

saturation of the cooled liquor, in temperature units, is approximately $(T_{pi} - T_{po})$; the maximum possible supersaturation is $(T_{pi} - T_s)$. The surface area required in the cooler is critically dependent on the ratio of these supersaturations, so that the ratio may be considered a "coefficient of performance." Prior to pilot plant experience, T_s will be unknown, so that a pilot plant cooler can be designed only for a given coefficient of performance. As a guide, maximum supersaturations are usually about 1° to 2° C. for inorganic salt solutions.

Appendix

The criteria necessary to achieve a constant scale temperature in a concurrent exchanger may be derived as follows: Consider a differential element of area dA , located somewhere in the exchanger. The quantity of heat, dQ , transferred to this element from the bulk of the process liquor, temperature T_p , at this point, to the scale surface is given by

$$dQ = h_p dA(T_p - T_s)$$

Likewise dQ is given from the rate of heat transfer from the scale surface to the bulk of the coolant, temperature T_c , by

$$dQ = (h_c^{-1} + h_s^{-1})^{-1} dA(T_s - T_c)$$

Thus

$$\frac{h_p}{h_c} + \frac{h_p}{h_s} = \frac{T_s - T_c}{T_p - T_s}$$

Applying this equation to the terminal conditions

$$\frac{h_p}{h_c} + \frac{h_p}{h_s} = \frac{T_s - T_{ci}}{T_{pi} - T_s} = \frac{T_s - T_{co}}{T_{po} - T_s}$$

By simple proportion

$$\frac{T_s - T_{co}}{T_{po} - T_s} = \frac{T_{co} - T_{ci}}{T_{pi} - T_{po}}$$

and also

$$Q = G_c \sigma_c (T_{co} - T_{ci}) = G_p \sigma_p (T_{pi} - T_{po})$$

Thus the criteria may be written

$$\frac{h_p}{h_c} + \frac{h_p}{h_s} = \frac{G_p \sigma_p}{G_c \sigma_c} = \frac{T_s - T_{ci}}{T_{pi} - T_s}$$

Nomenclature

A = process side heat transfer area, L^2
 G = mass feed rate, $M\theta^{-1}$
 h = heat transfer coefficient, based on A , $Q T^{-1} L^{-2} \theta^{-1}$
 Q = cooler heat load, $Q\theta^{-1}$
 T = temperature, T
 σ = specific heat, $Q^{-1} M T^{-1}$
 L = length
 M = mass
 θ = time

SUBSCRIPTS

c = coolant
 i = inlet
 o = outlet
 p = process liquor
 s = process side of heat transfer surface

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CORRELATION FOR BOILING HEAT TRANSFER TO SATURATED FLUIDS IN CONVECTIVE FLOW

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An additive mechanism of micro- and macroconvective heat transfer was formulated to represent boiling heat transfer with net vapor generation to saturated, nonmetallic fluids in convective flow. Interaction of the two mechanisms was taken into account by means of two dimensionless functions: an effective two-phase Reynolds number function, F , and a bubble-growth suppression function, S . F was obtained as a function of the Martinelli parameter by both empirical correlation of heat transfer data and a momentum-analogy analysis. S was obtained as an empirical function of the two-phase Reynolds number. The correlation was tested with available data for water and organic fluids. The average deviation between calculated and measured boiling coefficients for all (over 600) data points from ten experimental cases was $\pm 12\%$.

THE complexity of the boiling heat transfer phenomenon is well known. It is especially evident in the case of convective boiling with net generation of vapor. Under these conditions, the heat transfer is affected by the various flow parameters and the vapor quality, as well as by the parameters which are pertinent in pool boiling. A wide range of local conditions and different flow regimes can exist

along the length of a boiler for such a case. In the subcooled region, both fluid and wall temperatures increase along the boiler as the liquid gains sensible heat. At the point where nucleate boiling starts, the wall temperature begins to decrease. The fluid temperature continues to increase until it reaches its saturation value, from which point on it decreases gradually, corresponding to the decreasing pressure. As the

fluid progresses into the annular flow regime with continuously increasing vapor qualities, the wall temperature decreases, reflecting an increasing heat transfer coefficient. This condition often exists into the fog-flow region. Finally, liquid deficiency occurs and there is a sudden decrease in the heat transfer coefficient with a corresponding rise in the wall temperature.

In view of the widely different conditions that could exist at various points along a boiler, it is this author's opinion that it would be entirely unprofitable either to measure over-all "average" coefficients or to attempt to correlate them. The only hope of obtaining results which can be generalized is to work with local heat transfer coefficients. Furthermore, since the mechanisms of heat transfer may be different for the various flow regimes, it seems that any analytical study or correlation could be expected to apply strictly to only one of the possible flow regimes. It is therefore necessary to define the region of applicability clearly for a particular analytical study or correlation. The region of interest in this study is defined by the following conditions:

- Saturated, two-phase fluid in convective flow.
- Vertical, axial flow.
- Stable flow.
- No slug flow.
- No liquid deficiency.
- Heat flux less than critical flux.

These conditions usually occur with annular flow or annular-mist flow in the quality range of approximately 1 to 70%.

Background

A number of experimental studies in this field have reported data on systems with water, organic fluids, and even liquid metals. In this present analysis, we are concerned with only the first two classes of fluids. In some of these studies, the authors presented equations correlating their own data, and in almost all such cases, implied that the correlations could be generalized to apply to other systems.

One of the first major works in this area was that of Dengler and Addoms (2), who in 1952 obtained local boiling coefficients for water in upward vertical flow through a 1-inch tube. They proposed the following empirical correlation for their data,

$$h = 3.5 \frac{F_{DA}}{(X_{tt})^{0.5}} \times h_L' \quad (1)$$

where h_L' is evaluated by the conventional Dittus-Boelter equation,

$$h_L' = 0.023 \frac{k_L}{D} (DG/\mu_L)^{0.8} (C_{pL}\mu_L/k_L)^{0.4}$$

In the above correlation, F_{DA} represents the correction factor to be applied for points where nucleate boiling exists.

In 1956, Guerrieri and Talty (5) reported local boiling coefficients for a number of organic liquids in single-tube vertical boilers. They were able to correlate 88% of their data to within $\pm 20\%$ by the following equations.

$$h = 0.187(r^*/\delta)^{-5/9} \times 3.4 \frac{h_L}{(X_{tt})^{0.45}} \quad (2)$$

$$h_L = (1 - x)^{0.8} h_L'$$

$$r^* = \frac{2RT_s^2\sigma}{P_s\lambda(T_w - T_s)}$$

$$\delta = \frac{10\mu_L}{\rho_L} \left[\frac{4\rho_L}{(dP/dL)Dg_c} \right]^{1/2}$$

In 1959, Bennett and coworkers (7) reported data for water flowing vertically in annular test sections. They compared their data with both the correlation of Dengler and Addoms and that of Guerrieri and Talty but, in each case, found considerable scatter. They then proposed their own correlation where the local heat flux, q/A , was used as a multiplicative correction factor.

$$h = 0.64 \frac{1}{(X_{tt})^{0.74}} \times h_L \times (q/A)^{0.11} \quad (3)$$

In 1960, Sani (8) presented data for water in vertical flow through a tube. He compared his results with the available correlations but found the deviations to be substantial. No new correlation was proposed.

In 1962, Shrock and Grossman (9) published a report on force-convective boiling of water in tubes. They introduced the boiling number, B_o , as an extra variable and obtained a correlation of the form,

$$h = 7400[B_o + 1.5 \times 10^{-4} (1/X_{tt})^{2/3}] h_L' \quad (4)$$

$$B_o = (q/A)/(1/G\lambda)$$

They reported that this equation correlated their data to within $\pm 35\%$. However, when Sani (8) compared his results with this correlation, he discovered that the experimentally measured coefficients were almost twice as great as those predicted.

There are a number of other correlations reported in the literature. Most of them, however, deal with either subcooled boiling or boiling with horizontal flow and therefore are beyond the scope of the present study.

Comparison of Existing Correlations

There appeared to be a need for testing the various correlations against a representative body of available data. This was carried out with the aid of a digital computer and some of the results are shown in Figures 1, 2, 3, and 4, where the predicted values of the boiling coefficients have been plotted against the experimental values, the 45° line representing perfect correlation. Only a representative fraction of the 665 data points used in this check are shown in these figures. The range of experimental conditions covered by the data in this comparison is summarized in Table I.

Figure 1 shows that while the correlation of Dengler and Addoms (2) represents the authors' own data fairly well, it predicts coefficients which are too high when compared with data from other sources. Figure 2 shows that the equation proposed by Guerrieri and Talty (5) correlates their own results well but predicts coefficients much lower than those measured by Dengler and Addoms and by Sani. This correlation, however, appears to predict coefficients of the right order of magnitude when compared with the data of Schrock and Grossman. Figure 3 shows that Bennett's (7) correlation appears to predict reasonable values for the majority of the water data, although the scatter is great. However, the predicted coefficients are approximately 50% too low when compared with the organic data. In general, the correlation of Schrock and Grossman (9), as shown in Figure 4, appears to be better than the other three. With respect to the organic data, this correlation gives values which are approximately

30% too low. The correlation appears to bisect the water data, although the scatter ranges from +30 to -50%.

In general, none of the existing correlations appears entirely satisfactory for general use. This study was undertaken in hope of developing a more generally applicable correlation, based on the concepts discussed below.

Derivations of Equations

It is postulated that two basic mechanisms take part in the heat transfer process for the boiling of saturated fluids with flow: the ordinary macroconvective mechanism of heat transfer which normally operates with flowing fluids, and the microconvective mechanism associated with bubble nucleation and growth. It is further postulated that these two mechanisms are additive in their contributions to total heat transfer. The concept of additive contributions is not new. Rohsenow (7) in 1952 first suggested that the heat transfer associated with convection and with boiling can be added. Gambill (4) recently applied the additive concept to correlate boiling burnout data. In both cases, subcooled nucleate boiling was treated, and the authors found that the convective and the boiling contributions could be superimposed without modifications. In this present study, we are interested in saturated boiling with net vapor generation, and it is felt that for this case the two mechanisms would not have their normal effects but would be modified by effects of quality and by the interaction of one mechanism on the other. Under this general premise, equations describing these two mechanisms were formulated as follows.

Considering first the macroconvective mechanism, it was recognized that at the two limits of 0 and 100% quality, the macroconvective heat transfer should be described by the Dittus-Boelter type of correlation. It was then postulated that in the two-phase region where both liquid and vapor are present, the macroconvective heat transfer should still be described by a modified form of the Dittus-Boelter equation.

$$h_{\text{mac}} = 0.023(\text{Re})^{0.8}(\text{Pr})^{0.4}(k/D) \quad (5)$$

In this equation, the Prandtl and the Reynolds numbers and the thermal conductivity represent effective values associated with the two-phase fluid. We may define three parameters as ratios of these quantities divided by the liquid quantities.

$$\beta = \text{Pr}/\text{Pr}_L \quad (6)$$

$$\gamma = k/k_L \quad (7)$$

$$F = (\text{Re}/\text{Re}_L)^{0.8} = \left(\text{Re} \times \frac{\mu_L}{DGz} \right)^{0.8} \quad (8)$$

In the case of ordinary fluids—i.e., not liquid metals—the Prandtl numbers of the liquid and of the vapor are normally of the same magnitude. The Prandtl number of the two-phase fluid should therefore also be of the same magnitude. Furthermore, since the heat is transferred through an annular film of liquid adhering to the wall, it is expected that the liquid properties would have the dominant effect. For these reasons it is reasonable to assume that β and γ may be taken to be unity as a first approximation. Equation 5 may then be rewritten as

$$h_{\text{mac}} = 0.023(\text{Re}_L)^{0.8}(\text{Pr}_L)^{0.4}(k_L/D)F \quad (9)$$

The only unknown function in Equation 9 is F , the ratio of the two-phase Reynolds number to the liquid Reynolds number, based on the liquid fraction of the flow. This ratio is strictly a flow parameter, and therefore by analogy to momentum transfer in two-phase flow, it may be assumed that F is a func-

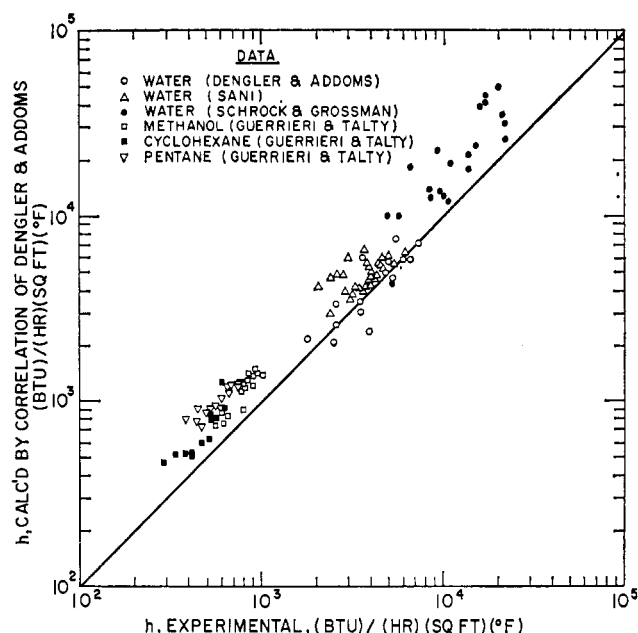


Figure 1. Comparison of correlation of Dengler and Addoms with experimental results from various sources

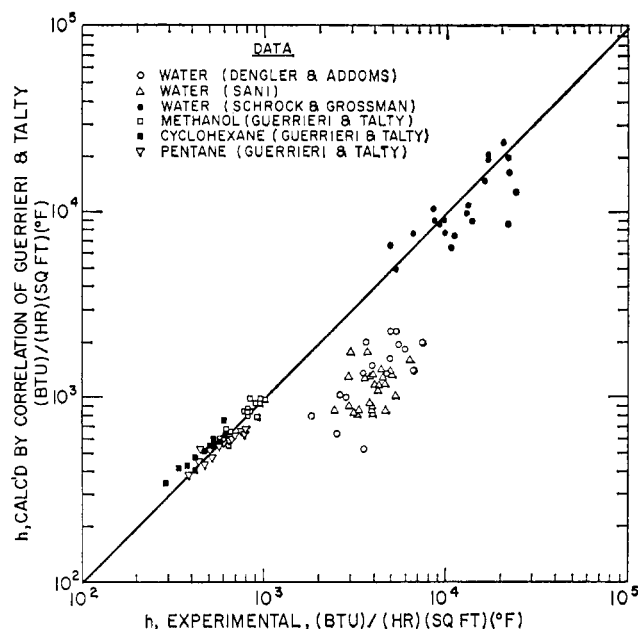


Figure 2. Comparison of correlation of Guerrieri and Talty with experimental results from various sources

tion of the Martinelli parameter, X_{tt} . This postulate has been verified and the actual dependence of F on X_{tt} obtained by comparison with experimental data.

The analysis of Forster and Zuber (3) was taken as a basis for the formulation of a microconvective mechanism of heat transfer. The Forster and Zuber analysis was derived for the case of pool boiling and resulted in the following equations for the boiling Nusselt number.

$$\begin{aligned} \text{Nu}_b &= 0.0015(\text{Re}_b)^{0.62}(\text{Pr}_L)^{0.33} = (h_b r_b)/k_L \\ r_b &= \frac{\Delta T}{\lambda \rho_v} \left(\frac{2\pi k_L \rho_L C_{pL} \sigma}{\Delta P} \right)^{0.5} \left(\frac{\rho_L}{g_c \Delta P} \right)^{0.25} \\ \text{Re}_b &= \left(\frac{\pi k_L C_{pL}}{\mu_L} \right) \left(\frac{\rho_L \Delta T}{\rho_v \lambda} \right)^2 \end{aligned} \quad (10)$$

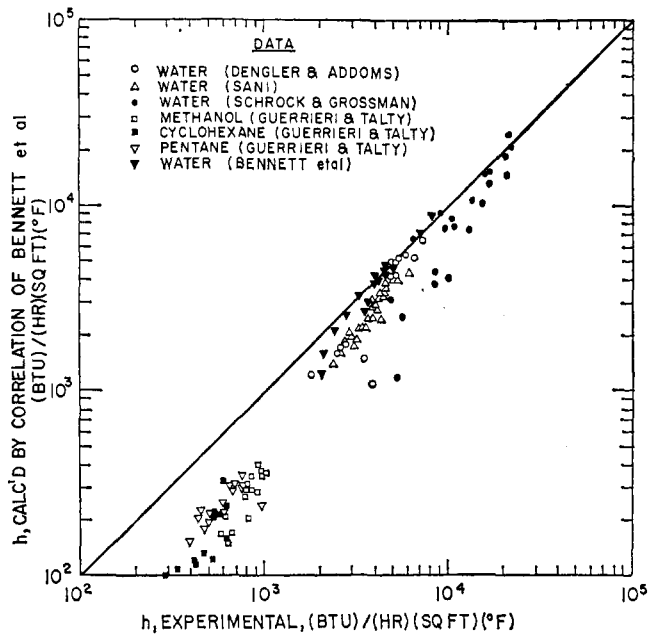


Figure 3. Comparison of correlation of Bennett et al. with experimental results from various sources

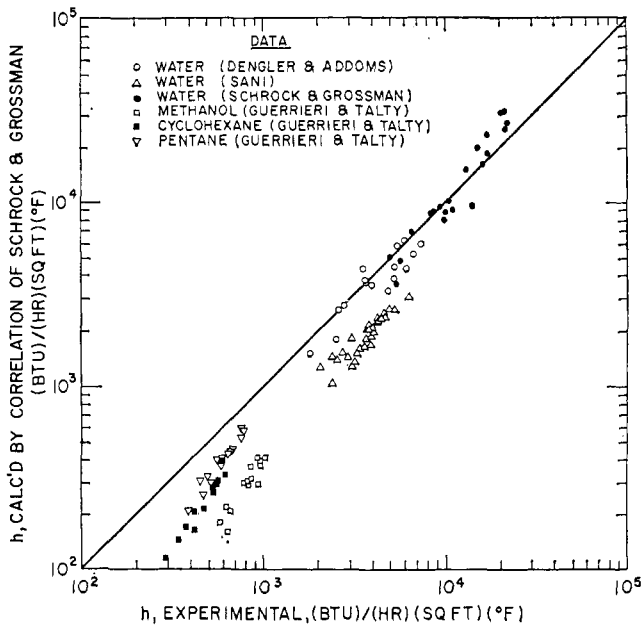


Figure 4. Comparison of correlation of Schrock and Grossman with experimental results from various sources

These equations are based on the premise that the Reynolds number for microconvective boiling heat transfer is governed by the growth rate of bubbles. This rate is described by the Rayleigh equation, which Forster and Zuber solved to show that the product of the bubble radius and the bubble growth rate is a constant for a specific superheat.

$$r_b \frac{dr_b}{d\theta} = \frac{\pi k_L \rho_L C_{pL}}{(\lambda \rho_v)^2} (\Delta T)^2 \quad (11)$$

As illustrated in Figure 5 for the case of boiling from a superheated wall, whether in pool boiling or convective boiling, the degree of superheat actually is not constant across the boundary layer. Therefore, the Reynolds number used in the Forster-Zuber equation actually is an effective Reynolds number representative of some mean, effective superheat. The

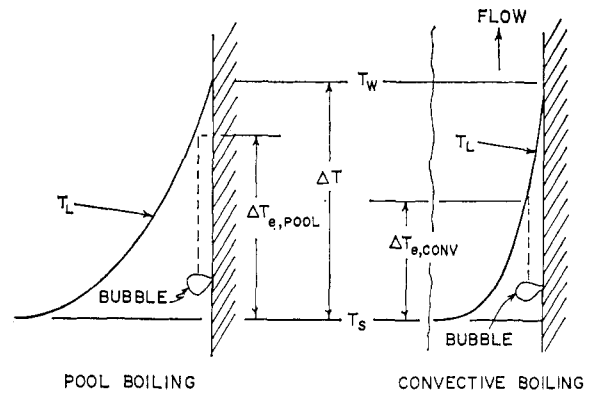


Figure 5. Temperature profiles for pool boiling and for convective boiling with same total superheat

difference between this effective superheat and the wall superheat is small for the case of pool boiling and was neglected in Forster and Zuber's derivation. However, it cannot be neglected in the case of convective boiling, since the temperature gradient is now dependent on flow rate and vapor quality and would generally be much steeper than in the corresponding pool boiling case with the same wall superheat.

In using Forster and Zuber's formulation to represent microconvective heat transfer in convective boiling, this effect was taken into account by writing Equation 10 in terms of effective ΔT and ΔP .

$$h_{mic} = 0.00122 \left(\frac{k_L^{0.79} C_{pL}^{0.45} \rho_L^{0.49} g_c^{0.25}}{\sigma^{0.5} \mu_L^{0.29} \lambda^{0.24} \rho_v^{0.24}} \right) (\Delta T_e)^{0.24} (\Delta P_e)^{0.75} \quad (12)$$

Then a suppression factor, S , was defined as the ratio of the effective superheat to the total superheat of the wall:

$$S = (\Delta T_e / \Delta T)^{0.99} \quad (13)$$

The ratio is arbitrarily taken to the 0.99 power in order that S may appear to the first power in the final equations.

By the Clausius and Clapeyron equation,

$$\begin{aligned} (\Delta T_e)^{0.99} &= \left(\frac{T_e}{\lambda \rho_v j} \right)_{T_e}^{0.75} (\Delta T_e)^{0.24} (\Delta P_e)^{0.75} \\ (\Delta T)^{0.99} &= \left(\frac{T}{\lambda \rho_v j} \right)_T^{0.75} (\Delta T)^{0.24} (\Delta P)^{0.75} \end{aligned} \quad (14)$$

For $\Delta T / T \ll 1$, we can write

$$\left(\frac{T_e}{\lambda \rho_v j} \right)_{T_e} \cong \left(\frac{T}{\lambda \rho_v j} \right)_T \quad (15)$$

and

$$S = (\Delta T_e / \Delta T)^{0.24} (\Delta P_e / \Delta P)^{0.75} \quad (16)$$

Combining with Equation 12, we then obtain an expression for the microconvective coefficient in terms of the suppression factor and the total superheat.

$$h_{mic} = 0.00122 \left(\frac{k_L^{0.79} C_{pL}^{0.45} \rho_L^{0.49} g_c^{0.25}}{\sigma^{0.5} \mu_L^{0.29} \lambda^{0.24} \rho_v^{0.24}} \right) (\Delta T)^{0.24} (\Delta P)^{0.75} S \quad (17)$$

The suppression factor, S , approaches unity at zero flow rate and zero at infinite flow rate. It was postulated, subject to experimental verification, that in all ranges of flow S can be

represented as a function of the local two-phase Reynolds number.

The total heat transfer coefficient is then obtained as the sum

$$h = h_{mic} + h_{mac} \quad (18)$$

The two unknowns in the above equations are the Reynolds number factor, F , and the suppression factor, S . These two functions were determined empirically from experimental data by the following procedure.

A first estimate of F was obtained by plotting the ratio of experimental two-phase heat transfer coefficient divided by the liquid coefficient as a function of the reciprocal Martinelli parameter. Using this estimate of F , values of h_{mic} were obtained from Equations 17 and 18 by subtracting the estimated values of h_{mac} from the experimental data. The results were then plotted against two-phase Reynolds number and a best-fit curve was obtained for S . Using this first estimate of the S function, a second estimate of F was calculated from the experimental data and compared with the original estimate of F . This iterative procedure was repeated for a total of 10 trials, at the end of which the two functions were found to remain relatively constant from one trial to the next and were assumed to have converged.

The results are shown in Figures 6 and 7, where the shaded areas represent the scatter of the data around the two correlating functions. F increases with increasing values of the reciprocal Martinelli parameter, indicating that the effective turbulence associated with two-phase flow increases with increasing vapor fraction. The suppression factor, S , approaches unity at low values of the two-phase Reynolds number and decreases asymptotically to zero at high values of the two-phase Reynolds number. These results are reasonable in indicating that at low flow rates and low fraction vapor the boiling mechanism plays a relatively important part, whereas at high flow rates and high fraction vapor, the boiling is suppressed and the macroconvective heat transfer mechanism becomes increasingly important.

The final correlation is given by Equations 9, 17, and 18 with the two correlating functions, F and S , represented graphically in Figures 6 and 7.

An alternative method of determining the F function, by means of heat-momentum transfer analogy, is presented in the Appendix. The resultant curve for the F function is found to be in good agreement with the empirically determined curve shown in Figure 6.

Comparison with Experimental Data

This correlation was tested against available data for both water and organic fluids. The comparisons are shown in Figures 8 and 9. This correlation is seen to fit the data from all, rather than from just one or two, of the sources. It also reduces the scatter substantially. The points presented in Figure 8 include data for water, methanol, cyclohexane, and pentane. The mechanism and resultant equations postulated above appear to correlate all these data well. The average deviation for Dengler and Addoms' water data was 14.7%; for Schrock and Grossman's water data, 15.1%; for Guerrieri and Talty's methanol data, 11.3%; for their cyclohexane data, 13.6%; and for their pentane data, 11.9%. One of the sets best correlated by this formulation is the data of Sani for water in downflow through a tube, where the deviation is only 8.5%. This indicates that for the range of flow conditions encountered, the direction of vertical flow makes no appreciable difference. Previously, there was some uncertainty on this question, since Schrock and Grossman had not been able to demonstrate consistency between their upflow results and Sani's downflow results.

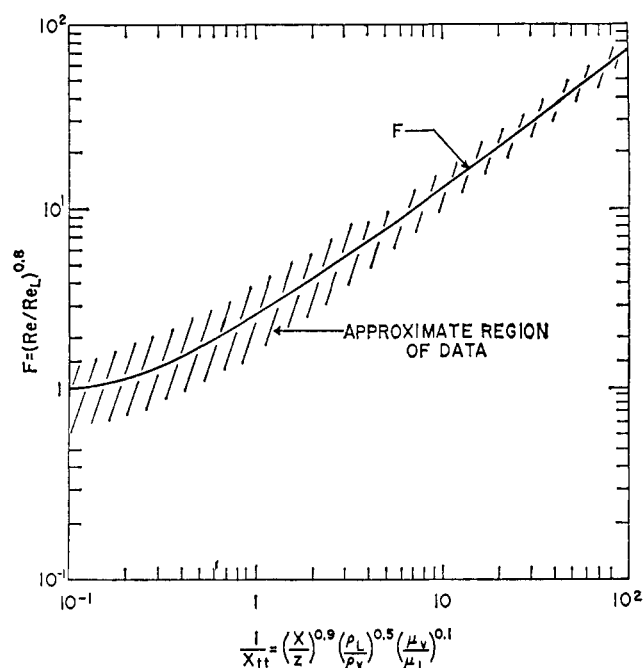


Figure 6. Reynolds number factor, F

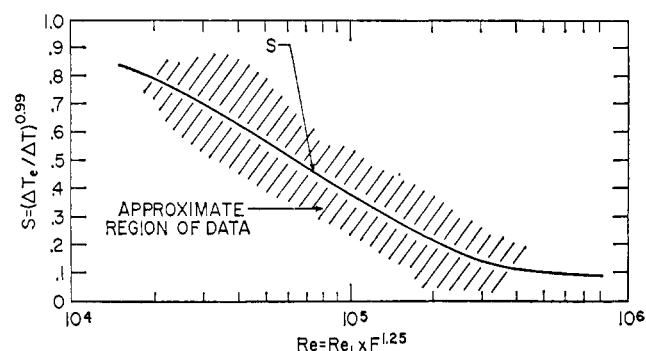


Figure 7. Suppression factor, S

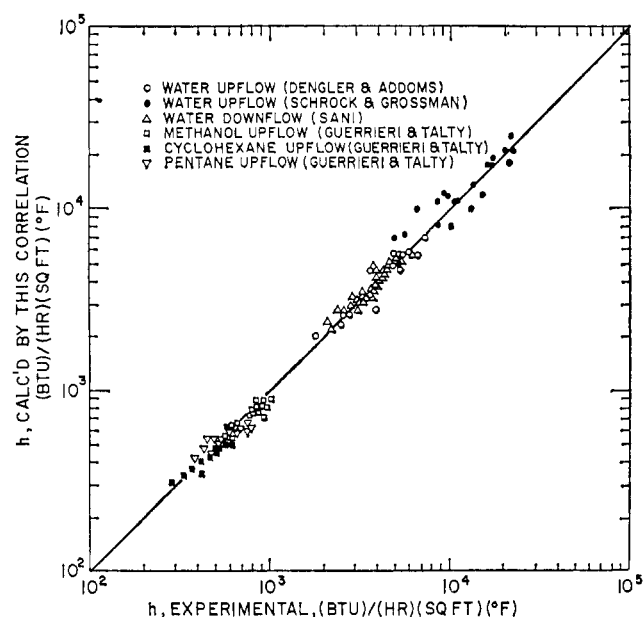


Figure 8. Comparison of this correlation with experimental results from various sources

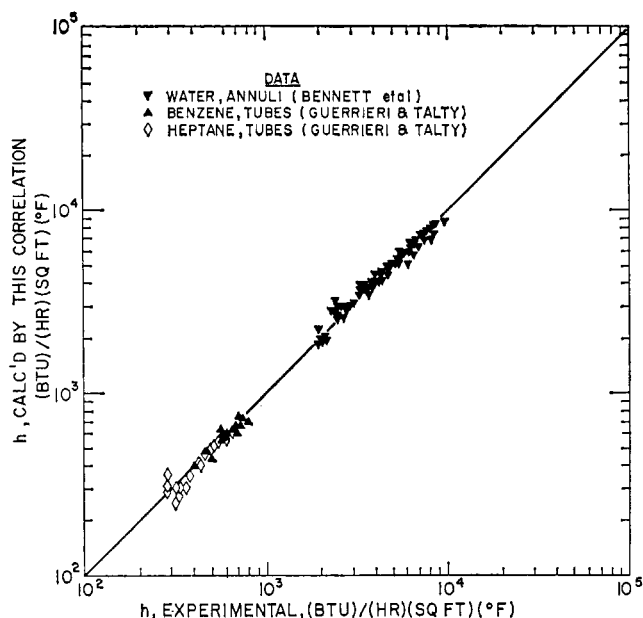


Figure 9. Further comparison of this correlation with experimental results from various sources

Figure 9 shows further comparisons of this correlation with experimental results. Results for benzene and heptane are presented here along with results for water taken by Bennett *et al.* Bennett's data can also be correlated by this formulation, in view of the fact that his coefficients were obtained on annuli whereas the other data were all obtained on round tubes. The equivalent diameter used to correlate Bennett's annular data was four times the cross-sectional area divided by the heated perimeter. The average deviations for Bennett's water data and for the benzene and heptane data of Guerrieri and Talty are 10.8, 6.3, and 11.0%, respectively.

Another method of testing this correlation is to see if it can predict correctly the wall temperature profile along the length of a boiler when all other conditions are given. Some results from such tests are shown in Figure 10. Figure 10, A, shows a comparison with data from one of Schrock and Grossman's runs in which the incoming liquid was subcooled. For the initial lengths of the boiler, the measured wall temperatures diverge from the calculated values. However, for later lengths of the boiler where vapor quality becomes greater than 2% and annular flow could exist, the calculated wall temperatures are verified by the experimental measurements. Figure 10, B, shows results of a comparison with data from one of Sani's runs. In this case, the fluid entered with 7% vapor quality, and annular flow probably existed through the entire length of the boiler. The calculated and measured wall temperatures are in good agreement.

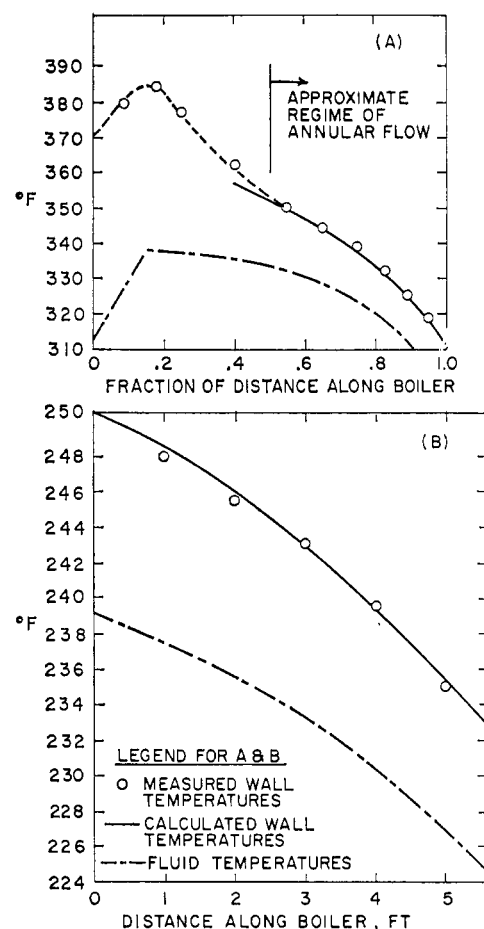


Figure 10. Comparison of calculated and measured wall temperatures

A. Data of Schrock and Grossman
B. Data of Sani

It is also of interest to see if this correlation can correctly predict the variation of the ratio h/h_L with Martinelli parameter X_{tt} . The majority of data available in the literature are presented as plots of h/h_L vs. $1/X_{tt}$. Many investigators (1, 5, 8) have found that at high values of $1/X_{tt}$ the data converge to a single line, whereas at low values of $1/X_{tt}$ they diverge in parametric curves. This divergence has not been satisfactorily reproduced by any of the previous theoretical equations. Figure 11 shows curves of h/h_L calculated by this correlation, which show the characteristic divergence at low values of $1/X_{tt}$; in fact, when compared with experimental data for identical conditions (curve 2), excellent agreement is obtained. Furthermore, the above analysis indicates that at high values of $1/X_{tt}$ the ratio h/h_L should converge to the ratio $(Re/Re_L)^{0.8}$ —i.e., the F function.

Table I. Range of Conditions for Data Used in Testing Correlation

Ref.	Fluid	Geometry	Flow	Pressure, P.S.I.A.	Flow Vel., Ft./Sec. Liq.	Quality, Wt. %	$Q/A \times 10^{-4}$, (B.t.u.)/(Hr.) (Sq. Ft.)
(2)	Water	Tube	Up	8-40	0.2-4.8	15-71	2.8-20
(9)	Water	Tube	Up	42-505	0.8-14.7	3-50	6.5-76
(8)	Water	Tube	Down	16-31	0.8-2.7	2-14	1.4-5.0
(7)	Water	Annulus	Up	15-35	0.2-0.9	1-59	3.2-1.6
(5)	Methanol	Tube	Up	15	1.0-2.5	1-4	0.7-1.7
(5)	Cyclohexane	Tube	Up	15	1.3-2.8	2-10	0.3-1.3
(5)	Pentane	Tube	Up	15	0.9-2.2	2-12	0.3-1.2
(5)	Heptane	Tube	Up	15	1.0-2.4	2-10	0.2-0.9
(5)	Benzene	Tube	Up	15	1.0-2.4	2-9	0.4-1.3
(70)	Water	Tube	Down	16-68	1.8-11.2	1-19	1.3-8.8

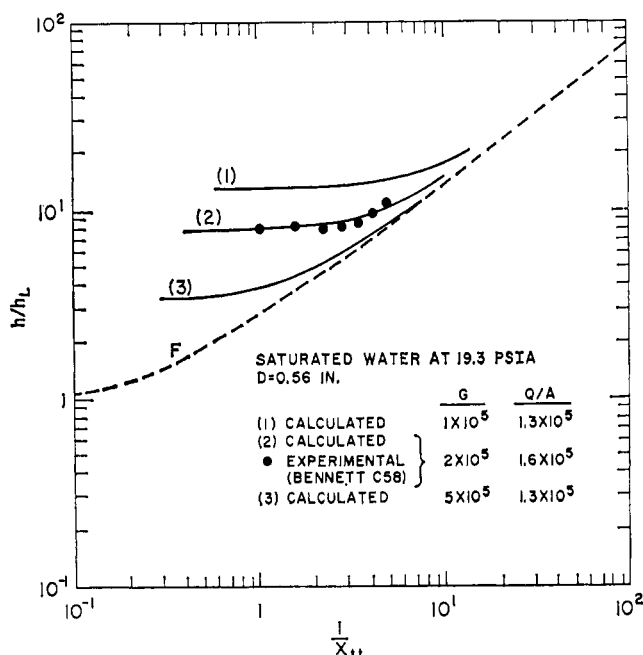


Figure 11. Calculated and measured ratios of h/h_L as functions of the Martinelli parameter

Table I lists the range of experimental conditions covered by the data used in the various comparisons above. Table II summarizes the average deviations for this and for the other correlations when tested against these data. The combined average deviation for this correlation is $\pm 11.6\%$, approximately one third the deviations for the other correlations. Thus for the range of conditions listed in Table I, this correlation markedly improves the chances of accurately predicting two-phase boiling coefficients.

Conclusions

The good agreement between experimentally measured coefficients and those calculated by this correlation appears to verify the postulated principle of interacting micro- and macro-convective contributions to boiling heat transfer, in the annular flow regime. This is further borne out by the excellent agreement in the F function as determined empirically and calculated by momentum transfer analogy. The exact functional

form of the correlation and the numerically determined values of the F and S functions may possibly be further improved as more data and better understanding of bubble dynamics become available. At present, this correlation appears to be more generally applicable and to give accuracy than any other existing correlation.

Appendix. Determination of F Function by Momentum Transfer Analogy

The F function is defined by Equation 8 as

$$F = (Re/Re_L)^{0.8} \quad (A-1)$$

where the two-phase Reynolds number may be written as

$$Re = \frac{DV_{TP}\rho_L}{\mu_L} \quad (A-2)$$

The velocity, V_{TP} , is an effective two-phase velocity, selected in such a way that the macroconvective heat transfer satisfies the usual relationship as expressed by Equation 9.

Considering the case of two-phase flow with heat transfer, the local heat flux and shear stress at any distance y from the wall are expressed by the usual differential relations

$$q = -(k + \rho C_p E_h) \frac{dt}{dy} \quad (A-3)$$

$$\tau g_c = (\mu + \rho E_m) \frac{du}{dy} \quad (A-4)$$

Following Reynolds' analysis, if it is assumed that the shear stress and heat flux profiles are similar,

$$\frac{q(y)}{q_w(y=0)} = \frac{\tau(y)}{\tau_w(y=0)} = f(y) \quad (A-5)$$

then

$$\frac{q_w \cdot f(y)}{\tau_w \cdot f(y)} = -C_p g_c \frac{\left[\left(\frac{k}{\rho C_p} \right) + E_h \right] dt}{\left[\left(\frac{\mu}{\rho} \right) + E_m \right] du} \quad (A-6)$$

Further, assuming equal total diffusivities for heat and momentum transfer, Equation A-6 reduces to

$$\frac{q_w}{C_p} du = -\tau_w g_c dt \quad (A-7)$$

In almost all cases, V_{TP} would correspond to the velocity at a point somewhere beyond the laminar boundary layer. The corresponding local temperature at that value of y would then be approximately equal to the bulk temperature, t_b (or T_s in the case of saturated convective flow). Therefore, Equation A-7 may be integrated to obtain

$$\int_0^{V_{TP}} \frac{q_w}{C_p} du = - \int_{t_w}^{t_b} \tau_w g_c dt$$

$$\frac{q_w}{C_p} V_{TP} = \tau_w g_c (t_w - t_b)$$

$$h_{mac} = \frac{q_w}{t_w - t_b} = \frac{\tau_w C_p g_c}{V_{TP}} \quad (A-8)$$

Considering the liquid fraction flowing alone in the pipe at a superficial velocity V_L , a similar expression is obtained

$$h_L = \frac{\tau_w L C_p g_c}{V_L}$$

Table II. Comparison of Correlations

Data	Average % Deviations for Correlations				
	Dengler and Addoms	Guerrieri and Talty	Bennett et al.	Schrock and Grossman	This correlation
Dengler and Addoms (water)	30.5	62.3	20.0	20.3	14.7
Schrock and Grossman (water)	89.5	16.4	24.9	20.0	15.1
Sani (water)	26.9	70.3	26.5	48.6	8.5
Bennett et al. (water)	17.9	61.8	11.9	14.6	10.8
Guerrieri and Talty (methanol)	42.5	9.5	64.8	62.5	11.3
Guerrieri and Talty (cyclohexane)	39.8	11.1	65.9	50.7	13.6
Guerrieri and Talty (benzene)	65.1	8.6	56.4	40.1	6.3
Guerrieri and Talty (heptane)	61.2	12.3	58.0	31.8	11.0
Guerrieri and Talty (pentane)	66.6	9.4	59.2	35.8	11.9
Wright (water)	24.0	75.8	30.4	51.7	15.4
Combined average for all data	36.0	47.5	32.3	34.7	11.6

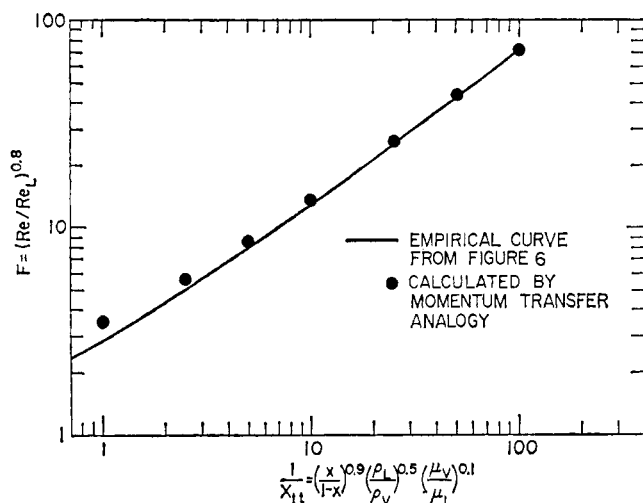


Figure 12. Comparison of F function calculated by momentum transfer analogy to empirical curve

so that the ratio of h_{mac} to h_L may be written

$$\frac{h_{mac}}{h_L} = \frac{\tau_{wTP}}{\tau_{wL}} \times \frac{V_L}{V_{TP}} = \frac{\tau_{wTP}}{\tau_{wL}} \times \frac{Re_L}{Re} \quad (A-9)$$

where

$$Re_L = \frac{DG(1-x)}{\mu_L} \quad (A-10)$$

Writing shear stress in terms of frictional pressure drop and utilizing Equation A-2, one obtains

$$\left(\frac{Re}{Re_L}\right)^{1.8} = \frac{(dP/dz)_{TP}}{(dP/dz)_L} \quad (A-11)$$

Equation A-11 can be used to calculate Re directly if the two-phase pressure drop is known.

Alternatively, any of the available correlations may be used to estimate $(dP/dz)_{TP}$. Thus, according to the Lockhart-Martinelli correlation (ϕ),

$$\frac{(dP/dz)_{TP}}{(dP/dz)_L} = \phi_{Ltt}^2 \quad (A-12)$$

where ϕ_{Ltt} is a known function of X_{tt} . The final results may be written in terms of the ratio

$$F = \left(\frac{Re_{TP}}{Re_L}\right)^{0.8} = \phi_{Ltt}^{0.89} \quad (A-13)$$

Values calculated by this equation are shown in Figure 12, compared to the empirical curve of Figure 6. Good agreement is obtained.

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Nomenclature

- B_o = boiling number, $q/A/\lambda G$
- C_p = heat capacity, (B.t.u.)/(lb.)(° F.)
- D = diameter, ft.
- dP/dL = pressure drop along boiler, p.s.f./ft.
- F = Reynolds number factor, $(Re/Re_L)^{0.8}$
- f = general function
- F_{DA} = Dengler and Addoms' correction factor, see Equation 1
- G = mass flow velocity, (lb.)/(hr.)(sq. ft.)
- g_c = gravitational constant
- h = two-phase heat transfer coefficient, (B.t.u.)/(hr.)(sq. ft.)(° F.)

- h_L' = total liquid heat transfer coefficient, see Equation 1
- h_L = fraction liquid heat transfer coefficient, see Equation 2
- j = Joule's constant, 778 (ft.)(lb.)/(B.t.u.)
- k = thermal conductivity, (B.t.u.)/(hr.)(ft.)(° F.)
- Nu_b = Nusselt number for bubble, see Equation 10
- P = pressure, p.s.f.
- Pr = Prandtl number
- q = local heat flux at y
- q/A = heat flux, (B.t.u.)/(hr.)(sq. ft.)
- r_b = radius of bubble, see Equation 10
- r^* = see Equation 2
- Re = effective Reynolds number for two-phase fluid
- Re_b = Reynolds number for bubble
- Re_L = Reynolds number for liquid fraction = DGz/μ_L
- S = suppression factor, $(\Delta T_e/\Delta T)^{0.99}$
- t = temperature at y
- T = temperature, ° R.
- u = local velocity at y
- x = weight fraction vapor
- X_{tt} = Martinelli parameter, $(z/x)^{0.9}(\rho_v/\rho_L)^{0.5}(\mu_L/\mu_v)^{0.1}$
- y = distance from wall
- z = weight fraction liquid
- V = velocity
- β = Pr/Pr_L
- γ = k/k_L
- δ = see Equation 2
- ΔP = difference in vapor pressure corresponding to ΔT , (p.s.f.)
- ΔT = superheat, $T_w - T_s$
- ΔT_e = effective superheat with flow, see Equation 13
- λ = latent heat of vaporization, (B.t.u.)/lb.)
- μ = viscosity, (lb.)/(ft.)(hr.)
- ρ = density, lb./(cu. ft.)
- σ = vapor-liquid surface tension, lb./ft.
- τ = shear stress
- ϕ_{Ltt} = two-phase pressure drop factor, see Equation A-12

SUBSCRIPTS

- None = value for two-phase fluid
- b = value for bulk
- e = effective value with flow
- L = value for liquid
- s = value at saturation condition
- v = value for vapor
- w = value at wall condition

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