Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.

- Write all answers sequentially.

- Start answering a question (not the pat of the question) from the top of a new page.

- Write legibly and in orderly fashion maintaining all mathematical norms and rules.

- Start working right away based on whatever you know. **Do not wait for the last moment and ask for time extension**.

---

1. Consider the ollowing linear system,

$$\begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= 5 \\ 4x_1 + 5x_2 - 3x_3 + 6x_4 &= 9 \\ -2x_1 + 5x_2 - 2x_3 + 6x_4 &= 4 \\ 4x_1 + 11x_2 - 4x_3 + 8x_4 &= 2 \ . \end{aligned}$$

   (a) (2 marks) Explain how to find this system has a unique solution or not.
   (b) (1 mark) Write down the Augmented matrix for the system (*i.e.* write down $\text{Aug}(A)$).
   (c) (3 marks) Find the upper triangular matrix U.
   (d) (4 marks) Using the Gaussian elimination method, find the solution of the linear system. Show your works to get full credit.

2. A linear system is described by the following equations

$$\begin{aligned} x_1 + x_2 + x_3 &= 6 \\ 2x_1 + 3x_2 + 4x_3 &= 20 \\ 3x_1 + 4x_2 + 2x_3 &= 17 \ . \end{aligned}$$

   (a) (3 marks) Construct the Frobenius matrices $F^{(1)}$ and $F^{(2)}$ from this system.
   (b) (2 marks) Compute the unit lower triangular matrix L.
   (c) (5 marks) Now find the solution of the linear system using LU decomposition method. Use the unit lower triangular matrix found in the previous question.

3. A linear system is defined by three independent variables that obey the following equations

$$\begin{aligned} 3x_1 + 3x_2 + 4x_3 &= 1 \\ -x_3 &= 6 \\ -x_2 + 3x_3 &= 4 \ . \end{aligned}$$

   (a) (2 marks) Explain why the Gaussian elimination method fails to solve the system.
   (b) (3 marks) State how we can remove the problem and solve the system by Gaussian elimination method.

4. Answer the following questions:
   (a) (2 marks) If $x^\text{T}y = 0$, what should be the value of θ.
   (b) (2 marks) What are the conditions for a set to be orthonormal?
   (c) (2 marks) Find out whether the following set is orthogonal or not:

$$S = \left\{ (3, 5, 2)^\text{T} , \ (-2, 2, -2)^\text{T} \right\} \ .$$

   (d) (3 marks) Find out whether the following set is orthonormal or not:

$$S = \left\{ \frac{1}{\sqrt{36}} (-5, 3)^\text{T} , \frac{1}{\sqrt{36}} (3, 5)^\text{T} \right\} \ .$$

   (e) (6 marks) Find out whether the following set is orthonormal or not:

$$S = \left\{ \left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}} \right)^\text{T} , \left(-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)^\text{T} , \left(-\frac{1}{\sqrt{66}}, -\frac{4}{\sqrt{66}}, \frac{7}{\sqrt{66}} \right)^\text{T} \right\} \ .$$

☺☺☺ **Yes!!!!! Let the PHUN begin!!!** ☺☺☺