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New criterion proposal for transition from natural to forced convection (prescribed wall flux)

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Abstract

In this paper is presented a criterion for identification of heat transfer regime through convection (natural, forced or mixed) by making use of the Mathematica system symbolic computation capabilities. The criterion is based on a comparison of buoyancy and viscous forces. The analysis is realized at the interior of two vertical plates submitted to uniform heat flux density, in steady and laminar state in a fully developed flow. Thus, it was proposed the dimensionless numbers product *RiRe* to be the only representative parameter in order to identify the convective thermal regime, instead of the idea based only on the dimensionless number Richardson or Grashof.

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Keywords: Criterion transition; heat transfer; mixed convection; wall flux; buoyancy forces; viscous forces

1. Introduction

In mixed convection exists two extreme situations where buoyancy forces are negligible, compared to either pressure forces (forced convection) or are predominant (natural convection). In literature, mixed convection is defined by different parameters or dimensionless groups [1-10].

Studies about fully developed mixed convection have been performed by [2], [3], [4] and [5] between two parallel plates at uniform wall temperature, by [2], [6] and [7] between uniform temperature on a wall and a uniform

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wall heat flux on the opposite wall and by [8], [9] and [10] between uniform wall heat fluxes on both walls, but they do not provide selection criteria for identifying different thermal convection modes (natural, forced or mixed) between two vertical plates subjected to uniform heat flux density.

Nomenclature

a, b, c	buoyancy, pressure and viscous terms
g	gravity acceleration, $m \cdot s^{-2}$
q_v	volumetric flow rate, $m^3 \cdot s^{-1}$
Ri	Richardson number
Re	Reynolds number
$Ri.Re$	dimensionless numbers product
T	temperature, ^oC
U	fluid velocity component in x direction, $m \cdot s^{-1}$
V	fluid velocity component in y direction, $m \cdot s^{-1}$
V_d	average fluid velocity, $m \cdot s^{-1}$
x	vertical coordinate, m
y	transversal coordinate, m
β	isobaric coefficient of thermal expansion of fluid, K^{-1}
φ	heat flux density, $W \cdot m^{-2}$
ν	kinematic viscosity of fluid, $m^2 \cdot s^{-1}$
ρ	fluid density, $kg \cdot m^{-3}$

In a particular case, for a laminar flow established between two vertical plates subjected to uniform temperature, Aung and Worku [1] have shown that there is a downward flow along the wall, expressed by the condition of the dimensionless numbers product $RiRe>288$. For the same configuration, Padet [11] suggested a first criterion based on the comparison of the wall shear stresses. A difference lower than 10% for pure natural convection corresponds to $RiRe>5470$, while a difference lower than 10% in the case of forced convection corresponds to $RiRe<15.2$.

Later, three other selection criteria were proposed by Padet J *et al.* [12-16] and Padet C *et al.* [17-18] for uniform temperature imposed on the surface channel: ratio of quadratic means between viscosity and buoyancy forces, pressure and buoyancy forces and the ratio between buoyancy kinetic energy and the total kinetic energy. The first conclusion after the study is that the coefficient $RiRe$ is the only representative parameter of the various proposed criteria, which eliminates a judgment based only on dimensionless numbers Ri or Grashof ($Gr = Ri\, Re^2$). Secondly, the four relevant criteria have comparable orders of magnitude ($RiRe = 8$ up to 40 for "mixed / forced" transition and from 2300 up to 5500 for "mixed / natural" transition) except first, which enlarge the natural convection domain (starting from $RiRe \cong 500$). Moreover, these criteria include the value of $RiRe = 288$ translating the appearance of a recirculating flow on the cold wall, and $RiRe = 166.28$, characterizing the equilibrium between the pressure and buoyancy.

Since in literature the studies concern criteria only for uniform temperature imposed on the surfaces channels, the paper aims to establish a new criterion, based on comparison between buoyancy forces and viscous forces, in order to select the nature of the convective regime (natural, forced or mixed) considering a laminar and upward flow in a two parallel-plates vertical channel, subjected to uniform and constant heat flux density.

2. Problem formulation

A laminar flow is considered between vertical parallel plates, situated at y = 0 and y = e, subjected to uniform and equal heat flux density φ (Fig. 1). The fluid is isovolumic and the flow is ascendant with an average velocity Vd. This velocity is zero in the case of pure natural convection, where the upward flow near the hot wall is equal to downward flow near the cold wall. The boundary conditions of entry and exit of the fluid are carried forward indefinitely.

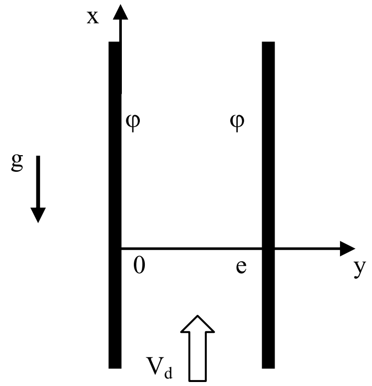

Fig. 1. Vertical channel subjected to heat flux densities on the walls

Velocity and temperature fields are thus independent of the vertical coordinate x ($\frac{\partial U}{\partial x}=0$ and $\frac{\partial T}{\partial x}=0$) . In particular, the driving pressure gradient $\frac{dp^*}{dx}=const$.

The regime is considered permanent and fluid is Newtonian, incompressible with thermophysical properties μ, β, λ and C_p independent of temperature.

The equations that describe the physical model are defined below in cartesian coordinates:

- the continuity equation:

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \tag{1}$$

- the momentum equation:

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=g\beta\left(T-\frac{T_1+T_2}{2}\right)-\frac{1}{\rho}\frac{\partial p^*}{\partial x}+\nu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y}+\nu\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right) \tag{3}$$

- the energy equation:

$$U\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial y}=a\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Considering the assumptions above, they can be simplified in:

- the continuity equation:

$$V = const = 0 \tag{5}$$

- the momentum equation:

$$0 = g\,\beta\left(T - T_m\right) - \frac{1}{\rho}\frac{dp^*}{dx} + \nu\frac{d^2U}{d\,y^2} \tag{6}$$

$$\frac{dp^*}{dy} = 0 \tag{7}$$

- the energy equation:

$$U\frac{\partial T}{\partial x} = a\frac{\partial^2 T}{\partial y^2} \tag{8}$$

The dimensionless velocity profile is given by the following equation [2]:

$$u^+(y^+) = a_1 \sinh(\omega\, y^+) + a_2 \cosh(\omega\, y^+) + a_3 \sin(\omega\, y^+) + a_4 \cos(\omega\, y^+) \tag{9}$$

where: $$\omega = \left(-\frac{1}{4}(RiRe)_e^*\right)^{1/4} \tag{10}$$

and the coefficients a_1, a_2, a_3 and a_4 are given by the equations:

$$a_1 = -\frac{\omega[-1 + \cosh(\omega) + \cos(\omega)(-1 + \cosh(\omega)) + \sin(\omega)\sinh(\omega)]}{4[-1 + \cos(\omega)\cosh(\omega)]}$$

$$a_2 = \frac{\omega[\sin(\omega)(1 + \cosh(\omega)) + \sinh(\omega)(1 + \cos(\omega))]}{4[-1 + \cos(\omega)\cosh(\omega)]}$$

$$a_3 = -\frac{\omega[1 + \cosh(\omega) - \cos(\omega)(1 + \cosh(\omega)) + \sin(\omega)\sinh(\omega)]}{4[-1 + \cos(\omega)\cosh(\omega)]}$$

$$a_4 = -\frac{\omega[\sin(\omega)(1 + \cosh(\omega)) + \sinh(\omega)(1 + \cos(\omega))]}{4[-1 + \cos(\omega)\cosh(\omega)]} \tag{11}$$

In Fig. 2 is shown the dimensionless velocity profile for different values of coefficient $(RiRe)_e^*$. Thus, it can be seen that the flow in the channel is always upward with maximum velocity in the center. When the coefficient $(RiRe)_e^*$ increases, it appears that velocity also increases in the center of the channel and decreases slightly towards the walls.

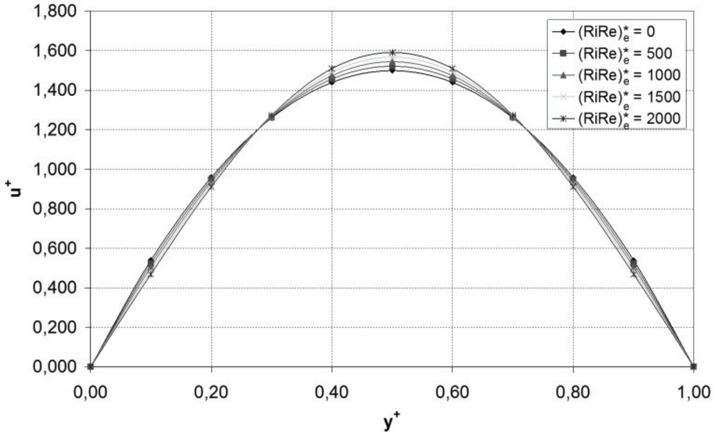

Fig. 2. Dimensionless velocity profile depending on the different numbers of $(RiRe)_e^*$

Knowing the variation of u^+ depending on the coefficient $(RiRe)_e^*$ for values ranges from *0* to *2000*, the velocity profile $u^+(y^+)$ can be expressed as parabolic:

$$u^+(y^+) = (0{,}0002(RiRe)_e^* + 6)\left(-y^{+2} + y^+ - \frac{1}{6}\right) + 1 \tag{12}$$

Replacing the dimensionless parameters below in the equation *12*:

$$u^+ = \frac{U}{V_d},\ y^+ = \frac{y}{e} \tag{13}$$

it obtain the velocity profile dimensioned according to the coefficient $(RiRe)_e^*$, characteristic to mixed convection:

$$U(y) = (0{,}0002(RiRe)_e^* + 6)\left(-\frac{y^2}{e^2} + \frac{y}{e} - \frac{1}{6}\right)V_d + V_d \tag{14}$$

Thus, in equation *14* it can be seen that the velocity profile depends essentially on the coefficient $(RiRe)_e^*$, characteristic to mixed convection.

Also, the dimensionless temperature profile is given by the equation:

$$T^+(y^+) = \frac{2\,[a_1 \sinh(\omega y^+) + a_2 \cosh(\omega y^+) - a_3 \sin(\omega y^+) - a_4 \cos(\omega y^+)]}{\omega^2} + \\ + \frac{2[\sin(\omega)\sinh(\omega) - \cos(\omega) + \cosh(\omega)]}{\omega^2\,[1 - \cos(\omega)\cosh(\omega)]} \tag{15}$$

Figure 3 shows that the temperature profile does not vary almost at all with the coefficient $(RiRe)_e^*$. The temperatures are maximum on the wall, decreasing towards the center of the channel, where one can observe a very small decrease as the coefficient $(RiRe)_e^*$ increases.

Thus, the temperature profile $T^+(y^+)$ can be defined according to the coefficient $(RiRe)_e^*$:

$$T^{+}(y^{+}) = (1{,}4.10^{-5}(RiRe)_e^{*} + 1{,}25)\left(y^{+2} - y^{+} + \frac{1}{6}\right) \tag{16}$$

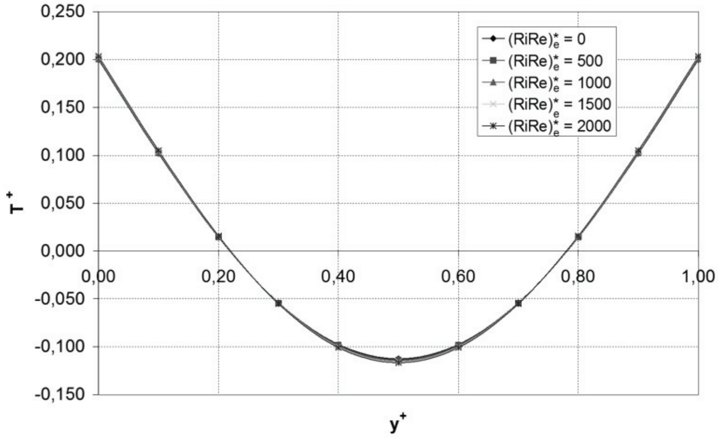

Fig. 3. Dimensionless temperature profile depending on the different numbers of $(RiRe)_e^{*}$

Replacing the dimensionless parameter T^{+}:

$$T^{+} = \frac{(T - T_m)\lambda}{e\varphi} \tag{17}$$

in the equation *16*, is obtained the temperature variation in the channel section:

$$T(y) = T_m + \frac{e\varphi}{\lambda}\left[(1{,}4.10^{-5}(Ri\,Re)_e^{*} + 1{,}25)\left(\frac{y^2}{e^2} - \frac{y}{e} + \frac{1}{6}\right)\right] \tag{18}$$

Thus, in order to distinguish different types of convection (forced, mixed or natural) may be proposed a criterion based on the comparison of buoyancy and viscous forces. Thus, the terms *a* and *b* of the equation *19* are compared:

$$\underbrace{0 = g\,\beta(T - T_m)}_{a} - \underbrace{\frac{1}{\rho}\frac{dp^{*}}{dx}}_{b} + \underbrace{\nu\frac{d^2U}{d\,y^2}}_{c} \tag{19}$$

3. Transition criterion

Substituting the expressions of velocity and temperature given by equations *18* and *14* in terms of *a* and *c* of equation *19*, is obtained:

$$a = \left[(1{,}4.10^{-5}(RiRe)_e^{*} + 1{,}25)\left(y^2 - e\,y + \frac{1}{6}e^2\right)\right]\frac{g\,\beta\varphi}{e\lambda} \tag{20}$$

$$c = (-12 - 0{,}0004(RiRe)_e^*)\frac{\nu V_d}{e^2} \tag{21}$$

Their means squares are respectively:

$$\overline{a^2} = 1{,}1.10^{-12}(89278{,}6 + (RiRe)_e^*)^2 \frac{e^2 g^2 \beta^2 \varphi^2}{\lambda^2} \tag{22}$$

$$\overline{c^2} = 1{,}6.10^{-7}(29993{,}5 + (RiRe)_e^*)^2 \frac{V_d^2 \nu^2}{e^4} \tag{23}$$

If P^2 is the ratio between the means squares $\overline{a^2}$ and $\overline{c^2}$, is obtained:

$$P^2 = \frac{\overline{a^2}}{\overline{c^2}} = \frac{6{,}8.10^{-6}(89278{,}6 + (RiRe)_e^*)^2}{(29993{,}5 + RiRe_e^*)^2} \frac{e^6 g^2 \beta^2 \varphi^2}{V_d^2 \lambda^2 \nu^2} \tag{24}$$

Knowing that the coefficient $(RiRe)_e^*$ is:

$$(RiRe)_e^* = \frac{8 e^3 g \beta \varphi}{V_d \lambda \nu} \tag{25}$$

the ratio P^2 becomes:

$$P^2 = 1{,}06.10^{-7} \frac{(89278{,}6 + (RiRe)_e^*)^2}{(29993{,}5 + RiRe_e^*)^2} (RiRe)_e^{*2} \tag{26}$$

In Fig. 4 it can be seen that P varies almost parabolic with coefficient $(RiRe)_e^*$, variation that can be expressed by a quadratic equation for values of $(RiRe)_e^*$ between *0* and *10 000*:

$$P = -1{,}58.10^{-8}\ (RiRe)_e^{*2} + 0{,}00094\ (RiRe)_e^* \tag{27}$$

Based on the equation *27,* $(RiRe)_e^*$ can be expressed according to P:

$$(RiRe)_e^* = 34058 \pm 8512{,}57\sqrt{16 - P} \tag{28}$$

If it is assumed that in forced convection the buoyancy forces are at least 5% of the viscous forces ($P<0.05$), and that in natural convection, they represent less than 95% ($P>0.95$), the values $(RiRe)_e^*$ for the transition mixed / natural convection and mixed / forced convection are:

$\boldsymbol{(RiRe)_e^*}$ **> 1026.1** for natural convection

$\boldsymbol{(RiRe)_e^*}$ **< 53.23** for forced convection

These values are calculated taking into account the "minus" sign into equation *28*, because with the second solution, the values of $(RiRe)_e^*$ exceed the validity of the domain:

$$(RiRe)_e^* = 34058 - 8512{,}57\sqrt{16 - P} \tag{29}$$

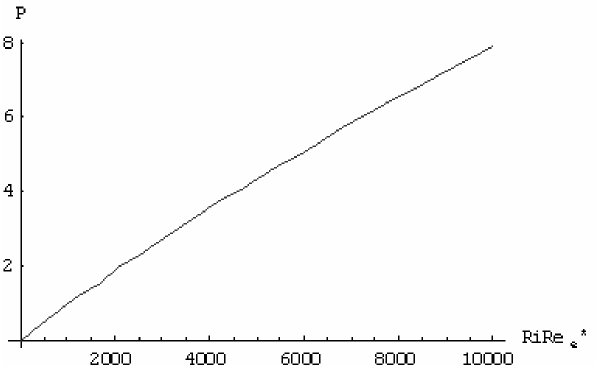

Fig. 4. Variation of *P* according to the coefficient $(RiRe)_e^*$

4. Conclusions

In this paper it was established a new selection criterion based on comparison between buoyancy forces and viscous forces in order to identify different regimes of heat transfer (natural, forced or mixed convection). The analysis is carried out for an internal laminar flow between two vertical flat plates subjected to uniform heat flux densities.

Thus, it was proposed the dimensionless numbers product RiRe to be the representative parameter used to identify the thermal convection instead a judgment based only on dimensionless numbers Richardson and Grashof (Gr = Ri.Re2). The results showed that the natural convective regime is established when $(RiRe)_e^*$ is bigger than 1026.1. When the values are between 1026.1 and 53.23 or smaller than the last one, the heat transfer regime is situated respectively in mixed and forced convection.

The selection criteria proposed and analyzed for heat flux densities imposed on a channel surfaces can be used to verify different physical situations for the predominance of natural/forced convection.

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