WOMEN'S INSTITUTE OF TECHNOLOGY AND INNOVATION

DIPLOMA IN COMPUTER SCIENCE

Course Name: Introduction to Mathematical Computing

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Task: **Group Work**

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GROUP ONE

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Question One

Please read and make notes on the following Topic plus its subtopics

Linear Algebra

- What is linear algebra? Why learn linear algebra?
- Vectors, combining, and scaling
- Transforming vectors and matrices
- System of linear equations and inverse matrices · Dot products
- Matrix decomposition

Question Two

Please write code example for the above subtopics in question one and link with me a link to your GitHub account

LINEAR ALGEBRA

1.What is linear Algebra?

Algebra:

In simple terms is the study of variables and other rules for manipulating these variables in formula, it's a unifying thread of almost all of mathematics.

Linear algebra.

- Linear Algebra is the branch of mathematics aimed at solving systems of linear equations with finite numbers of unknowns.
- Linear Algebra is the study of lines and plains, vector spaces and mappings that are required for linear transforms.

Linear algebra also has branches i.e.:

- Elementary
- Advanced
- Applied linear algebra

2.Why learn linear algebra?

- Linear algebra helps us find the solution of linear systems of differential equations.
- Techniques from linear algebra are also used in analytic geometry, engineering, physics, computer animation, and the social science (particularly in economics).
- In real life is for calculation of speed, distance, or time.
- It is also used for projecting a three-dimensional view into a two-dimensional plane, handled by linear maps.

VECTORS

Basis Vectors in Linear Algebra

In linear algebra, a basis vector refers to a vector that forms part of a basis for a vector space. A basis is a set of linearly independent vectors that can be used to represent any vector within that vector space. Basis vectors play a fundamental role in describing and analyzing vectors and vector spaces.
The basis of a vector space provides a coordinate system that allows us to represent vectors using numerical coordinates.

Some important Terminology

- **Vector Space (V):** Vector Space (V) is a mathematical structure of a set of vectors that can do addition and scalar multiplication. A set of vectors and operations that are defined on those vectors make up a mathematical structure called a vector space.

Example: $V = \{(x, y) \mid x, y \in \mathbb{R}\}$

- **Field (F):** Field is the name of the scalar field over which the vector space V is defined. It offers the coefficients used in linear vector combinations. Common examples of fields are real numbers (R), complex numbers (C), and rational numbers (Q).
- **Basis (B):** A collection of linearly independent vectors that span the entire vector space V is referred to as a basis for vector space V.

Example: The basis for the Vector space V = [x,y] having two vectors i.e x and y will be :

Basis Vector

In a vector space, if a set of vectors can be used to express every vector in the space as a unique linear combination of those vectors, and those vectors are linearly independent (meaning that none of them can be expressed as a linear combination of the others), then we refer to them as basis vectors for that vector space.

Properties of Basis vector:
Let V be the vector space of dimension n over the Field F and B as the basis for vector space V ie a set of vectors $\{v_1, v_2 \ldots v_n\}$, which have to satisfy the following two conditions:

1. **Basis vectors must be linearly independent of each other:**
   This means that no basis vector can be expressed as a linear combination of the others. If we take any two-basis vectors, such as v1 and v2, multiplying v_1 by any scalar should not yield v_2. This property ensures that each basis vector contributes unique information to the vector space
   .
2. **Basis vectors must span the whole space:**
   The entire vector space must be spanned by basis vectors. This means any vector in the space can be written as a linear combination of the basis vectors.

3. **Non-Uniqueness of Basis Vectors:**
   The non-uniqueness of basis vectors refers to the fact the choice of basis vectors is not unique, and different sets of vectors can form a basis for the same vector space.

   If you can write every vector in a given space as a linear combination of some vectors and these vectors are independent of each other then we call them basis vectors for that given space.

$\mathbb{R}^2$ Vector Space

Let us take an R-squared space which basically means that, we are looking at vectors in 2 dimensions. It means that there are 2 components in each of these vectors as we have taken in the above image. We can take many vectors. So, there will be an infinite number of vectors, which will be in 2 dimensions. So, the point is can we represent all of these vectors using some basic elements and then some combination of these basic elements?

Let's consider a simple example where we have a two-dimensional vector space.

Let's say we want to find the basis vectors for a given vector v = 2i+1j. To determine the coefficients of the linear combination of the basis vectors that form the vector v, we can set up the following equation:

$V = a_1$

where 'a_1' and 'a_2' are the coefficients we need to find.
To solve for 'a_1' and 'a_2', we can equate the corresponding components of both sides of the equation:
$A_1 = 2$

$A_2 = 1$

COMBINING

To concatenate two or more vectors in r we can use the combination function in R. Let's assume we have 3 vectors vec1, vec2, vec3 the concatenation of these vectors can be done as c(vec1, vec2, vec3). Also, we can concatenate different types of vectors at the same time using the same function.

Concatenate function:

- This is a function that combines its arguments.
- This method combines the arguments and results in a vector.
- All the arguments are converted to a common type that is the return value type.
- This function is also used to convert the array into vectors

Approach:

- Create a number of example vectors to concatenate.
- Concatenate the vectors by c function, c(vec1, vec2, vec3)

Example 1: Let's first create an example vector and then concatenate those vectors.
In this example, we have first created a vector vec1 and vec2. Then by using the concatenate function we have concatenated both the vectors to get the result.

Example 2:
Let's us now concatenate 3 vectors.
In this example, we have created 3 vectors vec1, vec2, vec3 using sample function and then concatenate the vectors into a single vector using the concatenate function

SCALING

This is a linear transformation that enlarges or shrinks of an object using a scale factor

a scale factor is usually a decimal which multiplies some quantity

for example

$y=cx$, c is the scale factor of x, it can also be called a coefficient of x.

scaling can also be represented by scaling matrix e.g to scale for a vector

v=(v_x, v_y v_z) where each point is p=(p_x p_y p_z) would be multiplied by a scaling matric

$$Sv = \begin{bmatrix} vx & 0 & 0 \\ 0 & vy & 0 \\ 0 & 0 & vz \end{bmatrix}$$

also shown below as

$$svp = \begin{bmatrix} vx & 0 & 0 \\ 0 & vy & 0 \\ 0 & 0 & vz \end{bmatrix} \begin{bmatrix} px \\ py \\ pz \end{bmatrix} = \begin{bmatrix} vx\,px \\ vy\,py \\ vz\,pz \end{bmatrix}$$

this kind of scaling changes a diameter of a n object by a factor between scale factors

the scaling is uniform if only and only if the scaling factors are equal. (vx=vy=vz)

n the case where $v_x = v_y = v_z = k$, scaling increases the area of any surface by a factor of k^2 and the volume of any solid object by a factor of k^3.

using homogeneous coordinates

in projective geometry, often used in computer graphics, points are represented using homogeneous coordinates. to scale an object by a vector $v = (v_x, v_y, v_z)$, each homogeneous coordinate vector $p = (p_x, p_y, p_z, 1)$ would need to be multiplied with this projective transformation matrix:

$$Sv = \begin{bmatrix} vx & 0 & 0 & 0 \\ 0 & vy & 0 & 0 \\ 0 & 0 & vz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

as shown below, the multiplication will give the expected result:

$$Svp = \begin{bmatrix} vx & 0 & 0 & 0 \\ 0 & vy & 0 & 0 \\ 0 & 0 & 0 & vz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} px \\ py \\ pz \\ 1 \end{bmatrix} = \begin{bmatrix} vx\,px \\ vy\,py \\ vz\,pz \\ 1 \end{bmatrix}$$

Since the last component of a homogeneous coordinate can be viewed as the denominator of the other three components, a uniform scaling by a common factor s (uniform scaling) can be accomplished by using this scaling matrix:

$$Sv = \begin{bmatrix} vx & 0 & 0 & 0 \\ 0 & vy & 0 & 0 \\ 0 & 0 & vz & 0 \\ 0 & 0 & 0 & 1/s \end{bmatrix}$$

For each vector $p = (p_x, p_y, p_z, 1)$ we would have

$$Svp = \begin{bmatrix} vx & 0 & 0 & 0 \\ 0 & vy & 0 & 0 \\ 0 & 0 & vz & 0 \\ 0 & 0 & 0 & 1/s \end{bmatrix} \begin{bmatrix} px \\ py \\ pz \\ 1 \end{bmatrix} = \begin{bmatrix} px \\ py \\ pz \\ 1/s \end{bmatrix}$$

which would be equivalent to

$$\begin{bmatrix} spx \\ spy \\ spz \\ 1 \end{bmatrix}$$

EXAMPLES OF SCALING

Example 1

Multiply the vector <9, 2> by 5.

We multiply each component by 5. The notation goes like this: 5<9, 2> = <5 · 9, 5 · 2> = <45, 10>.

Example 3:

In this example, we have created a character vector and a number vector and then combine both the vectors using the concatenate function of R library.

Check code

TRANSFORMING VECTORS AND MATRICES

Transformation matrix is a matrix that transforms one vector into another vector by the process of matrix multiplication. It alters Cartesian systems and maps the coordinates of the vectors to the new coordinates. The transformation matrix T of order mxn on multiplication with a vector A of n component represented as a column matrix transforms it into another matrix representing anew vector A'

However, to transform a vector, we need to multiply the transformation matrix with that vector, where the length or direction of the vector may be changed.

Example 1.

1. Finding the transformed vectors
   Apply the transformation matrix T to the following column vector x to find the transformed vector;

T = $\begin{pmatrix} -3 & 1 \\ 5 & 8 \end{pmatrix}, x = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

solution

Tx = $\begin{pmatrix} -3 & 1 \\ 5 & 8 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

= -3x2

=-6+1

=**-5**

=5x2

=10+8

=18

$\begin{pmatrix} -6+1 \\ 10+8 \end{pmatrix} = \begin{pmatrix} -5 \\ 18 \end{pmatrix}$NB. This. is a transformed vector

Example 2

Find the new vector formed for the vector 5i + 4j, with the help of the transformation matrix.$\begin{pmatrix} 2-3 \\ 1 \;\; 2 \end{pmatrix}$

solution

The given transformation matrix is T = $\begin{pmatrix} 2-3 \\ 1 \;\; 2 \end{pmatrix}$

The given vector A = 5i + 4j is written as a column matrix as $A = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

Let the new matrix transformation be B, and we have the transformation formula as TA = B

$$B = \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix} x \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$B = [2 \times 5 + (-3) \times 4]$

1x5+2x4

$$B = \begin{pmatrix} -2 \\ 13 \end{pmatrix}$$

B = -2i + 13j

Therefore, the new matrix on transformation -2 + 13j.

VECTORS.

Vector Spaces and Vector sub-space.

Vectors in n-spaces

The set of all n-tuples of real numbers, denoted by R^n, is called n-space.

Example. Let U = (2, -7, 1), V = (-3, 0, 4), and W = (0, 5, -8). Find the components of the vector x that satisfy 2U – V + X = 7x + W

Let $x = (x_1, x_2, x_3)$. Then

$2U – V + X = 2(2, -7, 1) – (-3, 0, 4) + (X_1, X_2, X_3)$,

$= (7 + X_1, -14 + X_2, -2 + X_3)$

Also, $7x + W = (7x_1, 7x_2 + 5, 7x_3 – 8)$

So 2U – V + X = 7x + W

$7 + X_1 = 7X_1$ or $X_1 = 7/6$,

$-4 + x_2 = 7x_2 + 5$ or $x_2 = -19/6$,

$-2 + x_3 = 7x_3 - 8$ or $x_3 = -1$

Therefore $x = \left(\frac{7}{6}, -\frac{19}{6}, -1\right)$

SYSTEM OF LINEAR EQUATIONS AND INVERSE MATRICES

Linear equations

These include variable where their highest power is 1.

They denoted as:

y=ax+b or ax+by+c=0

Where y is a dependent variable, x an independent variable and a,b and c are constants.

System of linear equations

This is a set of two or more equations containing two or more variables

For example;

x+y=5	OR	5X+15+56Z=35
2x+y=8		-4X-11Y-41Z=-26
		-X-3Y-11Z=-7

Matrix

This is an array of numbers inside square brackets

$$A = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -2 \\ 3 & 4 & -1 \end{bmatrix}$$

Inverse of a matrix

It is denoted as $\mathbf{A}^{-1}$ where;

A is the matrix

Formula

For $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

A^{-1}=1/det$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Where det is the determinant

Example

Calculate the inverse of a matrix $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

A^{-1}=1/det $\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$

Det =1*4-3*2

Det = -2

A^{-1}=1/-2$\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$

Therefore, inverse of a matrix:

A^{-1}= $\begin{bmatrix} -2 & 3/2 \\ 1 & -1/2 \end{bmatrix}$

Solving a System of Linear Equations Using the Inverse of a Matrix

Solving a system of linear equations using the inverse of a matrix requires the definition of two new matrices:

X

is the matrix representing the variables of the system, and

B

is the matrix representing the constants

Using matrix multiplication, we may define a system of equations with the same number of equations as variables as

AX=B

To solve a system of linear equations using an **inverse matrix**, let
A
be the **coefficient matrix**, let
X
be the variable matrix, and let
B
be the constant matrix. Thus, we want to solve a system
AX=B
. For example, look at the following system of equations.

a1x+b1y=c1

a2x+b2y=c2

A=$\begin{bmatrix} a1 & b1 \\ a2 & b2 \end{bmatrix}$

X=$\begin{bmatrix} x \\ y \end{bmatrix}$ B=$\begin{bmatrix} c1 \\ c2 \end{bmatrix}$

Therefore

$AX = B$ $= \begin{bmatrix} a1 & b1 \\ a2 & b2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ =$\begin{bmatrix} c1 \\ c2 \end{bmatrix}$

Formula for solving system of linear equations is;

$A^{-1}AX= A^{-1}B$

Example

Solve the given system of equations using the inverse of a matrix.

$$3x+8y=5$$

$$x+11y=7$$

Soln

From AX =B

A= $\begin{bmatrix} 3 & 8 \\ 1 & 11 \end{bmatrix}$ X= $\begin{bmatrix} x \\ y \end{bmatrix}$ B= $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$\begin{bmatrix} 3 & 8 \\ 1 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$A^{-1}AX= A^{-1}B$

Where $A^{-1} = \begin{bmatrix} 11 & -8 \\ -1 & 3 \end{bmatrix}$

$\begin{bmatrix} 11 & -8 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 8 \\ 1 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 & -8 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11(5) + (-8)7 \\ -4(5) + 3(7) \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

SOLN IS

X = (-1,1)

DOT PRODUCTS

The dot product is also called scalar product and shows how closely two vectors align in terms of the direction and point. It is represented by a dot between two vectors.

Dot product is defined as: A. B= |A|.|B|.cos θ

$$\#\cos\theta = \frac{A.B}{|A|.|B|}$$

Where |A| and |B| represents the magnitudes of the vectors A and B.

If two vectors A and B are given that; A= a_1+a_2 and B= b_1+b_2 ,the dot product for A and B will be written as;

A. B= $a_1 .b_1 + a_2 .b_2$

Example 1.[with two dimensions]:

Given vectors, A= 2i-3j and B= -4i+2j. find the dot product of the two vectors.

Solution.

A.B= (2i-3j).(-4i+2j)

=2(-4) +(-3)2

= -8-6 = -14

A.B= -14

Example 2.[with three dimensions]:

Vectors A and B are given that A= 2i-3j+7k, B= -4i+2j-4k. find the dot product of the two vectors.

Solution.

A.B = (2i-3j+7k) . (-4i+2j-4k)

=2(-4)+2(-3)+7(-4)

= -8-6-28

A .B = -42.

MAGNITUDE OF A VECTOR.

The magnitude of vector is the length of the vector and it is denoted as |a|. The formula of magnitude of a vector is given A= $a_1 + a_2$ is;

$$|A|=\sqrt{a_1^2+a_i^2}$$

Example 3.

Given vector A= 4i-3j . find |A|.

Solution.

$|A|= \sqrt{4^2+(-3)^2}$

$= \sqrt{25}$

$|A| =5$

Example 4. [with two dimensions].

Given Vectors A = 4i-5j and B = 7i + 3j, find the length of the two vectors.

Solution.

$|A|= \sqrt{4^2+(-5)^2}$

$|A| = \sqrt{41}$

$|B|= \sqrt{7^2+ 3^2}$

$|B| = \sqrt{58}$

THE ANGLE BETWEEN TWO VECTORS.

The formula for the angle between two vectors a and b is denoted as $\theta = \cos^{-1}(A.B/|A||B|)$,

Where A.B is the dot product, |A||B| is the magnitude of the two vectors.

Example.5

Given that angle between vectors has two dimension, determine the angle between A= 2i+2j and B=3j.

Solution.

$A.B =2(0) + 2(3)$

$A.B = 6$

$|A|=\sqrt{2^2 + 2^2}$

$|A| =2\sqrt{2}$

$|B|=\sqrt{3^2 + 0^2}$

$|B|= \sqrt{9}$

$|B| = 3.$

from

$\cos\theta = A.B/|A||B|$

$= 6/(2\sqrt{2}*3)$

$=6/6\sqrt{2}$

$\cos\theta =1/\sqrt{2}$

$\theta = cos^{-1}(1/\sqrt{2})$

$\theta = 1.32^0$

Example 6. [three dimension]

Determine the angle between A and b given that A = 2i+4j-5k , B = -i+3j-2k.

Solution.

A.B = 2(-1) +4(3) -5(-2)

A.B = -2 + 12+10

A.B = 20.

|A| =$\sqrt{2^2 + 4^2 + (-5)^2}$

|A| =$\sqrt{45}$.

|B| = $\sqrt{(-1)^2 + 3^2 + (-2)^2}$

|B| = $\sqrt{14}$

From

$\cos\theta =$ A.B/|A||B|

= 20/$\sqrt{45} * \sqrt{14}$)

$\theta = \cos^{-1}(2/21\sqrt{70})$

FOR ORTHOGONALS

If two vectors are orthogonal, this means that there dot product is zero.

Example 6.

Determine if the two vectors A=6i + 2j -k, and B = 2i-7j- 2k are orthogonal ?

Solution.

A.B = 6(2) + 2(-7) +(-1)(-2)

= 12 -14 +2A.B = 0,

Therefore A.B, is orthogonal

MATRIX DECOMPOSITION

1.1 Introduction to Matrices

A *matrix* is a rectangular array of numbers. An m ⇥ n (read as m by n) matrix of m ⇥ n numbers arranged in m horizontal rows and n vertical columns.

The numbers in the array are called *entries* of the matrix. A matrix is generally written as

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}$$

we write $A = (a_{ij})$ (or $[a_{ij}]$), where a_{ij} (or $[A]_{ij}$) is an entry in the i^{th} row and j^{th} column.
A matrix A is said to be a *square* matrix if it has the same number of rows and columns. That is an (m ⇥ n) matrix is a square matrix if $m = n$ where m is the number of rows and n is the number of columns. An m ⇥ n matrix is said to be of *size* or *order* m ⇥ n. If $m = n$, then it is of size (or order) n or m.

Two matrices A and B are defined to be equal if they have the same size and their corresponding entries are equal. In other words, the matrices $A = (a_{ij})$ and $B = (b_{ij})$, each of order m⇥ n, are said to be *equal* if their corresponding entries are the same, i.e., $a_{ij} = b_{ij}$ 8 i,j.

Given an m ⇥ n matrix $A = (a_{ij})$ the *transpose* of A, denoted by A^t or A^T or $A0$, is an n ⇥ m matrix that results from interchanging the rows and columns of A. So, $AT = (aij)T = (aji)$.

Example 1. If

$$A = \begin{pmatrix} 2 & 3 & 7 \\ 5 & 6 & 8 \\ 0 & 1 & 1 \end{pmatrix} \quad \text{then} \quad A^T = \begin{pmatrix} 2 & 5 & 0 \\ 3 & 6 & 1 \\ 7 & 8 & 1 \end{pmatrix}.$$

An m ⇥ n matrix $A = (a_{ij})$ is said to be a *zero* matrix if $a_{ij} = 0$, 8 i,j. That is to say, all its entries are zero. It is denoted by O_{mn} or simply O. The zero matrix has the property that for every matrix A of the same size as O it is true that $A + O = O + A = A$.

A square matrix A is said to be *symmetric* if $A^T = A$. A matrix A is *antisymmetric* or *skew-symmetric* if $A^T = A$.

Example. If

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}, \quad \text{then} \quad A^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}.$$

That is, $A^T = A$; so, A is symmetric.

Example. If

$$B = \begin{pmatrix} 0 & 2 & 3 \\ 2 & 0 & 5 \\ 3 & 5 & 0 \end{pmatrix}, \quad \text{then} \quad B^T = \begin{pmatrix} 0 & 2 & 3 \\ 2 & 0 & 5 \\ 3 & 5 & 0 \end{pmatrix}.$$

That is, $B^T =$ B; thus, B is anti-symmetric.

Theorem. Let A and B be matrices and α be a scalar in a field F. Then

Let A and B be matrices and of the same size in a field F and α and β be scalars. Then $(\alpha A + \beta B)^t = \alpha A^t + \beta B^t$.

A square matrix A = (a_{ij}) is said to be a *diagonal* matrix if $a_{ij} = 0 \ \forall\ i \neq j$. In other words, it is a square matrix in which all the entries off the main diagonal elements are zero. If $A = (a_{ij})$ is diagonal and of order n and $a_{ii} = 1, \forall i$, then A is called an *identity* matrix usually denoted by I or I_n. That is

$$I = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

The identity matrix has the property that for every square matrix A of the same order as I, it is true that $AI = IA = A$.

Example. The matrices

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

are diagonal matrices.

Example. The matrices

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}, \quad C = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ are

Idempotent.

Example;

Let A $=\begin{pmatrix}-1 & 2 & 3\\ -1 & 0 & 2\end{pmatrix}$ and B $=\begin{pmatrix}-1 & 5 & -2\\ 2 & 2 & -1\end{pmatrix}$, find

i) A + 2B = $\begin{pmatrix}-1 & 2 & 3\\ -1 & 0 & 2\end{pmatrix} + 2\begin{pmatrix}-1 & 5 & -2\\ 2 & 2 & -1\end{pmatrix} = \begin{pmatrix}-3 & 12 & -1\\ 3 & 4 & 0\end{pmatrix}$

(ii) 3A' – B' = $\begin{pmatrix}-3 & -3\\ 6 & 0\\ 9 & 6\end{pmatrix} - \begin{pmatrix}-1 & 2\\ 5 & 2\\ -2 & -1\end{pmatrix} = \begin{pmatrix}-2 & -5\\ 1 & -2\\ 11 & 7\end{pmatrix}$

(ii) (3A -B)' = $\left(\begin{pmatrix}-3 & 6 & 9\\ -3 & 0 & 6\end{pmatrix} - \begin{pmatrix}-1 & 5 & -2\\ 2 & 2 & -1\end{pmatrix}\right)'$

$= \begin{pmatrix}-2 & 1 & 11\\ -5 & -2 & 7\end{pmatrix}' = \begin{pmatrix}-2 & -5\\ 1 & -2\\ 11 & 7\end{pmatrix}'$

Solution: $a = 5$, $b =$ 3, $c = 4$, $d = 1$. 2

Example. Let $A = \begin{pmatrix}1 & 3\\ 2 & 6\end{pmatrix}$ and $B = \begin{pmatrix}7 & 4\\ 5 & 8\end{pmatrix}$. Find $A + B + (A + B)0$

Solution:

Note that $A + A' = \begin{pmatrix}2 & 1\\ 1 & 12\end{pmatrix}$ and $B + B' = \begin{pmatrix}14 & 1\\ 1 & 16\end{pmatrix}$. So, $A + B + (A + B)0 = A + A0 +$ $B + B' = \begin{pmatrix}16 & 2\\ 2 & 28\end{pmatrix}$. Alternatively, use $A + B = \begin{pmatrix}8 & 1\\ 3 & 14\end{pmatrix}$ so that $(A + B) + (A0 + B0) =$ $\begin{pmatrix}8 & 1\\ 3 & 14\end{pmatrix} + \begin{pmatrix}8 & 3\\ 1 & 14\end{pmatrix} = \begin{pmatrix}16 & 2\\ 2 & 28\end{pmatrix}$.

Note that $A + A0$, $B + B0$, and their sum are symmetric matrices.

Example. 2

Let $A = \begin{pmatrix}4 & 1\\ 0 & 2\end{pmatrix}$ and $B = \begin{pmatrix}1 & 4 & 2\\ 3 & 1 & 5\end{pmatrix}$

Then $AB = \begin{pmatrix}4 & 1\\ 0 & 2\end{pmatrix}\begin{pmatrix}1 & 4 & 2\\ 3 & 1 & 5\end{pmatrix} = \begin{pmatrix}1 & 15 & 3\\ 6 & 2 & 10\end{pmatrix}$

$$BB' = \begin{pmatrix}1 & 4 & 2\\ 3 & 1 & 5\end{pmatrix}\begin{pmatrix}1 & 3\\ 4 & 1\\ 2 & 5\end{pmatrix} = \begin{pmatrix}21 & 17\\ 17 & 35\end{pmatrix}.$$

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