Lecture 17

Eulerian Solution of the Radiation-Hydrodynamics Equations with Grey Radiation Diffusion

1 Introduction

The MUSCL-Hancock (MH) method is one of the most popular Eulerian hydrodynamics methods. The standard cell-centered diffusion discretization is one of the most popular diffusion discretization schemes in the nuclear engineering community. Our purpose is to define a radiation-hydrodynamics method that combines these two basic methods. We consider only slab geometry, but the generalization to spherical and cylindrical geometry is straightforward. The equations that we solve can be written as follows in terms of the conserved variables and the pressure:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0\,. \tag{1}$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}(\zeta u + p) = -\frac{1}{3}\frac{\partial \mathcal{E}}{\partial x}\,, \tag{2}$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}\left[E + p)u\right] = \sigma_a c\left(\mathcal{E} - aT^4\right) - \frac{1}{3}\frac{\partial \mathcal{E}}{\partial x}u\,, \tag{3}$$

$$\frac{\partial \mathcal{E}}{\partial t} - \frac{\partial}{\partial x}\left(\frac{c}{3\langle\sigma_t\rangle}\frac{\partial \mathcal{E}}{\partial x}\right) + \frac{4}{3}\frac{\partial \mathcal{E}}{\partial x}u = \sigma_a c\left(aT^4 - \mathcal{E}\right) + \frac{1}{3}\frac{\partial \mathcal{E}}{\partial x}u\,, \tag{4}$$

where the material momentum density is

$$\varsigma = \rho u \,, \tag{5}$$

The material total energy density is

$$E = \frac{1}{2}\rho u^2 + \rho e \,, \tag{6}$$

and where we assume a gamma-law closure for the pressure:

$$p = \rho e(\gamma - 1) \,. \tag{7}$$

The grid is composed of N contiguous spatial cells, indexed from $i = 1$ to $i = N$. The center of cell i has coordinate x_i, and the left and right edge coordinates for that cell are denoted by $x_{i-1/2}$ and $x^{i+1/2}$, respectively. The pressure and the conserved hydrodynamic unknowns, ρ, ξ, and E, are defined on each cell interior in terms of average values and interpolated slopes. The time integration is analagous to that of the Lagrangian scheme. It is a second-order accurate IMEX scheme with a predictor step and a corrector step. THe Crank-Nicholson method The hydrodynamics and radiation momentum deposition are explicit, but the radiation energy deposition and emission are implicit. The spatial discretizations are second-order accurate and cell centered for all but the radiation fluxes, which are located on the vertices. Auxilliary radiation intensities that are used to ensure continuity of the flux are also generated on the vertices.

2 Overview

We first give a high-level view of the algorithm.

2.1 The Predictor Step

1. Given initial values for the conserved variables, generate limited slopes for those variables and then calculate corresponding left and right values in each cell.

2. Use these quantities to calculate left and right values for p in each cell.

3. Use initial values to compute a maximum wave speed and a courant factor of 0.5 to compute a time step.

4. Ignoring all radiation terms, explicitly evolve the cell-centered variables a half time step using the fluxes evaluated with the left and right values of the conserved variables and the pressure within each cell. Note that this is not conservative.

5. Add the radiation momentum deposition to *zeta* using the initial vertex intensity values.

6. Add the corresponding change in kinetic energy to E.

7. Using initial values for $\mathcal{E}$ and u, generate limited slopes and edge values for the variable cEu.

8. Advect $\mathcal{E}$ over a half time step using the left and right values of cEu together with standard upwinding.

9. Linearize the emission terms in the material total energy equation and the radiation diffusion equation about $T^{n-1/2}$ and substitute $(e^n - e^{n-1/2})/C_V$ for $T^n - T^{n-1/2}$.

10. Obtain updated values for e and $\mathcal{E}$ by simultaneously solving the appropriately modified total energy and radiation diffusion equations.

11. Compute and save the mid-time edge radiation intensities and vertex fluxes for use in the corrector.

2.2 The Corrector Step

1. Given the mid-time predicted values for the conserved variables, compute limited slopes for those variables and then calculate corresponding left and right values in each cell.

2. Use these quantities to calculate left and right values for p in each cell.

3. Use the HLLC approximate Riemann solver to generate interface fluxes using adjacent cell-edge values for ρ, ζ, E, and p.

4. Ignoring the radiation contributions, evolve the conserved variables one full time step.

5. Add the radiation momentum deposition to *zeta* using the mid-time vertex intensity values.

6. Add the corresponding change in kinetic energy to E using the mid-time velocity values.

7. Using initial values for $\mathcal{E}$ and u, generate limited slopes and edge values for the variable cEu at the mid-time.

8. Advect $\mathcal{E}$ over a full time step using the left and right values of cEu together with standard upwinding.

9. Linearize the emission terms in the material total energy equation and the radiation diffusion equation about T^n and substitute $(e^{n+1} - e^n)/C_V$ for $T^{n+1} - T^n$.

10. Obtain updated values for e and $\mathcal{E}$ by simultaneously solving the appropriately modified total energy and radiation diffusion equations.

11. Compute and save the edge radiation intensities and vertex fluxes at $t^{n+1/2}$ for use in the next time step.