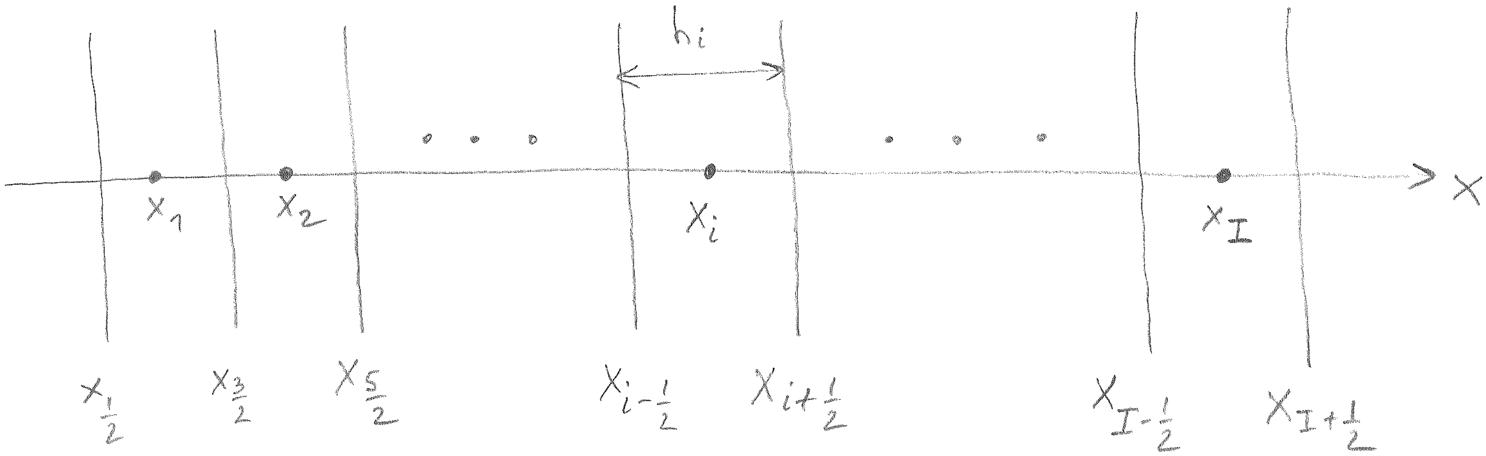
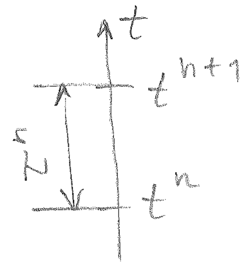
Mesh





$$\forall i: \quad h_i = x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}$$

$$\forall n: \quad \tau^n = t^{n+1} - t^n$$

Godunov type scheme:
We use finite volumes!

$$u_t + f(u)_x = 0$$

$$f(u) = cu$$

$$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \int_{t^n}^{t^{n+1}} u_t \, dt\, dx + \int_{t^n}^{t^{n+1}} \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(u)_x \, dx\, dt = 0$$

$$\Downarrow$$

$$\left[\underbrace{\frac{1}{h_i}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} u(x,t^{n+1})\, dx}_{\Rightarrow u_i^{n+1}} - \underbrace{\frac{1}{h_i}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} u(x,t^{n})\, dx}_{\Rightarrow u_i^{n}} \right] \cdot h_i +$$

$$+ \left[\underbrace{\frac{1}{\tau^n}\int_{t^n}^{t^{n+1}} f\left[u(x_{i+\frac{1}{2}},t)\right] dt}_{\Downarrow\; f_{i+\frac{1}{2}}^{n+\frac{1}{2}} \equiv f_{i+\frac{1}{2}}} - \underbrace{\frac{1}{\tau^n}\int_{t^n}^{t^{n+1}} f\left[u(x_{i-\frac{1}{2}},t)\right] dt}_{\Downarrow\; f_{i-\frac{1}{2}}^{n+\frac{1}{2}} \equiv f_{i-\frac{1}{2}}} \right] \cdot \tau^n = 0$$

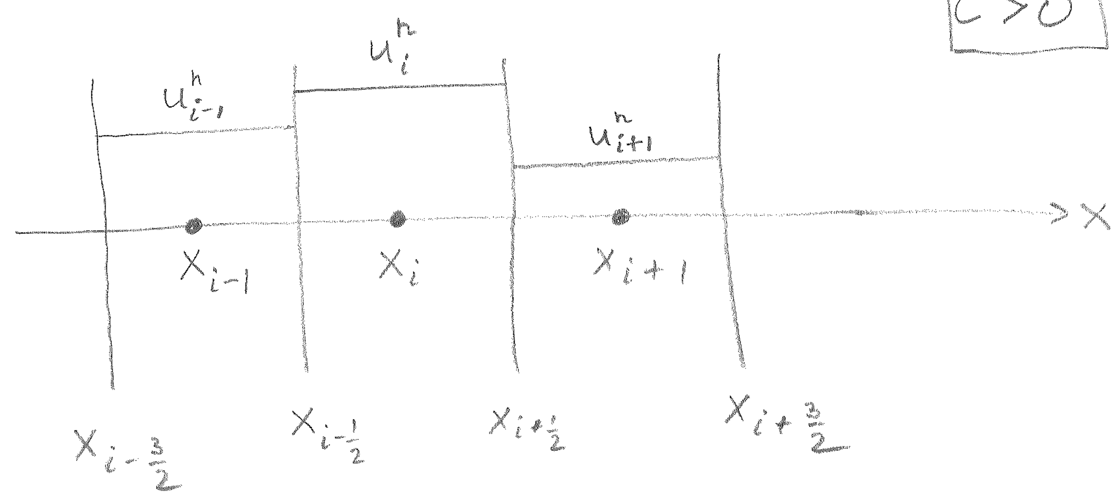
$$h_i u_i^{n+1} - h_i u_i^n + \tau^n f_{i+\frac{1}{2}} - \tau^n f_{i-\frac{1}{2}} = 0$$

The Godunov type scheme :

$$\boxed{u_i^{n+1} = u_i^n + \frac{\tau^n}{h_i}\left[f_{i-\frac{1}{2}} - f_{i+\frac{1}{2}}\right]} \quad !$$

Num. fluxes :

$\boxed{c>0}$



central differencing : $f_{i-\frac{1}{2}} = \frac{1}{2}\left[c u_{i-1}^n + c u_i^n\right]$

$$f_{i+\frac{1}{2}} = \frac{1}{2}\left[c u_i^n + c u_{i+1}^n\right]$$

$$\boxed{c > 0}$$

upwinding :

$$f_{i-\frac{1}{2}} = \underbrace{\frac{1}{2}\left[cu_{i-1}^n + cu_i^n\right]}_{\text{average}} + \frac{1}{2}c\underbrace{\left[u_{i-1}^n - u_i^n\right]}_{\text{jump}} = \boxed{cu_{i-1}^n}$$

upwinding

$$f_{i+\frac{1}{2}} = \underbrace{\frac{1}{2}\left[cu_i^n + cu_{i+1}^n\right]}_{\text{average}} + \frac{1}{2}c\underbrace{\left[u_i^n - u_{i+1}^n\right]}_{\text{jump}} = \boxed{cu_i^n}$$