Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- Write all answers sequentially.
- Start answering a question (not the pat of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules.
- Start working right away based on whatever you know. **Do not wait for the last moment and ask for time extension**.

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1. In the classes, there are three forms of floating number representation,

$$\begin{aligned} \text{Lecture Note Form} &: \quad F = \pm(0.d_1 d_2 d_3 \cdots d_m)_\beta\, \beta^e \ , \\ \text{Normalized Form} &: \quad F = \pm(1.d_1 d_2 d_3 \cdots d_m)_\beta\, \beta^e \ , \\ \text{Denormalized Form} &: \quad F = \pm(0.1 d_1 d_2 d_3 \cdots d_m)_\beta\, \beta^e \ , \end{aligned}$$

where $d_i, \beta, e \in \mathbb{Z}$, $0 \leq d_i \leq \beta - 1$ and $e_{\min} \leq e \leq e_{\max}$. Now, let's take, $\beta = 2$, $m = 4$ and $e \in \{-2, -1, 0, 1, 2\}$. Based on these, answer the following:

   (a) (3 marks) How many numbers in total can be represented by this system? Find this separately for each of the three forms above. Ignore negative numbers.

   (b) (3 marks) For each of the three forms, find the smallest, positive number and the largest number representable by the system.

   (c) (2 marks) For the IEEE standard (1985) for double-precision (64-bit) arithmetic, find the smallest, positive number and the largest number representable by a system that follows this standard. Do not find their decimal values, but simply represent the numbers in the following format:

$$\pm(0.1d_1 \ldots d_m)_\beta \cdot \beta^{e-\text{exponentBias}}$$

   Be mindful of the conditions for representing $\pm\,\text{inf}$ and ± 0 in this IEEE standard.

   (d) (2 marks) In the above IEEE standard, if the exponent bias were to be altered to exponentBias = 500, what would the smallest, positive number and the largest number be? Write your answers in the same format as in part (c). Note that the conditions for representing $\pm\infty$ and ± 0 are still maintained as before.

2. Given a system parameterized by $\beta = 2$, $m = 3$, and $e_{\min} = -1 \leq e \leq e_{\max} = 2$ where $e \in \mathbb{Z}$. For this system,

   (a) (3 marks) find the floating-point representation of the numbers $(6.25)_{10}$ and $(6.875)_{10}$ in the Normalized Form. That is, find fl[6.25] and fl[6.875].

   (b) (2 marks) what are the rounding errors δ_1, δ_2 in part (a)?

   (c) (2 marks) can the values $(6.25)_{10}$ and $(6.875)_{10}$ be represented in the Denormalized Form? If so, find the floating-point representations. If not, then concisely explain why?

   (d) (3 marks) find the upper bound of the rounding error for Lecture Note, Normalized and Denormalized Forms.

3. The following nodes come from the function $f(x) = \ln(5x + 9)$:

x	$f(x)$
-0.5	1.87
0	2.20
0.5	2.44

   (a) (4 marks) Using Newton's divided difference method, find the equation of a second degree polynomial which fits the above data points.

   (b) (5 marks) Expand the function $f(x) = \ln(5x + 9)$ using Taylor Series, centered at 0. Include till the x^2 term of the taylor series.

   (c) (1 mark) Should the equation which you found in part (a) and part (b) match? Comment on why, or why not.

☺☺☺ **Yes!!!!! Let the PHUN begin!!!** ☺☺☺

4. Consider the following nodes:

x	$f(x)$
0	5
3	9.5
6	5

(a) (1 mark) If an equation of a polynomial which fits through the above nodes is found using both the Vandermonde Matrix approach and the Lagrange approach, will both the equations match?

(b) (7 marks) Find the equation of a polynomial which fits through the above nodes using the Vandermonde matrix approach.

(c) (7 marks) Find the equation of a polynomial which fits through the above nodes using the Lagrange approach.

5. Consider the following data set:

x	$f(x)$	$f'(x)$
0.1	-0.620	3.585
0.2	-0.283	3.140

Answer the following based on the above data:

(a) (8 marks) Compute the Hermite bases: $h_0(x)$, $h_1(x)$, $\hat{h}_0(x)$ and $\hat{h}_1(x)$.

(b) (2 marks) Write the Hermite polynomial and find the value at $x = 0.15$.

6. (5 marks) During the class, the derivation of Eq.(2.17) for a_1 (which is the Example in the lecture notes on page-19) is shown in detail. However the derivation of Eq.(2.18) for a_2 has some missing steps (the dotted part in Eq.-2.18 in page-19 of the lecture note). Now, you are asked show the detail derivation of the following

$$a_2 \equiv f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} .$$