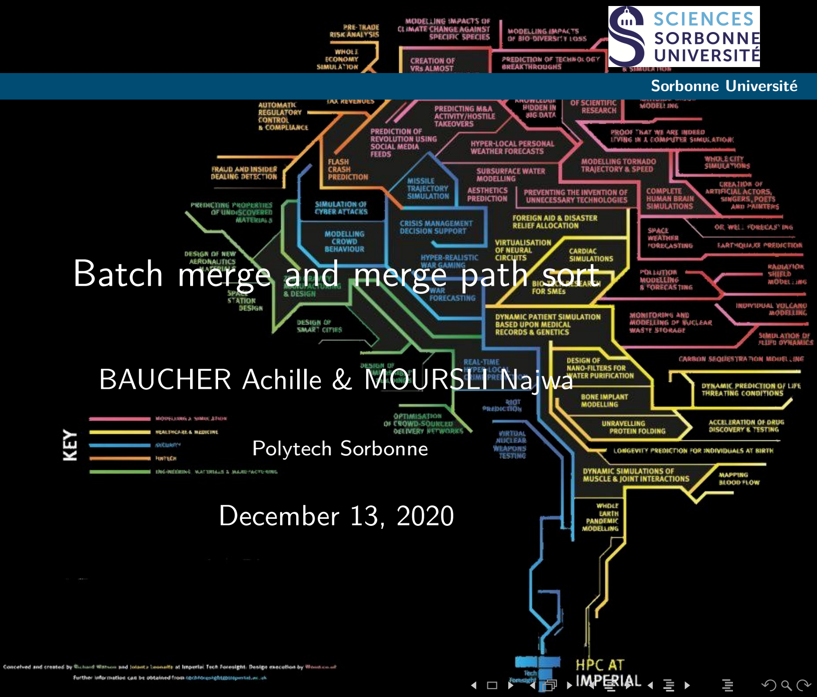
Batch merge and merge path sort

BAUCHER Achille & MOURSLI Najwa

Polytech Sorbonne

December 13, 2020


PRE-TRADE RISK ANALYSIS
MODELLING IMPACTS OF CLIMATE CHANGE AGAINST SPECIFIC SPECIES
MODELLING IMPACTS OF BIO-DIVERSITY LOSS
WHOLE ECONOMY SIMULATION
CREATION OF VRs ALMOST
PREDICTION OF TECHNOLOGY BREAKTHROUGHS
AUTOMATIC REGULATORY CONTROL & COMPLIANCE
TAX REVENUES
PREDICTING M&A ACTIVITY/HOSTILE TAKEOVERS
KNOWLEDGE HIDDEN IN BIG DATA
OF SCIENTIFIC RESEARCH
PREDICTION OF REVOLUTION USING SOCIAL MEDIA FEEDS
HYPER-LOCAL PERSONAL WEATHER FORECASTS
PROOF THAT WE ARE INDEED LIVING IN A COMPUTER SIMULATION
FRAUD AND INSIDER DEALING DETECTION
FLASH CRASH PREDICTION
SUBSURFACE WATER MODELLING
MODELLING TORNADO TRAJECTORY & SPEED
WHOLE CITY SIMULATIONS
MISSILE TRAJECTORY SIMULATION
AESTHETICS PREDICTION
PREVENTING THE INVENTION OF UNNECESSARY TECHNOLOGIES
COMPLETE HUMAN BRAIN SIMULATIONS
CREATION OF ARTIFICIAL ACTORS, SINGERS, POETS AND PAINTERS
PREDICTING PROPERTIES OF UNDISCOVERED MATERIALS
SIMULATION OF CYBER ATTACKS
CRISIS MANAGEMENT DECISION SUPPORT
FOREIGN AID & DISASTER RELIEF ALLOCATION
SPACE WEATHER FORECASTING
OIL WELL FORECASTING
EARTHQUAKE PREDICTION
MODELLING CROWD BEHAVIOUR
VIRTUALISATION OF NEURAL CIRCUITS
CARDIAC SIMULATIONS
DESIGN OF NEW AERONAUTICS MATERIALS
HYPER-REALISTIC WAR GAMING
RADIATION SHIELD MODELLING
POLLUTION MODELLING & FORECASTING
SPACE STATION DESIGN
& DESIGN
WAR FORECASTING
FOR SMEs
INDIVIDUAL VOLCANO MODELLING
DESIGN OF SMART CITIES
DYNAMIC PATIENT SIMULATION BASED UPON MEDICAL RECORDS & GENETICS
MONITORING AND MODELLING OF NUCLEAR WASTE STORAGE
SIMULATION OF FLUID DYNAMICS
REAL-TIME HYPER-LOCAL
DESIGN OF NANO-FILTERS FOR WATER PURIFICATION
CARBON SEQUESTRATION MODELLING
DYNAMIC PREDICTION OF LIFE THREATENING CONDITIONS
RIOT PREDICTION
BONE IMPLANT MODELLING
OPTIMISATION OF CROWD-SOURCED DELIVERY NETWORKS
UNRAVELLING PROTEIN FOLDING
ACCELERATION OF DRUG DISCOVERY & TESTING
VIRTUAL NUCLEAR WEAPONS TESTING
LONGEVITY PREDICTION FOR INDIVIDUALS AT BIRTH
DYNAMIC SIMULATIONS OF MUSCLE & JOINT INTERACTIONS
MAPPING BLOOD FLOW
WHOLE EARTH PANDEMIC MODELLING
KEY
MODELLING & SIMULATION
HEALTHCARE & MEDICINE
SECURITY
FINTECH
ENGINEERING, MATERIALS & MANUFACTURING
Conceived and created by Richard Watson and Jolanta Leonardi at Imperial Tech Foresight. Design execution by
Further information can be obtained from techforesight@imperial.ac.uk
Tech Foresight
HPC AT IMPERIAL

Summary

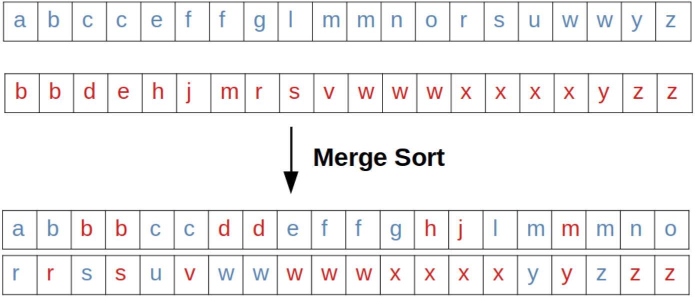
1 Introduction

2 Merge Path and Sort

3 Batch merge

4 Application in Cryptanalysis

Main Goal



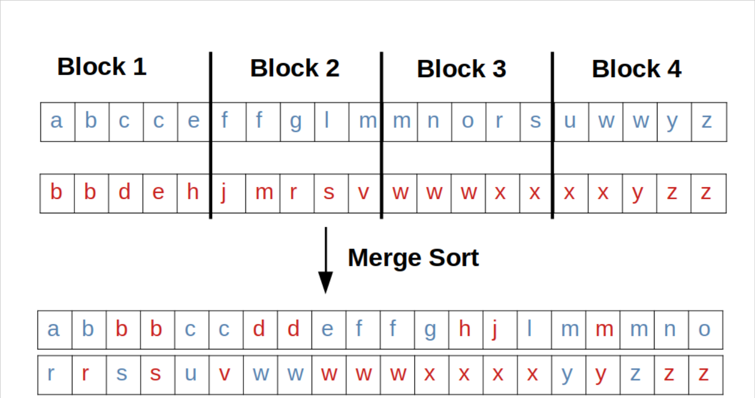
a	b	c	c	e	f	f	g	l	m	m	n	o	r	s	u	w	w	y	z

b	b	d	e	h	j	m	r	s	v	w	w	w	x	x	x	x	y	z	z

Merge Sort

a	b	b	b	c	c	d	d	e	f	f	g	h	j	l	m	m	m	n	o
r	r	s	s	u	v	w	w	w	w	w	x	x	x	x	y	y	z	z	z

Can be distributed

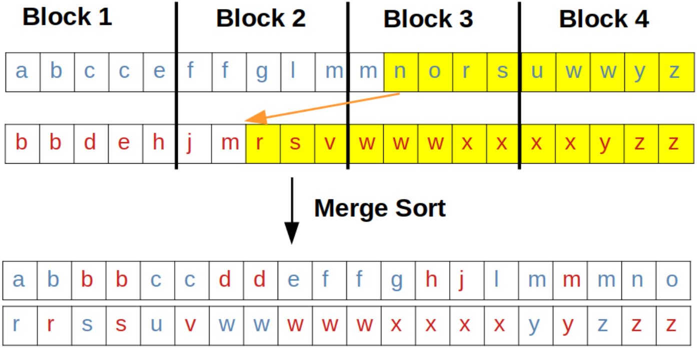


Block 1					Block 2					Block 3					Block 4				
a	b	c	c	e	f	f	g	l	m	m	n	o	r	s	u	w	w	y	z
b	b	d	e	h	j	m	r	s	v	w	w	w	x	x	x	x	y	z	z

Merge Sort

a	b	b	b	c	c	d	d	e	f	f	g	h	j	l	m	m	m	n	o
r	r	s	s	u	v	w	w	w	w	w	x	x	x	x	y	y	z	z	z

Task distribution



Block 1					Block 2					Block 3					Block 4				
a	b	c	c	e	f	f	g	l	m	m	n	o	r	s	u	w	w	y	z
b	b	d	e	h	j	m	r	s	v	w	w	w	x	x	x	x	y	z	z

Merge Sort

a	b	b	b	c	c	d	d	e	f	f	g	h	j	l	m	m	m	n	o
r	r	s	s	u	v	w	w	w	w	w	x	x	x	x	y	y	z	z	z

Summary

1 Introduction

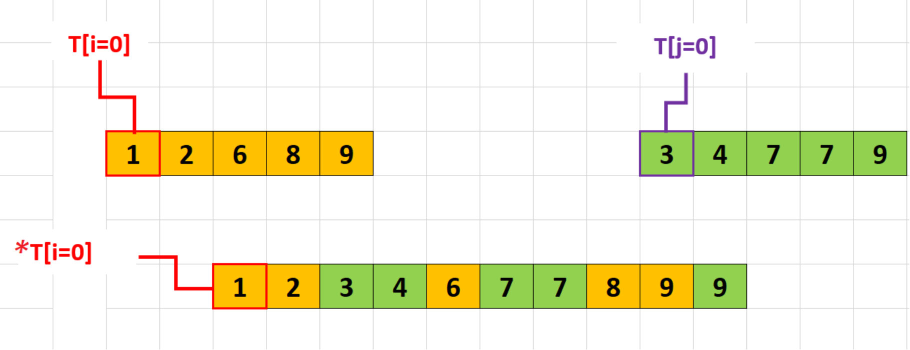
2 Merge Path and Sort

3 Batch merge

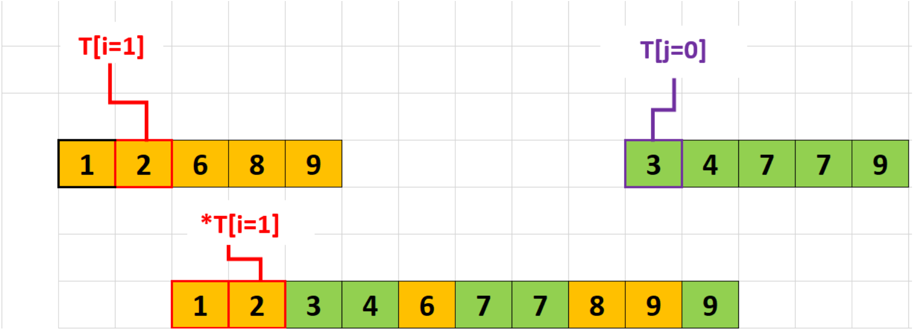
4 Application in Cryptanalysis

Serial Merge Path

The most common idea is to use two pointers and increment one of them whenever it has the lowest value among both, while copying its value to the output array:



Serial Merge Path



T[i=1]

T[j=0]

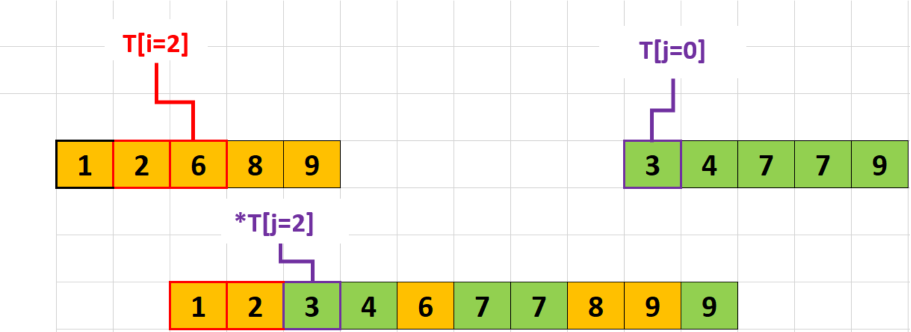
1 2 6 8 9

3 4 7 7 9

*T[i=1]

1 2 3 4 6 7 7 8 9 9

Serial Merge Path



T[i=2]

1	2	6	8	9

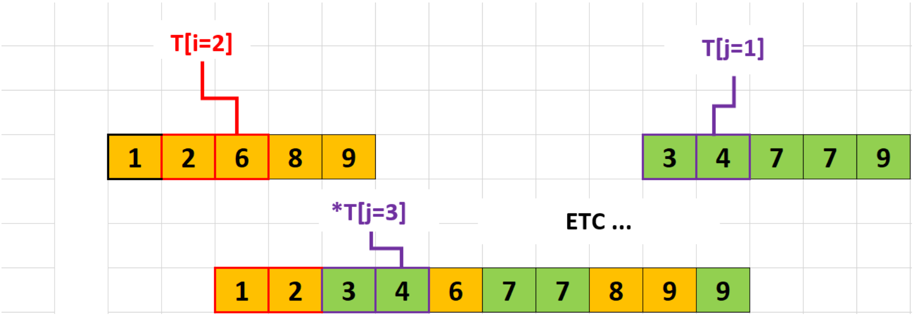
T[j=0]

3	4	7	7	9

*T[j=2]

1	2	3	4	6	7	7	8	9	9

Serial Merge Path



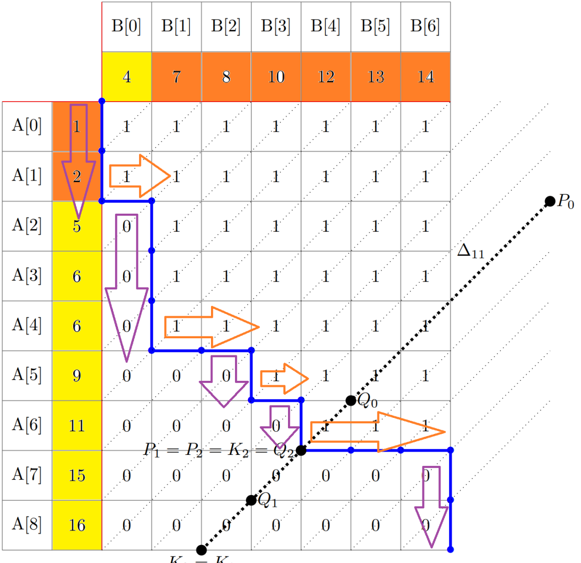
On our example

⇨ *A[i]<B[j]*

⇩ *A[i]>B[j]*
A[i']>B[j]
i'⩽i

🟨 *0 marked*

🟧 *1 marked*



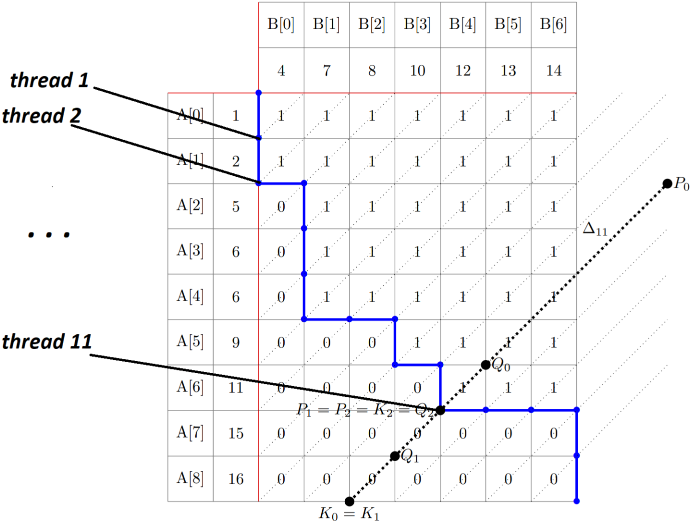
		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]
		4	7	8	10	12	13	14
A[0]	1	1	1	1	1	1	1	1
A[1]	2	1	1	1	1	1	1	1
A[2]	5	0	1	1	1	1	1	1
A[3]	6	0	1	1	1	1	1	1
A[4]	6	0	1	1	1	1	1	1
A[5]	9	0	0	0	1	1	1	1
A[6]	11	0	0	0	0	1	1	1
A[7]	15	0	0	0	0	0	0	0
A[8]	16	0	0	0	0	0	0	0

On our example

- It is simple to implement
- It has always the same linear complexity $\Theta(|A| + |B|)$: no bad surprise
- In its original form, it is impossible to parallelize the algorithm.
- An average linear complexity is not optimal

Blocks and memories use

For one block :



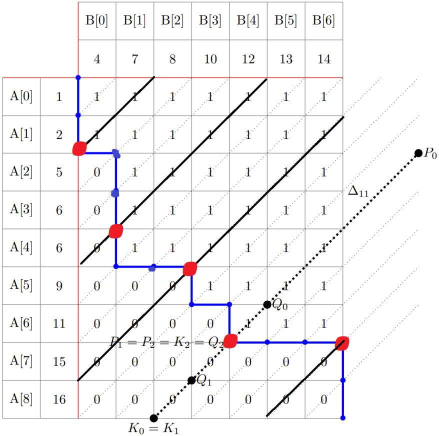
		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]
		4	7	8	10	12	13	14
A[0]	1	1	1	1	1	1	1	1
A[1]	2	1	1	1	1	1	1	1
A[2]	5	0	1	1	1	1	1	1
A[3]	6	0	1	1	1	1	1	1
A[4]	6	0	1	1	1	1	1	1
A[5]	9	0	0	0	1	1	1	1
A[6]	11	0	0	0	0	1	1	1
A[7]	15	0	0	0	0	0	0	0
A[8]	16	0	0	0	0	0	0	0

Several Blocks and memories use

- **Parallel Partition**
  ⇒ Map exactly the necessary inputs into each tile.
- **Merge Path**
  ⇒ Cross diagonal binary search **GLOBAL MEMORY**
- **MergeSort of the Merge Path** ⇒ Placing the Merge Path on the **SHARED MEMORY** ⇒ Blocksort

Blocks and memories use

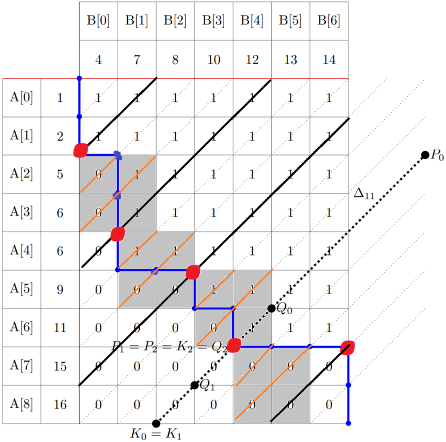
Diagonals divide the work among the blocks.Points of intersection using binary searches on the cross diagonals by comparing elements from A and B on **GLOBAL MEMORY**.



		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]
		4	7	8	10	12	13	14
A[0]	1	1	1	1	1	1	1	1
A[1]	2	1	1	1	1	1	1	1
A[2]	5	0	1	1	1	1	1	1
A[3]	6	0	1	1	1	1	1	1
A[4]	6	0	1	1	1	1	1	1
A[5]	9	0	0	0	1	1	1	1
A[6]	11	0	0	0	0	1	1	1
A[7]	15	0	0	0	0	0	0	0
A[8]	16	0	0	0	0	0	0	0

Blocks and memories use

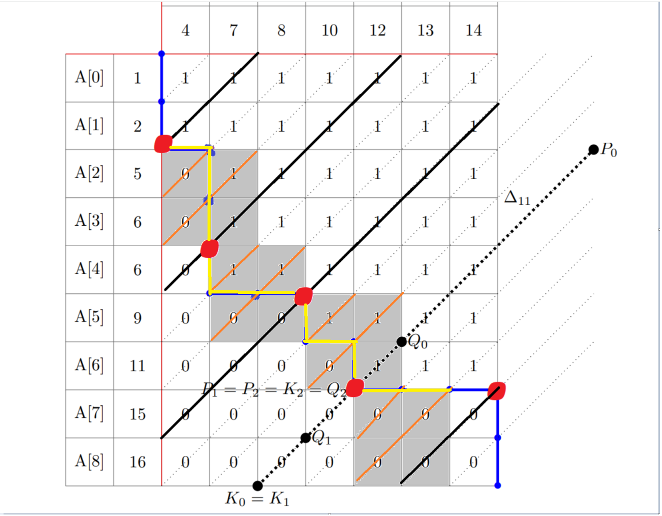
Take a window consisting of the Z largest elements of each of the partitions and place them in local **SHARED MEMORY**.



		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]
		4	7	8	10	12	13	14
A[0]	1	1	1	1	1	1	1	1
A[1]	2	1	1	1	1	1	1	1
A[2]	5	0	1	1	1	1	1	1
A[3]	6	0	1	1	1	1	1	1
A[4]	6	0	1	1	1	1	1	1
A[5]	9	0	0	0	1	1	1	1
A[6]	11	0	0	0	0	1	1	1
A[7]	15	0	0	0	0	0	0	0
A[8]	16	0	0	0	0	0	0	0

Blocks and memories use

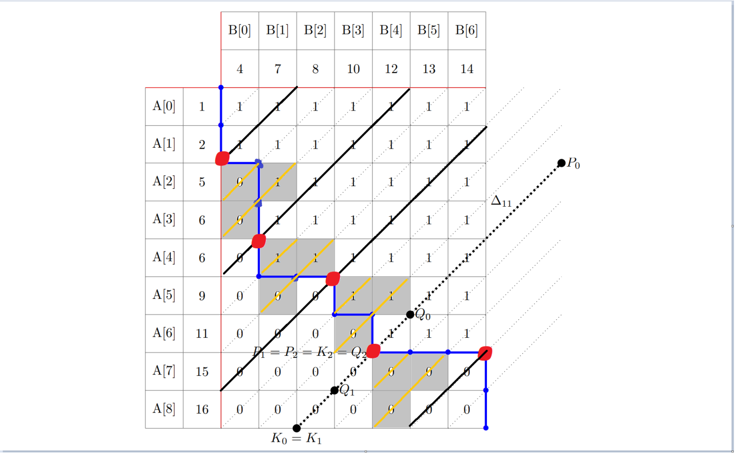
Write the Merge Path found on **GLOBAL MEMORY**.



		4	7	8	10	12	13	14
A[0]	1	1	1	1	1	1	1	1
A[1]	2	1	1	1	1	1	1	1
A[2]	5	0	1	1	1	1	1	1
A[3]	6	0	1	1	1	1	1	1
A[4]	6	0	1	1	1	1	1	1
A[5]	9	0	0	0	1	1	1	1
A[6]	11	0	0	0	0	1	1	1
A[7]	15	0	0	0	0	0	0	0
A[8]	16	0	0	0	0	0	0	0

Blocks and memories use

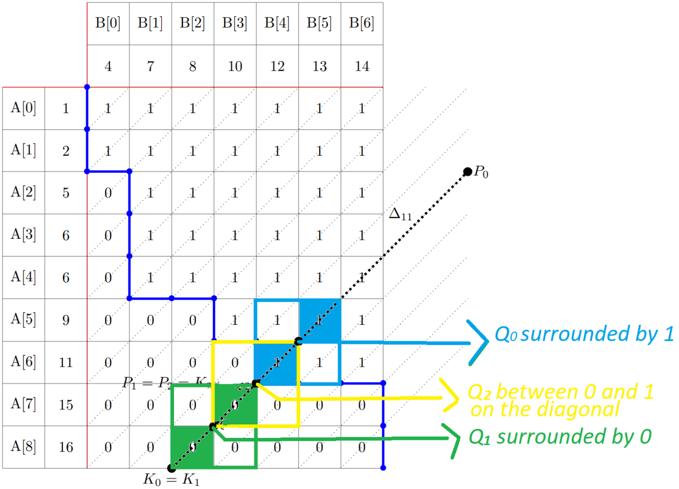
Several cycles in parallel and Place the Merge Path on the **SHARED MEMORY**⇒ MERGE.



		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]
		4	7	8	10	12	13	14
A[0]	1	1	1	1	1	1	1	1
A[1]	2	1	1	1	1	1	1	1
A[2]	5	0	1	1	1	1	1	1
A[3]	6	0	1	1	1	1	1	1
A[4]	6	0	1	1	1	1	1	1
A[5]	9	0	0	0	1	1	1	1
A[6]	11	0	0	0	0	1	1	1
A[7]	15	0	0	0	0	0	0	0
A[8]	16	0	0	0	0	0	0	0

Blocks and memories use

3 different cases illustration:



		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]
		4	7	8	10	12	13	14
A[0]	1	1	1	1	1	1	1	1
A[1]	2	1	1	1	1	1	1	1
A[2]	5	0	1	1	1	1	1	1
A[3]	6	0	1	1	1	1	1	1
A[4]	6	0	1	1	1	1	1	1
A[5]	9	0	0	0	1	1	1	1
A[6]	11	0	0	0	0	1	1	1
A[7]	15	0	0	0	0	0	0	0
A[8]	16	0	0	0	0	0	0	0

Q_0 surrounded by 1

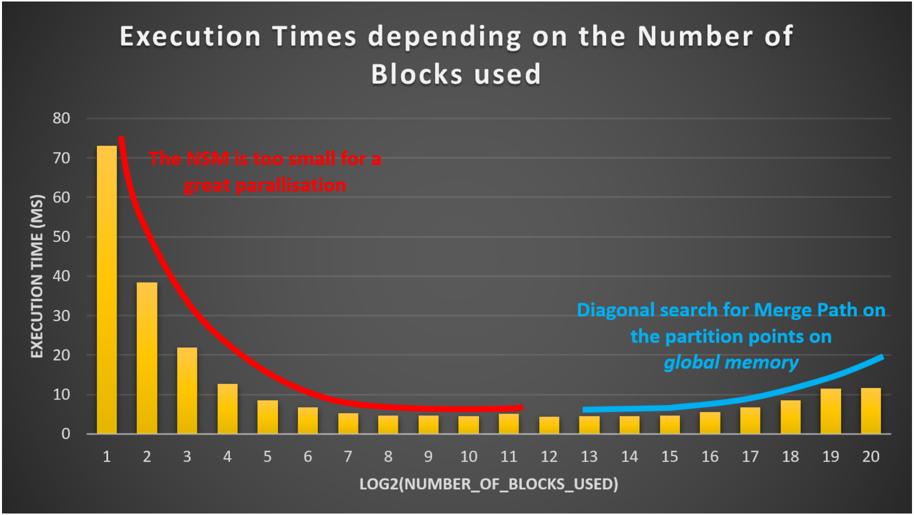
Q_2 between 0 and 1 on the diagonal

Q_1 surrounded by 0

Figure 1: An example of Merge Path procedure

Performances for our MergeSort

Optimal number of blocks

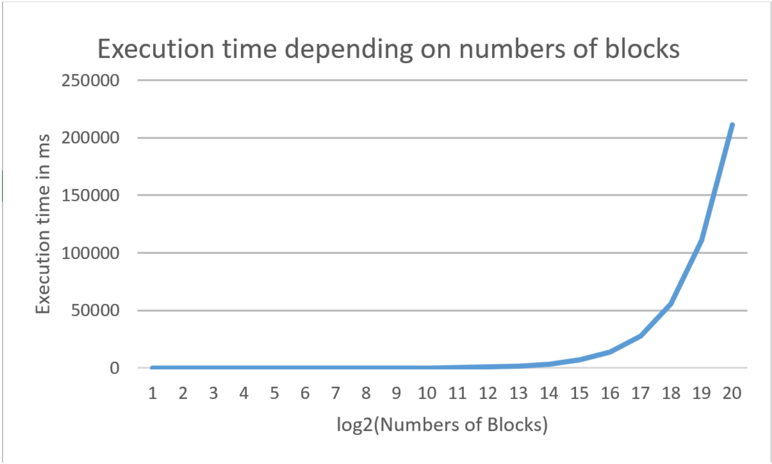


Execution Times depending on the Number of Blocks used

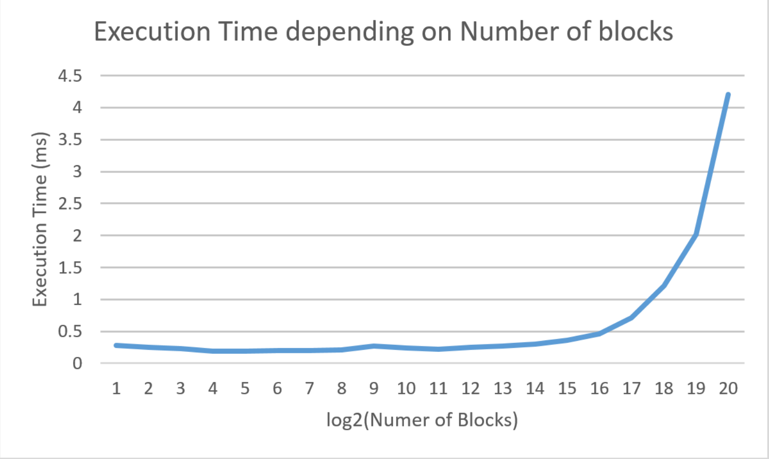
The NSM is too small for a great parallisation

Diagonal search for Merge Path on the partition points on *global memory*

Performances of 1.3



Performances of 1.2

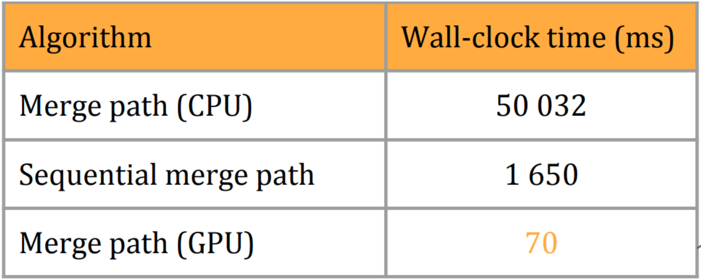


Performances: Wall Clock Time

- We decided to compare the wall-clock time of merging randomly generated vectors of integers for the three algorithms. For the GPU algorithm, we also monitored the wall-clock time for sort initialization ⇒**CUDA EVEN TIMER**
- Sanity check We implemented a sanity check (on CPU for the three versions) to check if the array is really sorted in the end of the program.

Performances: Wall Clock Time

We ran 100 simulations of the three algorithms, gathering the average wall-clock time of the merge part only. The task consisted in merging $2^{20} = 1{,}048{,}576$ arrays of size 2



Algorithm	Wall-clock time (ms)
Merge path (CPU)	50 032
Sequential merge path	1 650
Merge path (GPU)	70

$\approx \times 20$

Summary

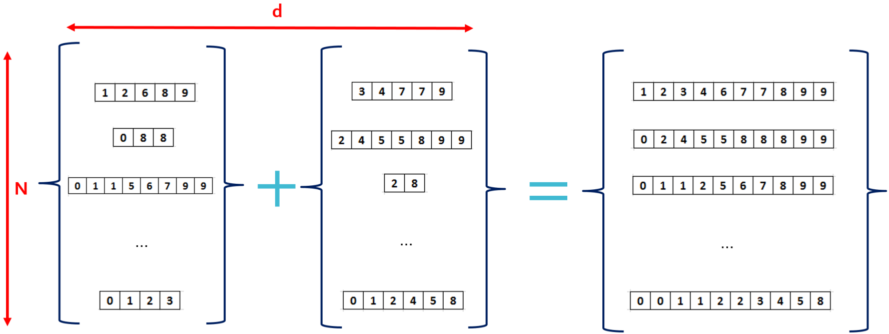
1 Introduction

2 Merge Path and Sort

3 Batch merge

4 Application in Cryptanalysis

Definition

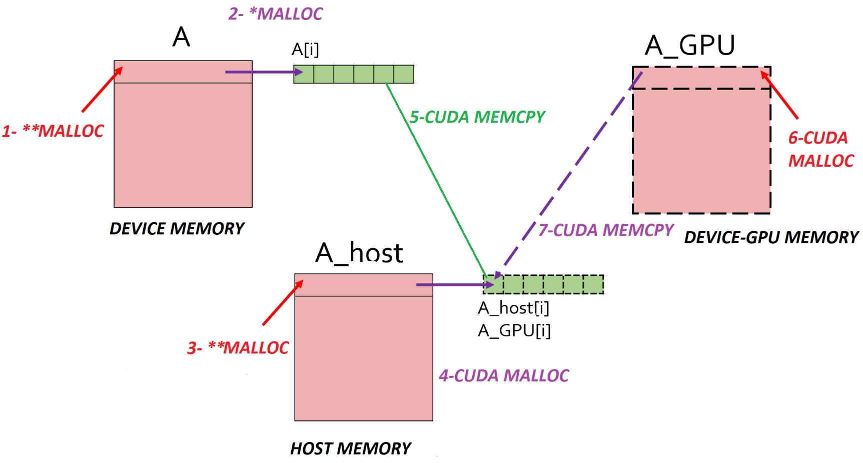


1 2 6 8 9
0 8 8
0 1 1 5 6 7 9 9
...
0 1 2 3

3 4 7 7 9
2 4 5 5 8 9 9
2 8
...
0 1 2 4 5 8

1 2 3 4 6 7 7 8 9 9
0 2 4 5 5 8 8 8 9 9
0 1 1 2 5 6 7 8 9 9
...
0 0 1 1 2 2 3 4 5 8

Procedure to allocate on the DEVICE an array of arrays:



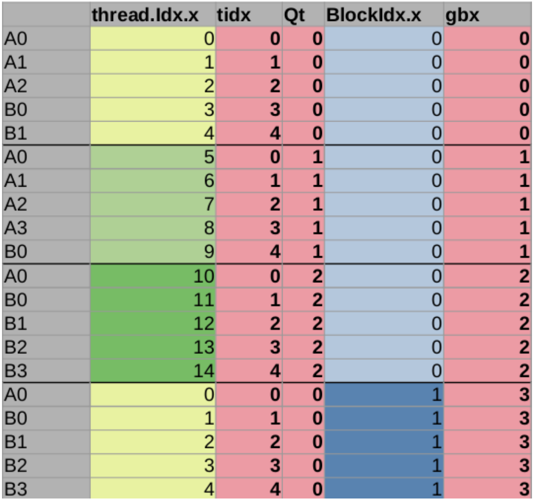
Indexes Importance

d: number of elements in a pair of array

m: number of pairs of array mergesorted in a single block

1. int tidx = threadIdx.x%d ⇒ Enumeration from 0 to d-1 of the elements of each of the m pairs of arrays which are going to be sorted in a single block
2. int Qt = (threadIdx.x-tidx)/d ⇒ Local enumeration from 0 to m-1 of the pairs of arrays in a single block
3. int gbx = Qt + blockIdx.x*(blockDim.x/d) ⇒ Global enumeration from 0 to N-1 of the pairs of arrays in all the blocks

Indices



	thread.Idx.x	tidx	Qt	BlockIdx.x	gbx
A0	0	0	0	0	0
A1	1	1	0	0	0
A2	2	2	0	0	0
B0	3	3	0	0	0
B1	4	4	0	0	0
A0	5	0	1	0	1
A1	6	1	1	0	1
A2	7	2	1	0	1
A3	8	3	1	0	1
B0	9	4	1	0	1
A0	10	0	2	0	2
B0	11	1	2	0	2
B1	12	2	2	0	2
B2	13	3	2	0	2
B3	14	4	2	0	2
A0	0	0	0	1	3
B0	1	1	0	1	3
B1	2	2	0	1	3
B2	3	3	0	1	3
B3	4	4	0	1	3

Figure: Indices for $d = 5$, $N = 4$, BlockDim.x $= 15$

Execution time

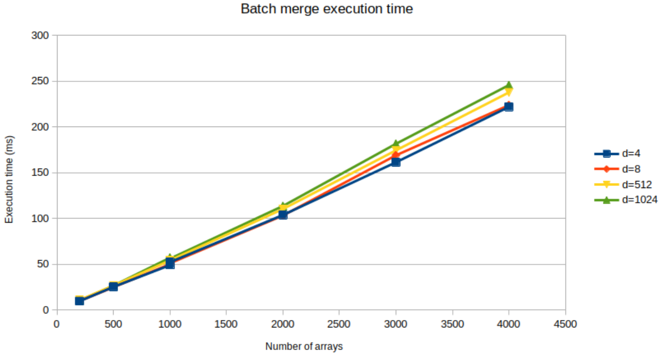


Figure: Batch merge execution times in function of N with various values of *d*

Execution time

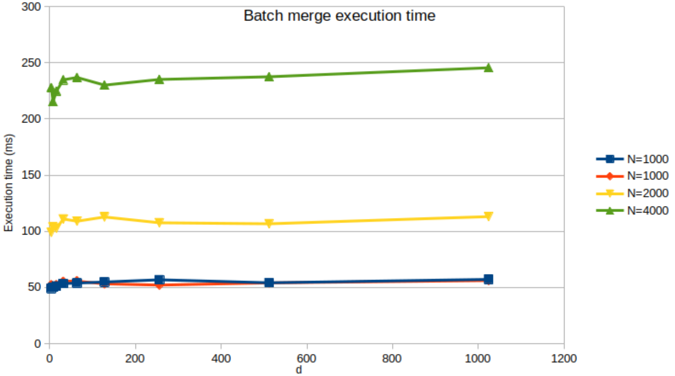


Figure: Batch merge execution times in function of *d* with various values of *N*

Execution time

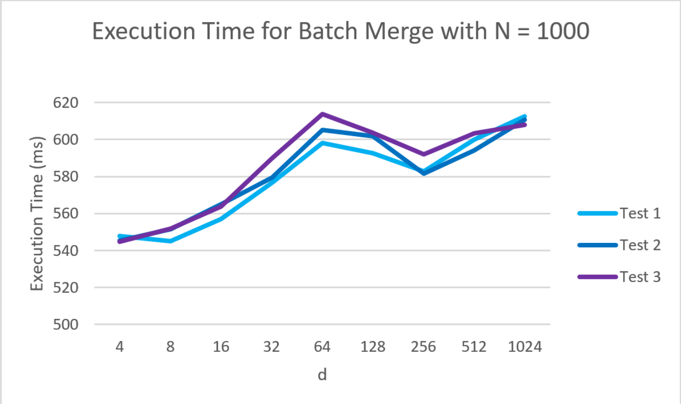


Figure: Batch merge execution times in function of $N = 10^4$ with various values of d

Execution time

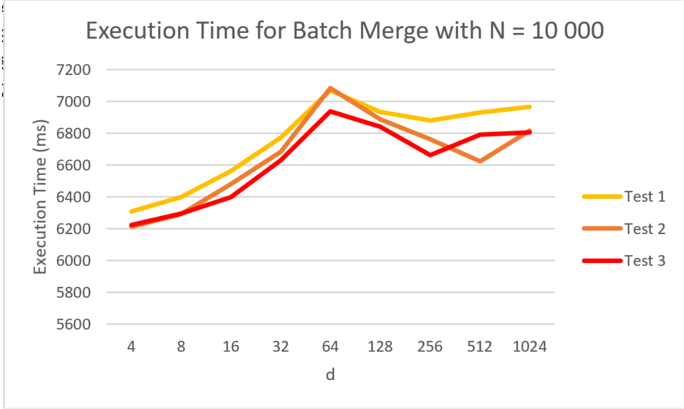


Figure: Batch merge execution times in function of $N = 10^5$ with various values of d

Execution time

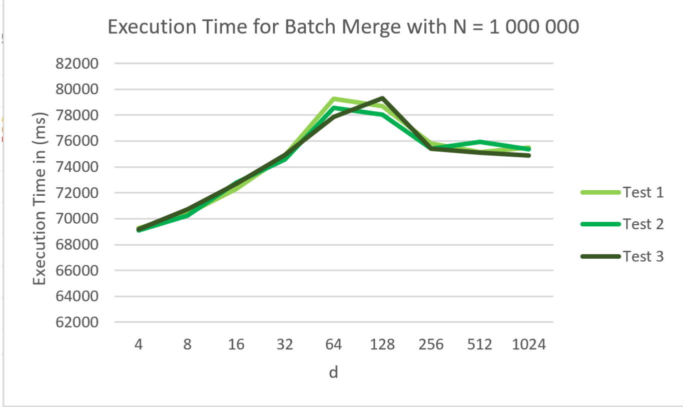


Figure: Batch merge execution times in function of $N = 10^6$ with various values of d

Summary

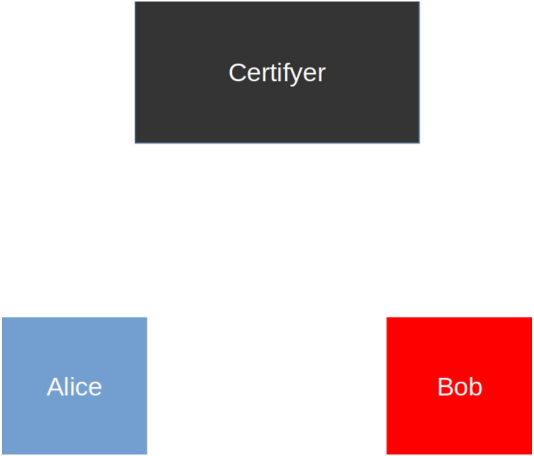
1 Introduction

2 Merge Path and Sort

3 Batch merge

4 Application in Cryptanalysis

Authentification protocol



Certifyer

Alice

Bob

Figure: Alice and Bob wan't to get access to the certifyer

Authentification protocol

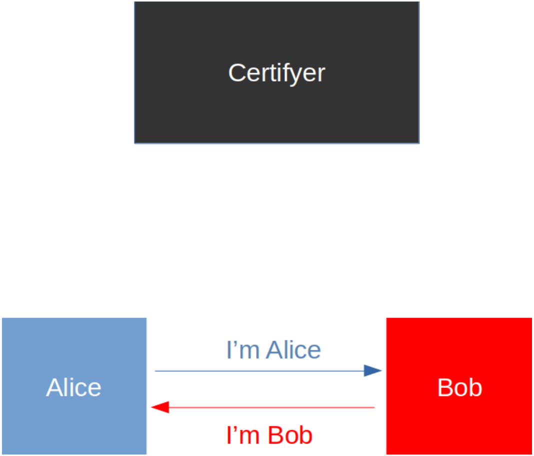


Figure: Alice and Bob recognise each other

Authentification protocol

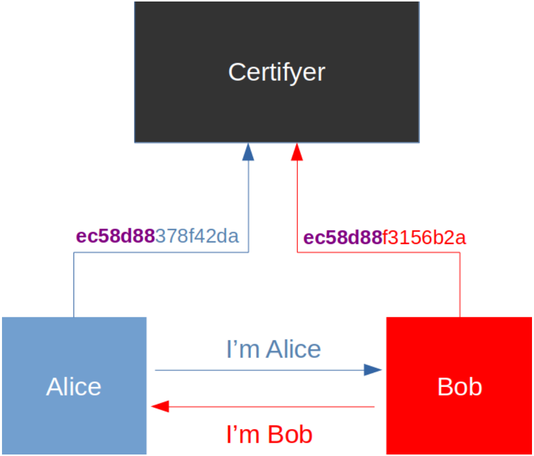
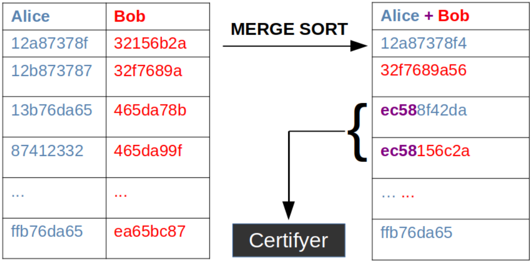


Figure: Alice and Bob send their hashed message

Hackage method

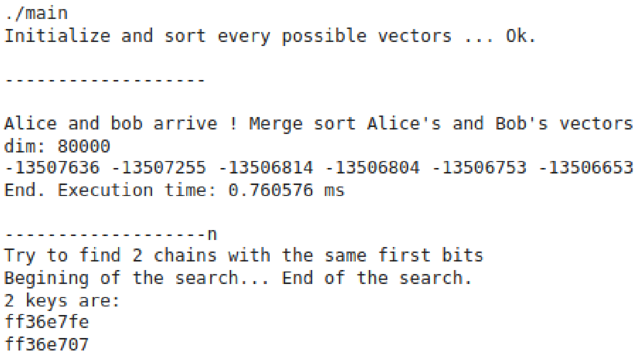


Alice	Bob
12a87378f	32156b2a
12b873787	32f7689a
13b76da65	465da78b
87412332	465da99f
...	...
ffb76da65	ea65bc87

Alice + Bob
12a87378f4
32f7689a56
ec588f42da
ec58156c2a
... ...
ffb76da65

Figure: Merge sort and find keys to get access

Result



```
./main
Initialize and sort every possible vectors ... Ok.

--------------------

Alice and bob arrive ! Merge sort Alice's and Bob's vectors
dim: 80000
-13507636 -13507255 -13506814 -13506804 -13506753 -13506653
End. Execution time: 0.760576 ms

--------------------n
Try to find 2 chains with the same first bits
Begining of the search... End of the search.
2 keys are:
ff36e7fe
ff36e707
```

Figure: Output

Conclusion

THANK YOU
FOR
YOUR
ATTENTION!
ANY QUESTIONS?

Reference: O. Green, R. McColl and D. A. Bader «GPU Merge Path, A GPU Merging Algorithm» 26th ACM International Conference on Supercomputing (ICS), San Servolo Islan, Venice, Italy, June 25-29, 2012