TECHNICAL REPORT:
Introduction to Reduced Order Modelling in *ChiTech*

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Contents

Page

1 Introduction **2**

List of Figures

1 Introduction

Suppose we have heat diffusion in a 1D space:

$$-\nabla(k\nabla T) + s = 0 \tag{1}$$

Using a finite difference approximation of the derivatives we obtain:

$$-\frac{1}{\Delta x_i}\left[k_{i+\frac{1}{2}}\frac{T_{i+1}-T_i}{\Delta x_{i+\frac{1}{2}}} - k_{i-\frac{1}{2}}\frac{T_i - T_{i-1}}{\Delta x_{i-\frac{1}{2}}}\right] + s_i = 0$$

If the unknowns T_i do not coincide with region boundary points, we obtain $k_{i\pm\frac{1}{2}}$ from the harmonic mean:

$$k_{i+\frac{1}{2}} = \left(\frac{1}{k_i} + \frac{1}{k_{i+1}}\right)^{-1} = \frac{k_i.k_{i+1}}{k_i + k_{i+1}}$$

Alternatively, we can define the unknowns T_i to coincide with region boundary points so that:

$$k_{i+\frac{1}{2}} = \frac{k_{i\to i+\frac{1}{2}} + k_{i+\frac{1}{2}\to i+1}}{2} = k_i$$

Similarly:

$$k_{i-\frac{1}{2}} = k_{i-1}$$

With equation 1 we can now construct a linear system $AT = b$,

$$A = \begin{bmatrix} \frac{k_{i+1}}{\Delta x_i \Delta x_{i+\frac{1}{2}}} & -\frac{k_{i+1}}{\Delta x_i \Delta x_{i+\frac{1}{2}}} & 0 & \dots & \dots \\ \vdots & \ddots & \dots & \dots & \dots \\ \vdots & -\frac{k_{i-1}}{\Delta x_i \Delta x_{i-\frac{1}{2}}} & \frac{k_{i-1}}{\Delta x_i \Delta x_{i-\frac{1}{2}}} + \frac{k_{i+1}}{\Delta x_i \Delta x_{i+\frac{1}{2}}} & -\frac{k_{i+1}}{\Delta x_i \Delta x_{i+\frac{1}{2}}} & \dots \end{bmatrix}$$

Furthermore we can write $A = f(\mu)$. Now suppose that we made enough calculations to support an emulated function which uses a set of basis vectors $(\Psi_i)_{(1\leq i\leq r)}$ and expansion coefficients $c(\mu)$ such that:

$$\begin{aligned} T^r(\mu) &= [\Psi_1^T \ \Psi_2^T \ .. \ \Psi_r^T] \ c(\mu) \\ T^r(\mu) &= U^r c(\mu) \end{aligned} \tag{2}$$

References

[1] *Two-Phase Flow Patterns and Flow-Pattern Maps: Fundamentals and Applications*, ASME, Applied Mechanics Reviews, 2008.

[2] Shieh, A.S., Ransom, V.H., Krishnamurthy, R., *RELAP5/MOD3 Code Manual Volume 6: Validation of numerical techniques in RELAP5/MOD3*, NUREG/CR-5535/Rev 1.