SI 670 Problem Set 2

Q1 (15 points). Suppose that you are working as a data scientist at the sales department for an activewear clothing company, say like Lululemon or Nike, and you would like to train a machine learning model using sales records from last year to forecast sales for this year.

(1) Is your model a classifier or a regressor? Why?
It is a regressor. Because we want to predict the number of sales of this year but not classify something into different categories.

(2) How would your training data be similar to testing data? List three ways you expect your training data to be similar to testing data, and discuss the reasons.

Similar features: The training and test dataset should contain same predictors or features so that models can make prediction or test accuracy.

Similar distribution in features: We would expect the distributions of the features to be similar to ensure our prediction model to be consistent. Otherwise we need to do some feature scaling or manipulation

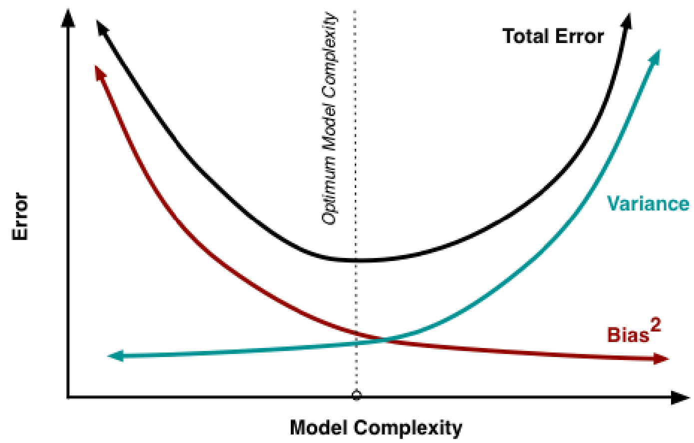
Similar DGP: We would expect the underlying data generating process to remain similar so that we can use the same set of features from last year to predict this year's sales. Otherwise, we would expect larger prediction error.

(3) How would your training data be different from your testing data? List three ways you expect your training data to be different from testing data, and discuss the reasons.

Different representativeness: training data and testing data may be different samples of the population.

Different economic situation from last years.
Different marketing strategies used by the team from last year's.

Q2 (16 points). We learned about Ridge vs Lasso regularizations in class. Recall that "alpha" (α) is a hyperparameter that determines the impact of the regularization term. The following figure illustrates the bias-variance tradeoff. Use it to answer the first two questions.



(1) As model complexity increases, what happens to the bias and variance of the model?
As complexity increases, bias decreases and variance increases. Because complex model can capture more patterns in the data, the prediction error will be smaller. But the model is more sensitive to the sample data, so the variance is larger.
(2) In ridge regression, what happens if we set $\alpha = 0$? What happens as α approaches ∞?
If $\alpha = 0$, there will be no regularization. The model will be equivalent to the linear regression model. If alpha term approaches infinity, the coefficients will approach to zero.

(3) If we have a large number of features (10,000 +) and we suspect that only a handful of features are useful, which type of regression (Lasso vs Ridge) would be more helpful in identifying useful features?
Lasso regression would be more helpful. Because we only want to keep the important features and minimize the coefficients of other variables to zero.

(4) What are the benefits of using Ridge regression compared to standard linear regression (minimizing RSS)?
Ridge regression includes regularization term can help control the overfitting problem which gives us smaller RSS and better prediction in test data.

Q3 (12 points). We learned about k-NN regression and linear regression in class.

(1) Can you think about a real-life situation where k-NN regression would work better than linear regression? Describe the situation and explain why k-NN regression is better.

Predicting housing prices. The pattern of house prices is more likely to be non-linear. Also, there are more local patterns in the house prices.

(2) Can you think about a real-life situation where linear regression would work better than k-NN regression? Describe the situation and explain why linear regression is better.

Predicting overall GPA using weekly assignment grades. The relationship between assignment grades, course grades, and GPA is mostly linear.

(3) Summarize what are the advantages and disadvantages of k-NN/linear regression, based on your examples above.

k-NN: advantages: better at capturing local and non-linear patterns, better with data including multiple similar data points (similar houses or houses in the neighboring region), flexible (without linearity assumption)

disadvantages: worse in generalization (if the testing house is very different from the houses in training data, larger error), computationally expensive in high dimensional data, sensitive to the choice of number of neighbors (k)

Linear regression: better when relationship is linear (course final grades and weekly assignment grades), easier to interpret the relationships between features and outcomes, better to deal with high-dimensional large scale datasets.

Disadvantages: too simplifying the model leads to underfitting, missing local patterns (hard to capture the local pattern in house prices if no relevant predictors are included in the model)

Q4-Q7 in si670f25_hw2.ipynb

SI 670 Applied Machine Learning, HW 2 (Due Tuesday Sep 16, 2025 11:59 PM)

IMPORTANT: please name your submitted file `si670f25_hw2_youruniqname.ipynb` when submitting on Gradescope

As a reminder, the notebook code you submit must be your own work. Feel free to discuss general approaches to the homework with classmates. If you end up forming more of a team discussion on multiple questions, please include the names of the people you worked with at the top of your notebook file.

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Collaborators (if any):

Questions 1-3 are in SI 670 - Problem Set 2

Problem 4 (13 points)

Your goal for this question is to run a simple linear regression on the data, and investigate its performance. Note that this question has multiple parts.

First you need to run the following block, which sets up the training and test datasets to be used for this question.

In [1]:
```
import numpy as np

# We fix the state of the random number generator here in order to maintain consist
rs = np.random.RandomState(0)
X_train = rs.uniform(size = (100, 1))
y_train = 5 * X_train + 8
X_test = rs.uniform(size = (20, 1))
y_test = 7 * X_test + 1
```

Now, write a function that takes in training data as input, and fits a line to the data using a linear regression model. Your function should return the linear regression model.

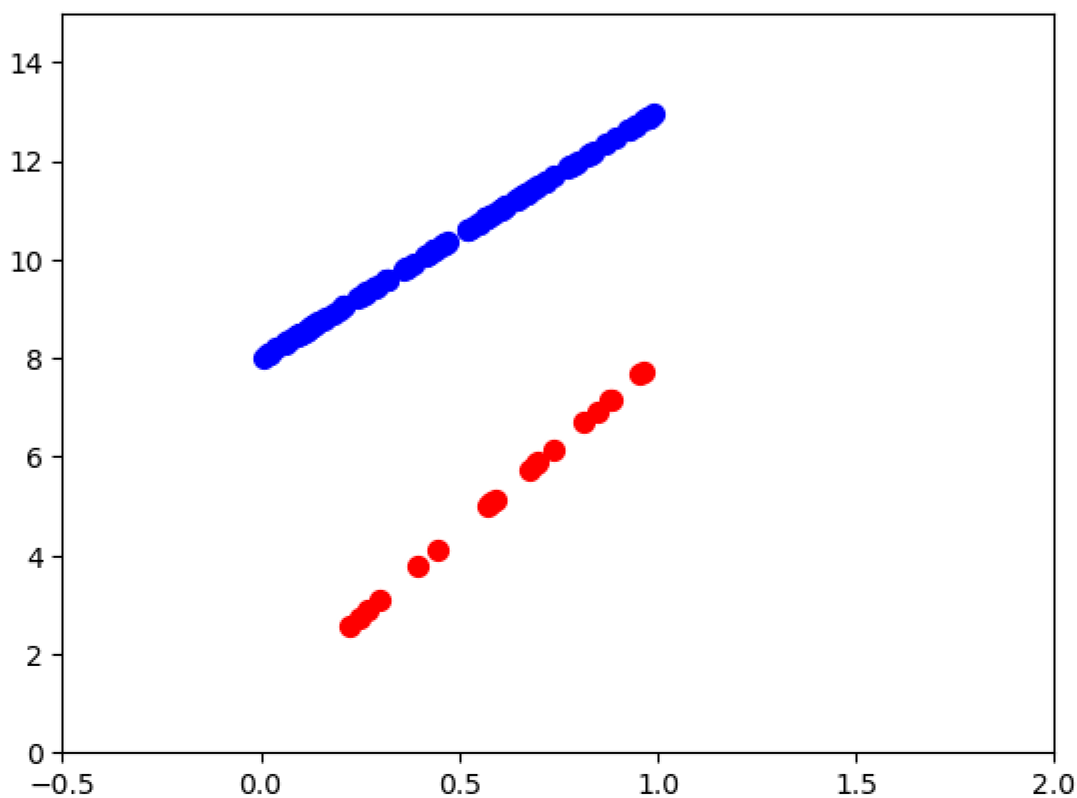
In [2]:
```
def fit_line(X_train, y_train):
    from sklearn.linear_model import LinearRegression

    # Your code: fit a linear regression model to the input data, name it `linreg'
    linreg = LinearRegression().fit(X_train, y_train)
    # Return the linear regression model you created
    return linreg
```

Run the following block of code in order to (1) compute the the R^2 (coefficient of determination) regression score on both training and test datasets, and then (2) plot the training and test datasets in the same figure, marked by different colors (blue for training data, red for test data).

In [3]:
```
# Fit a line to the dataset we generated at the beginning of this problem using you
linreg = fit_line(X_train, y_train)

# Compute R^2 score of your model on training and test sets
print('R-squared score (training): {:.3f}'
     .format(linreg.score(X_train, y_train)))
print('R-squared score (test): {:.3f}'
     .format(linreg.score(X_test, y_test)))

# Plot the training and test datasets in the same figure marked by different colors
import matplotlib.pyplot as plt
fig = plt.figure()
plt.xlim(-0.5, 2)
plt.ylim(0, 15)
plt.scatter(X_train, y_train, c='blue', marker='o', s=50) # Blue points are from tr
plt.scatter(X_test, y_test, c='red', marker='o', s=50) # Red points are from test a
```

```
R-squared score (training): 1.000
R-squared score (test): -11.533
```

Out[3]: `<matplotlib.collections.PathCollection at 0x28487a9c690>`



What does the R^2 score of your model on the two datasets tell you about the performance of your model? Using the plot of the two datasets, why do you think we get these R^2 scores? Please provide your answer in the text cell below.

Your answer to problem 4 here: We have a overfitting problem in our model: the training accuracy is very high and the test accuracy is extremely low (even lower than 0). It is very likely due to the very different data generating processes in the training and testing datasets. And both the DGPs are fully linear. The training model can hardly be generalized to other DGPs.

Question 5 (20 points) Bias and Variance

(a) 10 points

Your task is to investigate the influence of different regularization parameters on the coefficients of a ridge regression model.

Given 10 points around the function $y = x^2 + 2x + 1$. You are asked to train a linear ridge regression with degree 6 polynomial features. And run it with different regularization parameters $\alpha \in \{0, 0.1, 1, 10, 100\}$.

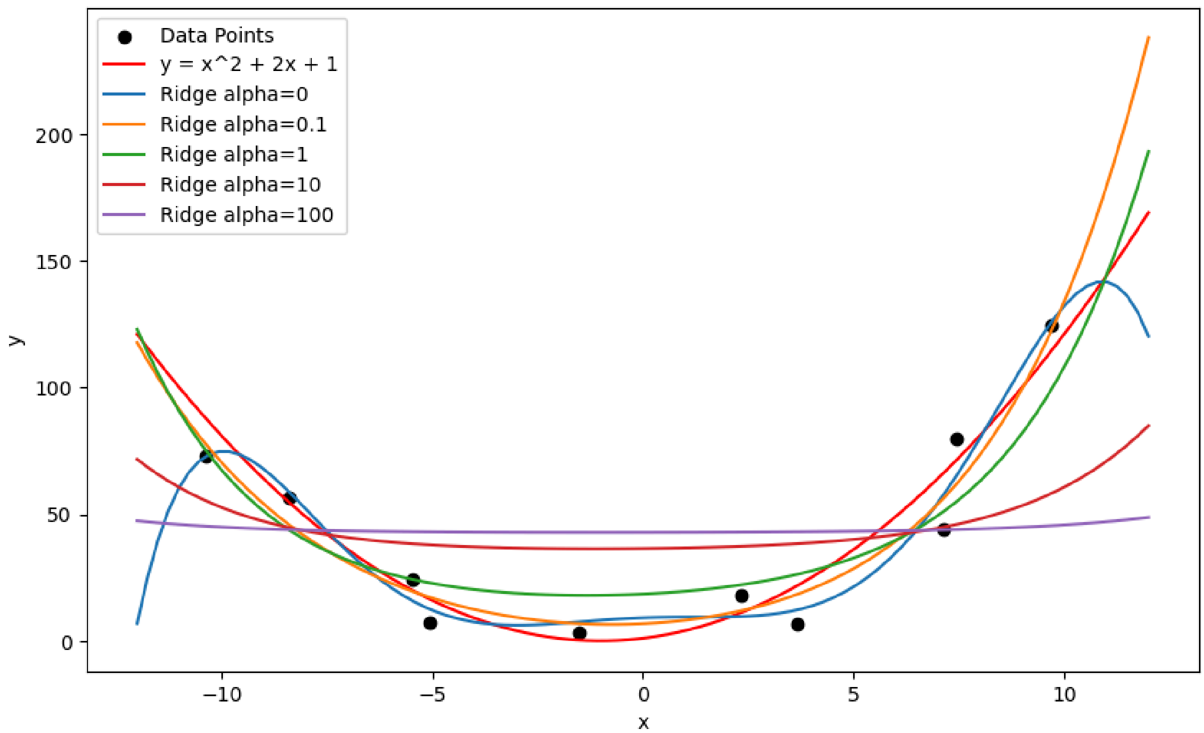
Plot the polynomials from your regression results corresponding to each regularization parameter in one figure.

Hint: One way to plot the polynomial from your regressions is to create a dataset with a bunch of data points for the x values, then use your model to predict the outputs for these x values. Finally, use plt.plot(my_x_values, predicted_y_values) to show the polynomial predicted by the model.

In [4]:

```
import numpy as np
from sklearn.linear_model import Ridge
from sklearn.preprocessing import PolynomialFeatures, StandardScaler, MinMaxScaler
from sklearn.pipeline import make_pipeline
import matplotlib.pyplot as plt

def generate_polynomials_for_diff_alphas():
    x_list = np.array([-10.38446879, -8.38394902, -5.47700112, -5.04337481, -1.5054
    y_list = np.array([73.21995367, 56.4250573, 24.15989601, 7.14325154, 3.45955269

    # Generate original x values
    x_original = np.linspace(-12, 12, 100).reshape(-1, 1)

    # Generate y values based on the function y = x^2 + 2x + 1
    y_original = x_original**2 + 2*x_original + 1

    # Plot the original function and the data points
    plt.figure(figsize=(10, 6))
    plt.scatter(x_list, y_list, label='Data Points', color='black')
    plt.plot(x_original, y_original, label='y = x^2 + 2x + 1', color='red')

    ### your code here
```

```python
    alphas = [0, 0.1, 1, 10, 100]
    poly_pipeline = make_pipeline(
        PolynomialFeatures(degree=6),
        MinMaxScaler()
    )
    x_list_scaled = poly_pipeline.fit_transform(x_list)
    x_original_scaled = poly_pipeline.transform(x_original)

    for alpha in alphas:
        model = Ridge(alpha=alpha)
        model.fit(x_list_scaled, y_list)
        y_pred = model.predict(x_original_scaled)
        plt.plot(x_original, y_pred, label=f'Ridge alpha={alpha}')


    plt.xlabel('x')
    plt.ylabel('y')
    plt.legend()
    plt.show()

generate_polynomials_for_diff_alphas()
```



(b) 10 points

We then give you a data generator that gives 10 data points around the function $y = x^2 + 2x + 1$ each time you run **data_gen()**.

We would like you to consider the bias and variance of a 6-degree polynomial classifier. In particular, for each of the regularization parameters $\alpha \in \{0, 0.1, 1, 10, 100\}$ we would like you to calculate the bias and variance over 1000 generated data sets from **data_gen()**.

To do so, you will need to calculate the bias and variance with respect to a particular data point. Please use $x^* = 0, y^* = 1$ for this data point.

For more context on how to calculate bias and variance, refer back to the slides from lecture 5.

In addition, briefly describe how the bias and variance change when the regularization parameter increases. If you are unable to finish the coding portion, please explain how you would expect bias and variance to change.

In [5]:
```python
def data_gen():
    # Number of data points
    n = 10

    # Generate original x values: you can choose the range and number of points
    x_original = np.linspace(-10, 10, n)

    # Generate y values based on the function y = x^2 + 2x + 1
    y_original = x_original**2 + 2*x_original + 1

    # Add some noise to x and y values
    noise_strength_x = 1.0  # Control the noise strength for x
    noise_strength_y = 5.0  # Control the noise strength for y

    x_noisy = x_original + noise_strength_x * np.random.randn(n)
    y_noisy = y_original + noise_strength_y * np.random.randn(n)

    return x_noisy, y_noisy


def calculate_bias_var():
    import numpy as np
    from sklearn.linear_model import LinearRegression

    bias_list = []
    variance_list = []

    # your code here
    alphas = [0, 0.1, 1, 10, 100]
    datasets = [data_gen() for _ in range(1000)]
    pipeline = make_pipeline(
        PolynomialFeatures(degree=6),
        MinMaxScaler()
    )
    for alpha in alphas:
        model_bias = []
        model_variance = []
        for dataset in datasets:
            x_original, y_original = dataset
            x_original = x_original.reshape(-1, 1)
            x_original_scaled = pipeline.fit_transform(x_original)
            model = Ridge(alpha=alpha)
            model.fit(x_original_scaled, y_original)
            y_pred = model.predict(x_original_scaled)
```

```
            bias = np.mean((y_original - y_pred) ** 2)
            variance = np.var(y_pred)
            model_bias.append(bias)
            model_variance.append(variance)
        bias_list.append(np.mean(model_bias))
        variance_list.append(np.mean(model_variance))
    return bias_list, variance_list

calculate_bias_var()
```

Out[5]:

```
([np.float64(44.20316901280138),
  np.float64(103.02619660175057),
  np.float64(264.58572600335214),
  np.float64(948.8996804679945),
  np.float64(1397.8952112837346)],
 [np.float64(1430.3065773507665),
  np.float64(1228.243680495409),
  np.float64(705.988741204077),
  np.float64(75.29230159152863),
  np.float64(1.3066799725672673)])
```

Your answer to Problem 5 here. As the regularization term increases, the bias increases and variance decreases.

Question 6 (15 points) Regression to the mean

Imagine you are working on a regression problem with six features.

You will:

1. Generate synthetic data.
2. Split data into training, validation, and test sets, with a ratio of 6:2:2
3. Train two different linear regression models (one with Ridge regularization term with $\alpha = 0.2$ and the other without).
4. Evaluate both models on the validation data with Mean Squared Error.
5. Suppose that for each time you will apply the "better" model according to the MSE on the validation data, evaluate it on the test data.

Repeat this 1000 times. Each time calculate the average of the error of the "better" model on validation data and test data respectively (so you will only record the error of the better model on the validation data, and the error of this same model on the test data.

Which average error is smaller? Why do you expect this (or not expect this)?

In [6]:

```
import numpy as np
from sklearn.linear_model import LinearRegression, Ridge
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error
```

```
def answer_3():
    val_mse_list = []
    test_mse_list = []
    for _ in range(1000):

        # Generate synthetic data
        X = 2 * np.random.rand(50, 6)  # 50 samples, 6 features
        y = 10 + np.dot(X, np.array([3, 5, 2, 0, 0, 0])) + 2 * np.random.randn(50)

        # Your code here
        # Split data into training, validation, and test sets, with a ratio of 6:2:
        X_train, X_temp, y_train, y_temp = train_test_split(X, y, test_size=0.4, ra
        X_val, X_test, y_val, y_test = train_test_split(X_temp, y_temp, test_size=0

        # Train two models
        # Model 1: Linear Regression
        linear_model = LinearRegression().fit(X_train, y_train)

        # Model 2: Ridge Regression with alpha = 0.2
        ridge_model = Ridge(alpha=0.2).fit(X_train, y_train)

        # Evaluate on validation set
        val_mse1 = mean_squared_error(y_val, linear_model.predict(X_val))
        val_mse2 = mean_squared_error(y_val, ridge_model.predict(X_val))

        print(f"Validation MSE for Linear Regression: {val_mse1}")
        print(f"Validation MSE for Ridge Regression: {val_mse2}")

        # Choose the "better" model based on validation MSE
        better_model = linear_model if val_mse1 < val_mse2 else ridge_model
        val_mse_list.append(min(val_mse1, val_mse2))
        # Evaluate 'better' model on the test set
        test_mse = mean_squared_error(y_test, better_model.predict(X_test))
        test_mse_list.append(test_mse)
        #print(f"Test MSE for the 'better' model: {test_mse}")
    return np.mean(val_mse_list), np.mean(test_mse_list)

answer_3()
```

```
Validation MSE for Linear Regression: 2.577636033795739
Validation MSE for Ridge Regression: 2.3968194797708433
Validation MSE for Linear Regression: 2.8131857999119623
Validation MSE for Ridge Regression: 2.84640274189133
Validation MSE for Linear Regression: 6.556697378245781
Validation MSE for Ridge Regression: 5.868087644616071
Validation MSE for Linear Regression: 16.969987046796565
Validation MSE for Ridge Regression: 17.411523512059812
Validation MSE for Linear Regression: 7.514409066626444
Validation MSE for Ridge Regression: 7.7904388633179495
Validation MSE for Linear Regression: 4.195162257461351
Validation MSE for Ridge Regression: 4.176729845843388
Validation MSE for Linear Regression: 5.2591657902767
Validation MSE for Ridge Regression: 4.924293035066554
Validation MSE for Linear Regression: 6.0785484039247155
Validation MSE for Ridge Regression: 5.961771430021715
Validation MSE for Linear Regression: 7.496063066908538
Validation MSE for Ridge Regression: 7.505359618596164
Validation MSE for Linear Regression: 4.559269124011912
Validation MSE for Ridge Regression: 4.602392861266818
Validation MSE for Linear Regression: 7.709422107088265
Validation MSE for Ridge Regression: 7.306937360021314
Validation MSE for Linear Regression: 4.547254738155814
Validation MSE for Ridge Regression: 4.452151141598338
Validation MSE for Linear Regression: 2.9279111550230117
Validation MSE for Ridge Regression: 2.9773208831778875
Validation MSE for Linear Regression: 3.386046322237523
Validation MSE for Ridge Regression: 3.285279842299316
Validation MSE for Linear Regression: 8.655163675766254
Validation MSE for Ridge Regression: 8.5653671895649
Validation MSE for Linear Regression: 7.299970459305134
Validation MSE for Ridge Regression: 6.884070178251031
Validation MSE for Linear Regression: 12.697586511652377
Validation MSE for Ridge Regression: 12.511577819958855
Validation MSE for Linear Regression: 6.1589327284250395
Validation MSE for Ridge Regression: 6.175341908749803
Validation MSE for Linear Regression: 0.40246198904534103
Validation MSE for Ridge Regression: 0.41918331330206
Validation MSE for Linear Regression: 6.858040359602699
Validation MSE for Ridge Regression: 6.747622762756682
Validation MSE for Linear Regression: 5.621477678788123
Validation MSE for Ridge Regression: 5.4201897297097945
Validation MSE for Linear Regression: 2.725081287934419
Validation MSE for Ridge Regression: 2.7067796689398795
Validation MSE for Linear Regression: 6.9370605504028475
Validation MSE for Ridge Regression: 6.937146764802508
Validation MSE for Linear Regression: 7.044117464106695
Validation MSE for Ridge Regression: 6.520809102761698
Validation MSE for Linear Regression: 5.389227839727392
Validation MSE for Ridge Regression: 5.331372367610003
Validation MSE for Linear Regression: 11.49332794058304
Validation MSE for Ridge Regression: 11.615754443002427
Validation MSE for Linear Regression: 9.930181590759585
Validation MSE for Ridge Regression: 9.82453903167307
Validation MSE for Linear Regression: 4.556189764547357
Validation MSE for Ridge Regression: 4.348491011726741
```

```
Validation MSE for Linear Regression: 4.540677424548243
Validation MSE for Ridge Regression: 4.419050294290736
Validation MSE for Linear Regression: 4.406685328730525
Validation MSE for Ridge Regression: 4.46934534864145
Validation MSE for Linear Regression: 5.582114859286176
Validation MSE for Ridge Regression: 5.148118109920721
Validation MSE for Linear Regression: 3.585334051204692
Validation MSE for Ridge Regression: 3.5112335606886385
Validation MSE for Linear Regression: 5.381769561925455
Validation MSE for Ridge Regression: 5.173235135641899
Validation MSE for Linear Regression: 3.8999377486284197
Validation MSE for Ridge Regression: 3.851270482870215
Validation MSE for Linear Regression: 8.370429197506528
Validation MSE for Ridge Regression: 8.00436367128234
Validation MSE for Linear Regression: 3.21940606511568
Validation MSE for Ridge Regression: 3.3847476347864704
Validation MSE for Linear Regression: 5.565641852928186
Validation MSE for Ridge Regression: 5.443310547681034
Validation MSE for Linear Regression: 7.808812705817243
Validation MSE for Ridge Regression: 7.8435091945314195
Validation MSE for Linear Regression: 5.319074779201565
Validation MSE for Ridge Regression: 5.205181938622751
Validation MSE for Linear Regression: 7.7659301010396105
Validation MSE for Ridge Regression: 7.552987906085017
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Validation MSE for Ridge Regression: 3.748713225822604
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Validation MSE for Ridge Regression: 4.028235136183939
Validation MSE for Linear Regression: 4.384884906673974
Validation MSE for Ridge Regression: 4.290328132700722
Validation MSE for Linear Regression: 3.320016524925458
Validation MSE for Ridge Regression: 3.4909992253686566
Validation MSE for Linear Regression: 7.007594160748367
Validation MSE for Ridge Regression: 7.086622545590368
Validation MSE for Linear Regression: 1.8760746512577342
Validation MSE for Ridge Regression: 1.5856998839089989
Validation MSE for Linear Regression: 3.4193465262507177
Validation MSE for Ridge Regression: 3.2415856782003845
Validation MSE for Linear Regression: 6.43501433008345
Validation MSE for Ridge Regression: 6.150932369846186
Validation MSE for Linear Regression: 3.5116940295673813
Validation MSE for Ridge Regression: 3.600835264701516
Validation MSE for Linear Regression: 6.20879284298981
Validation MSE for Ridge Regression: 6.211308461258015
```

```
Validation MSE for Linear Regression: 7.526687206998064
Validation MSE for Ridge Regression: 7.342384133635763
Validation MSE for Linear Regression: 2.2674705311515417
Validation MSE for Ridge Regression: 2.31220796458907
Validation MSE for Linear Regression: 2.3052986807846407
Validation MSE for Ridge Regression: 2.322857552255025
Validation MSE for Linear Regression: 8.840806134371675
Validation MSE for Ridge Regression: 8.597498051082233
Validation MSE for Linear Regression: 5.713430940091741
Validation MSE for Ridge Regression: 5.807880832867324
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Validation MSE for Linear Regression: 5.910503559585157
Validation MSE for Ridge Regression: 5.93152466252811
Validation MSE for Linear Regression: 4.414352600581242
Validation MSE for Ridge Regression: 4.546147279407227
```

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Validation MSE for Ridge Regression: 5.647677157273707
Validation MSE for Linear Regression: 6.511809332239818
Validation MSE for Ridge Regression: 6.486245819475092
Validation MSE for Linear Regression: 4.634583419130891
Validation MSE for Ridge Regression: 4.3624091127417755
Validation MSE for Linear Regression: 2.5184945899486264
Validation MSE for Ridge Regression: 2.273352673921382
Validation MSE for Linear Regression: 2.2395829945066046
Validation MSE for Ridge Regression: 2.214126697758396
Validation MSE for Linear Regression: 1.6987469431168933
Validation MSE for Ridge Regression: 1.7789488519998577
Validation MSE for Linear Regression: 8.425406084052607
Validation MSE for Ridge Regression: 8.490893406144332
Validation MSE for Linear Regression: 3.83144173134621
Validation MSE for Ridge Regression: 3.8608306728981447
Validation MSE for Linear Regression: 5.44077734995323
Validation MSE for Ridge Regression: 5.3978262413950215
Validation MSE for Linear Regression: 7.6988546824135
Validation MSE for Ridge Regression: 7.75917755178903
Validation MSE for Linear Regression: 3.5281086662799233
Validation MSE for Ridge Regression: 3.73173007002258
Validation MSE for Linear Regression: 2.5595381385542093
Validation MSE for Ridge Regression: 2.4964409703480106
Validation MSE for Linear Regression: 5.386682102066743
Validation MSE for Ridge Regression: 5.286736993806422
Validation MSE for Linear Regression: 5.458474177752276
Validation MSE for Ridge Regression: 5.437467835852253
Validation MSE for Linear Regression: 4.403320450353755
Validation MSE for Ridge Regression: 4.431786684987406
Validation MSE for Linear Regression: 3.847212337240424
Validation MSE for Ridge Regression: 3.7907881298459722
Validation MSE for Linear Regression: 2.875590870243537
Validation MSE for Ridge Regression: 2.7947716946413577
Validation MSE for Linear Regression: 3.1953439528856356
Validation MSE for Ridge Regression: 3.2904351230078817
Validation MSE for Linear Regression: 6.353183468309
Validation MSE for Ridge Regression: 5.945835622167836
```

```
Validation MSE for Linear Regression: 2.374650662654075
Validation MSE for Ridge Regression: 2.1656158651781974
Validation MSE for Linear Regression: 4.428281123716354
Validation MSE for Ridge Regression: 4.427975611163049
Validation MSE for Linear Regression: 5.854055192258263
Validation MSE for Ridge Regression: 5.338438417092259
Validation MSE for Linear Regression: 9.152586784749705
Validation MSE for Ridge Regression: 8.460553295674346
Validation MSE for Linear Regression: 3.640107295695631
Validation MSE for Ridge Regression: 3.769460158006325
Validation MSE for Linear Regression: 10.678047512874148
Validation MSE for Ridge Regression: 10.887627759150792
Validation MSE for Linear Regression: 9.568222338756996
Validation MSE for Ridge Regression: 9.460519153574229
Validation MSE for Linear Regression: 4.542459081752392
Validation MSE for Ridge Regression: 4.590643379816404
Validation MSE for Linear Regression: 4.162130605048054
Validation MSE for Ridge Regression: 4.169307525270149
Validation MSE for Linear Regression: 2.7728004248158964
Validation MSE for Ridge Regression: 2.671441778471862
Validation MSE for Linear Regression: 3.0200129640256264
Validation MSE for Ridge Regression: 3.0700832804434746
Validation MSE for Linear Regression: 6.001734255929742
Validation MSE for Ridge Regression: 5.986649681029367
Validation MSE for Linear Regression: 7.368406762067179
Validation MSE for Ridge Regression: 7.321711196659159
Validation MSE for Linear Regression: 8.865350406493159
Validation MSE for Ridge Regression: 8.590421971782638
Validation MSE for Linear Regression: 6.488107952512264
Validation MSE for Ridge Regression: 6.554642433481829
Validation MSE for Linear Regression: 5.205303121470534
Validation MSE for Ridge Regression: 5.264152326203016
Validation MSE for Linear Regression: 1.9602911100418843
Validation MSE for Ridge Regression: 1.8108879074574493
Validation MSE for Linear Regression: 4.954203395822721
Validation MSE for Ridge Regression: 5.082124551509816
Validation MSE for Linear Regression: 3.583731792679489
Validation MSE for Ridge Regression: 3.4963317937793454
Validation MSE for Linear Regression: 6.379033796764462
Validation MSE for Ridge Regression: 6.378612002099165
```

Out[6]:
```
(np.float64(5.276546419406722), np.float64(5.257389690129041))
```

Preliminary for Q7 and Q8

Before continuitng on to the next question, run the following block to set up the variables needed for later sections. It generates training and test data that we're going to use to fit a non-linear function (as opposed to the traditional linear regression problems you saw in the previous questions).

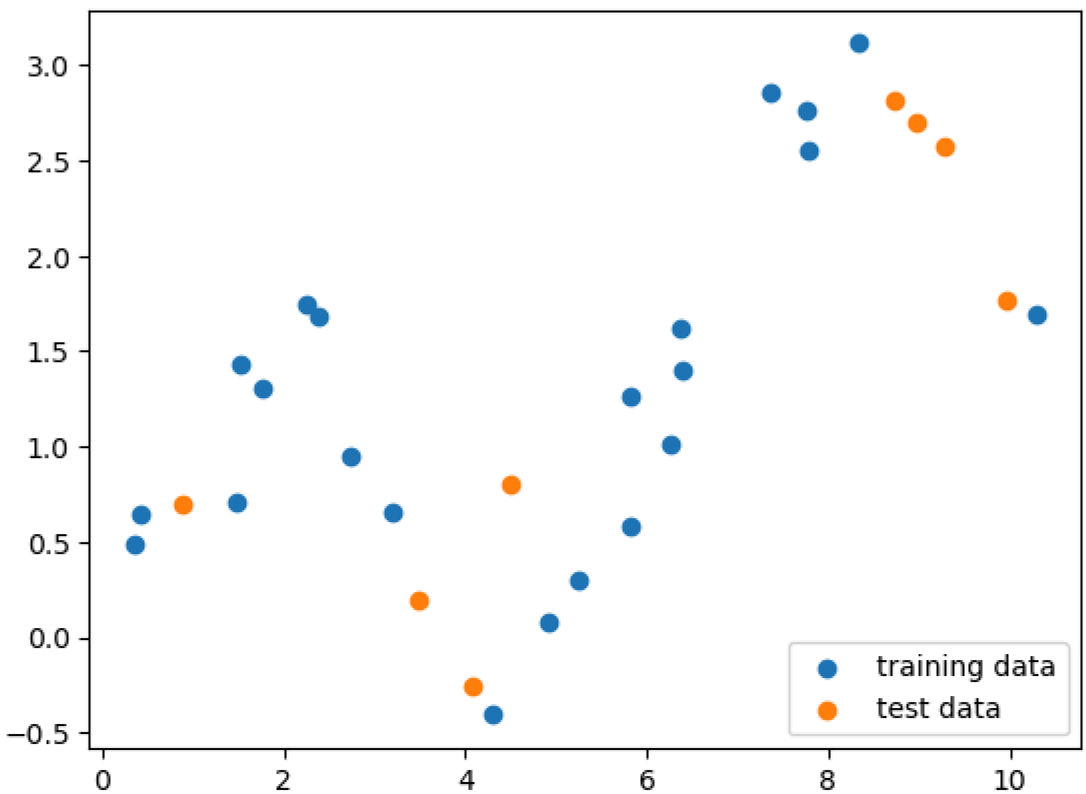
In [7]:
```
import pandas as pd
from sklearn.model_selection import train_test_split
import numpy as np
```

```python
np.random.seed(0)
n = 30
x = np.linspace(0, 10, n) + np.random.randn(n) / 5
y = np.sin(x) + x / 4 + np.random.randn(n) / 3


X_train, X_test, y_train, y_test = train_test_split(x, y, random_state=0)

# You can use this function to help you visualize the dataset by
# plotting a scatterplot of the data points
# in the training and test sets.
def part1_scatter():
    import matplotlib.pyplot as plt
    %matplotlib inline
    plt.figure()
    plt.scatter(X_train, y_train, label='training data')
    plt.scatter(X_test, y_test, label='test data')
    plt.legend(loc=4);


part1_scatter()  # let's plot the training and test data for the upcoming questions
```



Question 7 (10 points)

Write a function that fits a polynomial curve to the data, by using a polynomial LinearRegression model on the *training data* `X_train` for degrees 1, 3, 6, and 9. (Recall that we use PolynomialFeatures in sklearn.preprocessing to create the polynomial features and then fit a linear regression model to those extended features). For each model, find 100 predicted values over the interval x = 0 to 10 (e.g. `np.linspace(0,10,100)`) and store this in a numpy array. The first row of this array should correspond to the output from the model trained on degree 1, the second row degree 3, the third row degree 6, and the fourth row degree 9.

Note that you can run the `plot_two()` function to see the fitted curves on top of the dataset.

This function should return a numpy array with shape `(4, 100)`

In [8]:

```python
def answer_two():
    from sklearn.linear_model import LinearRegression
    from sklearn.preprocessing import PolynomialFeatures

    answer_array = []
    degree_list = [1, 3, 6, 9]
    X_pred = np.linspace(0, 10, 100).reshape(-1, 1)  # Generate predictions once

    for degree in degree_list:
        pipeline = make_pipeline(
            PolynomialFeatures(degree=degree),
            MinMaxScaler()
        )
        X_train_poly = pipeline.fit_transform(X_train.reshape(-1, 1))
        model = LinearRegression().fit(X_train_poly, y_train)
        X_pred_poly = pipeline.transform(X_pred)
        y_pred = model.predict(X_pred_poly)
        answer_array.append(y_pred)

    return np.vstack(answer_array)

answer_two()
```

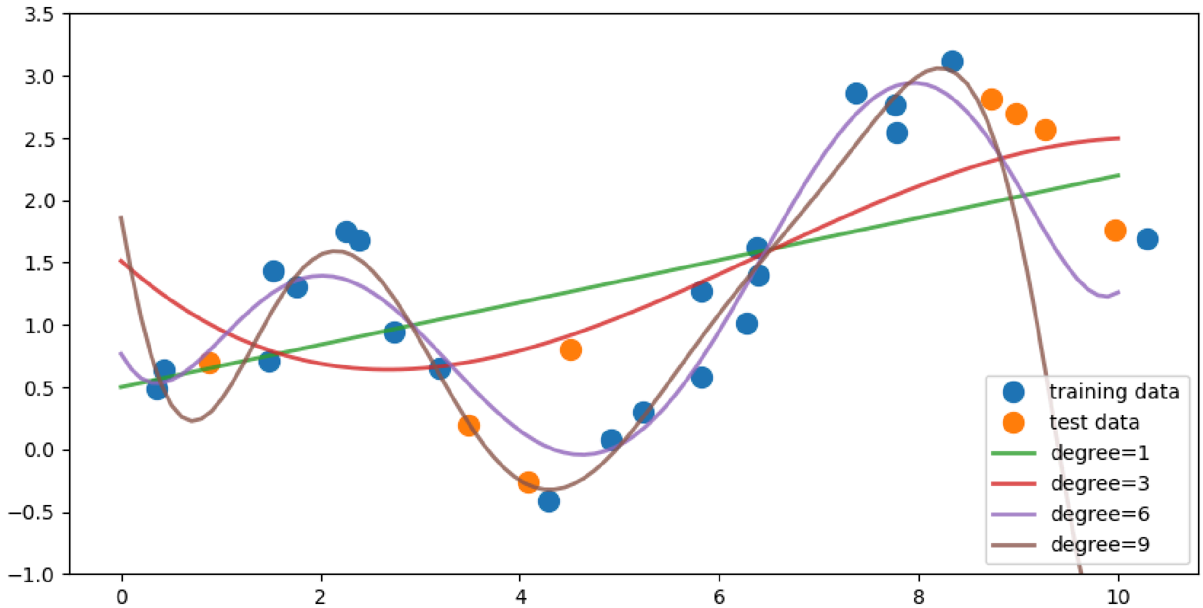
```
Out[8]:  array([[ 0.49947299,  0.51663261,  0.53379223,  0.55095185,  0.56811148,
          0.5852711 ,  0.60243072,  0.61959034,  0.63674996,  0.65390958,
          0.6710692 ,  0.68822883,  0.70538845,  0.72254807,  0.73970769,
          0.75686731,  0.77402693,  0.79118655,  0.80834617,  0.8255058 ,
          0.84266542,  0.85982504,  0.87698466,  0.89414428,  0.9113039 ,
          0.92846352,  0.94562315,  0.96278277,  0.97994239,  0.99710201,
          1.01426163,  1.03142125,  1.04858087,  1.06574049,  1.08290012,
          1.10005974,  1.11721936,  1.13437898,  1.1515386 ,  1.16869822,
          1.18585784,  1.20301747,  1.22017709,  1.23733671,  1.25449633,
          1.27165595,  1.28881557,  1.30597519,  1.32313482,  1.34029444,
          1.35745406,  1.37461368,  1.3917733 ,  1.40893292,  1.42609254,
          1.44325216,  1.46041179,  1.47757141,  1.49473103,  1.51189065,
          1.52905027,  1.54620989,  1.56336951,  1.58052914,  1.59768876,
          1.61484838,  1.632008  ,  1.64916762,  1.66632724,  1.68348686,
          1.70064648,  1.71780611,  1.73496573,  1.75212535,  1.76928497,
          1.78644459,  1.80360421,  1.82076383,  1.83792346,  1.85508308,
          1.8722427 ,  1.88940232,  1.90656194,  1.92372156,  1.94088118,
          1.9580408 ,  1.97520043,  1.99236005,  2.00951967,  2.02667929,
          2.04383891,  2.06099853,  2.07815815,  2.09531778,  2.1124774 ,
          2.12963702,  2.14679664,  2.16395626,  2.18111588,  2.1982755 ],
        [ 1.51170901,  1.4413091 ,  1.37428932,  1.31059585,  1.25017485,
          1.19297248,  1.13893491,  1.08800829,  1.0401388 ,  0.99527259,
          0.95335584,  0.9143347 ,  0.87815535,  0.84476393,  0.81410663,
          0.7861296 ,  0.760779  ,  0.73800101,  0.71774178,  0.69994748,
          0.68456427,  0.67153832,  0.66081579,  0.65234284,  0.64606565,
          0.64193037,  0.63988316,  0.6398702 ,  0.64183764,  0.64573165,
          0.6514984 ,  0.65908405,  0.66843475,  0.67949669,  0.69221601,
          0.70653889,  0.72241148,  0.73977996,  0.75859049,  0.77878923,
          0.80032234,  0.82313599,  0.84717635,  0.87238957,  0.89872182,
          0.92611927,  0.95452808,  0.98389441,  1.01416443,  1.0452843 ,
          1.07720019,  1.10985826,  1.14320467,  1.17718559,  1.21174719,
          1.24683562,  1.28239705,  1.31837764,  1.35472357,  1.39138099,
          1.42829606,  1.46541496,  1.50268384,  1.54004887,  1.57745621,
          1.61485203,  1.6521825 ,  1.68939377,  1.72643201,  1.76324338,
          1.79977405,  1.83597019,  1.87177795,  1.9071435 ,  1.942013  ,
          1.97633263,  2.01004853,  2.04310689,  2.07545385,  2.10703559,
          2.13779827,  2.16768805,  2.1966511 ,  2.22463359,  2.25158166,
          2.2774415 ,  2.30215926,  2.32568111,  2.34795321,  2.36892173,
          2.38853283,  2.40673267,  2.42346742,  2.43868324,  2.4523263 ,
          2.46434276,  2.47467878,  2.48328054,  2.49009419,  2.49506589],
        [ 0.76559896,  0.64347011,  0.56881935,  0.53395569,  0.53187029,
          0.55620316,  0.60121069,  0.66173379,  0.73316679,  0.81142705,
          0.89292527,  0.97453654,  1.05357206,  1.12775162,  1.19517674,
          1.25430458,  1.30392252,  1.34312347,  1.37128188,  1.3880305 ,
          1.3932378 ,  1.38698614,  1.36955063,  1.34137872,  1.30307049,
          1.2553597 ,  1.19909543,  1.13522459,  1.06477504,  0.98883944,
          0.90855982,  0.8251129 ,  0.73969605,  0.65351401,  0.56776633,
          0.48363548,  0.40227574,  0.32480271,  0.2522836 ,  0.18572826,
          0.12608081,  0.07421212,  0.03091287, -0.00311255, -0.02725154,
         -0.04098799, -0.04390634, -0.03569486, -0.01614824,  0.01483046,
          0.05722873,  0.11092406,  0.17568425,  0.25116837,  0.33692839,
          0.43241165,  0.53696389,  0.6498331 ,  0.77017404,  0.89705349,
          1.0294562 ,  1.16629155,  1.30640093,  1.44856585,  1.59151672,
          1.73394237,  1.87450032,  2.01182766,  2.14455274,  2.27130757,
          2.39074081,  2.50153167,  2.60240435,  2.69214327,  2.76960901,
          2.83375495,  2.88364465,  2.91846988,  2.93756942,  2.94044859,
```

```
        2.92679937,  2.89652144,  2.84974371,  2.78684673,  2.7084857 ,
        2.61561429,  2.50950911,  2.39179488,  2.26447036,  2.129935  ,
        1.99101621,  1.85099748,  1.71364707,  1.58324757,  1.46462599,
        1.36318472,  1.28493315,  1.23651994,  1.2252661 ,  1.25919873],
      [ 1.85727762,  1.4228108 ,  1.05006601,  0.74658273,  0.51502014,
        0.35419514,  0.25998742,  0.22612251,  0.24484331,  0.30747988,
        0.40492699,  0.52803822,  0.66794494,  0.81630817,  0.96551059,
        1.10879576,  1.24036097,  1.35540986,  1.45017043,  1.52188362,
        1.56876744,  1.58996102,  1.58545264,  1.55599564,  1.50301541,
        1.42851065,  1.3349516 ,  1.22517773,  1.1022969 ,  0.96958809,
        0.83040907,  0.68811046,  0.54595733,  0.40705909,  0.27430851,
        0.15033008,  0.03743826, -0.06239454, -0.14756016, -0.21682739,
       -0.26934546, -0.30463892, -0.32259438, -0.32343998, -0.30771822,
       -0.2762531 , -0.23011262, -0.17056732, -0.09904625, -0.01709109,
        0.07369028,  0.17166985,  0.27524415,  0.38287633,  0.493135  ,
        0.6047285 ,  0.71653416,  0.82762146,  0.93726853,  1.0449714 ,
        1.15044557,  1.25361945,  1.35461961,  1.45374767,  1.55144904,
        1.64827386,  1.74483049,  1.84173237,  1.9395392 ,  2.03869349,
        2.13945395,  2.24182738,  2.34550098,  2.44977726,  2.5535141 ,
        2.65507276,  2.75227698,  2.84238657,  2.92208948,  2.98751631,
        3.03428194,  3.05755926,  3.05219011,  3.01283944,  2.93419871,
        2.81124505,  2.63956348,  2.41573944,  2.13782974,  1.80592048,
        1.42278068,  0.99462135,  0.53196987,  0.05067027, -0.42697946,
       -0.87094147, -1.24199118, -1.49009666, -1.55261095, -1.35226302]])
```

In [9]:

```
# feel free to use the function part1_scatter() to replicate the figure
# from the prompt once you have completed question one
def plot_two(degree_predictions):
    import matplotlib.pyplot as plt
    %matplotlib inline
    plt.figure(figsize=(10,5))
    plt.plot(X_train, y_train, 'o', label='training data', markersize=10)
    plt.plot(X_test, y_test, 'o', label='test data', markersize=10)
    for i,degree in enumerate([1,3,6,9]):
        plt.plot(np.linspace(0,10,100), degree_predictions[i], alpha=0.8, lw=2, lab
    plt.ylim(-1,3.5)
    plt.legend(loc=4)

plot_two(answer_two())
```



Question 8 (10 points)

Write a function that fits a polynomial LinearRegression model on the training data `X_train` for degrees 0 through 9. For each model compute the R^2 (coefficient of determination) regression score on the training data as well as the the test data, and return both of these arrays in a tuple.

Based on the R^2 scores above (degree levels 0 through 9), what degree level corresponds to a model that is underfitting? What degree level corresponds to a model that is overfitting? What choice of degree level would provide a model with good generalization performance on this dataset? Note: there may be multiple correct solutions to this question.

(Hint: you can plot the r2_train and r2_test vs the degree to investigate underfitting/overfitting.)

This function should return one tuple of numpy arrays `(r2_train, r2_test, Underfitting, Good_Generalization, Overfitting)`*, where both* `r2_train` *and* `r2_test` *should have shape* `(10,)`*, and* `Underfitting, Good_Generalization, Overfitting` *should be three lists of int partitioning 0-9.*

In [10]:

```
def answer_three():
    from sklearn.linear_model import LinearRegression
    from sklearn.preprocessing import PolynomialFeatures
    r2_train, r2_test = [], []
    Underfitting, Good_Generalization, Overfitting = [], [], []

    # Your code here
    for degree in range(10):
        pipeline = make_pipeline(
            PolynomialFeatures(degree=degree),
            MinMaxScaler()
```

```
        )
        X_train_poly = pipeline.fit_transform(X_train.reshape(-1, 1))
        X_test_poly = pipeline.transform(X_test.reshape(-1, 1))
        model = LinearRegression().fit(X_train_poly, y_train)
        r2_train_score = model.score(X_train_poly, y_train)
        r2_test_score = model.score(X_test_poly, y_test)
        r2_train.append(r2_train_score)
        r2_test.append(r2_test_score)
        Underfitting = [0, 1, 2, 3]
        Good_Generalization = [4, 5, 6, 8]
        Overfitting = [7, 9]
    return (r2_train, r2_test, Underfitting, Good_Generalization, Overfitting)

answer_three()
```

Out[10]:

```
([0.0,
  0.2643975500223753,
  0.32742357618463,
  0.3722012635797515,
  0.8673604496714555,
  0.8714478477112108,
  0.913357440948628,
  0.9365234357860431,
  0.9384067915708731,
  0.9406850610895372],
 [-0.010639689448016743,
  0.5404084766379272,
  0.6426655069520221,
  0.7638109083466661,
  0.6592259101695964,
  0.7573712228485813,
  0.7713032363645259,
  0.38779951683191305,
  0.8104206433151161,
  -0.5929433096153114],
 [0, 1, 2, 3],
 [4, 5, 6, 8],
 [7, 9])
```

Your answer here When R^2_{train} and R^2_{test} are both low, the model is likely to underfit the data. When both R^2 are high, around or above 80% I chose as the threshold, the model is fitting the data well. When R^2_{train} is pretty high and much higher than R^2_{test}, the model is overfitting the data as our model is bad at generalizing and is catching too much variance in the training dataset.

Disclosure

If you used ChatGPT for any of the questions above, please disclose which questions you used it for, and how you used it.

Disclosure here: