Mach-Zehnder Interferometry on IBM Quantum Computer

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1 Mach-Zehnder interferometer

Mach-Zehnder Interferometer is a device which compose with two beam splitters and one phase shifter as shown in figure 1. We will assume that the incoming photon is in $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ which we can write in Bloch sphere as $(u, v, w) = (0, 0, 1)$ as shown in Figure 2.

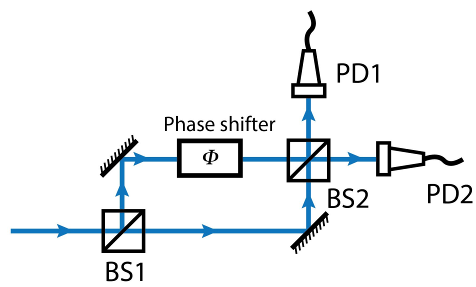

Figure 1. Mach-Zehnder Interferometer.

2 Beam splitter

In this case we will assume that the phase shift due to transmission and reflection by a beam splitter are $\Delta\phi_T = \pi/2$ and $\Delta\phi_R = 0$. The unitary operator for the beam splitter can be written as

$$\mathbf{B} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\phi_R} & e^{i\phi_T} \\ e^{i\phi_T} & e^{i\phi_R} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}, \tag{1}$$

which we can also write this operator in a form of Puali matrix as

$$\mathbf{B} = \frac{1}{2} \left(\mathbb{1} + i\hat{\sigma}_x \right). \tag{2}$$

The form of operator $\mathbf{B}$ in equation (2) suggested us that applying the operator $\mathbf{B}$ to the Bloch vector is equal to a rotation of the vector around x-axis by $\pi/2$. In this case, it will be the same as U Gate in Qiskit where $U(\theta, \phi\lambda) = U(\pi/2, \pi/2, -\pi/2)$ [1].

3 Phase shifter

The reflected beam from beam splitter BS1 will passes through the phase shifter $\boldsymbol{\phi}$. For this reason, the operator for the phase shifter can be written as

$$\boldsymbol{\phi} = \begin{bmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{bmatrix}, \tag{3}$$

which is same as a phase gate in Qiskit, and we can also write this operator in a form of Pauli matrix as

$$\boldsymbol{\phi} = \frac{1}{2} \left[(e^{i\phi} + 1)\mathbb{1} - (e^{i\phi} - 1)\hat{\sigma}_z \right], \tag{4}$$

which equivalent to a single-qubit rotation about the Z axis.

4 Mach-Zehnder evolution

The evolution of $|0\rangle$ throught the Mach-Zehnder interferometer can be writen in a density matrix form. After the beam passing through the first beam splitter, the state become

$$\mathbf{B} |0\rangle \langle 0| \mathbf{B}^\dagger = \mathbf{B} \left[\frac{1}{2}(\mathbb{1} + \hat{\sigma}_z) \right] \mathbf{B}^\dagger = \frac{1}{2}(\mathbb{1} + \hat{\sigma}_y). \tag{5}$$

Then, after passing through the phase shifter,the state become

$$\boldsymbol{\phi}\mathbf{B} |0\rangle \langle 0| \mathbf{B}^\dagger \boldsymbol{\phi}^\dagger = \boldsymbol{\phi} \left[\frac{1}{2}(\mathbb{1} + \hat{\sigma}_y) \right] \boldsymbol{\phi}^\dagger = \frac{1}{2} \left[\mathbb{1} + \sin(\phi)\hat{\sigma}_x + \cos(\phi)\hat{\sigma}_y \right] \tag{6}$$

Finally, the beam passing through the second beam splitter and the state become

$$
\begin{aligned}
\mathbf{B}\boldsymbol{\phi}\mathbf{B}\left|0\right\rangle\left\langle 0\right|\mathbf{B}^{\dagger}\boldsymbol{\phi}^{\dagger}\mathbf{B}^{\dagger} &= \mathbf{B}\left[\frac{1}{2}(\mathbb{1}+\sin(\phi)\hat{\sigma}_x+\cos(\phi)\hat{\sigma}_y)\right]\mathbf{B}^{\dagger} \\
\rho_f &= \frac{1}{2}\left(\mathbb{1}+\sin(\phi)\hat{\sigma}_x-\cos(\phi)\hat{\sigma}_z\right).
\end{aligned}
\tag{7}
$$

We can see that after the state $|0\rangle$ pass through Mach-Zehnder interferometer, the Bloch vector will evole as $(u,v,w)=(0,0,1)\to(0,1,0)\to(\sin(\phi),\cos(\phi),0)\to(\sin(\phi),0,-\cos(\phi))$, respectively. We can also found that the probability to finding the qubit in state $|0\rangle$

$$
\begin{aligned}
P(0|\left|0\right\rangle,\phi) &= Tr\left(\left|0\right\rangle\left\langle 0\right|\rho_f\right) \\
&= \frac{(1-\cos(\phi))}{2},
\end{aligned}
\tag{8}
$$

and the probability to finding the qubit in state $|1\rangle$

$$
\begin{aligned}
P(1|\left|0\right\rangle,\phi) &= Tr\left(\left|1\right\rangle\left\langle 1\right|\rho_f\right) \\
&= \frac{(1+\cos(\phi))}{2}
\end{aligned}
\tag{9}
$$

5 Qiskit programming

5.1 Visualization of Mach-Zehnder interferometer in quantum circuit

We need to define our initial state which is the input of the beam splitter BS1. The initial state is virtualized on Bloch sphere as shown in Figure 2

```python
# Implement Mach-Zehnder interferometer in quantum circuit, 'mz'
from qiskit import *
from qiskit import QuantumCircuit # Initial state
q = QuantumRegister(q)
c = ClassicalRegister(c)
mz = QuantumCircuit(q,c) # Define qubit
mz.draw(output='mpl') # Visualize the circuit
state = Statevector.from_instruction(mz)
plot_bloch_multivector(state, reverse_bits=False)
```

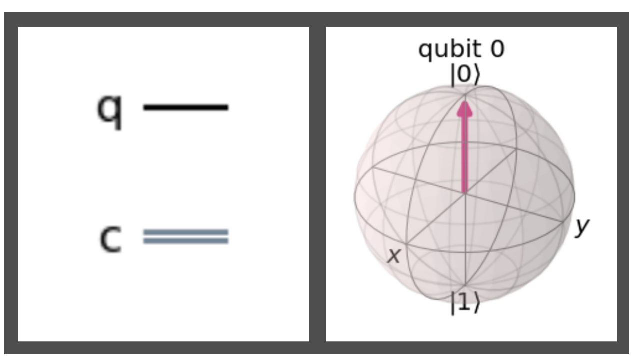

Figure 2. Initial state $|0\rangle$.

After that the beam will pass through the BS1 result in rotation on Bloch vector by $\pi/2$ along x-axis as shown in Figure 3

```python
# Operate U3-gates (Beam Splitter, BS1) on a |0> qubit
mz.u3(pi/2,pi/2,-pi/2,q)
mz.draw(output = 'mpl')
state = Statevector.from_instruction(mz)
plot_bloch_multivector(state, reverse_bits=False)
```

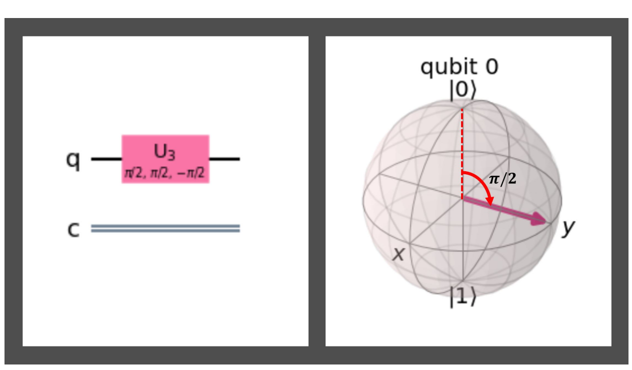

Figure 3. The state after passing through BS1.

Later on, the beam will pass through the phase shifter. In this case, I will assume that the phase shift is $\pi/4$ and I will use phase gate in Qiskit which has a little bit different in terms of matrix representation. In order to modify the matrix form of the phase gate, I will define ϕ as $-\phi$. The matrix representation of phase gate will become equation (3) with a factor of $e^{i\phi}$. We can treat this factor as a global phase that will not affect the measurement.

```
# Phase gate
phi = pi/4;
mz.p(-phi, q) # minus sign is used to change the form of matrix
mz.draw(output = 'mpl')
state = Statevector.from_instruction(mz)
plot_bloch_multivector(state, reverse_bits=False)
```

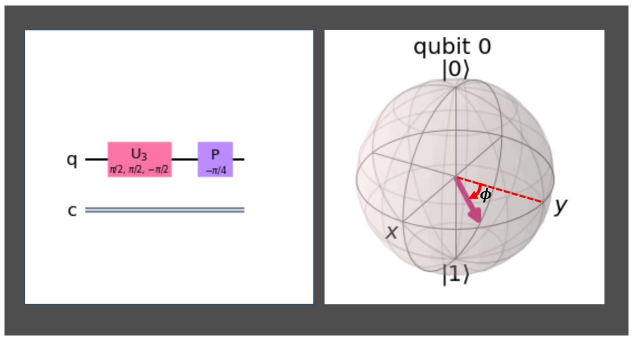

Figure 4. The state after passing through phase shifter.

Then, the beam will pass through BS2 and the state will be projected on xz plane as shown in Figure 5

```
# U3-gates (Beam Splitter, BS2)
mz.u3(pi/2,pi/2,-pi/2,q)
mz.draw(output = 'mpl')
state = Statevector.from_instruction(mz)
plot_bloch_multivector(state, reverse_bits=False)
```

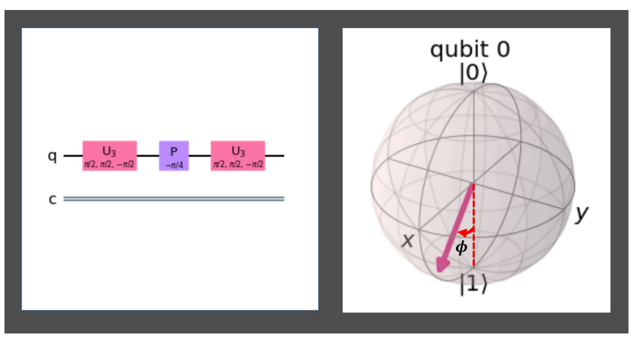

Figure 5. The state after passing through BS2.

Finally, we will perform the measurement. In this case ($\phi = pi/4$) we found that probability of the final state to be in $|0\rangle$ and $|1\rangle$ are 0.182 and 0.818, respectively, as shown in Figure 6

```
# perform measurement after BS2
mz.measure([0],[0])
mz.draw(output = 'mpl')
job = execute(mz, backend=device, shots=1000)
result = job.result()
print(result.get_counts())
plot_histogram(result.get_counts())
```

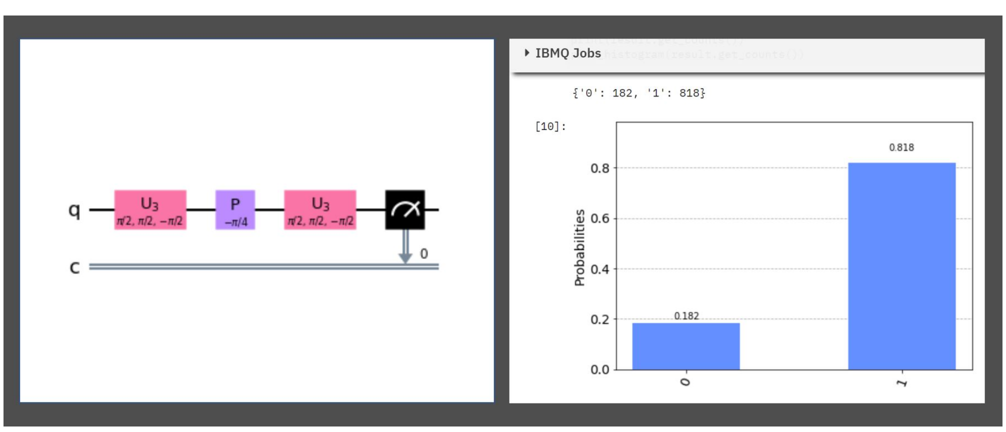

Figure 6. The state after passing through BS2.

5.2 Fringe visibility of Mach-Zehnder interferometer

In this section, we will do the section 5.1 again by varying the phase of the phase shifter. The scanning range of phase shift ranged from $-\pi/15$ to $5\pi/2$ where the number of repetitions of each circuit is 500. The code is run 10 times to calculate the standard deviation. The fringe of the Mach-Zehnder interferometer is shown in figure 7. The error bar represents a standard deviation of the data. The color lines represent the fits of the data using a curve fit function in Python where the fits have R-squared of 0.9972 in both cases.

```
from qiskit import *
from qiskit import IBMQ
import qiskit.tools.jupyter
%qiskit_job_watcher
IBMQ.load_account()
provider = IBMQ.get_provider(hub='ibm-q')
provider.backends()
```

```
# get the least-busy backend at IQX, this step may take up to one minute
from qiskit.providers.ibmq import least_busy
device = least_busy(provider.backends(filters=lambda b: b.configuration().n_qubits >= 4
and not b.configuration().simulator and b.status().operational==True))device
```

```
from numpy import *
from qiskit import *
import matplotlib.pyplot as plt
from qiskit.providers.aer import QasmSimulator
from qiskit.visualization import plot_bloch_multivector, plot_histogram
from qiskit.quantum_info import Statevector
from qiskit.visualization import plot_bloch_multivector
state_0=array([]); # Probability to get |0>
state_1=array([]); # Probability to get |1>
phase_data=array([]); # Phase of phase shifter
N_phase=20; # Step number of phase
shots=500; # Number of repetitions of each circuit
q = QuantumRegister(1)
c = ClassicalRegister(1)
```

```
for i in range(N_phase):
   mz = QuantumCircuit(q,c)
   phase = 2*(i-1)*pi/15; # Step width of the phase = pi/15
   phase_data = append(phase_data,phase);
   # Perform Mach-Zehnder interferometer
   mz.u(pi/2,pi/2,-pi/2,q); # Beam splitter BS1
   mz.p(-phase, q); # Phase shifter
   mz.u(pi/2,pi/2,-pi/2,q); # Beam splitter BS2
   mz.measure([0],[0]) # perform measurement
   job = execute(mz, backend=device, shots=500)
   result = job.result()
   data = result.get_counts();
   state_0=append(state_0,data['0']/shots);
   state_1=append(state_1,data['1']/shots);

```

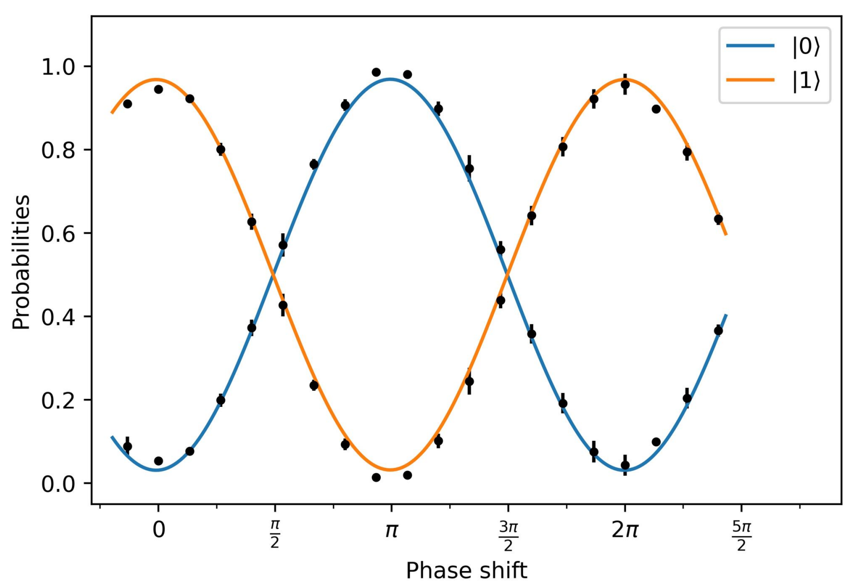

Figure 7. Fringe visibility of Mach-Zehnder interferometer.

In order to find visibility of the interferometer, we need to find the maximum and minimum probability to measure the state $|0\rangle$ and $|1\rangle$. The visibility of measuring the state $|i\rangle$ is defined as

$$V_i = \frac{P(i|\,|0\rangle\,,\phi_{max}) - P(i|\,|0\rangle\,,\phi_{min})}{P(i|\,|0\rangle\,,\phi_{max}) + P(i|\,|0\rangle\,,\phi_{min})}, \tag{10}$$

where i are 0 and 1 which corresponded to $|0\rangle$ and $|1\rangle$, respectively.

However, it is very difficult to find the proper phase shift which can maximize or minimize the probability of measurement because the step size of the phase shift is not infinitesimal. By looking at maximum and minimum values by looking at the fit curve, this problem can be solved. The other reason for doing this is to prevent overestimating and underestimating the visibility due to the non-infinitesimal step size.

6 Conclusion and discussion

In this experiment, we found that the visibility of measuring the state $|0\rangle$ and $|1\rangle$ are 0.9375 and 0.9361, respectively. The imperfection of the fringe visibility is caused by interactions between the quantum system and the environment which lead to decoherence and quantum noise during the computation in various forms.

Quantum noise can occur in quantum systems through through a quantum channel. The common quantum noise and decoherence channel are the following [2, 3]:

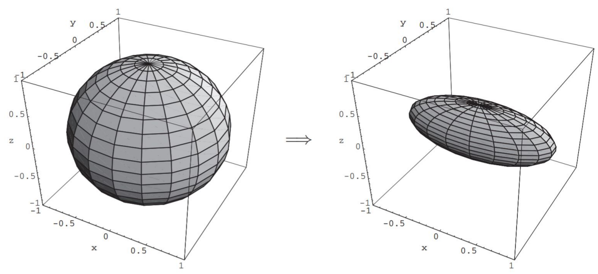

Figure 8. Effect of bit flip on a Bloch sphere. This figure is obtained from Ref [2].

- **Bit flip channnel**
  Bit flip channnel is the cahnnel which flip the state from $|0\rangle$ to $|1\rangle$ with probability of $1-p$. This channel can be represented by the following operators

$$E_0 = \sqrt{p}\mathbb{1}$$
$$E_0 = \sqrt{1-p}\hat{\sigma}_x,$$

  this channel will deform a pure state Bloch sphere by contracting the sphere in $y-z$ plane and turning the sphere into ellipsoid.

- **Phase flip channnel**
  Phase flip channnel is the cahnnel which flip the sign of the state (e.g., $|1\rangle \to -|1\rangle$) with probability of $1-p$. This channel can be represented by the following operators

$$E_0 = \sqrt{p}\mathbb{1}$$
$$E_0 = \sqrt{1-p}\hat{\sigma}_Z,$$

  simillarly, this channel will deform a pure state Bloch sphere by contracting the sphere in $x-y$ plane.

- **Depolarizing channel**
  Depolarizing channel is the cahnnel which can depolarized and change our desired qubit and turn it into a completely mixed state $\mathbb{1}/2$ with probability p. The state after this channel will become

$$\epsilon(\rho) = p\frac{\mathbb{1}}{2} + (1-p)\rho,$$

  this channel will contract the entire pure state Bloch.

As we can see, all of the quantum noise channels can deform the pure state Bloch sphere. This deforming results in turning pure states into mixed states while the evolution of state through the interferometer and deceasing of the visibility quality.

For example, when the qubit is depolarized the probability in equatuion (8) will be modified and become

$$P(0|\,|0\rangle\,, \phi) = \frac{p}{2} + \frac{(1-p)}{2}[1+\cos(x)],$$

where the maximum and minimum probability are $1-p/2$ and $p/2$, respectively. The effect of depolarizing channel on visibility fringe visibility of Mach-Zehnder interferometer is shown in figure 9.

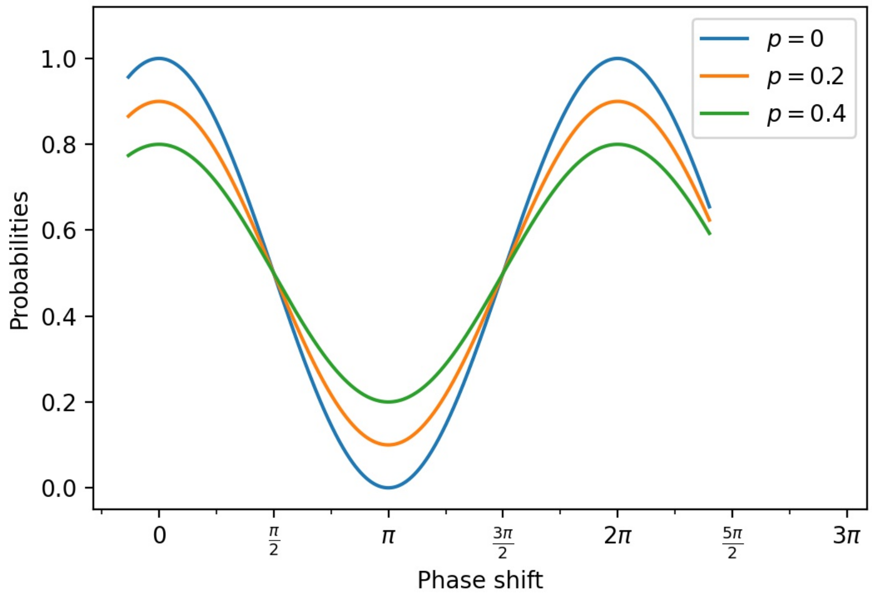

Figure 9. The effect of depolarizing channel on visibility fringe visibility of Mach-Zehnder interferometer where $p = 0$ repesents the fringe without the depolarizing.

References

[1] "Summary of Quantum Operations." `https://qiskit.org/documentation/tutorials/circuits/3_summary_of_quantum_operations.html`, 2021. [Online; accessed 30-October-2021].

[2] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition*. Cambridge University Press, 1 2011.

[3] K. Georgopoulos, C. Emary, and P. Zuliani, "Modelling and simulating the noisy behaviour of near-term quantum computers," *arXiv preprint arXiv:2101.02109*, 2021.