
General Information

Detailed information about the lecture, tutorials and homework assignments can be found on the lecture website¹. Solutions have to be submitted to Moodle². Upload your solutions as a single file named 'hw06.ml' to moodle. Since submissions are tested automatically, solutions that do not compile or do not terminate within a given time frame cannot be graded and thus result in 0 Points. If you do not manage to get all your stuff compiling and/or terminating, comment the corresponding parts of the file! Use Piazza³ to ask questions and discuss with your fellow students.

Functional Programming

Since this is a course about functional programming, we restrict ourselves to the functional features of OCaml. In other words: The imperative and object oriented subset of OCaml must not be used.

Assignment 6.5 (H) Peano Arithmetic

[3 Points]

The natural numbers can be defined recursively as follows:

- 0 is a natural number.
- if n is a natural number, then so is the successor of n

We can easily represent this in OCaml using corresponding constructors:

```
type nat = Zero | Succ of nat
```

Implement the following functions for natural numbers:

1. `int_to_nat : int -> nat` converts an integer to natural.
2. `nat_to_int : nat -> int` converts a natural to integer.
3. `add : nat -> nat -> nat` adds two natural numbers.
4. `mul : nat -> nat -> nat` multiplies two natural numbers.
5. `pow : nat -> nat -> nat` a call `pow a b` computes a^b .
6. `leq : nat -> nat -> bool` a call `leq a b` computes $a \leq b$.

You are **not** allowed to use the first two functions to implement the rest!

¹<https://www.in.tum.de/i02/lehre/wintersemester-1819/vorlesungen/functional-programming-and-verification/>

²<https://www.moodle.tum.de/course/view.php?id=44932>

³<https://piazza.com/tum.de/fall2018/in0003/home>

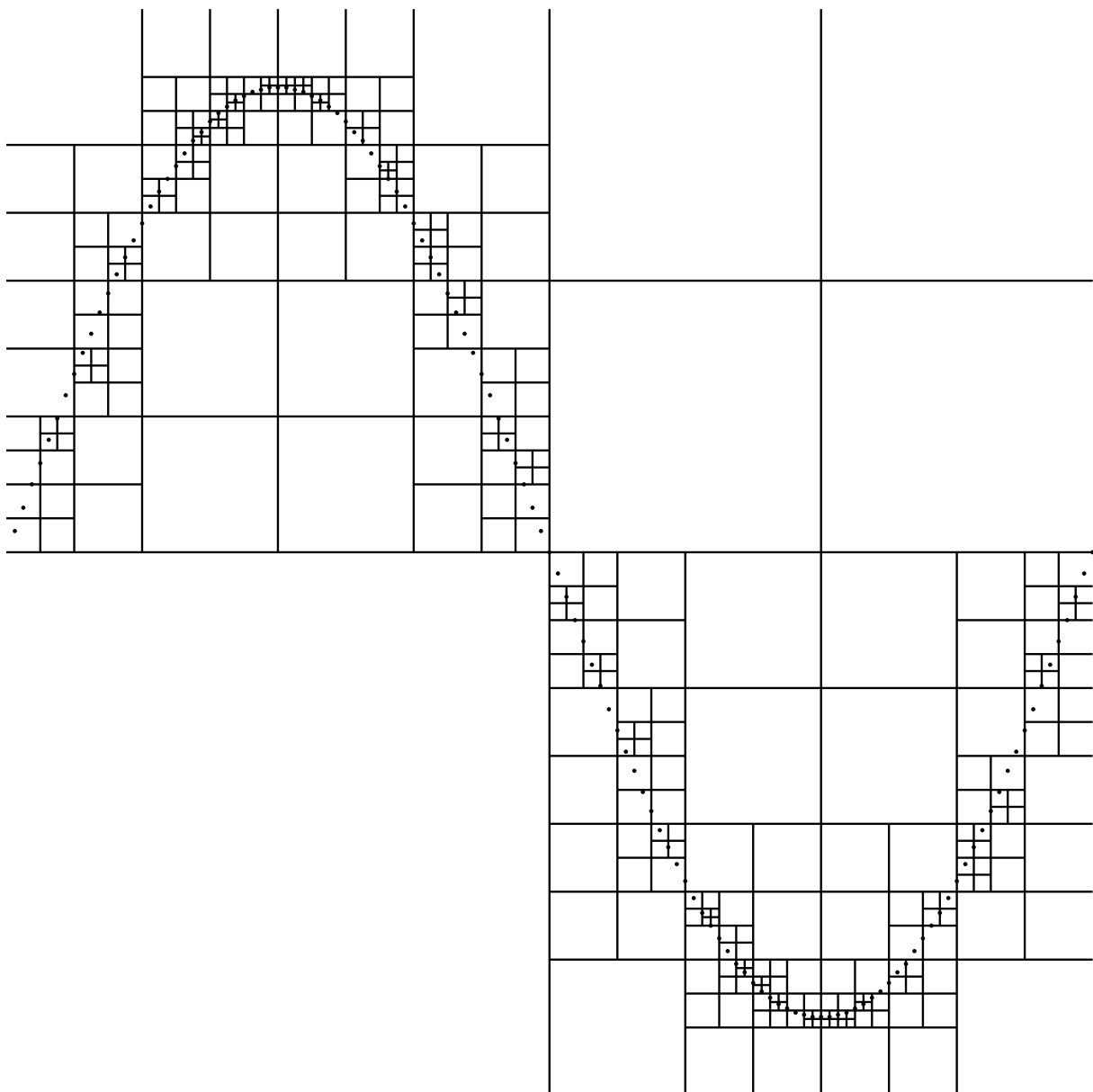
Assignment 6.6 (H) Quadtrees

[6 Points]

The quadtree data structure stores points by recursively partitioning a 2-dimensional region.

Let $Rect(x_1, y_1, x_2, y_2)$ denote the rectangular region with lower left corner (x_1, y_1) and upper right corner (x_2, y_2) . We define that all points (x, y) with $x_1 \leq x < x_2$ and $y_1 \leq y < y_2$ belong to this region. Every internal node represents a rectangular region $Rect(x_1, y_1, x_2, y_2)$ and has exactly four children, which represent the equally sized subregions $Rect(x_1, y_1, x_c, y_c)$, $Rect(x_1, y_c, x_c, y_2)$, $Rect(x_c, y_1, x_2, y_c)$ and $Rect(x_c, y_c, x_2, y_2)$, respectively, where $(x_c, y_c) = (\frac{x_2-x_1}{2}, \frac{y_2-y_1}{2})$ is the nodes center point. Leafs are either empty or contain a single point.

At all times, every subtree rooted at node N has the minimal height required to store all points that are inside the region corresponding to N . Every point (x, y) is stored at most once, so if the same point is inserted a second time, the tree is unchanged. The following figure shows a quadtree of size 512x512 that stores 100 points along the sine wave:



We represent a quadtree node by the following sum type:

```
type quadtree_node = NoPoint
  | Point of int * int
  | Node of quadtree_node (* bottom left *)
    * quadtree_node (* top left *)
    * quadtree_node (* bottom right *)
    * quadtree_node (* top right *)
```

and the entire quadtree is then represented by the `quadtree` type:

```
type quadtree = { width:int; height:int; root:quadtree_node }
```

Note, that we do not store the region inside the individual nodes, but just in the tree itself, so node regions must be computed during tree traversal.

Implement the function `insert : point -> quadtree -> quadtree` that inserts a point into the tree.

Hint: You may assume that region sizes are always a power of 2.

Hint: The function `print_quadtree` is provided to save an svg-image of your tree.

Assignment 6.7 (H) Expression Evaluation

[4 Points]

We define expressions over rationals using these OCaml types:

```
type rat = int * int (* num, denom *)
type unary_op = Neg
type binary_op = Add | Sub | Mul | Div
type expr = Const of rat
  | UnOp of unary_op * expr
  | BinOp of binary_op * expr * expr
```

Implement a function `eval_expr : expr -> rat` that evaluates the given expression. The resulting fraction need not be simplified, so $(-3,2)$, $(3,-2)$, $(-6,4)$, $(12,-8)$, ... are all accepted. Example:

```
eval_expr (BinOp (Mul, BinOp (Sub, Const (3,8), Const (2,4)), Const (6,-3)))
evaluates to (1,4) (or (2,8), ...).
```

Assignment 6.8 (H) Crawling on Trees

[7 Points]

Once again, consider binary trees, which we define as:

```
type tree = Empty | Node of int * tree * tree
```

In this assignment you are supposed to implement a crawler that walks along binary trees and performs different operations. At any time, the crawler “sits” on a particular node of the tree (this includes the **Empty**-leaf). In the following we refer to this node as the *current node*. Furthermore, the crawler uses a stack to store trees. Initially, the crawler is positioned at the input tree’s root and is then instructed using the commands

```
type command = Left | Right | Up | New of int | Delete | Push | Pop
```

with the following meaning:

- **Left** moves the crawler to the current node’s left child.
- **Right** moves the crawler to the current node’s right child.
- **Up** moves the crawler up to the current node’s parent node.
- **New** *x* replaces the current node (including all children) with a new node with value *x*.
- **Delete** removes the current node (including all children) leaving behind an **Empty**-leaf.
- **Push** pushes the subtree rooted at the current node onto the stack. The tree stays unchanged.
- **Pop** replaces the subtree rooted at the current node with the topmost tree of the stack. The tree is then popped from the stack.

Implement a function `crawl : command list -> tree -> tree` that executes a list of crawler commands on the given tree. You may assume that the list of commands is always valid, so there is no **Left** or **Right** when the crawler is already at a leaf, no **Up** when it is on the root and no **Pop** when the stack is empty.

*Hint: The tricky part is to get the **Up** command right. If you do not manage to implement this correctly, leave it out and you will still get some points for the rest.*

Hint: The function `print_tree` is provided, that can be used to dot⁴-export your tree.

⁴[https://en.wikipedia.org/wiki/DOT_\(graph_description_language\)](https://en.wikipedia.org/wiki/DOT_(graph_description_language))