$ and $R_3$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$ multiply $R_2$ by $-1/4$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$ multiply by $-1$ & add to $R_2$.

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -9 & 11/4 \end{bmatrix}$

$$[T]_B = \begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 1 & 0 & 1\end{bmatrix}$$

apply row operation,

$$[T]_B = \begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 0 & 0 & 0\end{bmatrix}$$

$\rho(T) = 2$ & nullity $= 1$.

3) $A = \begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}$ $\quad |A - \lambda I| = 0$.

$$\left|\begin{bmatrix}2-\lambda & -1\\ -1 & 2-\lambda\end{bmatrix}\right| \quad \Rightarrow (2-\lambda)^2 - 1$$

$\Rightarrow \lambda^2 - 4\lambda + 3$

$\Rightarrow (\lambda-1)(\lambda-3) = 0$

$\Rightarrow \lambda = 1, 3$

Eigen value of $\Rightarrow \lambda = 1, 3$.

for $\lambda_1 = 1$.

$$(A - \lambda_1 I)\, v_1 = \left(\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix} \cdot \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)$$

$\times R_3$ by $-1/3$.

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -9 & 17/4 \end{bmatrix}$$

$\times R_4$ by 3 and add to $R_3$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 5/4 \end{bmatrix} \quad R_4 \longrightarrow R_4 \times 4/5.$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Non-zero row = ~~4~~ 3

$\rho(A) = 3$.

Q// $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

Standard basis of $P_2$ be –

$$B' = \{1, x, x^2\}$$

RANKA
DATE
PAGE

5) $x+3y+2z=0$

$2x-y+3z=0$

$3x-5y+4z=0$

$x+17y+4z=0$

$[A:B]=$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \to 2R_1 - R_2$ ; $R_3 \to 3R_1 - R_3$ ; $R_4 \to R_4 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_4$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 14 & 2 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & -7 & -1 & 0 \end{array}\right]$$

$R_2 \to \cancel{3} -1R_3 + R_2$

$R_4 \to R_3 + R_4$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 28 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$

$\rho(A:B) = 2$

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$v_1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ $\therefore v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2 = 3$.

$(A - \lambda_2 I) v_2 = \left( \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right) v_3$

$\Rightarrow \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} v_2 = 0$

$v_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

So,
for $A^{-1}$ the eigen value $= 1$ & $\frac{1}{3}$ = eigen vector

for,
$A + 4I = \lambda + 4 = 1 + 4$ and $3 + 4$
$= 5$ $= 7$

4) $3x - 0.1y - 0.2z = 7.85$.
$0.1x + 7y - 0.3z = -19.3$
$0.3x - 0.2y + 10z = 71.4$

$x(0) = y(0) = z(0) = 0$.

Iteration-1

$= ((a_1+a_2)+1)+((b_1+b_2)+1)x+((c_1+c_2)+1)x^2$

$\Rightarrow T(u+v) = T(u)+T(v)$

ii) <u>Homogenity:</u>

let $u = a+bx+cx^2$

& $d$ be any scalar

$T(cu) = T(da+dbx+dcx^2)$

$= ((da+1)+(db+1)x+(dc+1)x^2)$

$dT(u) = (d(a+1)+(b+1)x+(c+1)x^2)$

$= (da+d)+(db+d)x+(dc+d)x$

$T(du) \neq dT(u)$.

$\therefore T: P_2 \longrightarrow P_2$ is not a linear transformation as homogenity is not satisfied.

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$\rho(A) = \rho(A:B) \Rightarrow$ consistent system

dimension $> \rho(A)$.
infinitely many solution.

let

$z = t$, so,
$y = 0$
& $x = -3t$
are the parametric solution

(6) To define linear transformation

i) Additivity

let $u = a_1 + b_1 x + c_1 x^2$
$v = a_2 + b_2 x + c_2 x^2$

$\Rightarrow T(u+v) = T((a_1+a_2) + (b_1+b_2)x + (c_1+c_2)x^2)$

$\Rightarrow ((a_1+a_2)+1) + ((b_1+b_2)+1)x + ((c_1+c_2)+1)x^2$

$\Rightarrow Tu + T(v) = (a_1+b) + (b_1+1)x + (c_1+1)x^2 +$
$(a_2+1) + (b_2+1)x^4 + (c_2+1)x^2$

$\Rightarrow ((a_1+1) + (a_2+1)) + ((b_1+1) + (b_2+1))x +$
$((c_1+1) + (c_2+1))x^2$

So values,

$x = 7.66$

$y = 15.64$

$z = \cancel{15}\ 6.46$.

<u>Second iteration:</u>

$x = \frac{26 + 6y - 2z}{3}$ $\quad y = 15.64,\ z = 6.46$

$x = 34.64$

$z = 6.46$.

then $y = 4x + z - 15 = 130.02$

$z = \frac{16 + 34 - x}{7} = 53.06$

values,

$x = 34.64 \quad y = 130.02,\ z = 53.06$

<u>Third iteration:</u>

$x = 23 + 6y - 2z \quad y = 130.02,\ z = 53.06$

$x = 232.33$.

$x = 232.33$ and $z = 53.06 \longrightarrow y = 4x + z - 15$.

$= 967.38$

7) Forming the equations,

$a+3b-2c=0$.
$2a+b+c=0$.
$3a+0b+3c=0$.

for $S=\{(1,2,3),(3,1,0),(-2,1,3)\}$

for S to be basis of $R^3$, the vectors must be a linear independent

$x(1,2,3)+y(3,1,0)+z(-2,1,3)=(0,0,0)$

$(x+3y-2z,\ 2x+y+z,\ 3x+3z)=(0,0,0)$

8/
8. $3x-6y+2z=23$
$-4x+y-z=-15$.
$x-3y+7z=16$

first iteration

$x=\frac{23+6y-2z}{3}=7.66$

$y=-15+z+4x=15.64$.

$z=\frac{16+3y-x}{7}$

$z=6.46$.

$y = 967.88$ , $x = 232.33$

Then,

$z = \frac{10+3y-x}{7} = 383.68$

so,

Sol^n

$x = 232.33$, $y = 967.38$, $z = 383.68$

Q 9// matrix operation are extensively used in image processing like transpose of matrix is used to rotate the image in various direction and the stus matrix is used to blur certain area of image.

Apart from this image are made up of matrix itself. Images are made up of pixels which are arranged in grid to produce image.

10/ linear transformation plays very important role in computer vision.

$$x^{(1)} = \frac{7.85 + 0.1y^{(0)} + 0.2z^{(0)}}{3} = \frac{7.85}{3} = 2.6167$$

$$y^{(1)} = \frac{-19.3 - (0.1)x^{(1)} + 0.3z^{(0)}}{7} = 2.7571$$

$$z^{(1)} = \frac{71.4 - 0.3x^{(1)} + 0.2y^{(1)}}{10} = 7.14$$

**Iteration-2**

$$x^{(2)} = \frac{7.85 + 0.1y^{(1)} + 0.2z^{(1)}}{3} = 2.8098$$

$$y^{(2)} = \frac{-19.3 - 0.1x^{(2)} + 0.3z^{(1)}}{7} = -2.9832$$

$$z^{(2)} = \frac{71.4 - 0.3x^{(2)} + 0.2y^{(2)}}{10} = 7.013$$

**Iteration-3**

$$x^{3} = \frac{7.85 + 0.1y^{(2)} + 0.2z^{(2)}}{3} = 2.8271$$

$$y^{3} = \frac{-19.3 - 0.1x^{(3)} + 0.3z^{(2)}}{7} = -2.996$$

$$z^{3} = \frac{71.4 - 0.3x^{(3)} + 0.2y^{(3)}}{10} = 7.003$$

Linear transformation is extensively used in extensively used in manipulating image for various purpose.

eg, is rotating image with $\theta$ angle about x-axis

For this purpose we use famous rotation matrix is drw to do this trail.

Here $T : V \longrightarrow W$,

where $T(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

if we have to rotate $(x, y)$ about $\theta$, then a new $x'$ and $y'$ are -

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

In this way we perform this basic operation for each pixel of the image and find the rotated image.