# SI 670 Applied Machine Learning, HW 2 (Due Tuesday Sep 16, 2025 11:59 PM)

**IMPORTANT**: please name your submitted file si670f25\_hw2\_youruniqname.ipynb when submitting on Gradescope

As a reminder, the notebook code you submit must be your own work. Feel free to discuss general approaches to the homework with classmates. If you end up forming more of a team discussion on multiple questions, please include the names of the people you worked with at the top of your notebook file.

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**Collaborators (if any):** 

Questions 1-3 are in SI 670 - Problem Set 2

### Problem 4 (13 points)

Your goal for this question is to run a simple linear regression on the data, and investigate its performance. Note that this question has multiple parts.

First you need to run the following block, which sets up the training and test datasets to be used for this question.

```
import numpy as np

# We fix the state of the random number generator here in order to maintain consist
rs = np.random.RandomState(0)
X_train = rs.uniform(size = (100, 1))
y_train = 5 * X_train + 8
X_test = rs.uniform(size = (20, 1))
y_test = 7 * X_test + 1
```

Now, write a function that takes in training data as input, and fits a line to the data using a linear regression model. Your function should return the linear regression model.

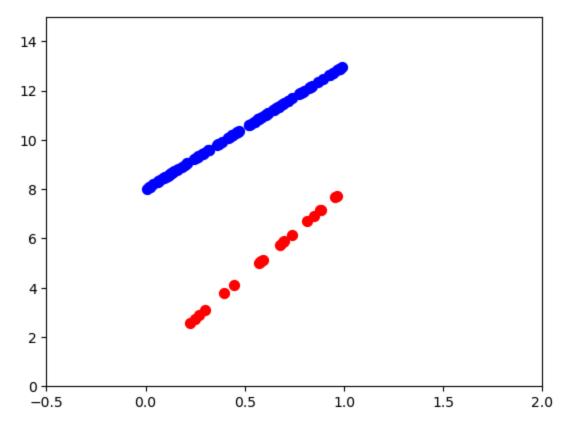
```
In [2]: def fit_line(X_train, y_train):
    from sklearn.linear_model import LinearRegression

# Your code: fit a linear regression model to the input data, name it `linreg'
linreg = LinearRegression().fit(X_train, y_train)
# Return the linear regression model you created
return linreg
```

Run the following block of code in order to (1) compute the the  $R^2$  (coefficient of determination) regression score on both training and test datasets, and then (2) plot the training and test datasets in the same figure, marked by different colors (blue for training data, red for test data).

```
In [3]: # Fit a line to the dataset we generated at the beginning of this problem using you
        linreg = fit_line(X_train, y_train)
        # Compute R^2 score of your model on training and test sets
        print('R-squared score (training): {:.3f}'
              .format(linreg.score(X_train, y_train)))
        print('R-squared score (test): {:.3f}'
             .format(linreg.score(X_test, y_test)))
        # Plot the training and test datasets in the same figure marked by different colors
        import matplotlib.pyplot as plt
        fig = plt.figure()
        plt.xlim(-0.5, 2)
        plt.ylim(0, 15)
        plt.scatter(X_train, y_train, c='blue', marker='o', s=50) # Blue points are from tr
        plt.scatter(X_test, y_test, c='red', marker='o', s=50) # Red points are from test d
       R-squared score (training): 1.000
       R-squared score (test): -11.533
```

Out[3]: <matplotlib.collections.PathCollection at 0x28487a9c690>



What does the  $\mathbb{R}^2$  score of your model on the two datasets tell you about the performance of your model? Using the plot of the two datasets, why do you think we get these  $\mathbb{R}^2$  scores? Please provide your answer in the text cell below.

**Your answer to problem 4 here**: We have a overfitting problem in our model: the training accuracy is very high and the test accuracy is extremely low (even lower than 0). It is very likely due to the very different data generating processes in the training and testing datasets. And both the DGPs are fully linear. The training model can hardly be generalized to other DGPs.

#### Question 5 (20 points) Bias and Variance

## (a) 10 points

Your task is to investigate the influence of different regularization parameters on the coefficients of a ridge regression model.

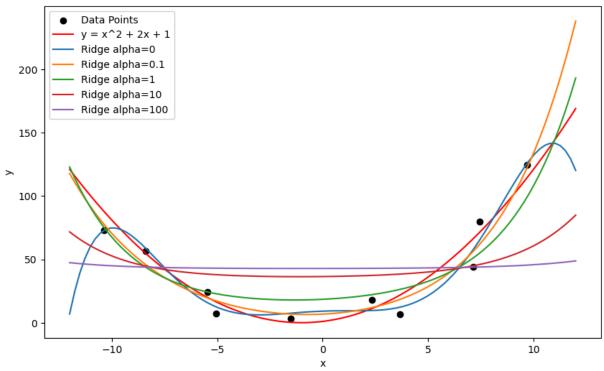
Given 10 points around the function  $y=x^2+2x+1$ . You are asked to train a linear ridge regression with degree 6 polynomial features. And run it with different regularization parameters  $\alpha \in \{0, 0.1, 1, 10, 100\}$ .

Plot the polynomials from your regression results corresponding to each regularization parameter in one figure.

**Hint:** One way to plot the polynomial from your regressions is to create a dataset with a bunch of data points for the x values, then use your model to predict the outputs for these x values. Finally, use plt.plot(my\_x\_values, predicted\_y\_values) to show the polynomial predicted by the model.

```
In [4]: import numpy as np
        from sklearn.linear_model import Ridge
        from sklearn.preprocessing import PolynomialFeatures, StandardScaler, MinMaxScaler
        from sklearn.pipeline import make pipeline
        import matplotlib.pyplot as plt
        def generate_polynomials_for_diff_alphas():
            x_{list} = np.array([-10.38446879, -8.38394902, -5.47700112, -5.04337481, -1.5054]
            y_list = np.array([73.21995367, 56.4250573, 24.15989601, 7.14325154, 3.45955269
            # Generate original x values
            x_{original} = np.linspace(-12, 12, 100).reshape(-1, 1)
            # Generate y values based on the function y = x^2 + 2x + 1
            y_original = x_original**2 + 2*x_original + 1
            # Plot the original function and the data points
            plt.figure(figsize=(10, 6))
            plt.scatter(x_list, y_list, label='Data Points', color='black')
            plt.plot(x_original, y_original, label='y = x^2 + 2x + 1', color='red')
            ### your code here
```

```
alphas = [0, 0.1, 1, 10, 100]
   poly_pipeline = make_pipeline(
        PolynomialFeatures(degree=6),
       MinMaxScaler()
   x_list_scaled = poly_pipeline.fit_transform(x_list)
   x_original_scaled = poly_pipeline.transform(x_original)
   for alpha in alphas:
       model = Ridge(alpha=alpha)
       model.fit(x_list_scaled, y_list)
       y_pred = model.predict(x_original_scaled)
        plt.plot(x_original, y_pred, label=f'Ridge alpha={alpha}')
   plt.xlabel('x')
   plt.ylabel('y')
   plt.legend()
   plt.show()
generate_polynomials_for_diff_alphas()
```



## (b) 10 points

We then give you a data generator that gives 10 data points around the function  $y = x^2 + 2x + 1$  each time you run **data\_gen()**.

We would like you to consider the bias and variance of a 6-degree polynomial classifier. In particular, for each of the regularization parameters  $\alpha \in \{0, 0.1, 1, 10, 100\}$  we would like you to calculate the bias and variance over 1000 generated data sets from **data\_gen()**.

To do so, you will need to calculate the bias and variance with respect to a particular data point. Please use  $x^* = 0$ ,  $y^* = 1$  for this data point.

For more context on how to calculate bias and variance, refer back to the slides from lecture 5.

**In addition**, briefly describe how the bias and variance change when the regularization parameter increases. If you are unable to finish the coding portion, please explain how you would expect bias and variance to change.

```
In [5]: def data_gen():
           # Number of data points
            # Generate original x values: you can choose the range and number of points
            x_original = np.linspace(-10, 10, n)
            # Generate y values based on the function y = x^2 + 2x + 1
            y_original = x_original**2 + 2*x_original + 1
            \# Add some noise to x and y values
            noise_strength_x = 1.0 # Control the noise strength for x
            noise_strength_y = 5.0 # Control the noise strength for y
            x_noisy = x_original + noise_strength_x * np.random.randn(n)
            y_noisy = y_original + noise_strength_y * np.random.randn(n)
            return x_noisy, y_noisy
        def calculate_bias_var():
            import numpy as np
            from sklearn.linear_model import LinearRegression
            bias_list = []
            variance_list = []
            # your code here
            alphas = [0, 0.1, 1, 10, 100]
            datasets = [data_gen() for _ in range(1000)]
            pipeline = make_pipeline(
                PolynomialFeatures(degree=6),
                MinMaxScaler()
            for alpha in alphas:
                model_bias = []
                model_variance = []
                for dataset in datasets:
                    x_original, y_original = dataset
                    x_original = x_original.reshape(-1, 1)
                    x_original_scaled = pipeline.fit_transform(x_original)
                    model = Ridge(alpha=alpha)
                    model.fit(x_original_scaled, y_original)
                    y_pred = model.predict(x_original_scaled)
```

```
bias = np.mean((y_original - y_pred) ** 2)
    variance = np.var(y_pred)
    model_bias.append(bias)
    model_variance.append(variance)
    bias_list.append(np.mean(model_bias))
    variance_list.append(np.mean(model_variance))
    return bias_list, variance_list

calculate_bias_var()
```

**Your answer to Problem 5 here.** As the regularization term increases, the bias increases and variance decreases.

#### Question 6 (15 points) Regression to the mean

Imagine you are working on a regression problem with six features.

You will:

- 1. Generate synthetic data.
- 2. Split data into training, validation, and test sets, with a ratio of 6:2:2
- 3. Train two different linear regression models (one with Ridge regularization term with lpha=0.2 and the other without).
- 4. Evaluate both models on the validation data with Mean Squared Error.
- 5. Suppose that for each time you will apply the "better" model according to the MSE on the validation data, evaluate it on the test data.

Repeat this 1000 times. Each time calculate the average of the error of the "better" model on validation data and test data respectively (so you will only record the error of the better model on the validation data, and the error of this same model on the test data.

Which average error is smaller? Why do you expect this (or not expect this)?

```
import numpy as np
from sklearn.linear_model import LinearRegression, Ridge
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error
```

```
def answer_3():
   val mse list = []
   test_mse_list = []
   for _ in range(1000):
        # Generate synthetic data
       X = 2 * np.random.rand(50, 6) # 50 samples, 6 features
       y = 10 + np.dot(X, np.array([3, 5, 2, 0, 0, 0])) + 2 * np.random.randn(50)
       # Your code here
       # Split data into training, validation, and test sets, with a ratio of 6:2:
       X_train, X_temp, y_train, y_temp = train_test_split(X, y, test_size=0.4, ra
       X_val, X_test, y_val, y_test = train_test_split(X_temp, y_temp, test_size=0
       # Train two models
       # Model 1: Linear Regression
       linear_model = LinearRegression().fit(X_train, y_train)
       # Model 2: Ridge Regression with alpha = 0.2
        ridge_model = Ridge(alpha=0.2).fit(X_train, y_train)
       # Evaluate on validation set
       val_mse1 = mean_squared_error(y_val, linear_model.predict(X_val))
       val_mse2 = mean_squared_error(y_val, ridge_model.predict(X_val))
        print(f"Validation MSE for Linear Regression: {val_mse1}")
        print(f"Validation MSE for Ridge Regression: {val_mse2}")
        # Choose the "better" model based on validation MSE
        better_model = linear_model if val_mse1 < val_mse2 else ridge_model
       val_mse_list.append(min(val_mse1, val_mse2))
       # Evaluate 'better' model on the test set
       test_mse = mean_squared_error(y_test, better_model.predict(X_test))
        test_mse_list.append(test_mse)
        #print(f"Test MSE for the 'better' model: {test_mse}")
   return np.mean(val_mse_list), np.mean(test_mse_list)
answer_3()
```

```
Validation MSE for Linear Regression: 2.577636033795739
Validation MSE for Ridge Regression: 2.3968194797708433
Validation MSE for Linear Regression: 2.8131857999119623
Validation MSE for Ridge Regression: 2.84640274189133
Validation MSE for Linear Regression: 6.556697378245781
Validation MSE for Ridge Regression: 5.868087644616071
Validation MSE for Linear Regression: 16.969987046796565
Validation MSE for Ridge Regression: 17.411523512059812
Validation MSE for Linear Regression: 7.514409066626444
Validation MSE for Ridge Regression: 7.7904388633179495
Validation MSE for Linear Regression: 4.195162257461351
Validation MSE for Ridge Regression: 4.176729845843388
Validation MSE for Linear Regression: 5.2591657902767
Validation MSE for Ridge Regression: 4.924293035066554
Validation MSE for Linear Regression: 6.0785484039247155
Validation MSE for Ridge Regression: 5.961771430021715
Validation MSE for Linear Regression: 7.496063066908538
Validation MSE for Ridge Regression: 7.505359618596164
Validation MSE for Linear Regression: 4.559269124011912
Validation MSE for Ridge Regression: 4.602392861266818
Validation MSE for Linear Regression: 7.709422107088265
Validation MSE for Ridge Regression: 7.306937360021314
Validation MSE for Linear Regression: 4.547254738155814
Validation MSE for Ridge Regression: 4.452151141598338
Validation MSE for Linear Regression: 2.9279111550230117
Validation MSE for Ridge Regression: 2.9773208831778875
Validation MSE for Linear Regression: 3.386046322237523
Validation MSE for Ridge Regression: 3.285279842299316
Validation MSE for Linear Regression: 8.655163675766254
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Validation MSE for Ridge Regression: 6.884070178251031
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Validation MSE for Linear Regression: 6.1589327284250395
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Validation MSE for Linear Regression: 0.40246198904534103
Validation MSE for Ridge Regression: 0.41918331330206
Validation MSE for Linear Regression: 6.858040359602699
Validation MSE for Ridge Regression: 6.747622762756682
Validation MSE for Linear Regression: 5.621477678788123
Validation MSE for Ridge Regression: 5.4201897297097945
Validation MSE for Linear Regression: 2.725081287934419
Validation MSE for Ridge Regression: 2.7067796689398795
Validation MSE for Linear Regression: 6.9370605504028475
Validation MSE for Ridge Regression: 6.937146764802508
Validation MSE for Linear Regression: 7.044117464106695
Validation MSE for Ridge Regression: 6.520809102761698
Validation MSE for Linear Regression: 5.389227839727392
Validation MSE for Ridge Regression: 5.331372367610003
Validation MSE for Linear Regression: 11.49332794058304
Validation MSE for Ridge Regression: 11.615754443002427
Validation MSE for Linear Regression: 9.930181590759585
Validation MSE for Ridge Regression: 9.82453903167307
Validation MSE for Linear Regression: 4.556189764547357
Validation MSE for Ridge Regression: 4.348491011726741
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Validation MSE for Linear Regression: 4.540677424548243
Validation MSE for Ridge Regression: 4.419050294290736
Validation MSE for Linear Regression: 4.406685328730525
Validation MSE for Ridge Regression: 4.46934534864145
Validation MSE for Linear Regression: 5.582114859286176
Validation MSE for Ridge Regression: 5.148118109920721
Validation MSE for Linear Regression: 3.585334051204692
Validation MSE for Ridge Regression: 3.5112335606886385
Validation MSE for Linear Regression: 5.381769561925455
Validation MSE for Ridge Regression: 5.173235135641899
Validation MSE for Linear Regression: 3.8999377486284197
Validation MSE for Ridge Regression: 3.851270482870215
Validation MSE for Linear Regression: 8.370429197506528
Validation MSE for Ridge Regression: 8.00436367128234
Validation MSE for Linear Regression: 3.21940606511568
Validation MSE for Ridge Regression: 3.3847476347864704
Validation MSE for Linear Regression: 5.565641852928186
Validation MSE for Ridge Regression: 5.443310547681034
Validation MSE for Linear Regression: 7.808812705817243
Validation MSE for Ridge Regression: 7.8435091945314195
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Validation MSE for Linear Regression: 6.20879284298981
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Validation MSE for Linear Regression: 2.740419730927431
Validation MSE for Ridge Regression: 2.7248944157486896
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Validation MSE for Linear Regression: 4.063765180976511
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Validation MSE for Linear Regression: 7.713898416221204
Validation MSE for Ridge Regression: 7.526010053426113
Validation MSE for Linear Regression: 6.222590589097791
Validation MSE for Ridge Regression: 5.993760961504425
Validation MSE for Linear Regression: 2.944099155077896
Validation MSE for Ridge Regression: 2.944709307744413
Validation MSE for Linear Regression: 7.8050519449278735
Validation MSE for Ridge Regression: 7.721461433728922
Validation MSE for Linear Regression: 1.6365293283333284
Validation MSE for Ridge Regression: 1.592170736679727
Validation MSE for Linear Regression: 4.7502688614362985
Validation MSE for Ridge Regression: 4.726914935907224
Validation MSE for Linear Regression: 4.380296994192476
Validation MSE for Ridge Regression: 4.304192470609536
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Validation MSE for Linear Regression: 3.583731792679489
Validation MSE for Ridge Regression: 3.4963317937793454
Validation MSE for Linear Regression: 6.379033796764462
Validation MSE for Ridge Regression: 6.378612002099165
```

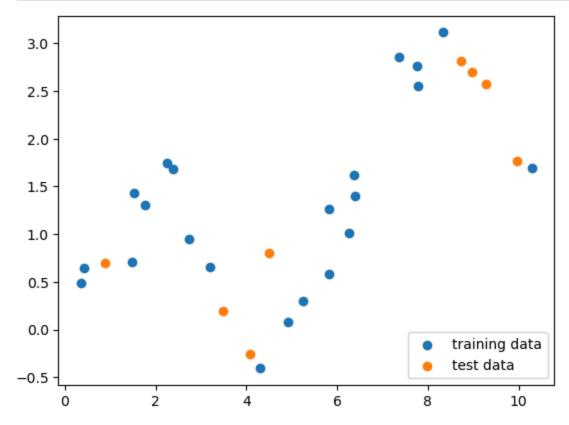
Out[6]: (np.float64(5.276546419406722), np.float64(5.257389690129041))

## Preliminary for Q7 and Q8

Before continuiting on to the next question, run the following block to set up the variables needed for later sections. It generates training and test data that we're going to use to fit a non-linear function (as opposed to the traditional linear regression problems you saw in the previous questions).

```
In [7]: import pandas as pd
from sklearn.model_selection import train_test_split
import numpy as np
```

```
np.random.seed(0)
n = 30
x = np.linspace(0, 10, n) + np.random.randn(n) / 5
y = np.sin(x) + x / 4 + np.random.randn(n) / 3
X_train, X_test, y_train, y_test = train_test_split(x, y, random_state=0)
# You can use this function to help you visualize the dataset by
# plotting a scatterplot of the data points
# in the training and test sets.
def part1_scatter():
   import matplotlib.pyplot as plt
   %matplotlib inline
   plt.figure()
   plt.scatter(X_train, y_train, label='training data')
   plt.scatter(X_test, y_test, label='test data')
   plt.legend(loc=4);
part1_scatter() # let's plot the training and test data for the upcoming questions
```



## Question 7 (10 points)

Write a function that fits a polynomial curve to the data, by using a polynomial LinearRegression model on the *training data*  $X_{train}$  for degrees 1, 3, 6, and 9. (Recall that we use PolynomialFeatures in sklearn.preprocessing to create the polynomial features and then fit a linear regression model to those extended features). For each model, find 100 predicted values over the interval x = 0 to 10 (e.g. np.linspace(0,10,100)) and store

this in a numpy array. The first row of this array should correspond to the output from the model trained on degree 1, the second row degree 3, the third row degree 6, and the fourth row degree 9.

Note that you can run the plot\_two() function to see the fitted curves on top of the dataset.

\*This function should return a numpy array with shape `(4, 100)`\*

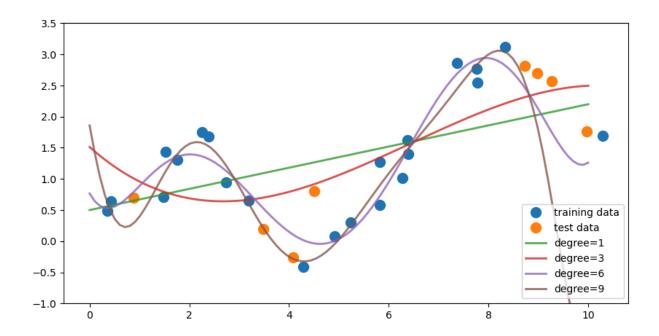
```
In [8]: def answer_two():
            from sklearn.linear_model import LinearRegression
            from sklearn.preprocessing import PolynomialFeatures
            answer_array = []
            degree_list = [1, 3, 6, 9]
            X_pred = np.linspace(0, 10, 100).reshape(-1, 1) # Generate predictions once
            for degree in degree_list:
                pipeline = make_pipeline(
                    PolynomialFeatures(degree=degree),
                    MinMaxScaler()
                X_train_poly = pipeline.fit_transform(X_train.reshape(-1, 1))
                model = LinearRegression().fit(X_train_poly, y_train)
                X_pred_poly = pipeline.transform(X_pred)
                y_pred = model.predict(X_pred_poly)
                answer_array.append(y_pred)
            return np.vstack(answer_array)
        answer_two()
```

```
Out[8]: array([[ 0.49947299,
                             0.51663261, 0.53379223, 0.55095185, 0.56811148,
                0.5852711 ,
                             0.60243072, 0.61959034, 0.63674996, 0.65390958,
                             0.68822883, 0.70538845,
                                                     0.72254807, 0.73970769,
                0.6710692 ,
                0.75686731, 0.77402693, 0.79118655, 0.80834617, 0.8255058,
                             0.85982504, 0.87698466,
                                                     0.89414428, 0.9113039,
                0.84266542,
                0.92846352, 0.94562315, 0.96278277, 0.97994239, 0.99710201,
                1.01426163, 1.03142125, 1.04858087, 1.06574049, 1.08290012,
                1.10005974, 1.11721936, 1.13437898, 1.1515386, 1.16869822,
                1.18585784, 1.20301747, 1.22017709, 1.23733671, 1.25449633,
                1.27165595, 1.28881557, 1.30597519,
                                                     1.32313482, 1.34029444,
                1.35745406, 1.37461368, 1.3917733, 1.40893292, 1.42609254,
                1.44325216, 1.46041179, 1.47757141, 1.49473103, 1.51189065,
                1.52905027, 1.54620989, 1.56336951, 1.58052914, 1.59768876,
                1.61484838, 1.632008 , 1.64916762, 1.66632724, 1.68348686,
                1.70064648, 1.71780611, 1.73496573, 1.75212535, 1.76928497,
                1.78644459, 1.80360421, 1.82076383, 1.83792346, 1.85508308,
                1.8722427 , 1.88940232 , 1.90656194 , 1.92372156 , 1.94088118 ,
                1.9580408 ,
                            1.97520043, 1.99236005, 2.00951967, 2.02667929,
                2.04383891, 2.06099853, 2.07815815, 2.09531778, 2.1124774,
                2.12963702, 2.14679664, 2.16395626, 2.18111588, 2.1982755],
               [ 1.51170901, 1.4413091 , 1.37428932, 1.31059585, 1.25017485,
                1.19297248, 1.13893491, 1.08800829, 1.0401388, 0.99527259,
                0.95335584, 0.9143347, 0.87815535, 0.84476393, 0.81410663,
                0.7861296 , 0.760779 , 0.73800101, 0.71774178, 0.69994748,
                0.68456427, 0.67153832, 0.66081579, 0.65234284, 0.64606565,
                0.64193037, 0.63988316, 0.6398702, 0.64183764, 0.64573165,
                             0.65908405, 0.66843475, 0.67949669, 0.69221601,
                0.6514984 ,
                0.70653889, 0.72241148, 0.73977996, 0.75859049, 0.77878923,
                0.80032234,
                             0.82313599, 0.84717635, 0.87238957, 0.89872182,
                0.92611927, 0.95452808, 0.98389441, 1.01416443, 1.0452843,
                1.07720019, 1.10985826, 1.14320467, 1.17718559, 1.21174719,
                1.24683562, 1.28239705, 1.31837764, 1.35472357, 1.39138099,
                1.42829606, 1.46541496, 1.50268384, 1.54004887, 1.57745621,
                1.61485203, 1.6521825, 1.68939377, 1.72643201, 1.76324338,
                1.79977405, 1.83597019, 1.87177795, 1.9071435, 1.942013,
                1.97633263, 2.01004853, 2.04310689, 2.07545385, 2.10703559,
                2.13779827, 2.16768805, 2.1966511, 2.22463359, 2.25158166,
                2.2774415 , 2.30215926, 2.32568111, 2.34795321, 2.36892173,
                2.38853283,
                             2.40673267, 2.42346742,
                                                     2.43868324, 2.4523263,
                2.46434276, 2.47467878, 2.48328054, 2.49009419, 2.49506589],
               [0.76559896, 0.64347011, 0.56881935, 0.53395569, 0.53187029,
                0.55620316, 0.60121069, 0.66173379, 0.73316679, 0.81142705,
                0.89292527,
                             0.97453654, 1.05357206, 1.12775162, 1.19517674,
                1.25430458,
                             1.30392252, 1.34312347, 1.37128188, 1.3880305,
                1.3932378 , 1.38698614, 1.36955063, 1.34137872, 1.30307049,
                1.2553597 , 1.19909543, 1.13522459, 1.06477504, 0.98883944,
                0.90855982, 0.8251129, 0.73969605, 0.65351401, 0.56776633,
                             0.40227574, 0.32480271, 0.2522836, 0.18572826,
                0.48363548,
                0.12608081, 0.07421212, 0.03091287, -0.00311255, -0.02725154,
                -0.04098799, -0.04390634, -0.03569486, -0.01614824, 0.01483046,
                0.05722873, 0.11092406, 0.17568425, 0.25116837, 0.33692839,
                0.43241165, 0.53696389, 0.6498331, 0.77017404, 0.89705349,
                1.0294562 , 1.16629155 , 1.30640093 , 1.44856585 , 1.59151672 ,
                1.73394237, 1.87450032, 2.01182766, 2.14455274, 2.27130757,
                2.39074081, 2.50153167, 2.60240435, 2.69214327, 2.76960901,
                2.83375495, 2.88364465, 2.91846988, 2.93756942, 2.94044859,
```

```
1.99101621, 1.85099748, 1.71364707, 1.58324757, 1.46462599,
                 1.36318472, 1.28493315, 1.23651994, 1.2252661, 1.25919873],
               [ 1.85727762, 1.4228108 , 1.05006601, 0.74658273, 0.51502014,
                 0.35419514, 0.25998742, 0.22612251, 0.24484331, 0.30747988,
                 0.40492699, 0.52803822, 0.66794494, 0.81630817, 0.96551059,
                 1.10879576, 1.24036097, 1.35540986, 1.45017043, 1.52188362,
                 1.56876744, 1.58996102, 1.58545264, 1.55599564, 1.50301541,
                 1.42851065, 1.3349516, 1.22517773, 1.1022969, 0.96958809,
                 0.83040907, 0.68811046, 0.54595733, 0.40705909, 0.27430851,
                 0.15033008, 0.03743826, -0.06239454, -0.14756016, -0.21682739,
                -0.26934546, -0.30463892, -0.32259438, -0.32343998, -0.30771822,
                -0.2762531 , -0.23011262, -0.17056732, -0.09904625, -0.01709109,
                 0.07369028, 0.17166985, 0.27524415, 0.38287633, 0.493135 ,
                 0.6047285 , 0.71653416 , 0.82762146 , 0.93726853 , 1.0449714 ,
                 1.15044557, 1.25361945, 1.35461961, 1.45374767, 1.55144904,
                 1.64827386, 1.74483049, 1.84173237, 1.9395392, 2.03869349,
                 2.13945395, 2.24182738, 2.34550098, 2.44977726, 2.5535141,
                 2.65507276, 2.75227698, 2.84238657, 2.92208948, 2.98751631,
                 3.03428194, 3.05755926, 3.05219011, 3.01283944, 2.93419871,
                 2.81124505, 2.63956348, 2.41573944, 2.13782974, 1.80592048,
                 1.42278068, 0.99462135, 0.53196987, 0.05067027, -0.42697946,
                -0.87094147, -1.24199118, -1.49009666, -1.55261095, -1.35226302]])
In [9]: # feel free to use the function part1_scatter() to replicate the figure
        # from the prompt once you have completed question one
        def plot_two(degree_predictions):
            import matplotlib.pyplot as plt
            %matplotlib inline
            plt.figure(figsize=(10,5))
            plt.plot(X_train, y_train, 'o', label='training data', markersize=10)
            plt.plot(X_test, y_test, 'o', label='test data', markersize=10)
            for i,degree in enumerate([1,3,6,9]):
                plt.plot(np.linspace(0,10,100), degree_predictions[i], alpha=0.8, lw=2, lab
            plt.ylim(-1,3.5)
            plt.legend(loc=4)
```

plot\_two(answer\_two())

2.92679937, 2.89652144, 2.84974371, 2.78684673, 2.7084857, 2.61561429, 2.50950911, 2.39179488, 2.26447036, 2.129935,



## Question 8 (10 points)

Write a function that fits a polynomial LinearRegression model on the training data  $X\_train$  for degrees 0 through 9. For each model compute the  $R^2$  (coefficient of determination) regression score on the training data as well as the the test data, and return both of these arrays in a tuple.

Based on the  $\mathbb{R}^2$  scores above (degree levels 0 through 9), what degree level corresponds to a model that is underfitting? What degree level corresponds to a model that is overfitting? What choice of degree level would provide a model with good generalization performance on this dataset? Note: there may be multiple correct solutions to this question.

(Hint: you can plot the r2\_train and r2\_test vs the degree to investigate underfitting/overfitting.)

This function should return one tuple of numpy arrays (r2\_train, r2\_test, Underfitting, Good\_Generalization, Overfitting), where both r2\_train and r2\_test should have shape (10,), and Underfitting, Good\_Generalization, Overfitting should be three lists of int partitioning 0-9.

```
In [10]: def answer_three():
    from sklearn.linear_model import LinearRegression
    from sklearn.preprocessing import PolynomialFeatures
    r2_train, r2_test = [], []
    Underfitting, Good_Generalization, Overfitting = [], [], []

# Your code here
for degree in range(10):
    pipeline = make_pipeline(
        PolynomialFeatures(degree=degree),
        MinMaxScaler()
```

```
    X_train_poly = pipeline.fit_transform(X_train.reshape(-1, 1))
    X_test_poly = pipeline.transform(X_test.reshape(-1, 1))
    model = LinearRegression().fit(X_train_poly, y_train)
    r2_train_score = model.score(X_train_poly, y_train)
    r2_test_score = model.score(X_test_poly, y_test)
    r2_train.append(r2_train_score)
    r2_test.append(r2_test_score)
    Underfitting = [0, 1, 2, 3]
    Good_Generalization = [4, 5, 6, 8]
    Overfitting = [7, 9]
    return (r2_train, r2_test, Underfitting, Good_Generalization, Overfitting)

answer_three()
```

```
Out[10]: ([0.0,
            0.2643975500223753,
            0.32742357618463,
            0.3722012635797515,
            0.8673604496714555,
            0.8714478477112108,
            0.913357440948628,
            0.9365234357860431,
            0.9384067915708731,
            0.9406850610895372],
           [-0.010639689448016743,
            0.5404084766379272,
            0.6426655069520221,
            0.7638109083466661,
            0.6592259101695964,
            0.7573712228485813,
            0.7713032363645259,
            0.38779951683191305,
            0.8104206433151161,
            -0.5929433096153114],
           [0, 1, 2, 3],
           [4, 5, 6, 8],
           [7, 9])
```

**Your answer here** When  $R^2_{train}$  and  $R^2_{test}$  are both low, the model is likely to underfit the data. When both  $R^2$  are high, around or above 80% I chose as the threshold, the model is fitting the data well. When  $R^2_{train}$  is pretty high and much higher than  $R^2_{test}$ , the model is overfitting the data as our model is bad at generalizing and is catching too much variance in the training dataset.

## **Disclosure**

If you used ChatGPT for any of the questions above, please disclose which questions you used it for, and how you used it.

Disclosure here: