

**Table 4 : CHI-SQUARE**  
**Significant Values  $\chi^2 (\alpha)$  of Chi-Square Distribution Right Tail Areas**  
**for Given Probability  $\alpha$ ,**

$$P = P_r(\chi^2 > \chi^2(\alpha)) = \alpha$$

**And is Degrees of Freedom (d.f.)**

Degree of freedom ( $\nu$ )	Probability (Level of significance)						
	0 = .99	0.95	0.50	0.10	0.05	0.02	0.01
1	.000157	.00393	.455	2.706	3.841	5.214	6.635
2	.0201	.103	1.386	4.605	5.991	7.824	9.210
3	.115	.352	2.366	6.251	7.815	9.837	11.341
4	.297	.711	3.357	7.779	9.488	11.668	13.277
5	.554	1.145	4.351	9.236	11.070	13.388	15.086
6	.872	2.635	5.348	10.645	12.592	15.033	16.812
7	1.239	2.167	6.346	12.017	14.067	16.622	18.475
8	1.646	2.733	7.344	13.362	15.507	18.168	20.090
9	2.088	3.325	8.343	14.684	16.919	19.679	21.669
10	2.558	3.940	9.340	15.987	18.307	21.161	23.209
11	3.053	4.575	10.341	17.275	19.675	22.618	24.725
12	3.571	5.226	11.340	18.549	21.026	24.054	26.217
13	4.107	5.892	12.340	19.812	22.362	25.472	27.688
14	4.660	6.571	13.339	21.064	23.685	26.873	29.141
15	4.229	7.261	14.339	22.307	24.996	28.259	30.578
16	5.812	7.962	15.338	23.542	26.296	29.633	32.000
17	6.408	8.672	15.338	24.769	27.587	30.995	33.409
18	7.015	9.390	17.338	25.989	28.869	32.346	34.805
19	7.633	10.117	18.338	27.204	30.144	33.687	36.191
20	8.260	10.851	19.337	28.412	31.410	35.020	37.566
21	8.897	11.591	20.337	29.615	32.671	36.343	38.932
22	9.542	12.338	21.337	30.813	33.924	37.659	40.289
23	10.196	13.091	22.337	32.007	35.172	38.968	41.638
24	10.856	13.848	23.337	32.196	36.415	40.270	42.980
25	11.524	14.611	24.337	34.382	37.65	41.566	44.314
26	12.198	15.379	25.336	35.363	38.885	41.856	45.642
27	12.879	16.151	26.336	36.741	40.113	41.140	46.963
28	13.565	16.928	27.336	37.916	41.337	45.419	48.278
29	14.256	17.708	28.336	39.087	42.557	46.693	49.588
30	14.933	18.493	29.336	40.256	43.773	47.962	50.892

**Note.** For degrees of freedom ( $\nu$ ) greater than 30, the quantity  $\sqrt{2\chi^2} - \sqrt{2\nu - 1}$  may be used as a normal variate with unit variance.

Math

## Chi-square test of goodness of fit.

(or)  $\chi^2$ -test

This test enables us to find if the deviations of the experiment from theory is just by chance or is it really due to the inadequacy of the theory to fit the observed data.

If  $O_i$  ( $i=1, 2, \dots, n$ ) is a set of observed (experimental) frequencies and  $E_i$  ( $i=1, 2, \dots, n$ ) is the corresponding set of expected (Theoretical) frequencies, then

Karl Pearson's chi-square, given by

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

Null hypothesis  $H_0$ : There is no significant difference between the observed and the theoretical values.

Note:

- 1) Get tabulated value of  $\chi^2$  for  $n-1$  degree of freedom.

2) (i) calculate value of  $\chi^2$  obtained is less than the corresponding tabulated value then we accept the null hypothesis.

3) (i) calculated value of  $\chi^2$  is greater than the tabulated value then we reject null hypothesis.

Ex 4) The demand for a particular spare part in a factory was found to vary from day to day. In a sample study the following information was obtained.

Days	mon	Tue	wed	Thu	Fri	Sat.
No. of parts demanded	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depend on the day of the week

Solution :

Null hypothesis  $H_0$ : The no. of parts demanded does not depend on the week. (or)

observed and expected values are same.

Expected frequencies of the spare part demanded on each of six days of the week is

$$= \frac{1}{6} (1124 + 1125 + 1110 + 1120 + 1126 + 1115)$$

Expected frequency = 1120.

Days	Frequency		$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
	observed $O_i$	Expected $E_i$		
mon	1124	1120	16	0.014
Tue	1125	1120	25	0.022
wed	1110	1120	100	0.089
Thu	1120	1120	0	0
Fri	1126	1120	36	0.032
Sat	1115	1120	25	0.022
Total	6720	6720		0.179

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 0.179$$

The number of degrees of freedom = 6 - 1 = 5

The tabulated value of  $\chi^2_{5\%}$  for 5 d.f. = 11.07.

$\chi^2$  calculated = 0.179 <  $\chi^2$  tabulated = 11.07.

Hence we accept the null hypothesis  $H_0$ .

i.e. The number parts demanded does not depend on the week.



2) The following figures show the distribution of digits in numbers chosen at random from a telephone directory:

digits :	0	1	2	3	4	5	6	7	8	9
Frequency :	1026	1107	997	966	1075	933	1107	972	964	853

Test whether the digits may be taken to occur equally frequently in the directory.

Solution :

Null hypothesis  $H_0$ : The digits occur equally frequently in the directory.

$$\text{Expected frequency} = \frac{1}{10} \{ 1026 + 1107 + 997 + 966 + 1075 + 933 + 1107 + 972 + 964 + 853 \} = 1000.$$

Digits	frequencies		$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
	Observed $O_i$	Expected $E_i$		
0	1026	1000	676	0.676
1	1107	1000	11449	11.449
2	997	1000	9	0.009
3	966	1000	1156	1.156
4	1075	1000	5625	5.625
5	933	1000	4489	4.489
6	1107	1000	11149	11.149
7	972	1000	784	0.784
8	964	1000	1296	1.296
9	853	1000	21609	21.609
Total	10,000	10,000		58.542

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 58.542.$$

Degrees of freedom =  $n - 1 = 10 - 1 = 9$ .

Tabulated  $\chi^2_{sy.}$  for 9 d.f. = 16.919.

$\chi^2$  calculated  $\geq 58.542 > \chi^2$  tabulated = 16.919

Hence we reject the null hypothesis.

- 3) A sample analysis of examination results of 200 students was made. It was found that 46 students had failed, 68 secured a third division, 62 secured a second division and the rest were placed in first division. Are these figures commensurate with the general examination result which is in the ratio of 4:3:2:1 for various categories respectively.

Solution:

Null hypothesis  $H_0$ : The given data commensurate with the general examination result.

General examination results

4	:	3	:	2	:	1
Failed		III-div.		II-div.		I-div.

category	frequencies		$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
	Observed ( $O_i$ )	Expected ( $E_i$ )		
Failed	46	$\frac{4}{10} \times 200 = 80$	1156	14.450
III Division	68	$\frac{3}{10} \times 200 = 60$	64	1.067
II Division	62	$\frac{2}{10} \times 200 = 40$	484	12.100
I Division	24	$\frac{1}{10} \times 200 = 20$	16	0.800
Total	200	200		28.417

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 28.417$$

$$\text{degrees of freedom} = 4 - 1 = 3$$

$$\chi^2_{5\% \text{ for } 3 \text{ d.f.}} = 7.815$$

$$\text{calculate } \chi^2 = 28.417 > \text{Tabulated } \chi^2 = 7.815$$

Hence we reject the null hypothesis.

(or) sample analysis of examination results are not commensurate with general examination results.