Batch merge and merge path sort

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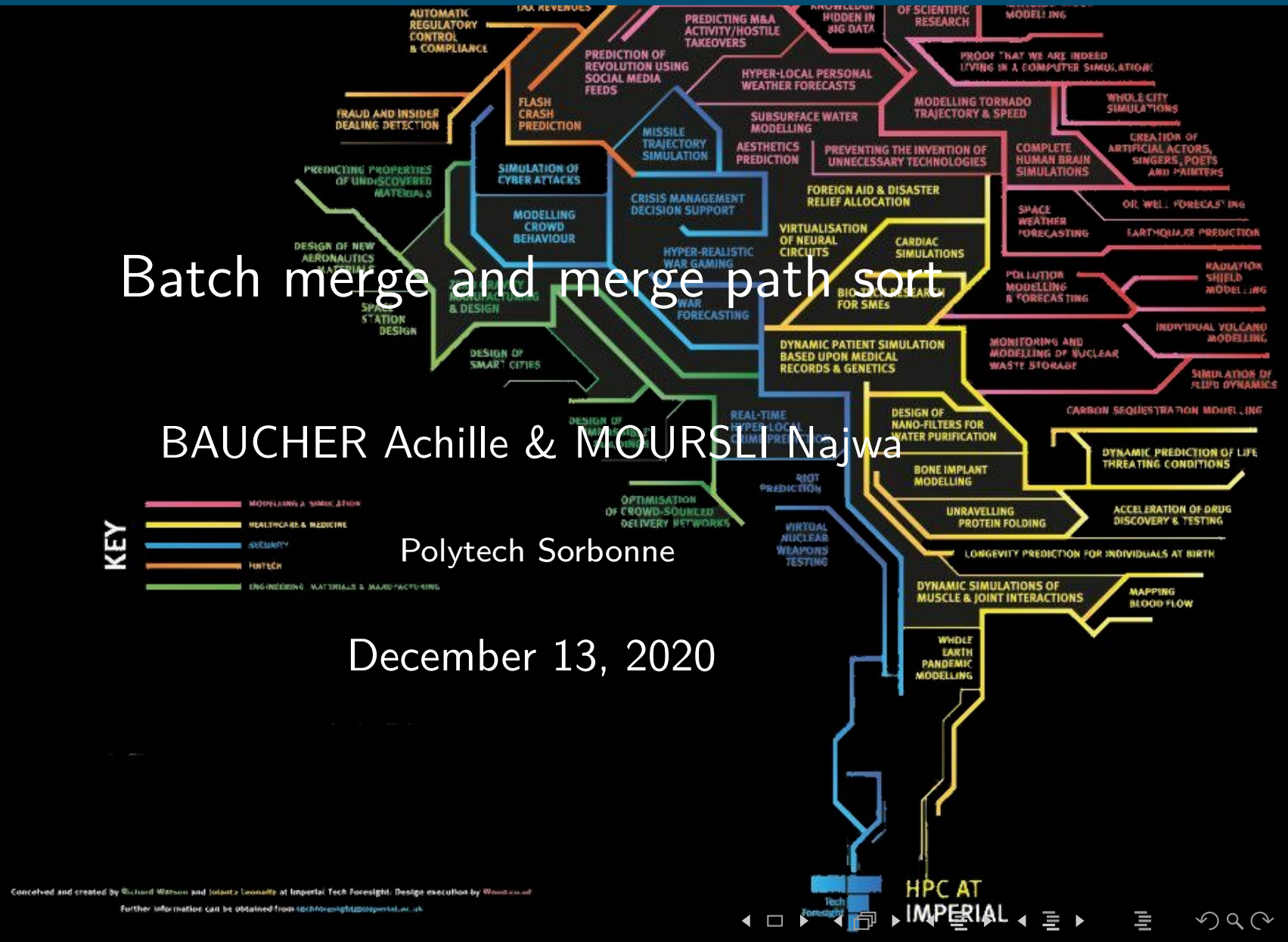
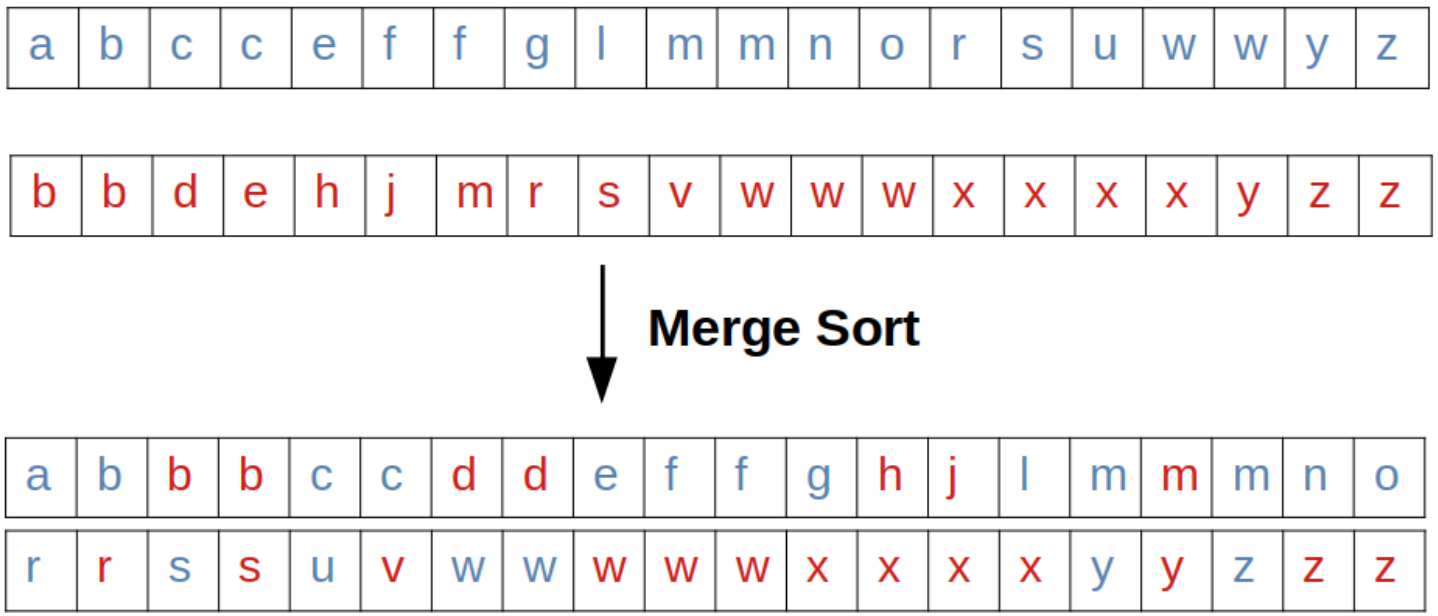
December 13, 2020

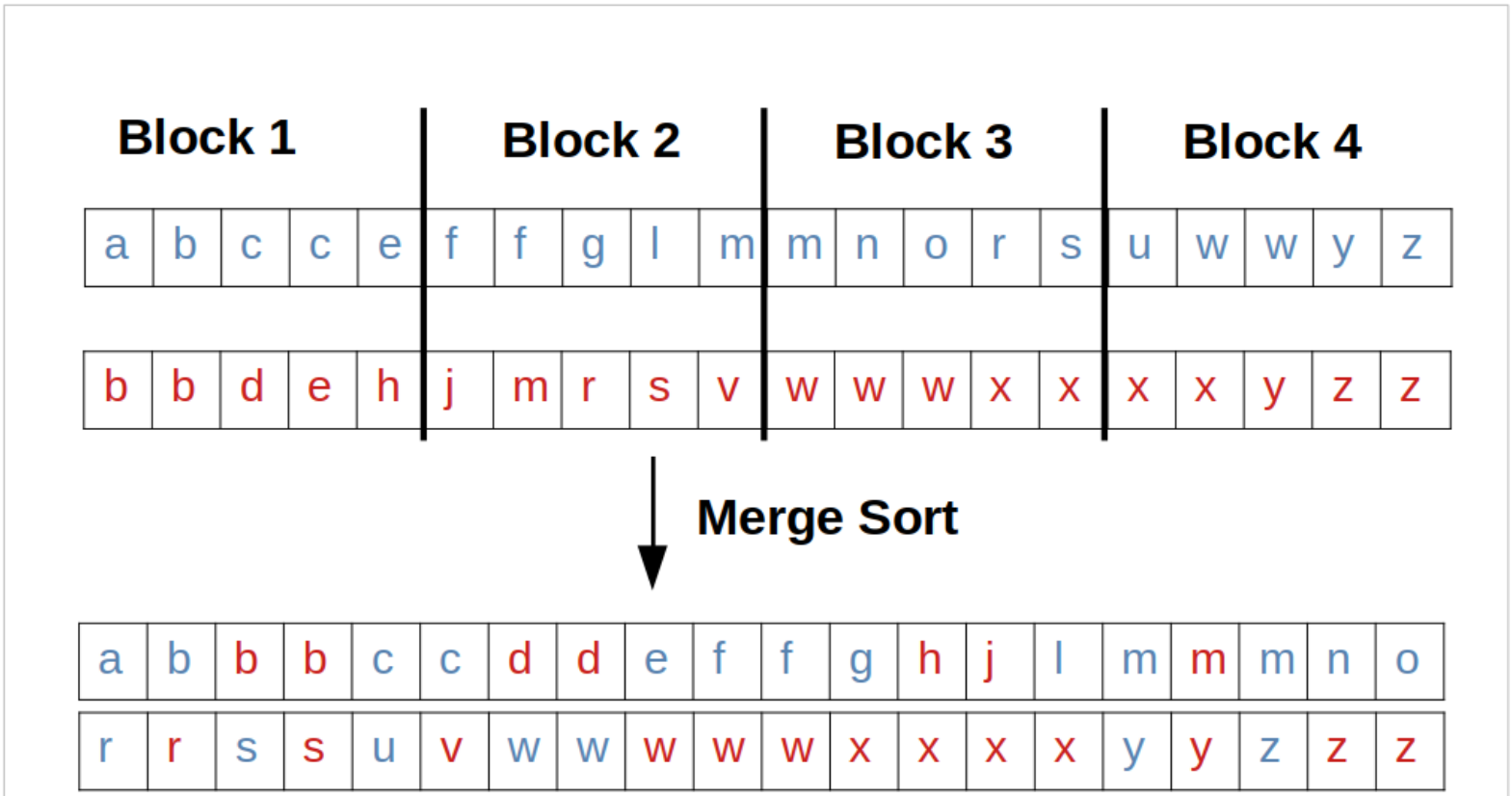


Summary

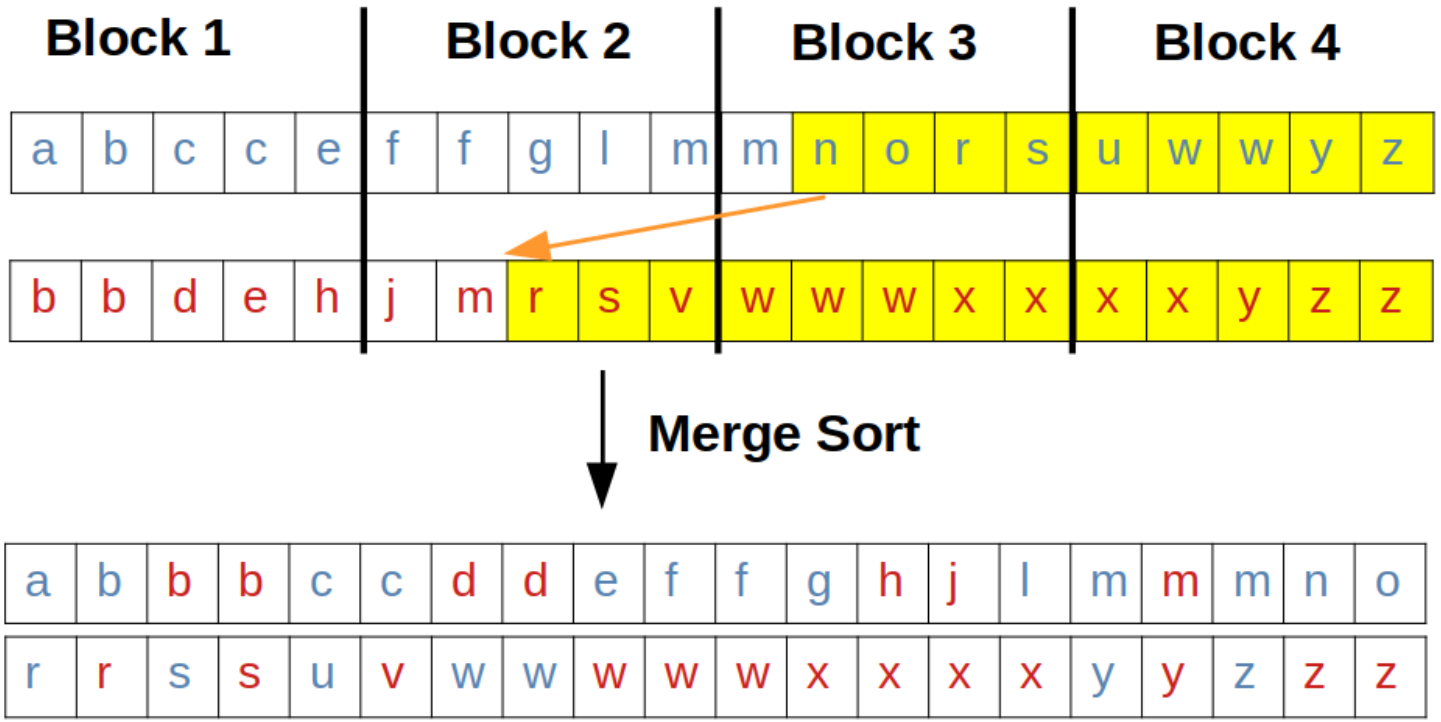
Main Goal


a b c c e f f g l m m n o r s u w w y z
b b d e h j m r s v w w w x x x x y z z
Merge Sort
a b b b c c d d e f f g h j l m m m n o
r r s s u v w w w w w x x x x y y z z z

Can be distributed



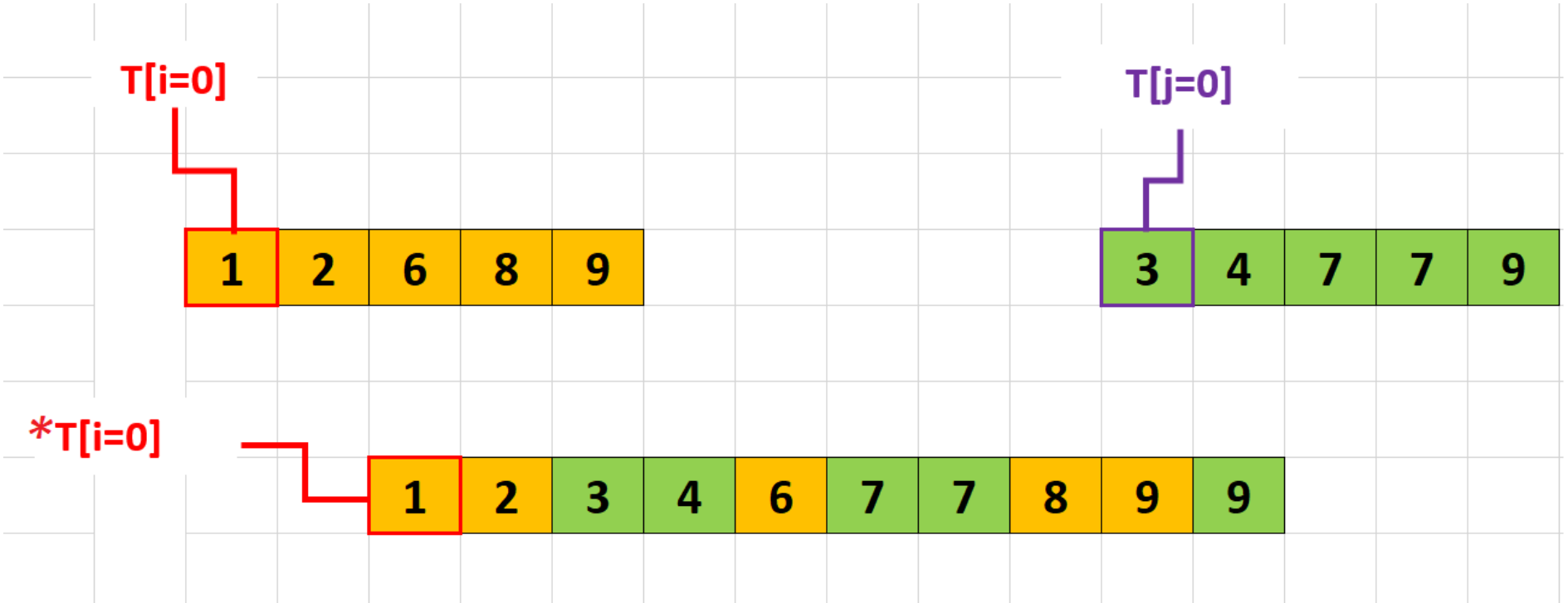
Task distribution



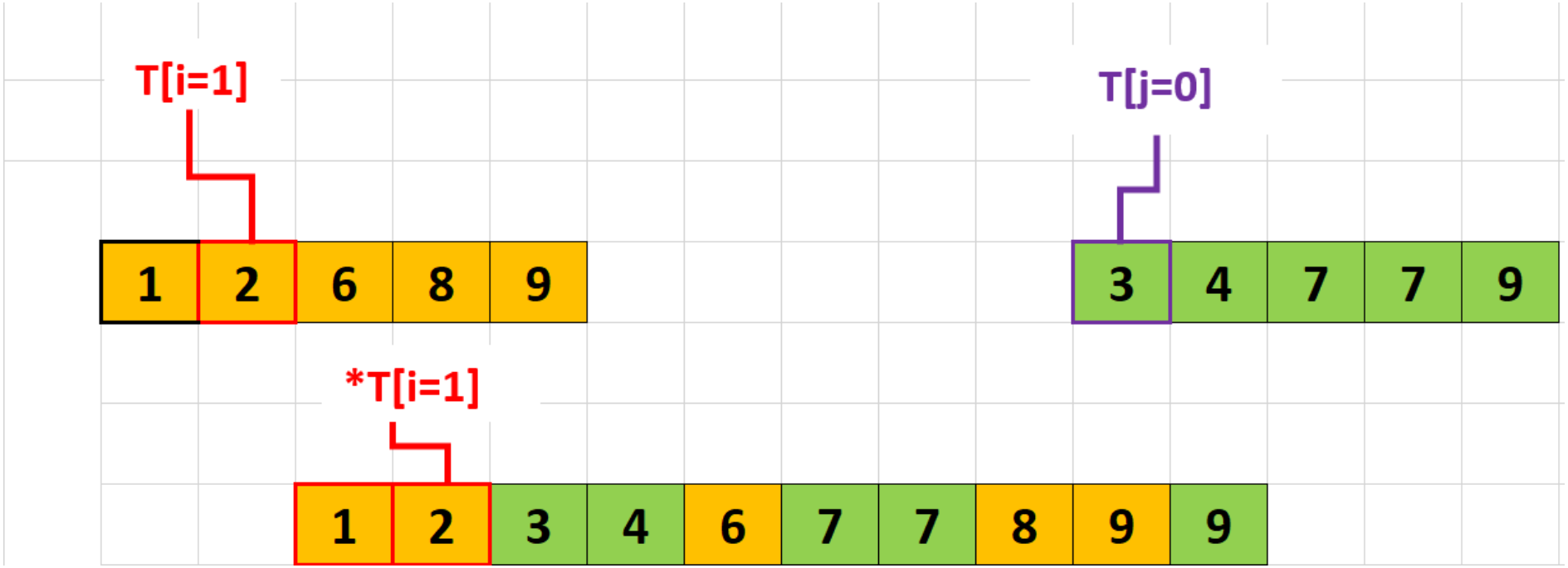
Summary

Serial Merge Path

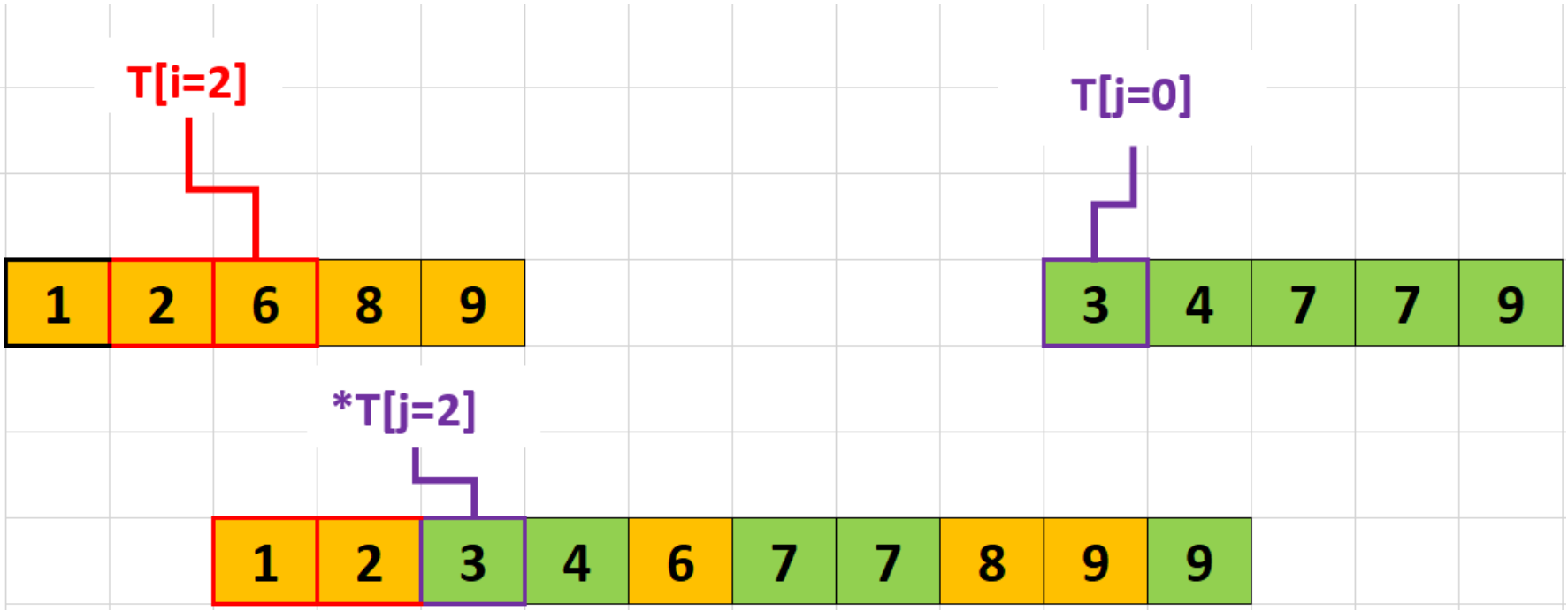
The most common idea is to use two pointers and increment one of them whenever it has the lowest value among both, while copying its value to the output array:



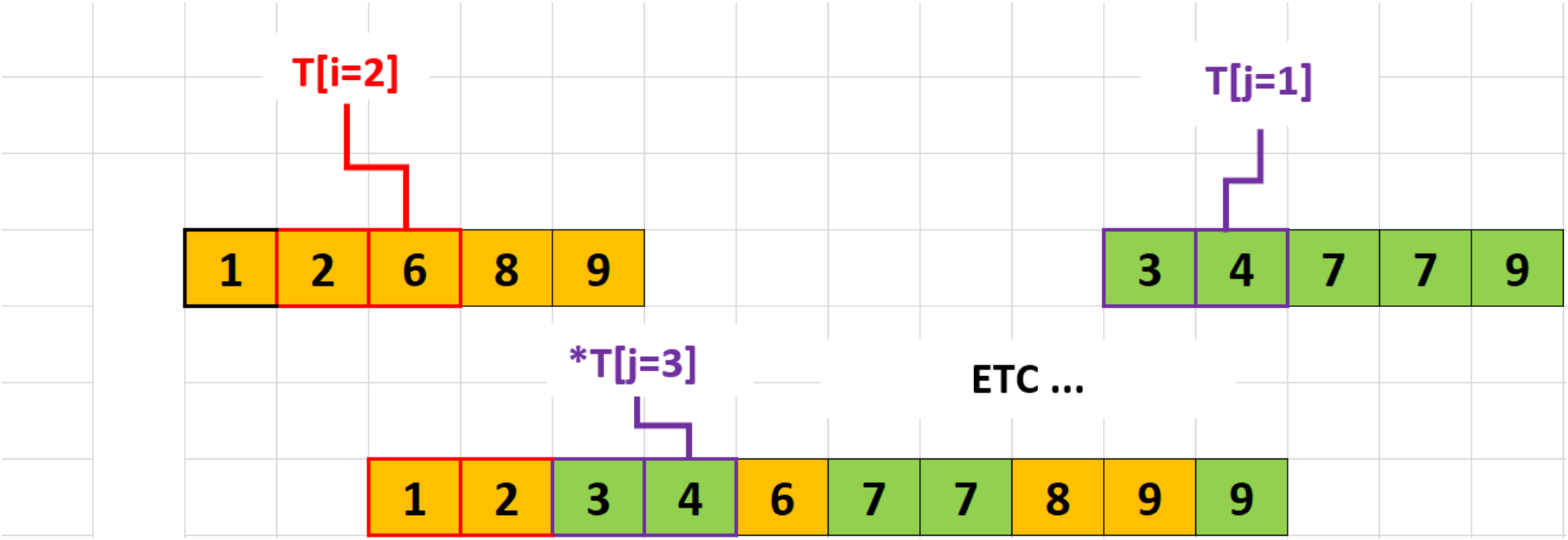
Serial Merge Path



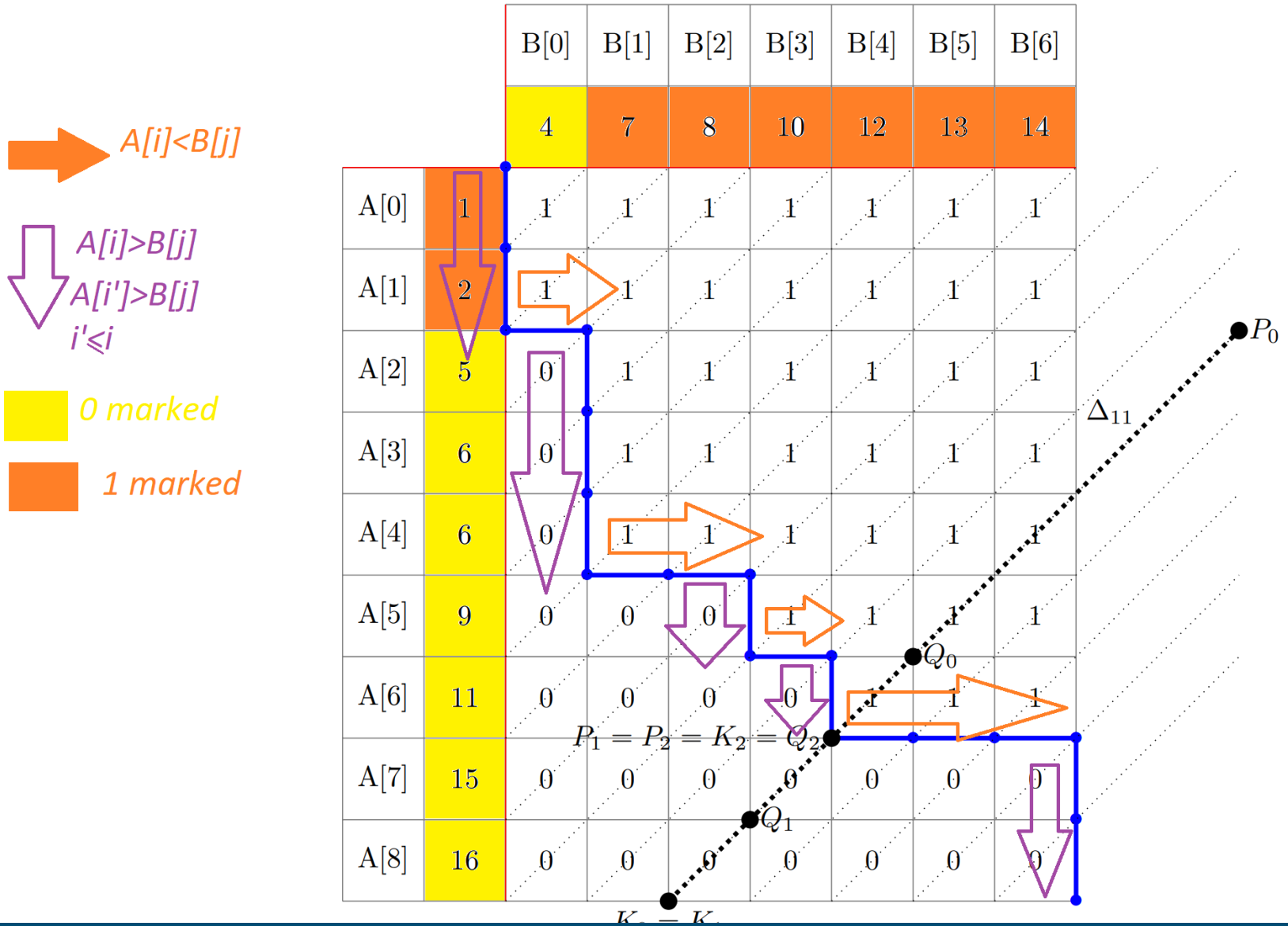
Serial Merge Path



Serial Merge Path



On our example

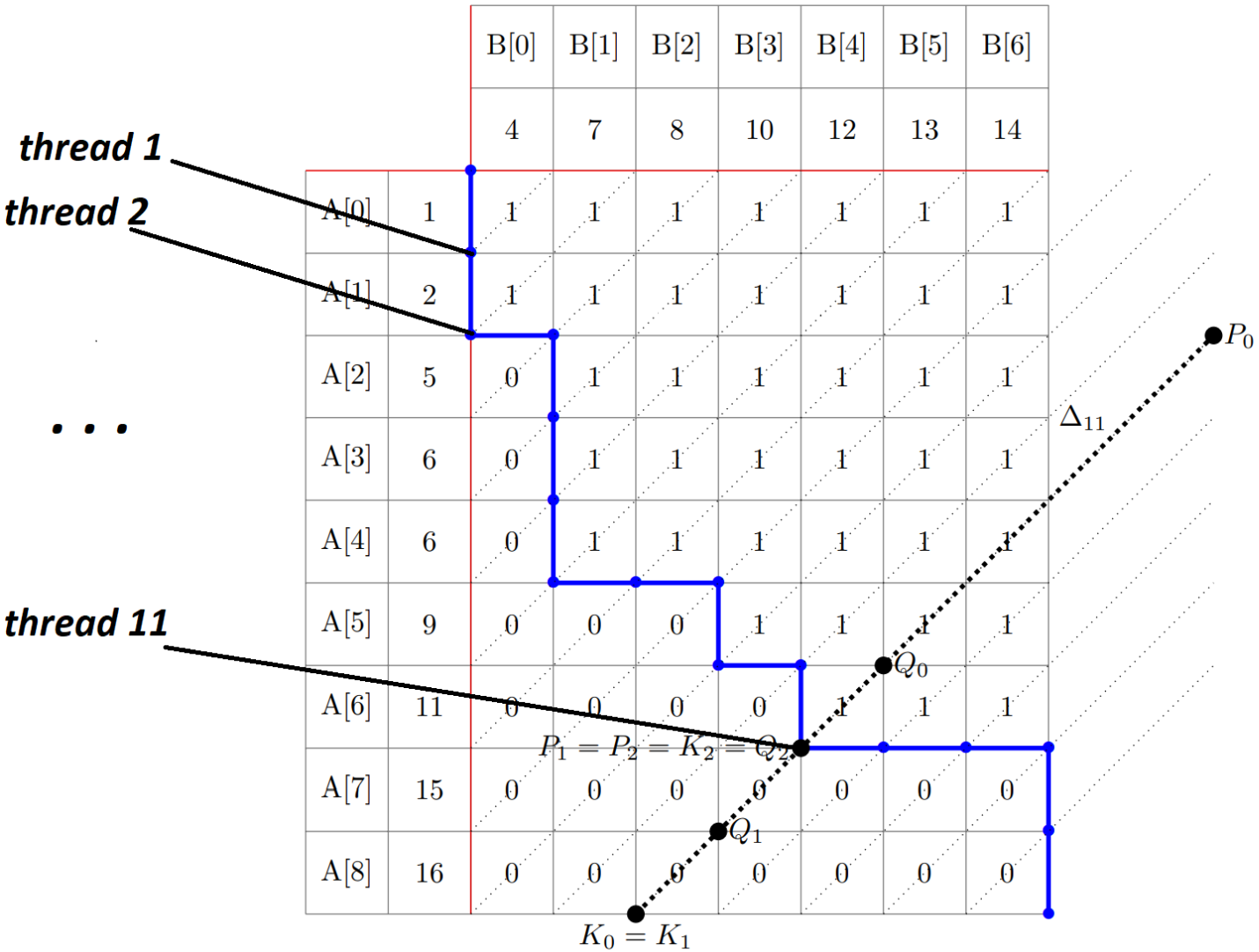


On our example

- It is simple to implement
- It has always the same linear complexity $\Theta(|A| + |B|)$: no bad surprise
- In its original form, it is impossible to parallelize the algorithm.
- An average linear complexity is not optimal

Blocks and memories use

For one block :

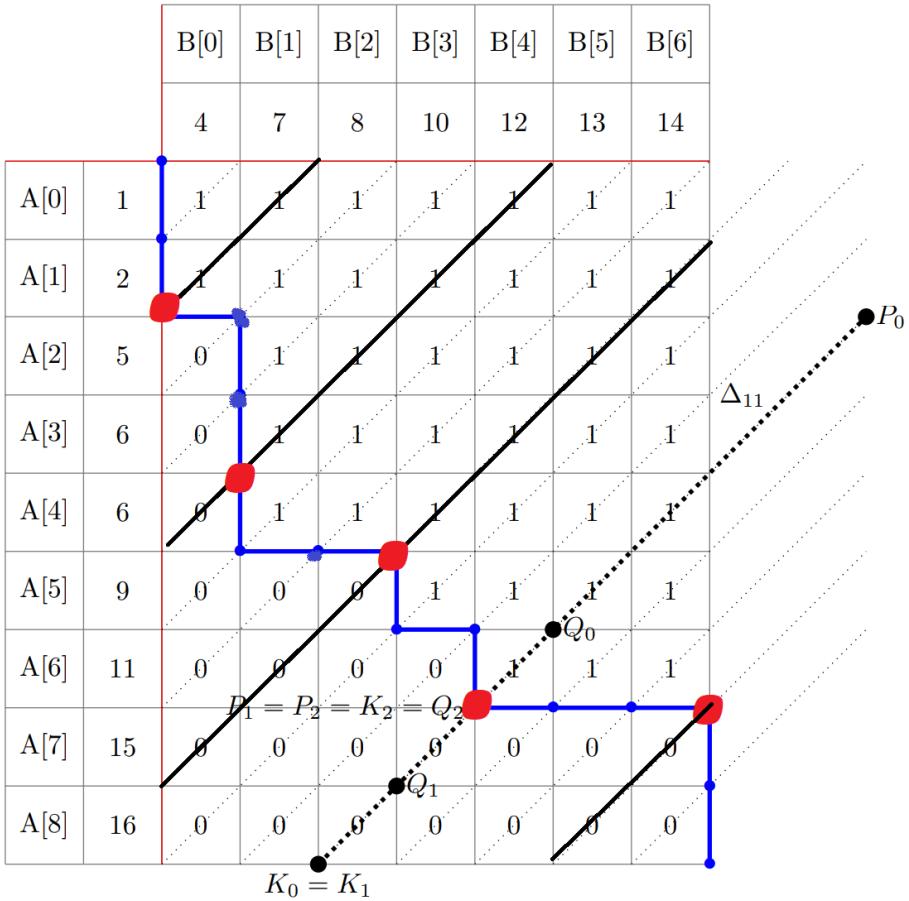


Several Blocks and memories use

- **Parallel Partition**
  ⇒ Map exactly the necessary inputs into each tile.
- **Merge Path**
  ⇒ Cross diagonal binary search **GLOBAL MEMORY**
- **MergeSort of the Merge Path** ⇒ Placing the Merge Path on the **SHARED MEMORY** ⇒ Blocksort

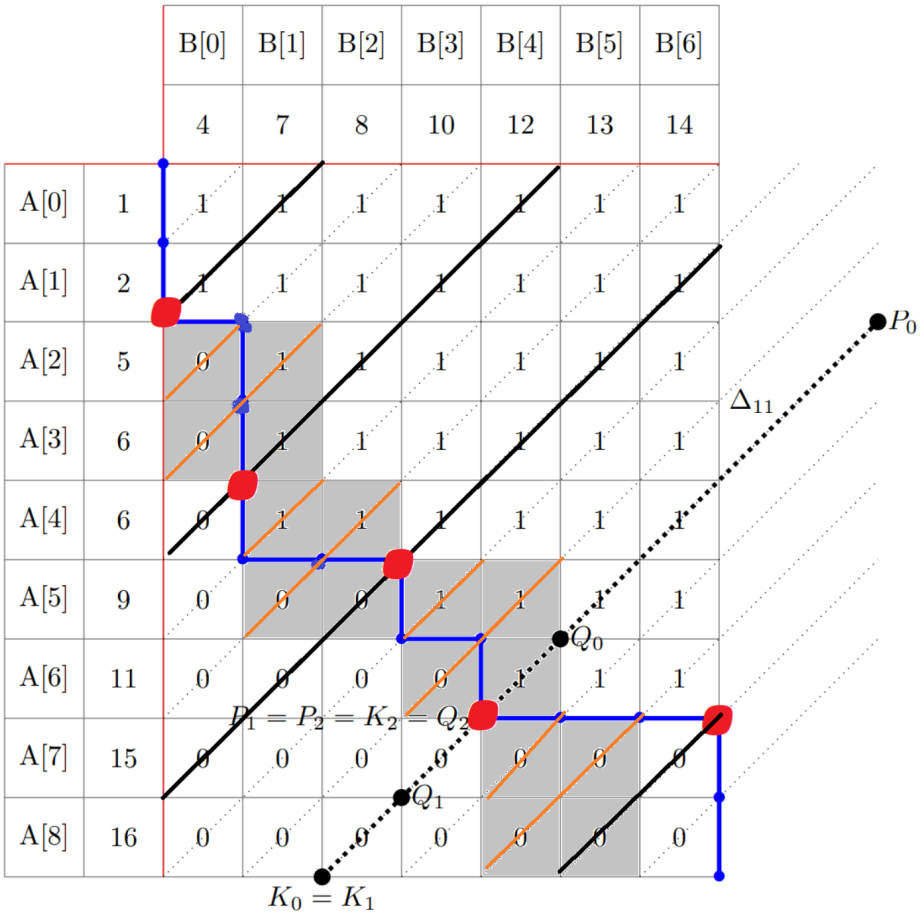
Blocks and memories use

Diagonals divide the work among the blocks.Points of intersection using binary searches on the cross diagonals by comparing elements from A and B on **GLOBAL MEMORY**.



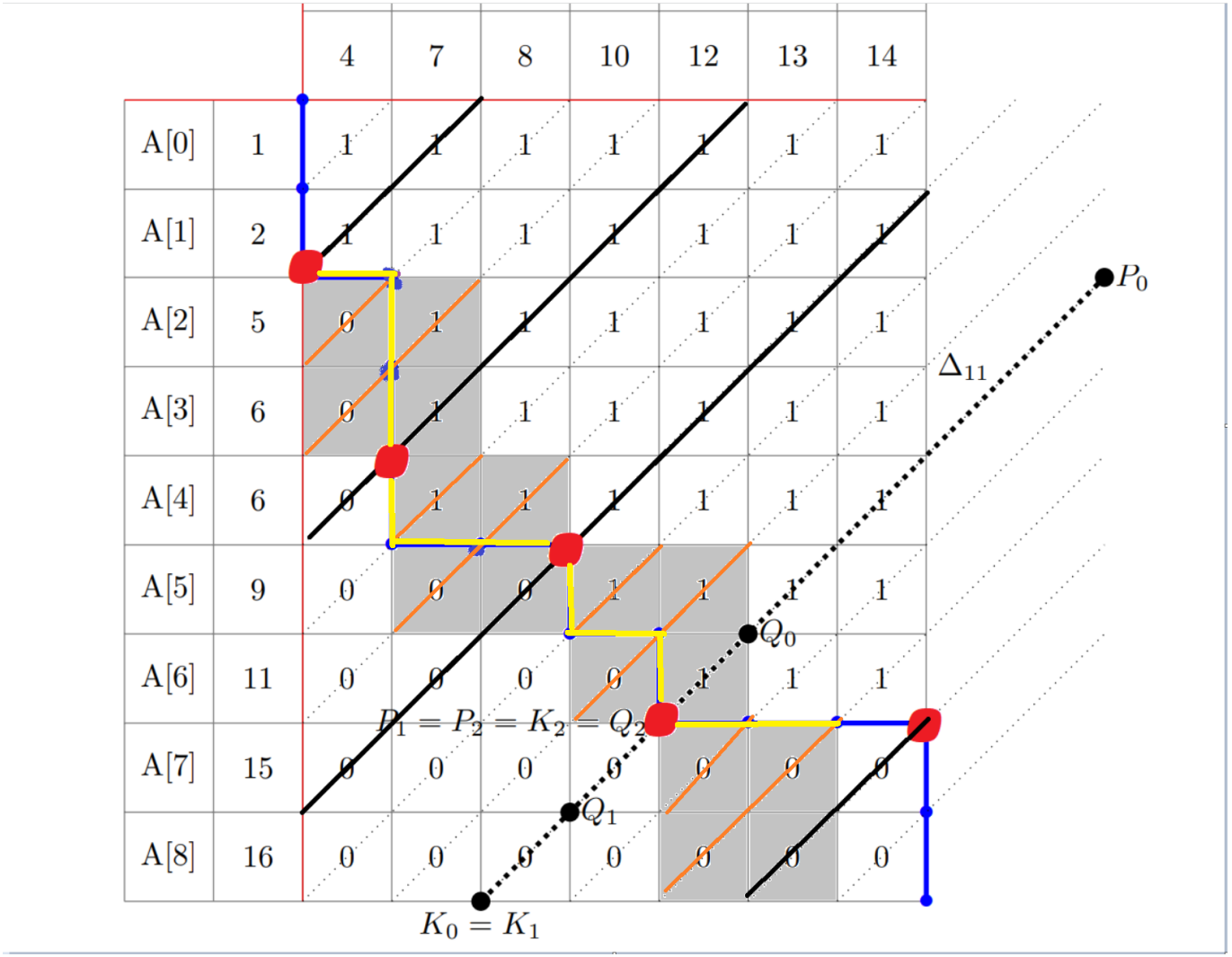
Blocks and memories use

Take a window consisting of the Z largest elements of each of the partitions and place them in local **SHARED MEMORY**.



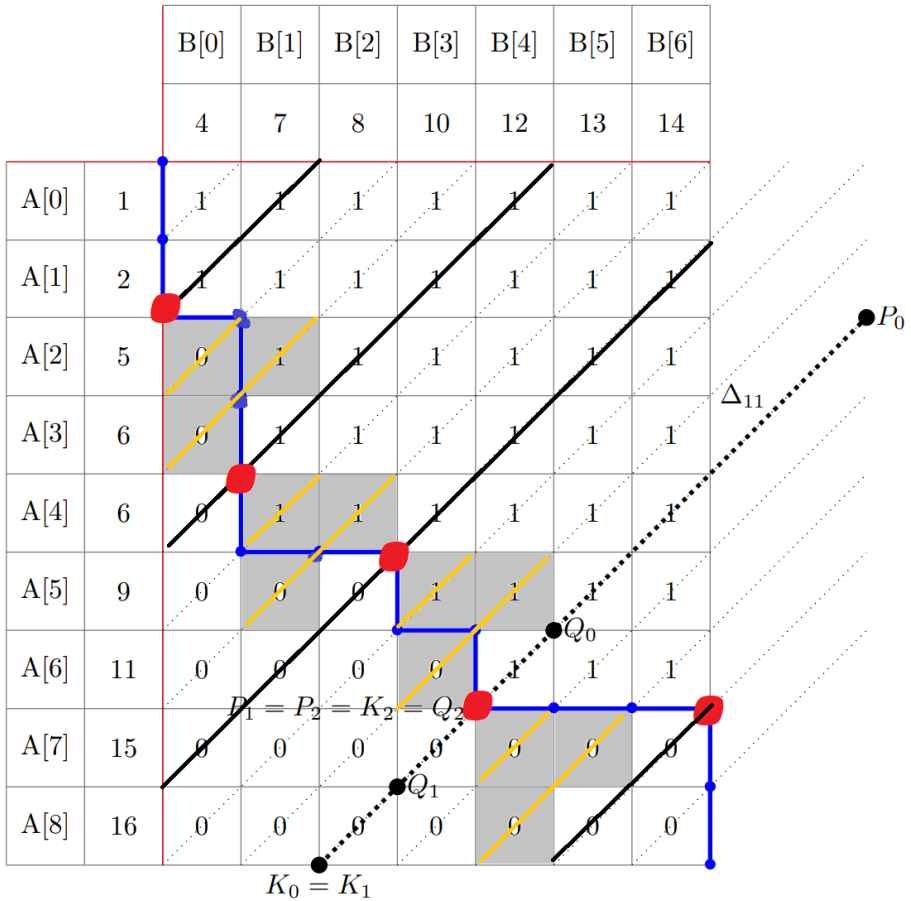
Blocks and memories use

Write the Merge Path found on **GLOBAL MEMORY**.



Blocks and memories use

Several cycles in parallel and Place the Merge Path on the **SHARED MEMORY**⇒ MERGE.



Blocks and memories use

3 different cases illustration:

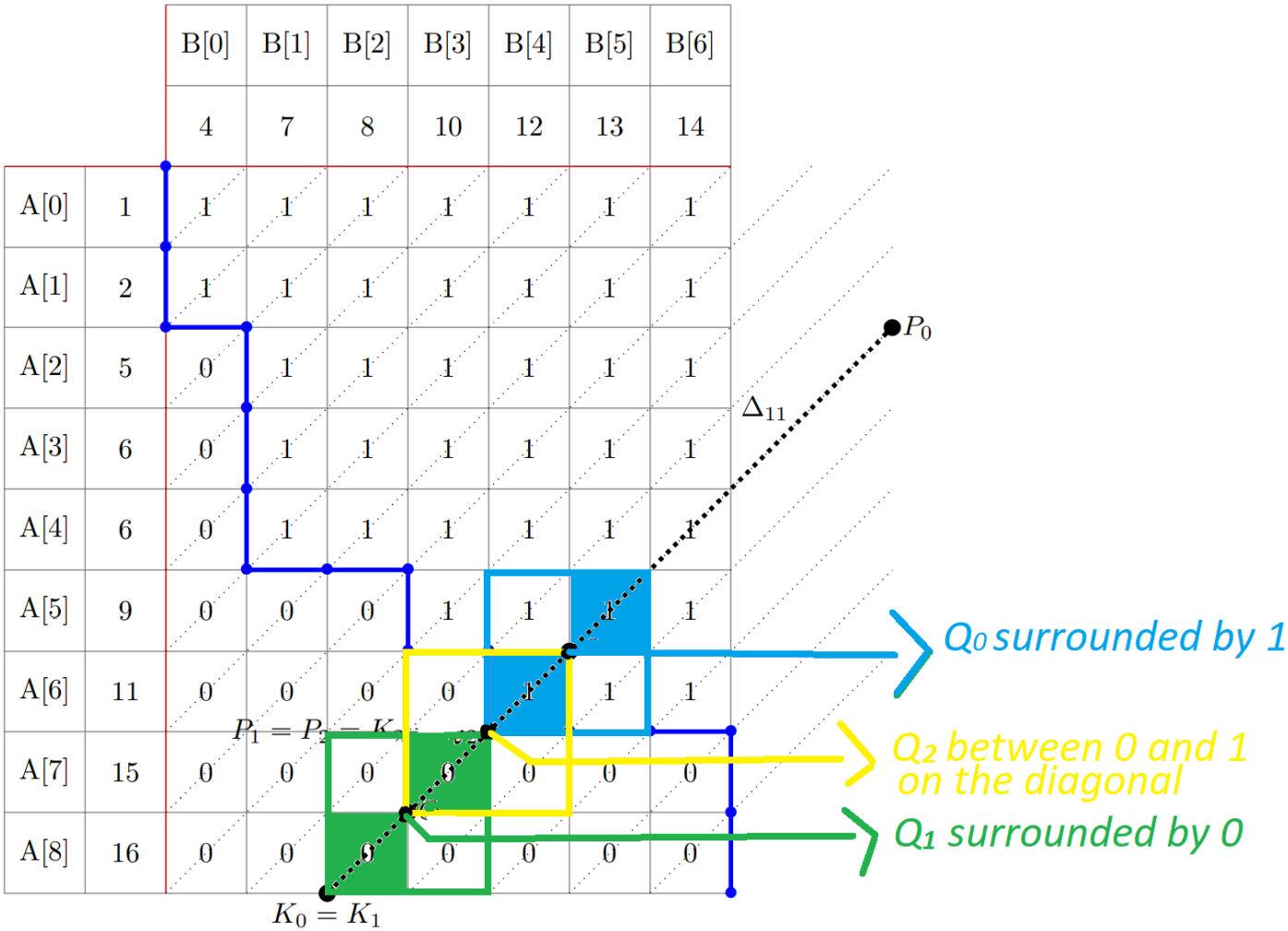
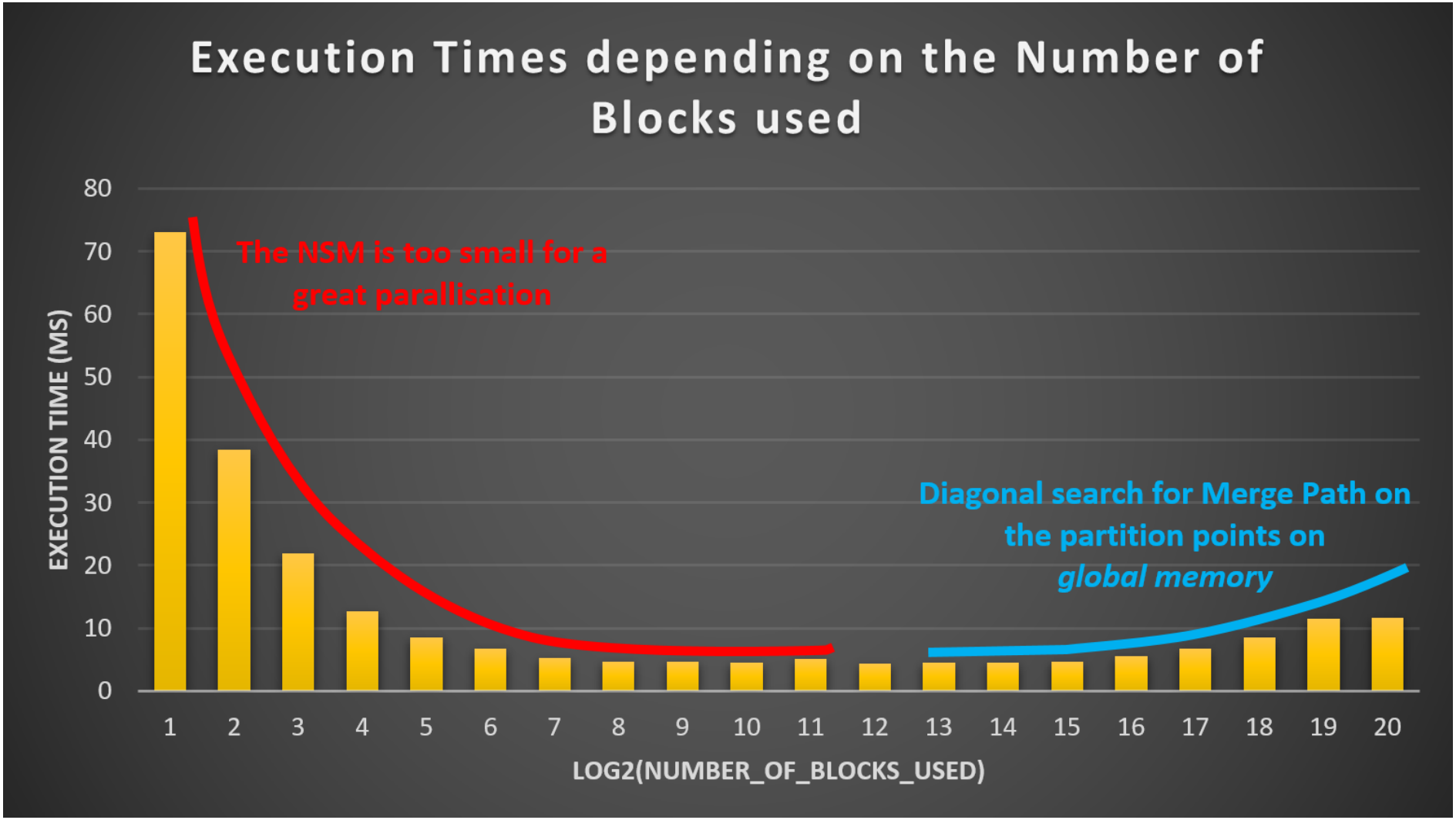


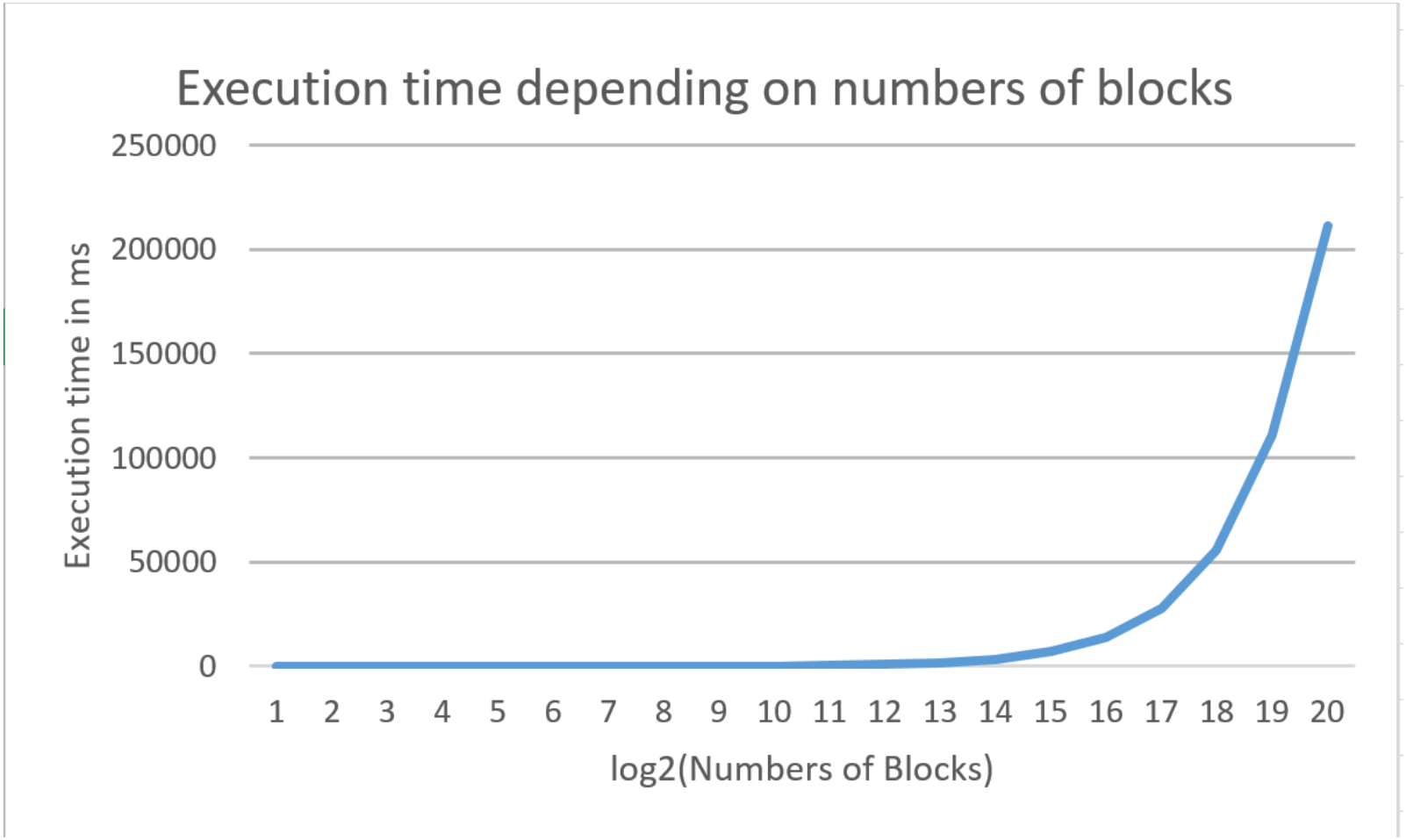
Figure 1: An example of Merge Path procedure

Performances for our MergeSort

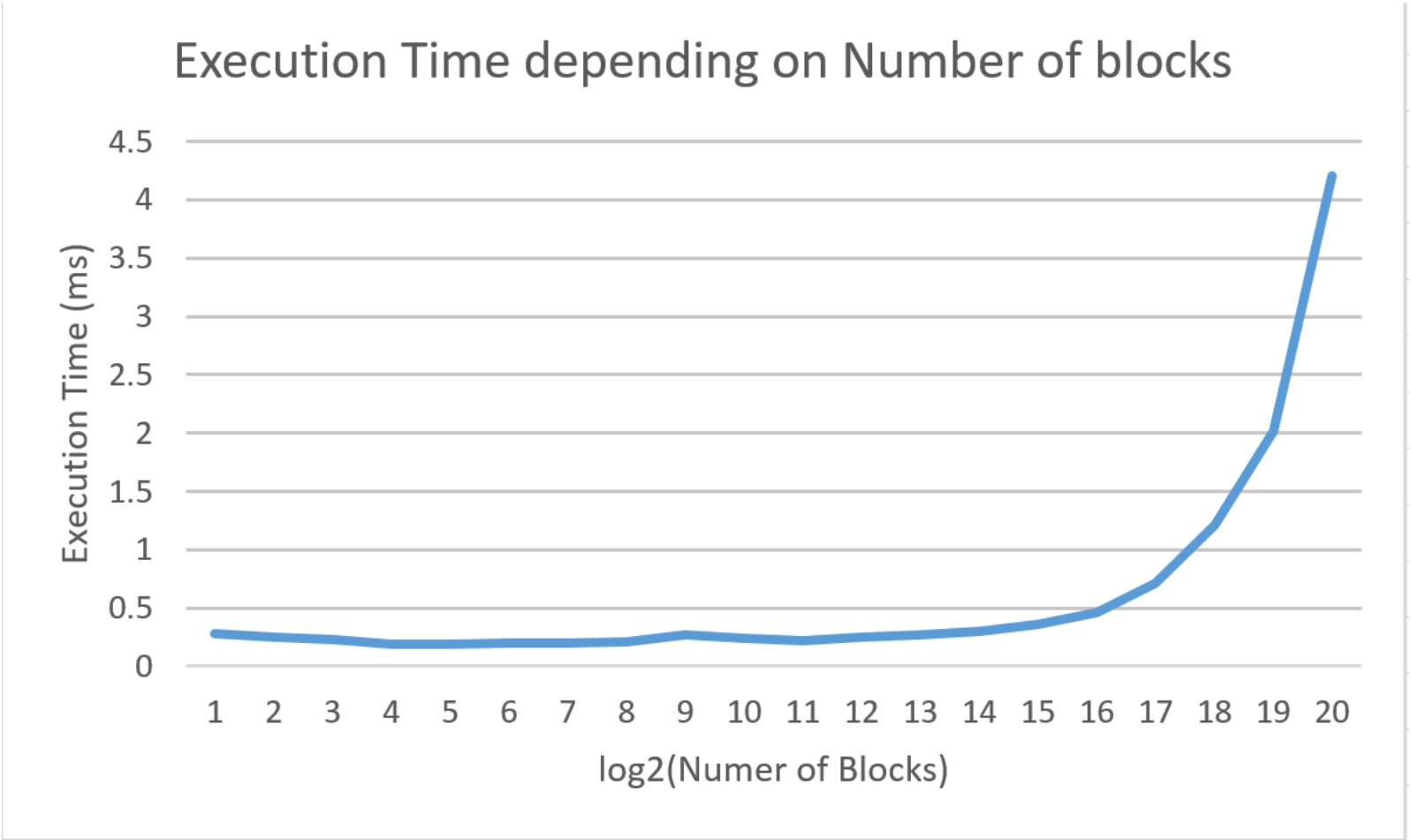
Optimal number of blocks



Performances of 1.3



Performances of 1.2



Performances: Wall Clock Time

- We decided to compare the wall-clock time of merging randomly generated vectors of integers for the three algorithms. For the GPU algorithm, we also monitored the wall-clock time for sort initialization $\Rightarrow$**CUDA EVEN TIMER**
- Sanity check We implemented a sanity check (on CPU for the three versions) to check if the array is really sorted in the end of the program.

Performances: Wall Clock Time

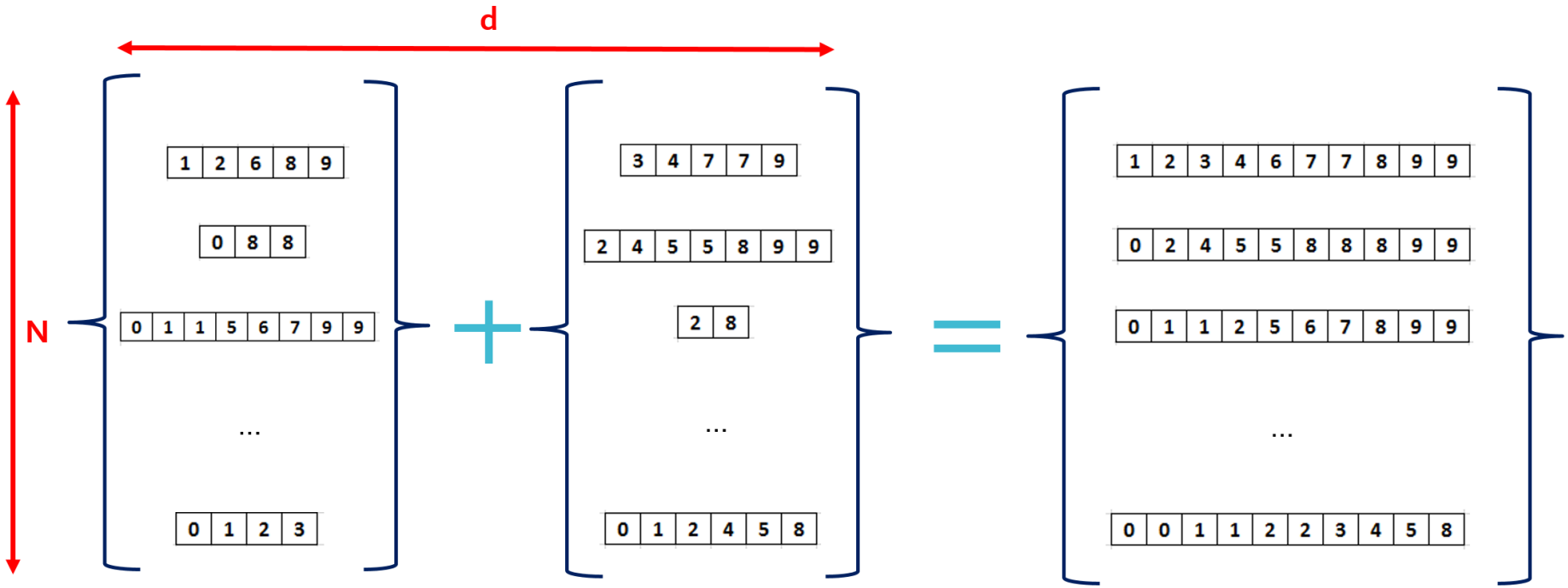
We ran 100 simulations of the three algorithms, gathering the average wall-clock time of the merge part only. The task consisted in merging 2^{20} = 1,048,576 arrays of size 2

Algorithm	Wall-clock time (ms)
Merge path (CPU)	50 032
Sequential merge path	1 650
Merge path (GPU)	70

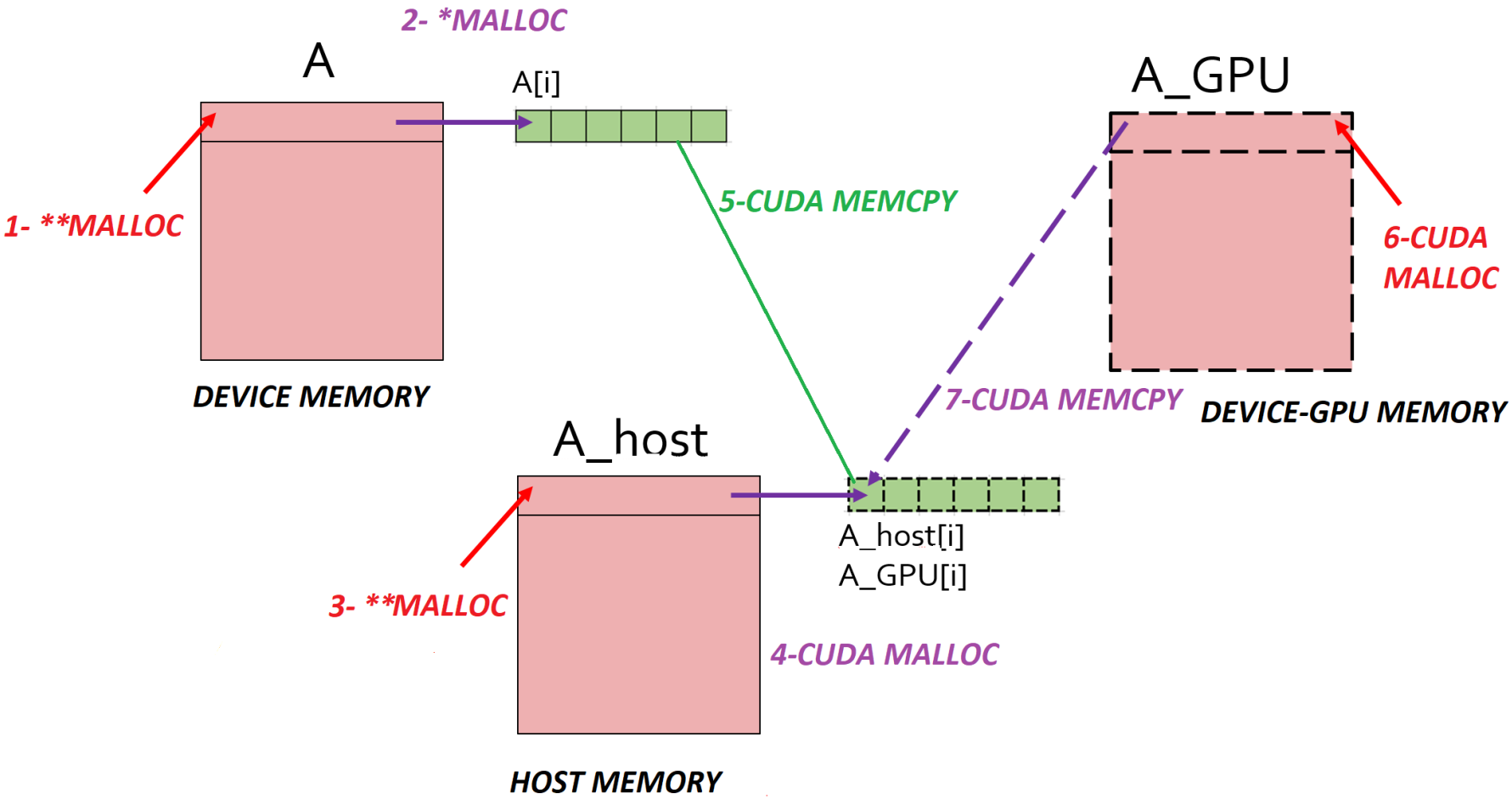
$\approx \times 20$

Summary

Definition


d
N
1 2 6 8 9
0 8 8
0 1 1 5 6 7 9 9
...
0 1 2 3
3 4 7 7 9
2 4 5 5 8 9 9
2 8
...
0 1 2 4 5 8
1 2 3 4 6 7 7 8 9 9
0 2 4 5 5 8 8 8 9 9
0 1 1 2 5 6 7 8 9 9
...
0 0 1 1 2 2 3 4 5 8

Procedure to allocate on the DEVICE an array of arrays:



Indexes Importance

d: number of elements in a pair of array

m: number of pairs of array mergesorted in a single block

1. int tidx = threadIdx.x%d ⇒ Enumeration from 0 to d-1 of the elements of each of the m pairs of arrays which are going to be sorted in a single block
2. int Qt = (threadIdx.x-tidx)/d ⇒ Local enumeration from 0 to m-1 of the pairs of arrays in a single block
3. int gbx = Qt + blockIdx.x*(blockDim.x/d) ⇒ Global enumeration from 0 to N-1 of the pairs of arrays in all the blocks

Execution time

Figure: Batch merge execution times in function of N with various values of *d*

Execution time

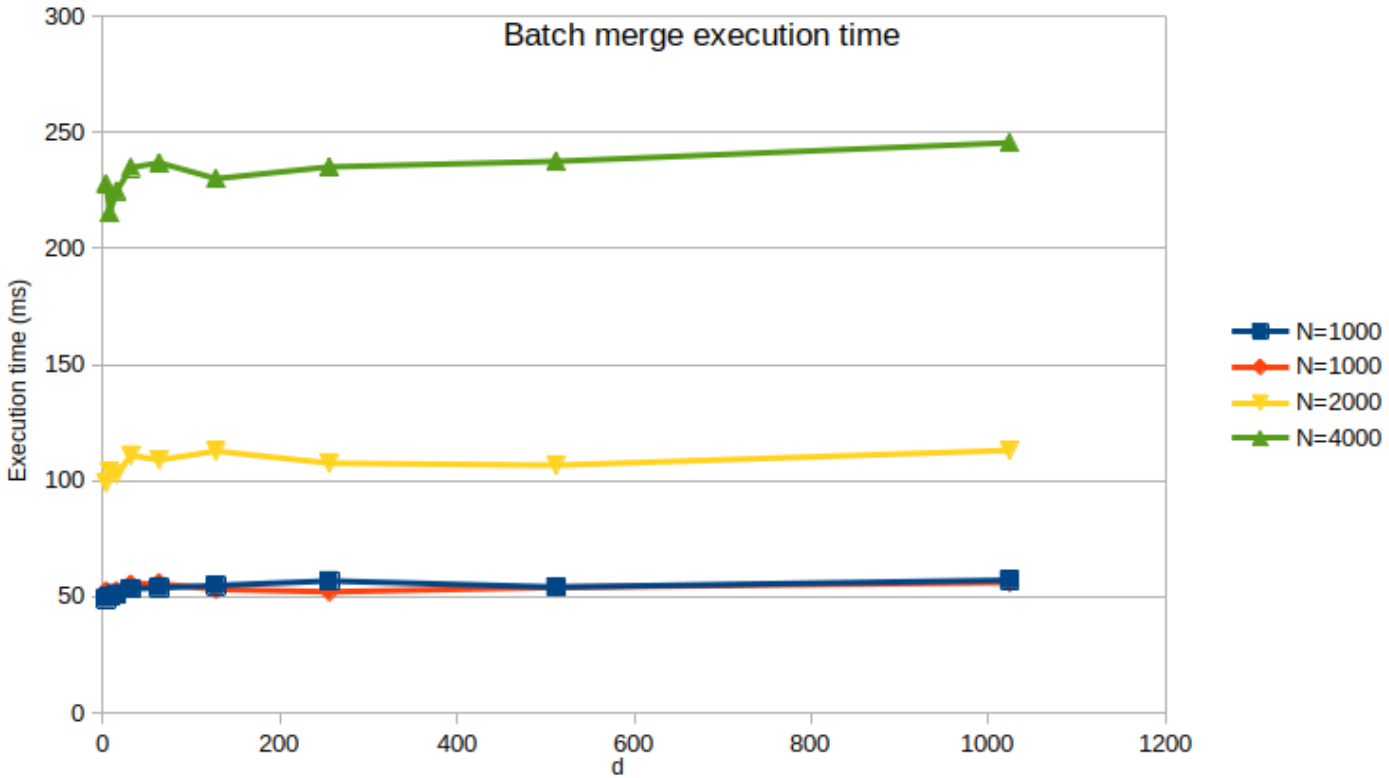


Figure: Batch merge execution times in function of d with various values of N

Execution time

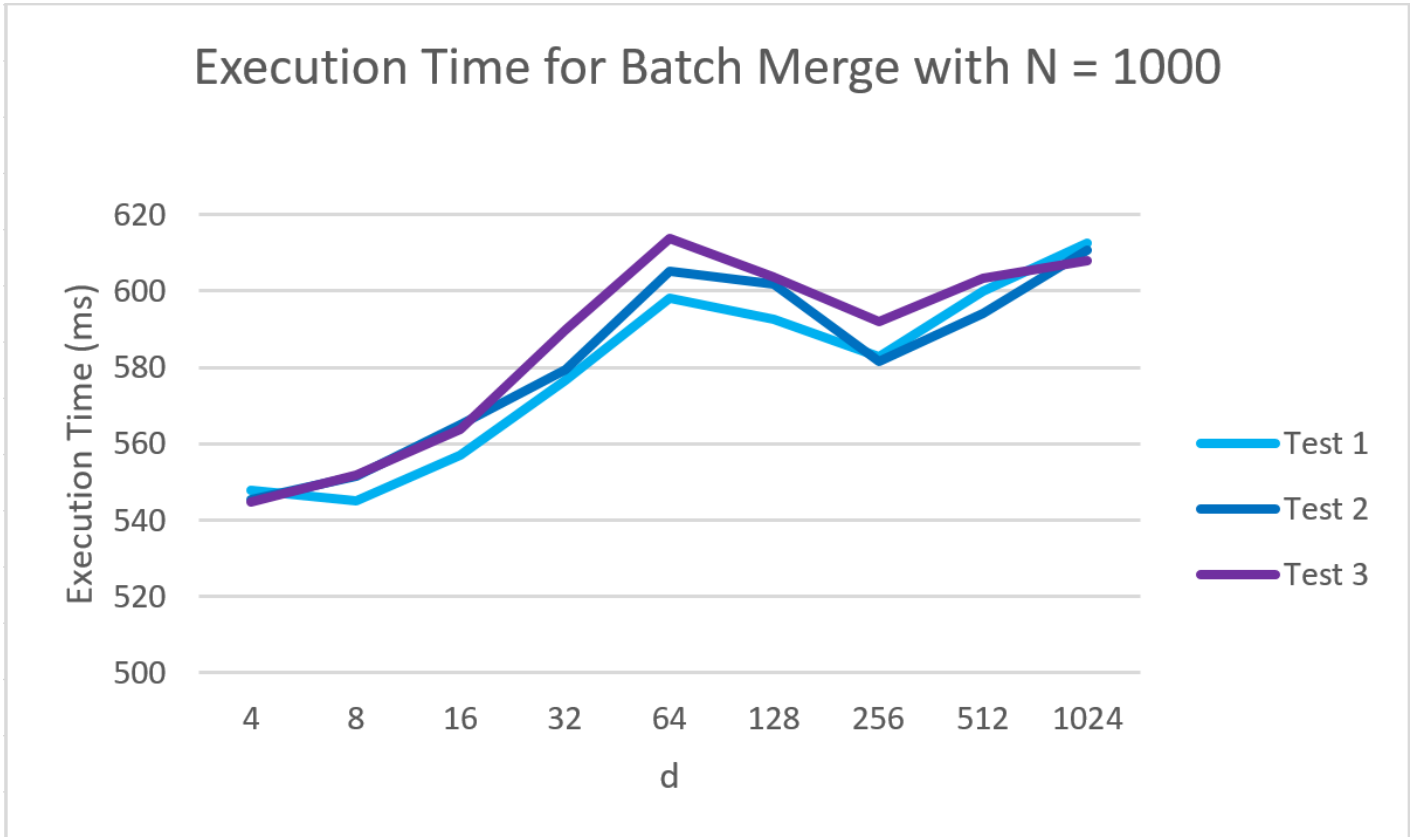


Figure: Batch merge execution times in function of $N = 10^4$ with various values of d

Execution time

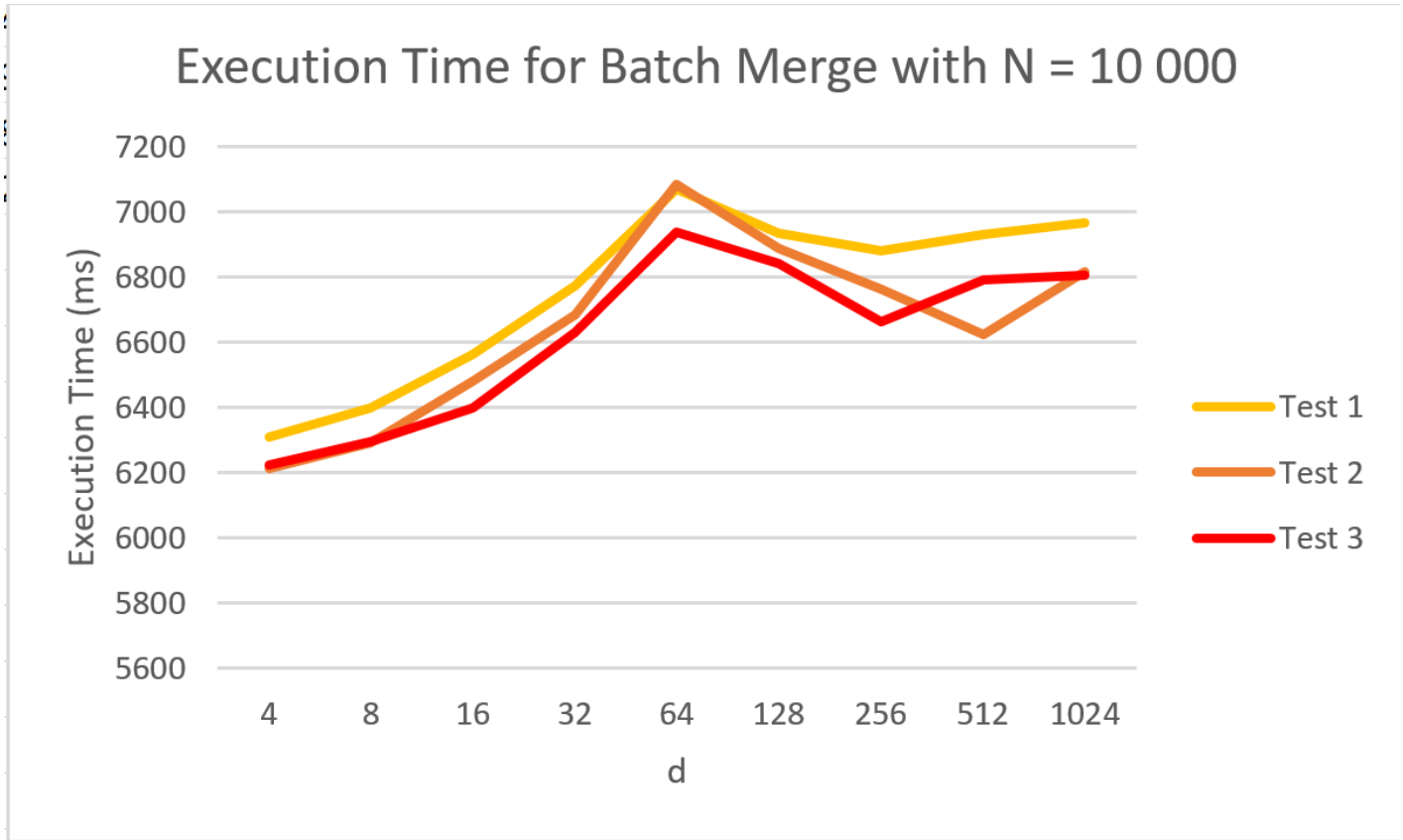


Figure: Batch merge execution times in function of $N = 10^5$ with various values of d

Execution time

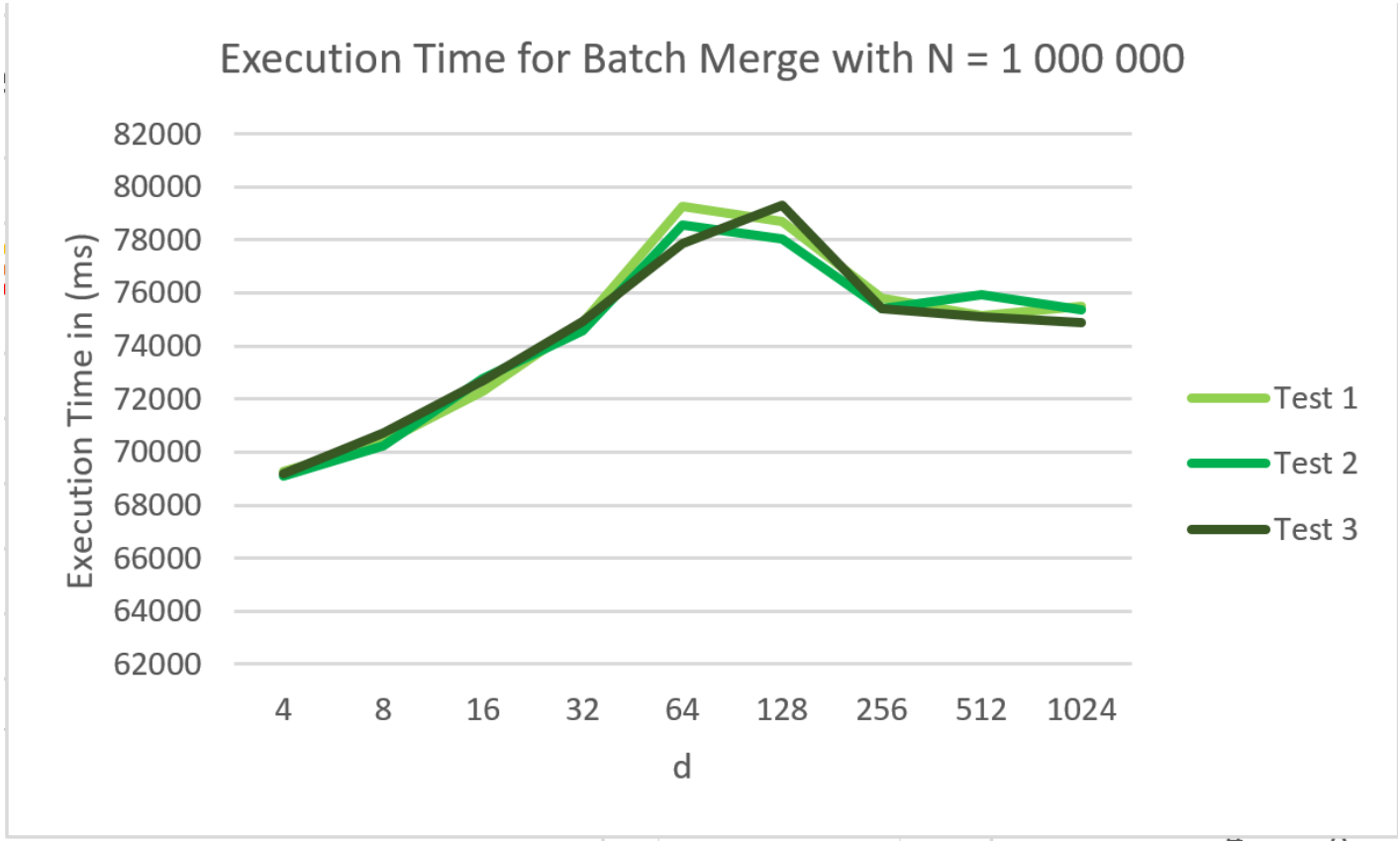


Figure: Batch merge execution times in function of $N = 10^6$ with various values of d

Summary

Authentification protocol

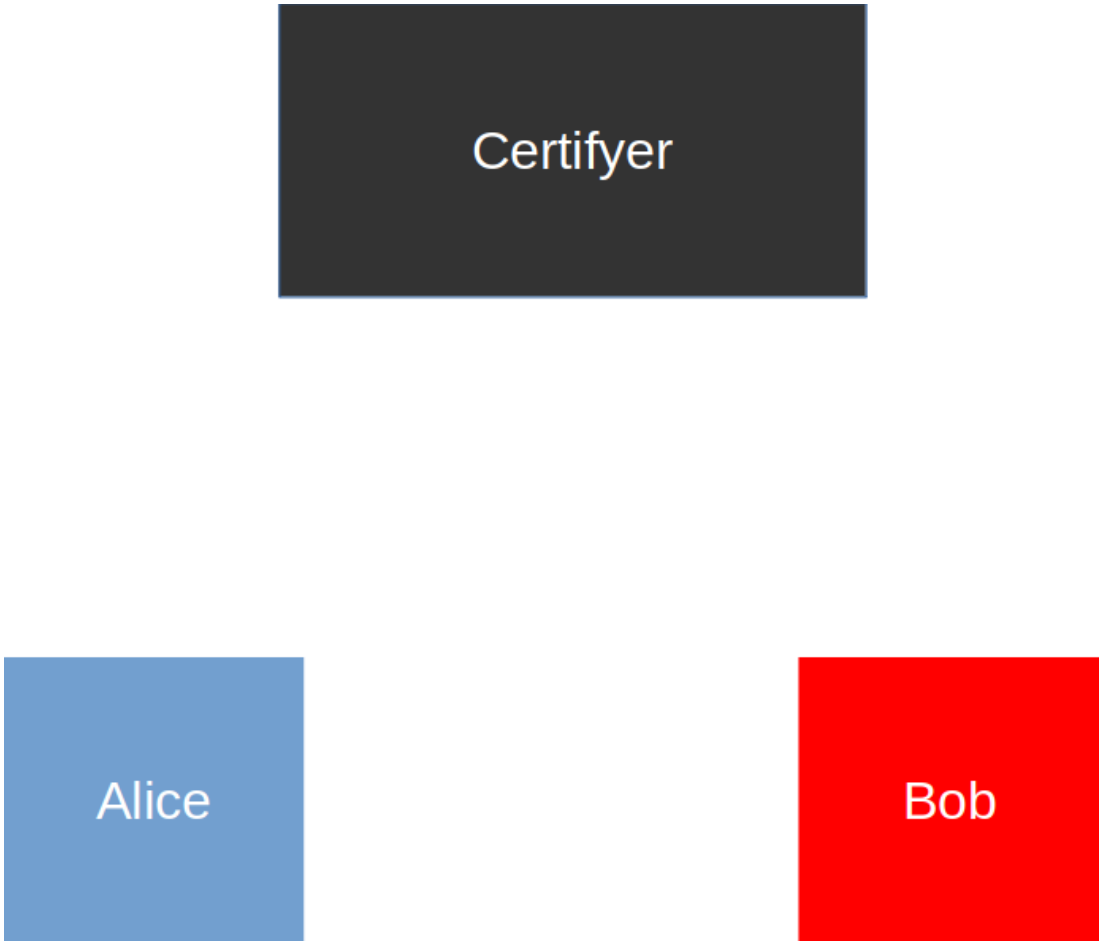


Figure: Alice and Bob wan't to get access to the certifyer

Authentification protocol

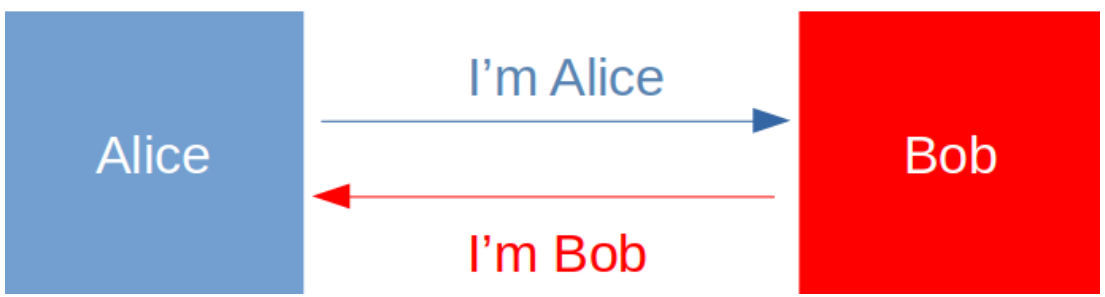


Figure: Alice and Bob recognise each other

Authentification protocol

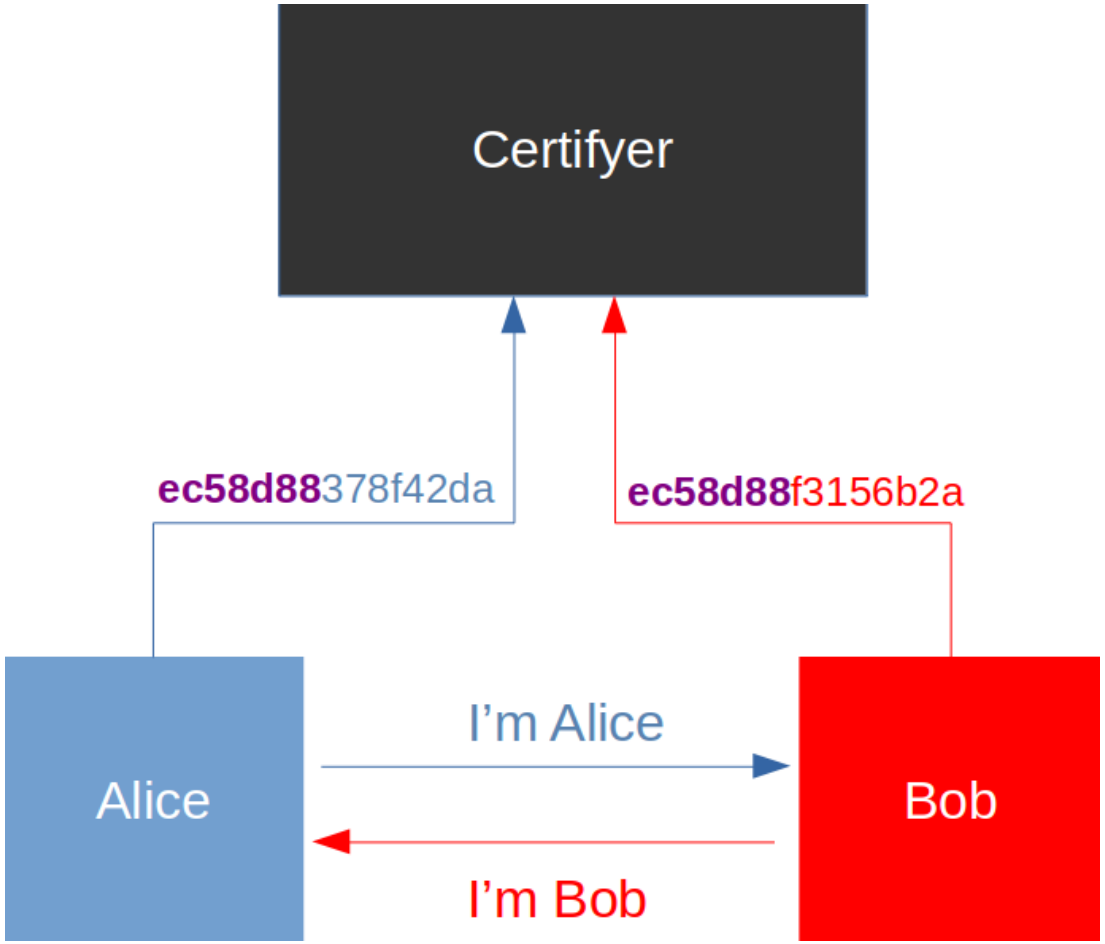


Figure: Alice and Bob send their hashed message

Hackage method

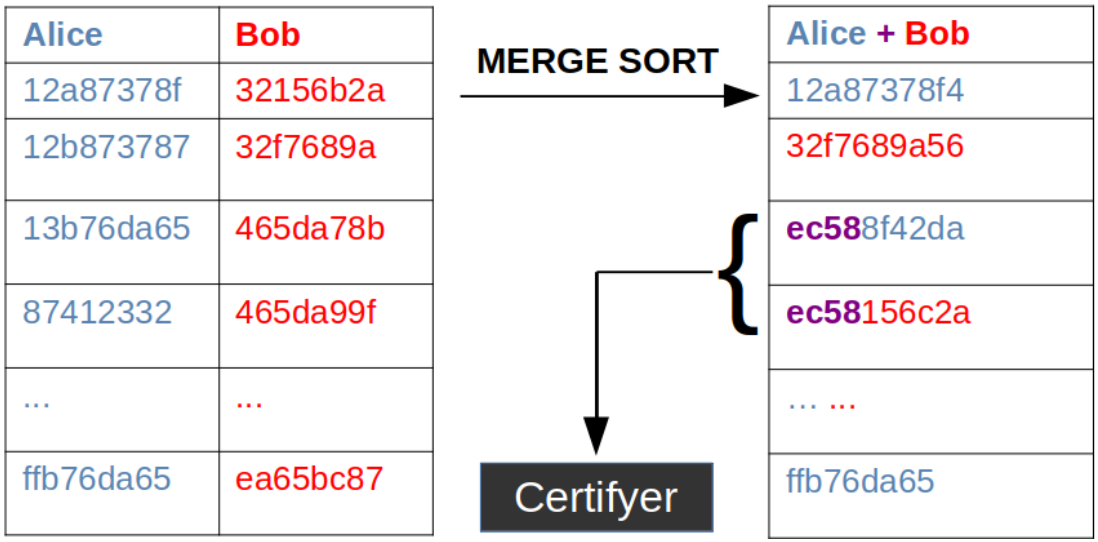


Figure: Merge sort and find keys to get access

Result

```
./main
Initialize and sort every possible vectors ... Ok.

------------------

Alice and bob arrive ! Merge sort Alice's and Bob's vectors
dim: 80000
-13507636 -13507255 -13506814 -13506804 -13506753 -13506653
End. Execution time: 0.760576 ms

-------------------n
Try to find 2 chains with the same first bits
Begining of the search... End of the search.
2 keys are:
ff36e7fe
ff36e707
```

Figure: Output

Conclusion

THANK YOU
FOR
YOUR
ATTENTION!
ANY QUESTIONS?

Reference: O. Green, R. McColl and D. A. Bader «GPU Merge Path, A GPU Merging Algorithm» 26th ACM International Conference on Supercomputing (ICS), San Servolo Islan, Venice, Italy, June 25-29, 2012