Numerical Study of two interconnected sparse random networks of neurons

Project available:
https://github.com/NajwaMoursli/Interconnected-Mean-Fields

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1 Context

Laboratory

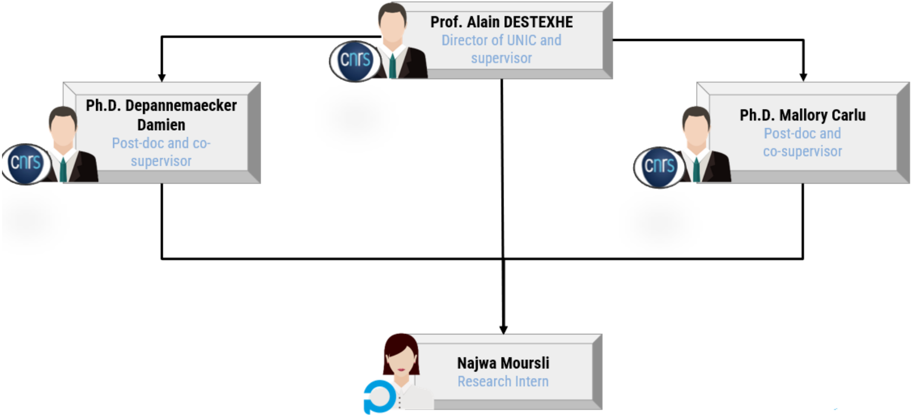

Figure 1: Organizational Chart of the research team

1 Context

Evolution

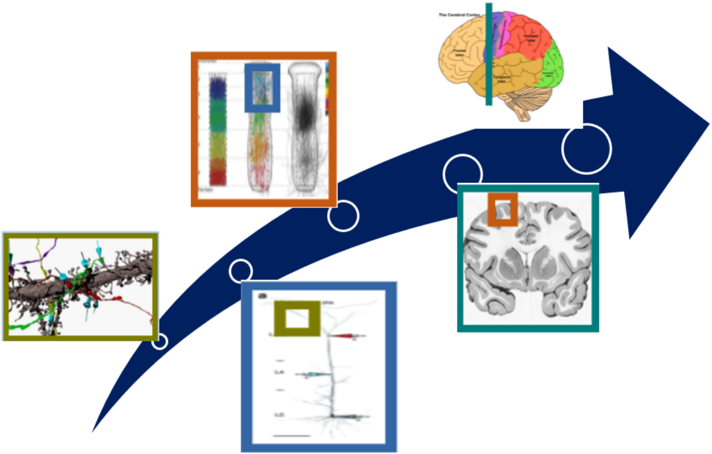

Figure 2: From single cells to the Connectome [1]

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Action Potential

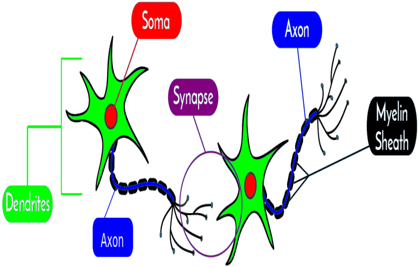

Figure 3: Representation of a single neuron [2, 3]

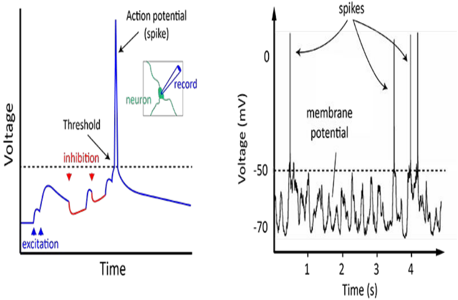

Figure 4: Action Potential Mechanism [4]

Cortical columns and states

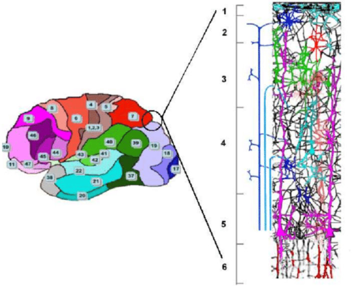

Figure 5: From cortex to column [5]

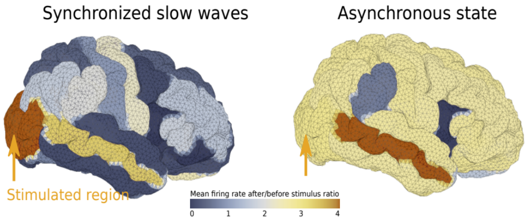

Figure 6: Mapping of different states for the brain activity [1]

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3 Realisation Plan

Project Outline

Relevant model

Pinpoint relevant parameters to vary and model architecture to undergo simulation

Time varying Parameters

Simulate over the time of the chosen parameters and configuration model

State Mapping

Delimit different states observed to construct a bifurcation map

New dynamics

Resulting in the discover of new dynamics and behaviors to interpret

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AdEx Mathematical Formalism

AdEx characteristic equations :

$$\begin{cases} C_m \frac{d\nu}{dt} = -g_l(\nu - E_l) + gl * Dt * e^{\frac{w-\nu_t}{D_t}} - w + I_{syn}) \\ \tau_w \frac{dw}{dt} = a(\nu - E_L) - w \end{cases} \quad (1)$$

Synaptic equations:

$$\begin{cases} \frac{dG_{syn_{i,e}}}{dt} = -\frac{G_{syn_{i,e}}}{T_{syn}} \\ I_{syn} = -G_{syn_e} * (\nu - E_e) - G_{syn_i} * (\nu - E_i) \\ G_{syn_{i,e}}(t) = Q_{i,e} \sum_{i,e.pre} \mathcal{H}(t - t_{sp}^{e,i}(k)) \times e^{\frac{t - t_{sp}^{e,i}(k)}{\tau_{i,e}}} \end{cases} \quad (2)$$

AdEx definition

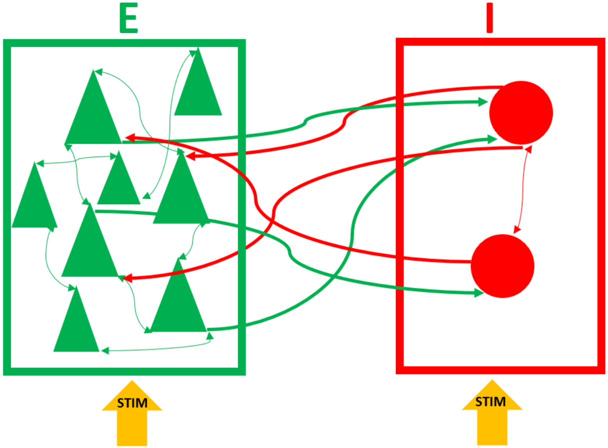

Figure 7: Schematic of the corresponding spiking AdEx neuron network with connections between and within both populations

Mean-field Definition

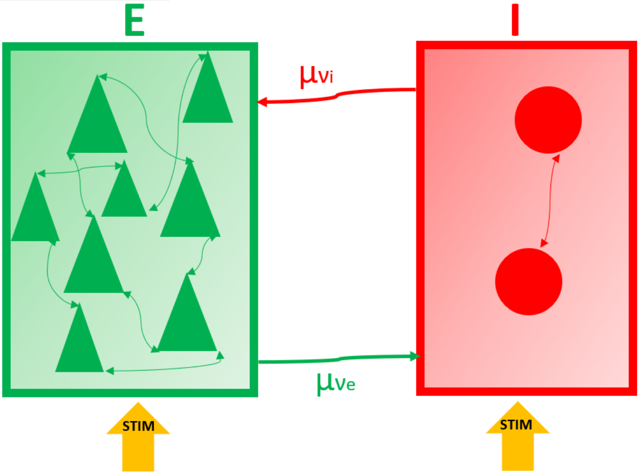

Figure 8: Mean-field neural mass model with synaptic feed forward and feedback connections. Each Rectangle represents a population

4 Methods

Mean-Field Formalism

$$\mathbb{P}_T(E_\alpha|E'_\gamma) = \binom{N_\alpha}{\nu_\alpha N_\alpha T} \times \mathbb{P}_\alpha(E'_\gamma)^{(\nu_\alpha N_\alpha T)} \times (1 - \mathbb{P}_\alpha(E'_\gamma))^{N_\alpha(1-\nu_\alpha N_\alpha T)} \tag{3}$$

$$W(\nu'|\nu) = \lim_{T\to 0} \frac{\prod_{\alpha=1,\dots,K} \mathbb{P}_T(E_\alpha|E'_\gamma)}{T} \tag{4}$$

$$\mathbb{P}_\alpha(E'_\gamma) = \mathbb{P}_\alpha(\nu) = \nu_\alpha(E'_\gamma) \times T \le 1 \tag{5}$$

$$\Longrightarrow \partial_t \mathbb{P}_t(\nu) = \int_0^{\frac{1}{T}} \partial\nu' \times \mathbb{P}(\nu') \times W(\nu|\nu') - \mathbb{P}(\nu) \times W(\nu'|\nu) \tag{6}$$

$\mathbb{P}(\nu') \times W(\nu|\nu')$ models the neurons flow entering in states E_α and $\mathbb{P}(\nu) \times W(\nu'|\nu)$, neurons flow leaving states E_α.

$$\begin{cases} T_{syn}\frac{d\nu_e(k)}{dt} = F_e(\nu_e^{input}(k), \nu_i(k)) - \nu_e(k) \\ T_{syn}\frac{d\nu_i(k)}{dt} = F_i(\nu_e^{input}(k), \nu_i(k)) - \nu_i(k) \\ \frac{dw(k)}{dt} = \frac{-w(k)}{\tau_w * b * \nu_e(k)} + a(\mu_\nu(\nu_e(k), \nu_i(k), w(k)) - E_l) \end{cases} \tag{7}$$

Numerical Integration

Weighted Inputs for $vsec_{vec}$

$(\mu_{nu_e}, \mu_{nu_i}, w)$

$\downarrow$

Integration of derivatives by TF through pointers

$F[i](*vsec_{vec}[i])$

$\downarrow$

Multi partial derivatives

$$\frac{\partial F_i}{\partial t} = \frac{1}{T}\left(\Delta_{1_{ik}} + \frac{1}{2} c_{2_{ijk}} \Delta_{2_{ikj}}\right)$$

$$\Delta_{1_{ik}} = \frac{\partial F_i}{\partial \mu_{v_{i_k}}} \qquad \Delta_{2_{ikj}} = \frac{\partial^2 F_i}{\mu^2_{\nu_{k_j}}} \Rightarrow \frac{\partial c_{2_{ijk}}}{\partial t} = \left(A_{ik} + \Delta_{1_{ik}} + c_{jk}\Delta_{2_{ijk}} + c_{ik}\Delta_{2_{ijk}} - 2 * c_{ij}\right)$$

$$\frac{\partial w_i}{\partial t} = \frac{w_i}{tau_{w_i}} b * F_i + a_i(\mu_F(\nu_i) - E_{l_i})$$

RK4 $\Rightarrow$

$$v_{out} = \begin{bmatrix} \mu_{v_{e1}} & \mu_{v_{e2}} & \mu_{v_{i1}} & \mu_{v_{i2}} \\ c_{e1e1} & c_{e1i1} & c_{e1e2} & c_{e1i2} \\ c_{i1e1} & c_{i1i1} & c_{i1e2} & c_{i1i2} \\ c_{e2e1} & c_{e2i1} & c_{e2e2} & c_{e2i2} \\ c_{i2e1} & c_{i2i1} & c_{i2e2} & c_{i2i2} \\ w_{e1} & w_{e2} & & \end{bmatrix}$$

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5 Simulations

Configuration

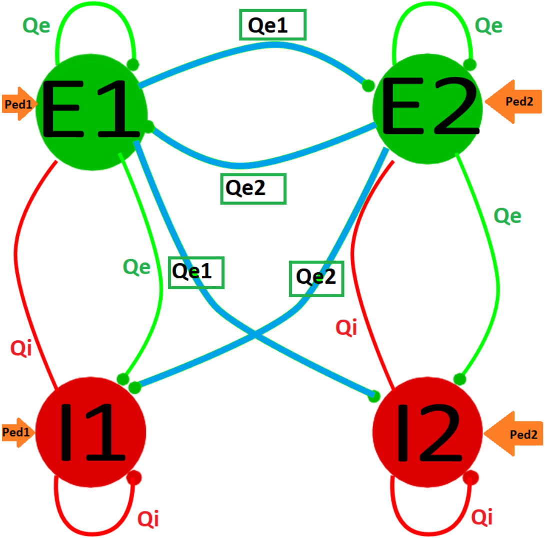


Figure 9: Configuration of the model : Delay

Population E1 & E2

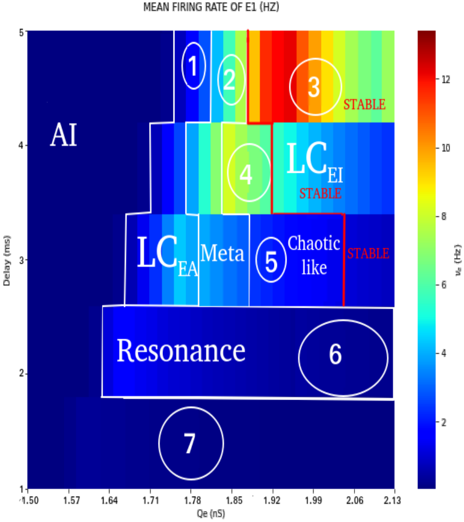


Figure 10: Bifurcation map of E1 states

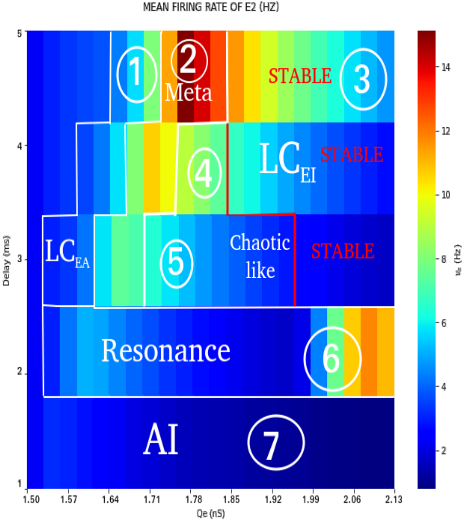
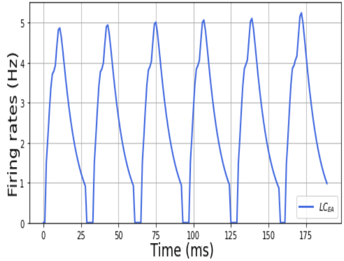
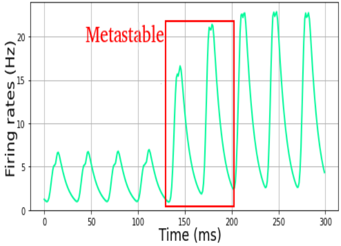
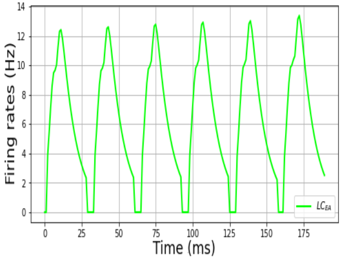
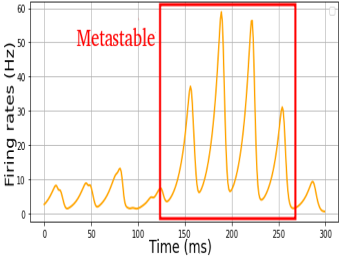


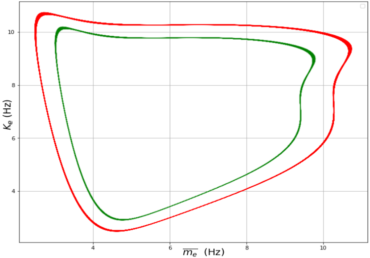
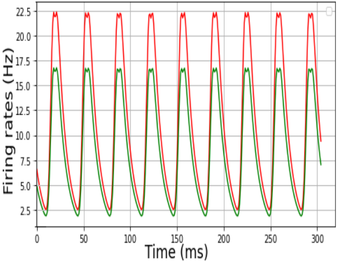
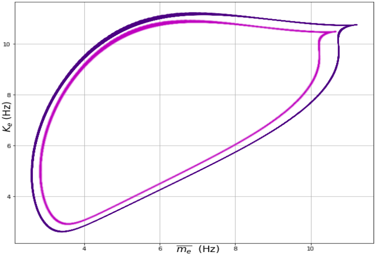
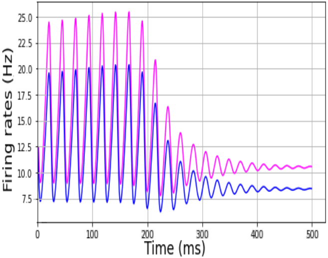
Figure 11: Bifurcation map of E2 states

5 Simulations

Simulation results

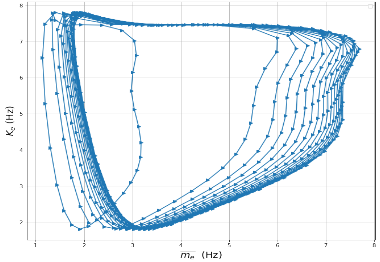
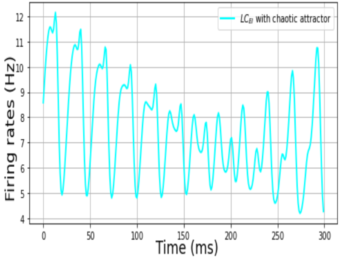
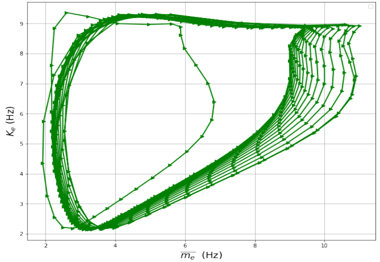
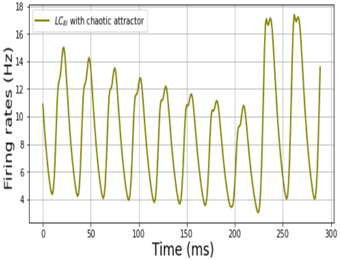
Pop	State 1 : LC_{EA}	State 2: Meta-stable
E1		
E2		

Simulation results

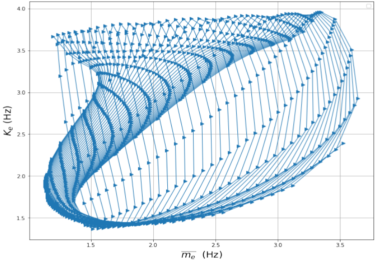
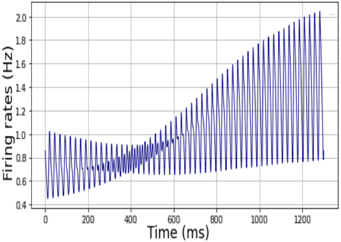
Pop	State 3 : Stable LC_{EI}	
E1		
E2		

5 Simulations

Simulation results

Pop	State 4 : Chaotic-like $State_1$	
E1		
E2		

Simulation results

<table>
<tr><th>Pop</th><th colspan="2">State 5 : Chaotic-like $State_2$</th></tr>
<tr><td>E1</td><td>
</td><td>
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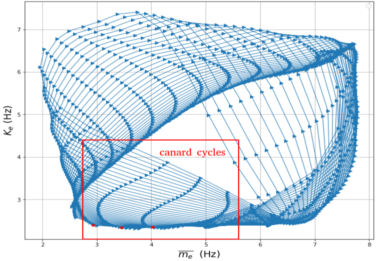
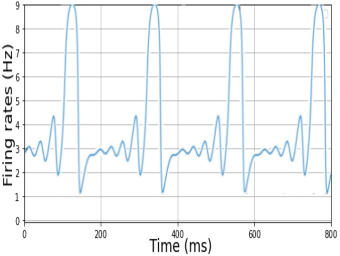
<tr><td>E2</td><td>
</td><td>
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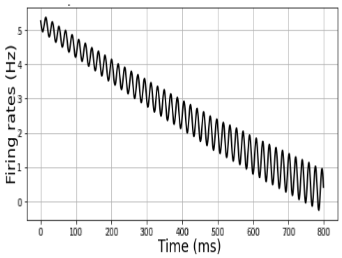
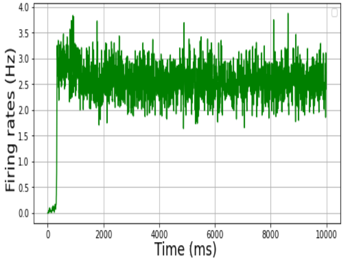
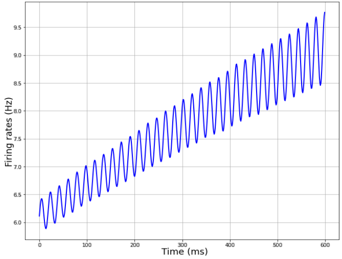
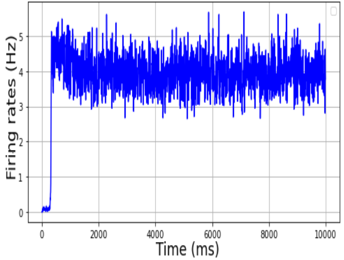
</table>

Simulation results

Pop	State 6: Resonance	State 7 : AI
E1		
E2		

5 Simulations

Population I1& I2

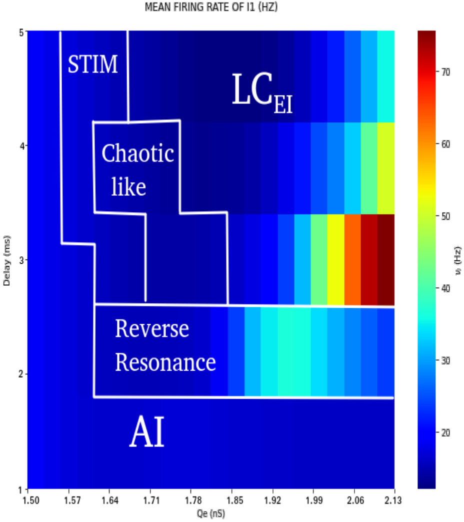


Figure 12: Bifurcation map of I1 states

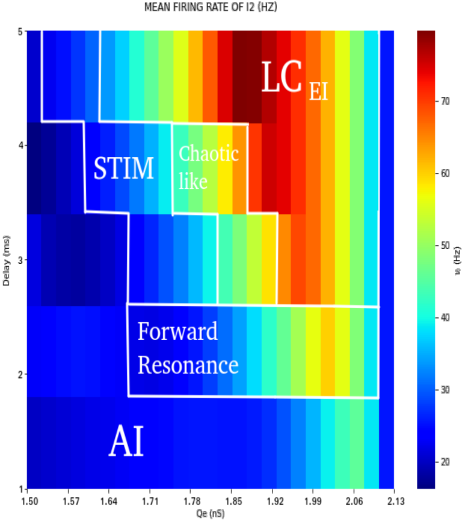


Figure 13: Bifurcation map of I2 states

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Results Outcomes

The outcomes of this study are the following ones

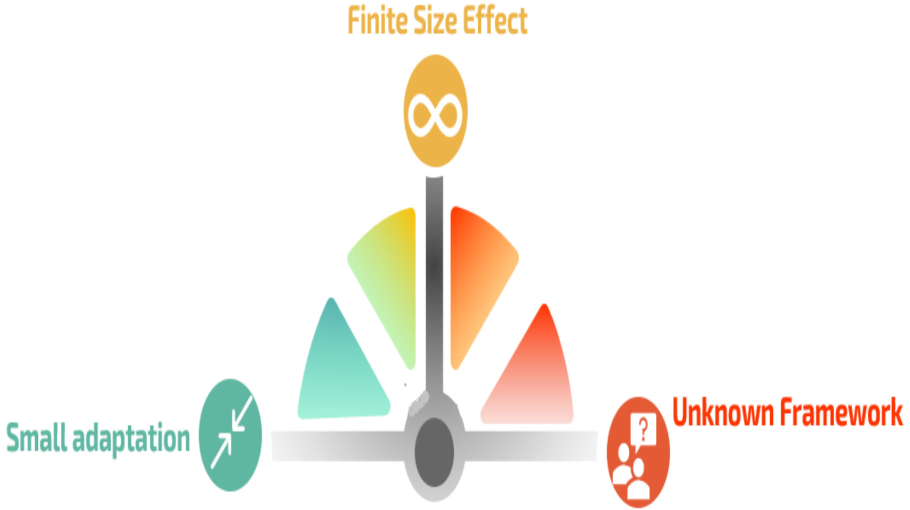
- **Mean field models seem to simulate AI states very well**
- **High delay => Chaos to LC_{EI} for lower Q_e values**

	Delay (ms)	g=$\frac{Q_i}{Q_e}$
E1	5	**2.75**
	4	2.6
	3	2.43
I1	5	2.99
	4	2.84
	3	2.7

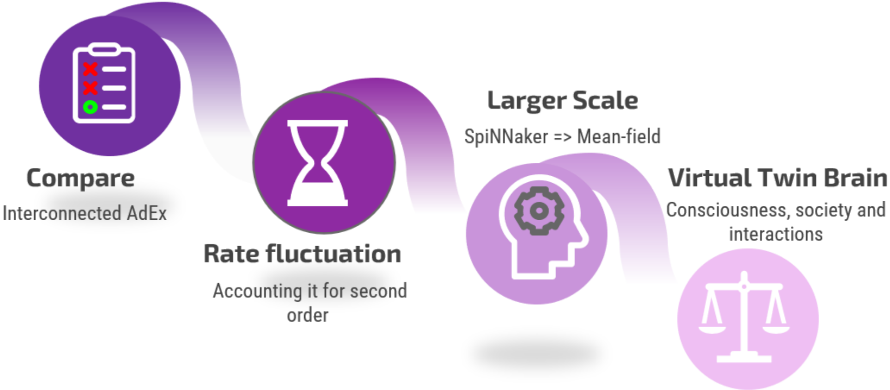
	Delay (ms)	g=$\frac{Q_i}{Q_e}$
E2	5	2.7
	4	2.7
	3	2.56
I2	5	**3.05**
	4	2.65
	3	2.65

- **New relation $\Longrightarrow$ The dominant inhibition state transition seems delay dependent when delay $\geq$ 5 ms**
- **Unexpected behaviors : LC_{EA} and θ-resonance**

Model Limitations



Improvement Perspectives



Presentation ending

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References I

[1] J. S. Goldman, L. Kusch, B. H. Yalcinkaya, D. Depannemaecker, T.-A. E. Nghiem, V. Jirsa, and A. Destexhe, "Brain-scale emergence of slow-wave synchrony and highly responsive asynchronous states based on biologically realistic population models simulated in the virtual brain," *bioRxiv*, 2020.

[2] A.-M. Oswald, B. Doiron, J. Rinzel, and A. Reyes, "Spatial profile and differential recruitment of gaba(b) modulate oscillatory activity in auditory cortex," *The Journal of neuroscience : the official journal of the Society for Neuroscience*, vol. 29, pp. 10321–34, 08 2009.

[3] Y. Zerlaut, S. Chemla, F. Chavane, and A. Destexhe, "Modeling mesoscopic cortical dynamics using a mean-field model of conductance-based networks of adaptive exponential integrate-and-fire neurons," *Journal of Computational Neuroscience*, vol. 44, pp. 45–61, Feb. 2018.

[4] P. Dayan and L. Abbott, *Theoretical Neuroscience: Computational and Mathematical Modeling of Neural Systems*, vol. 15. 01 2001.

[5] A. Rocha, "Toward a comprehensive understanding of eeg and its analyses," *SSRN Electronic Journal*, 01 2018.

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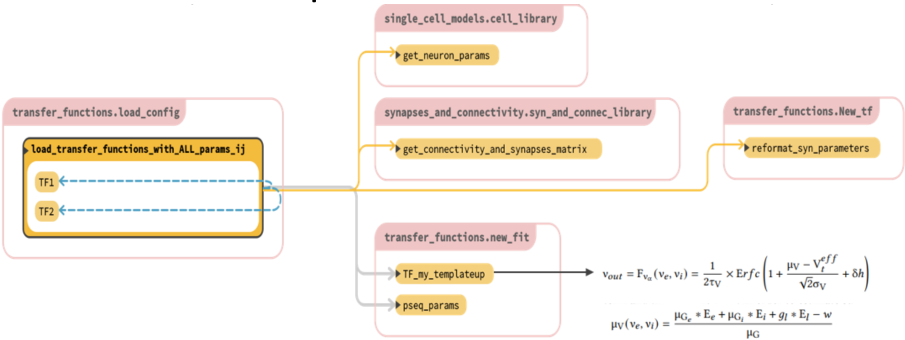
Transfer function template



$$\nu_{out} = F_{\nu_\alpha}(\nu_e, \nu_i) = \frac{1}{2\tau_V} \times Erfc\left(1 + \frac{\mu_V - V_t^{eff}}{\sqrt{2}\sigma_V} + \delta h\right)$$

$$\mu_V(\nu_e, \nu_i) = \frac{\mu_{G_e} * E_e + \mu_{G_i} * E_i + g_l * E_l - w}{\mu_G}$$

$$\mu_G = \mu_{G_e} + \mu_{G_i} + gl$$

$$\sigma_V(\nu_e, \nu_i) = \sqrt{\sum_s K_s \nu_s \frac{(U_s \tau_s)^2}{2(\tau_m^{eff} + \tau_s)}}$$

with $\tau_m^{eff} = \frac{c_m}{\mu_G}$ $\quad U_s = \frac{Q_s}{\mu_G}(E_s - \mu_V)$

$$\tau_V(\nu_e, \nu_i) = \sum_s K_s \nu_s (U_s \tau_s)^2 \times \frac{2(\tau_m^{eff} + \tau_s)}{\sum_s K_s \nu_s (U_s \tau_s)^2}$$

Figure 14: Python Call modules and functions representing the building of TF template [3]

8 Appendix

One column

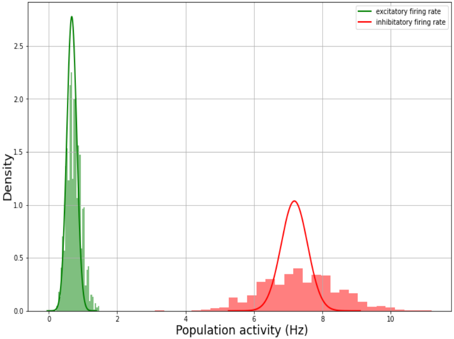


Figure 15: Firing rate distribution sampled from the spiking simulation and the MF Gaussian predictions of the population activities

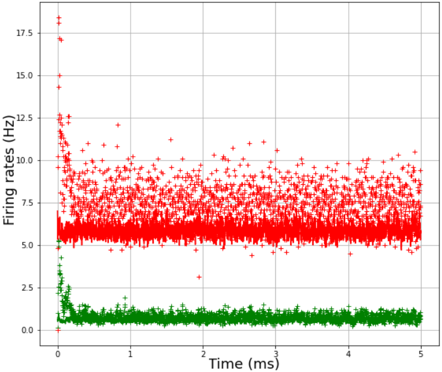


Figure 16: Time traces of the Firing rates for both models with the Ortein-Ulhenbeck noise added in the MF (+ represent the MF prediction)

Interconnected AdEx