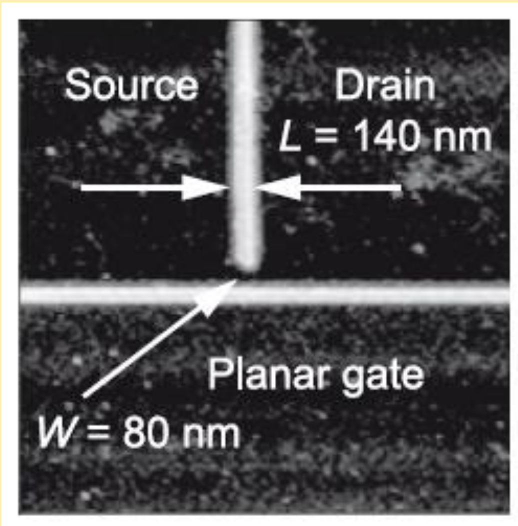
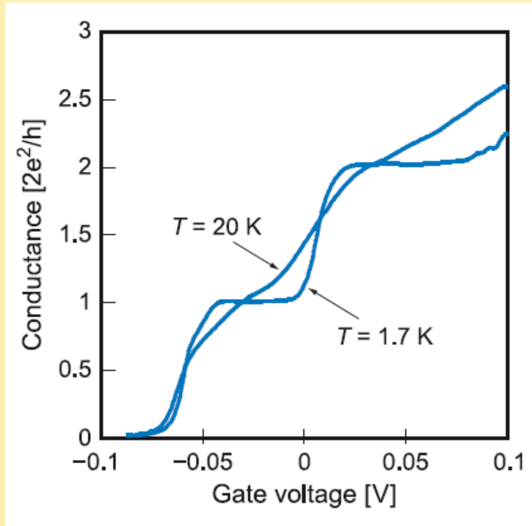
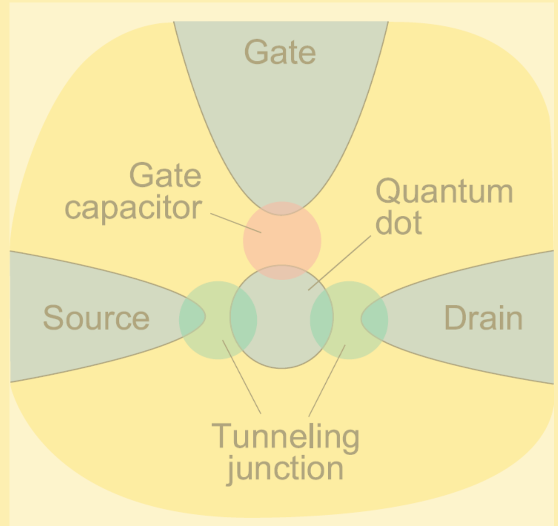
1 Mesoscopic electron transport







Transport in low dimensional structures

When is a system low dimensional?
When do we observe size effects?

Conductance quantization in ballistic 1D structures

Does the conductance diverge when there is no scattering?

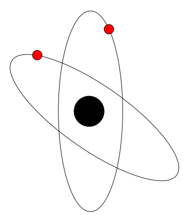
Coulomb blockade in quantum dot structures

Transferring single electrons across a quantum dot

Recap: Mesoscopic systems

mircoscopic < **mesoscopic** < macroscopic

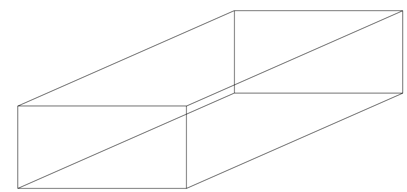
mircoscopic	**mesoscopic**	macroscopic
=	=	=
small	in between	large



atomic scale

too small
to be treated as bulk

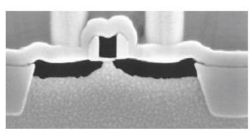
too large (complicated)
to be treated as single atoms



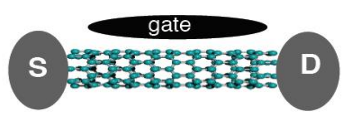
bulk

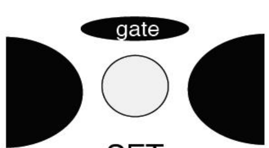
often "engineered" systems

MOSFET

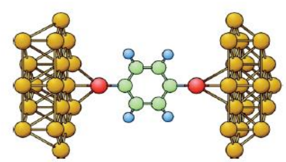


CNT FET

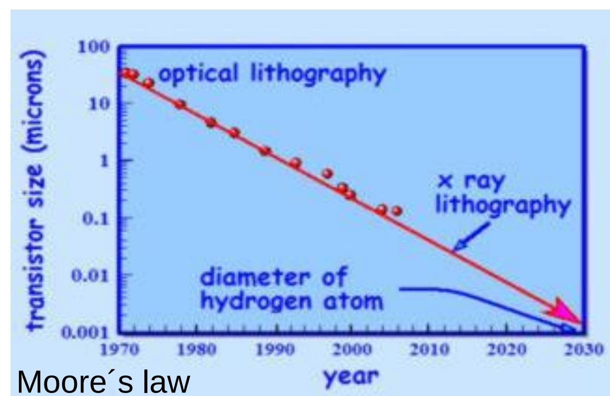




SET

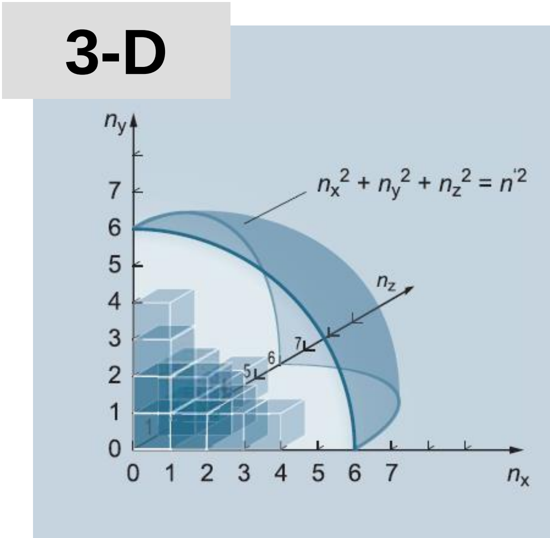


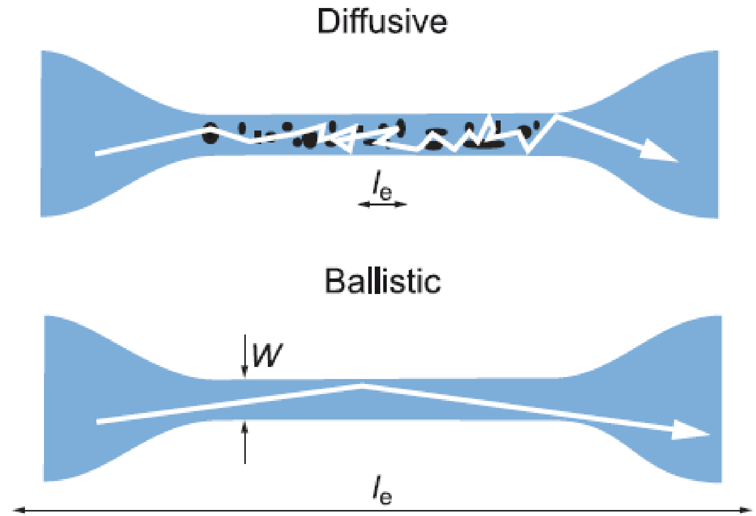
molecular electronics



Moore´s law

Recap: Size quantization and ballistic transport





$$L \leq \lambda_F$$

$$\ell_e \geq L$$

λ_F **is a measure of the length scale on which size quantization becomes important**

Systems with geometrical size smaller than the elastic scattering length (mean free path) are called *ballistic*

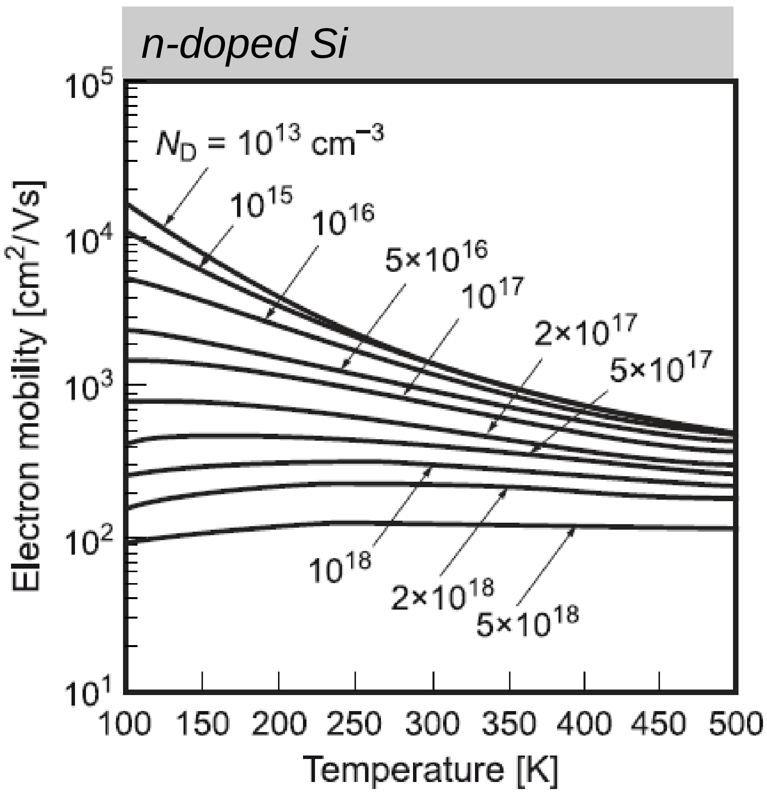
How to construct a ballistic sample?

How to get a ballistic sample from semiconductors?

Challenges in engineering ballistic systems from semiconductors

• *intrinsic* sc provide large ℓ_e, but they loose (all) carriers at low temperature

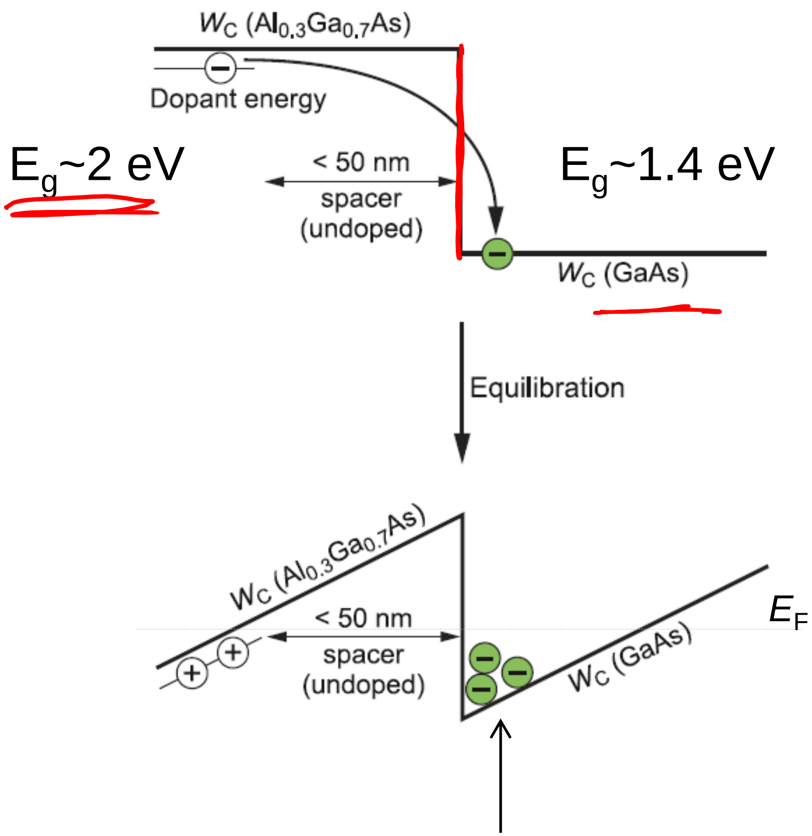
• *n-type* doped sc provide carriers, but the mobility drops



How to get a ballistic sample?

modulation doping – separate carriers and dopants

e.g. at AlGaAs/GaAs interface



2DEG
metal-like
quasi-free electrons

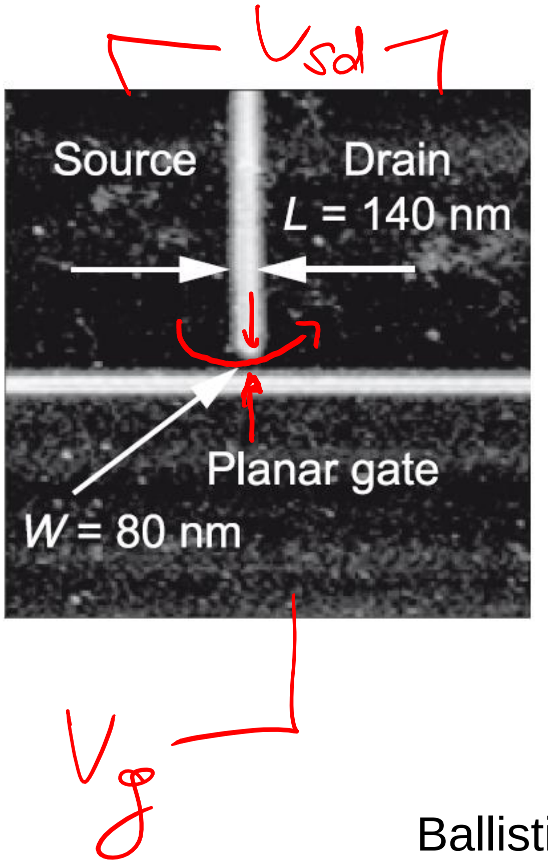
1. Engineer band gap by Ga-Al-ratio
2. Step in conduction band
3. Electrons diffuse to undoped GaAs
4. Emerging charge separation/electric field keeps electrons at the interface separated from positive ions(scatterers) forming 2D gas!

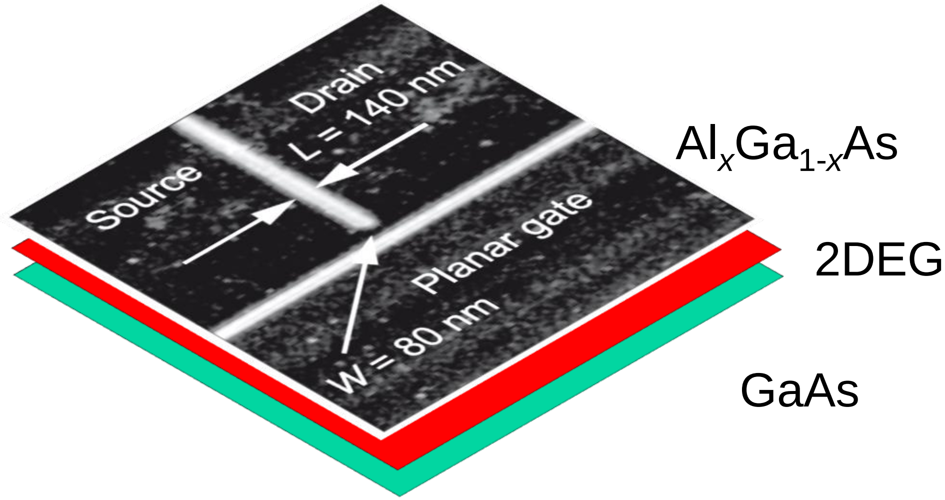
$$\ell_e = v_F \tau \approx 120\,\mu m$$

$$\mu \approx 1.4 \times 10^7 \frac{cm^2}{Vs} \qquad \text{at 4K}$$

$$\tau \approx 400\ ps$$

Electrical transport in ballistic wires (point contacts)

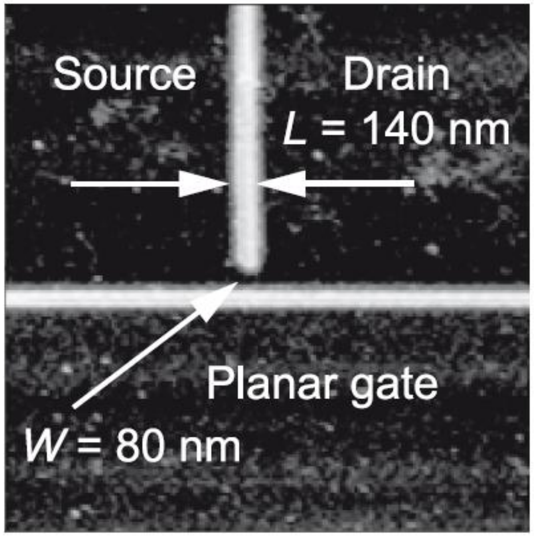


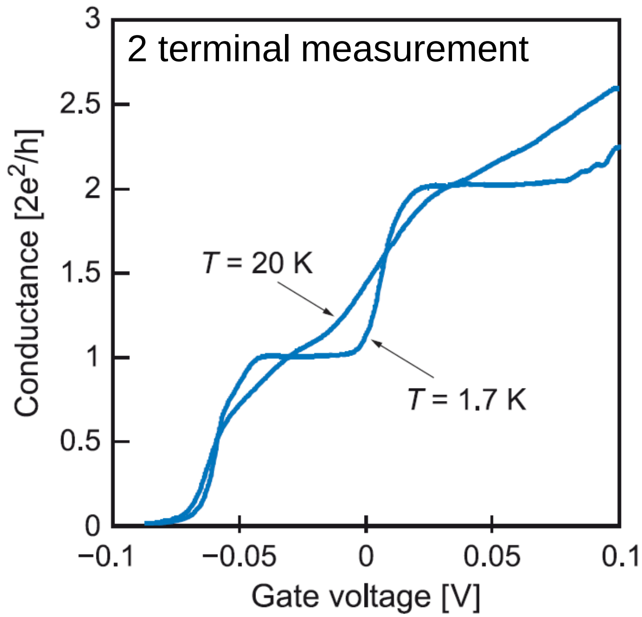


Ballistic channel
length ~ 140 nm << ℓ_e
width ~ 80 nm ~ λ_F

Such a wire is often called *(quantum) point contact (QPC)*

Electrical transport in ballistic wires (point contacts)





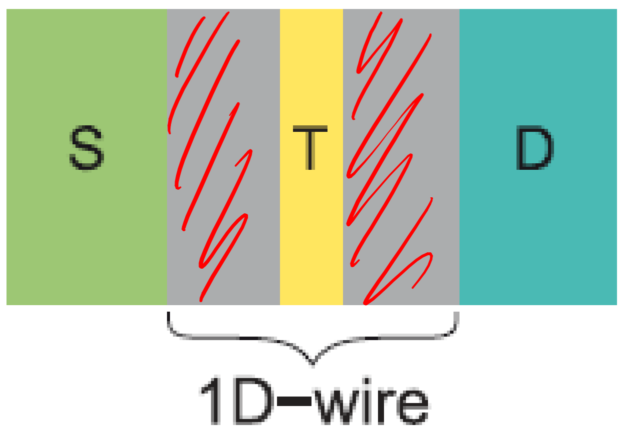
• at low temperatures the conductance of the quantum wire is a step-like function of gate voltage

$$G_0 = \frac{2e^2}{h} = \left(12.9\ \mathrm{k}\Omega\right)^{-1}$$ **conductance quantum**

1.3.3 Conductance quantization in ballistic quantum wires

Electrical transport in ballistic wires (point contacts)

How can we understand this phenomenon?
Where does the resistance come from?



$T = 1$ no transmission barrier

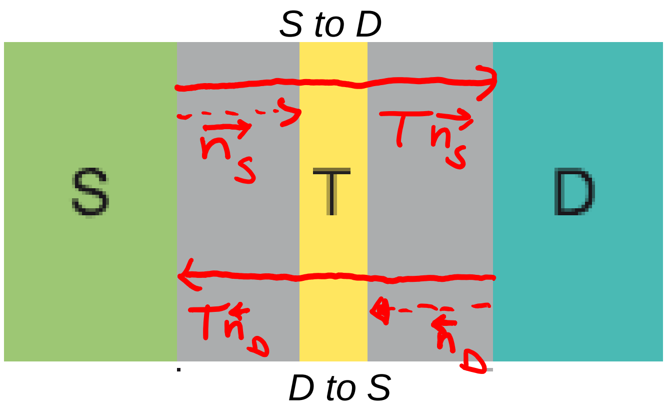
$T = 0$ no transmission

Simple but drastic model:

- source and drain are perfect metal reservoirs
- leads/transition region are ballistic, 1D quantum wires
- QPC is represented by barrier with transmission probability *T*

T=1 means the QPC is open

Electrical transport in ballistic wires (point contacts)

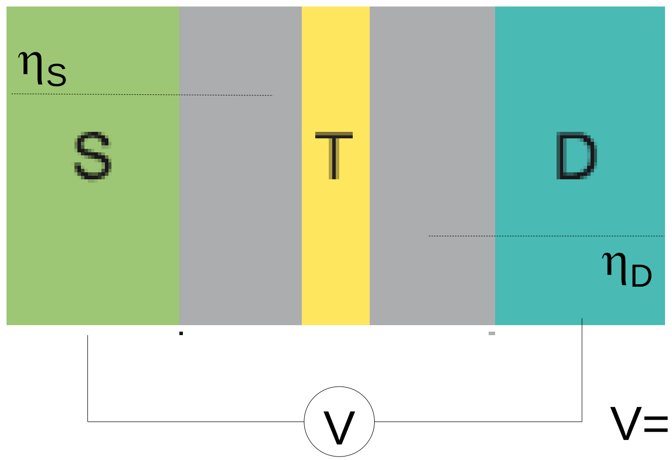


Current through QPC means electrons transverse barrier from left to right (S to D) or from right to left (D to S)

$$I = \int_0^\infty d\omega \; I(\omega)$$

$$I(\omega) = eT\left(\vec{n}_S \cdot v_S - \overleftarrow{n}_D \cdot v_D\right)$$

Electrical transport in ballistic wires (point contacts)



$V=(\eta_D-\eta_S)/e$

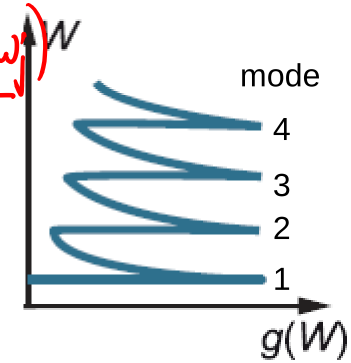
$$\vec{n}_S(W) = \vec{g}_{1D}(W)\, f(W-\eta_S)$$

$$\overleftarrow{n}_D(W) = \overleftarrow{g}_{1D}(W)\, f(W-\eta_D)$$

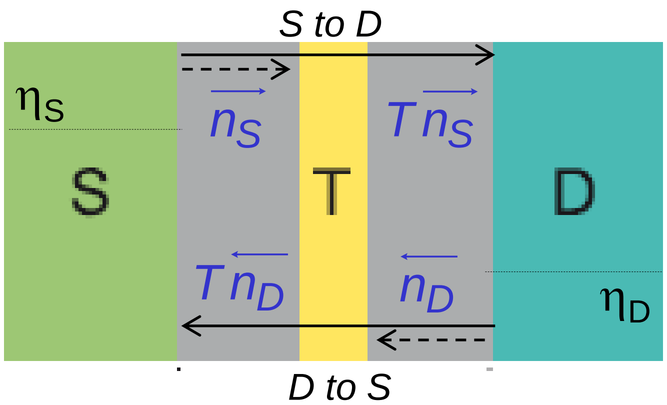
- reservoirs will fill up the 1D levels up to their chemical potential, η_S and η_D
- density of states in wires is 1D

$$g_{1D}(W) = \sum_j \sqrt{\frac{2m^*}{\pi^2\hbar^2}}\,(W-W_j)^{-\frac{1}{2}} \quad ; \quad v_j = \sqrt{\frac{2(W-W_j)}{m^*}}$$

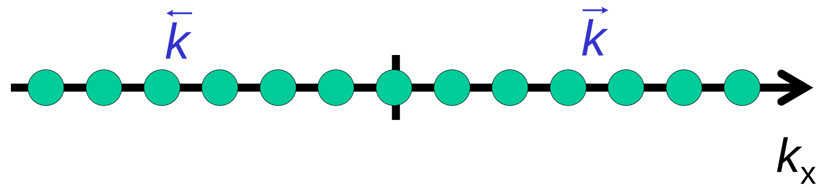
no of activated modes (in quantized dimensions)



Electrical transport in ballistic wires (point contacts)



$$\overrightarrow{g_{1D}}(W) = \overleftarrow{g_{1D}}(W) = \tfrac{1}{2} g_{1D}(W)$$

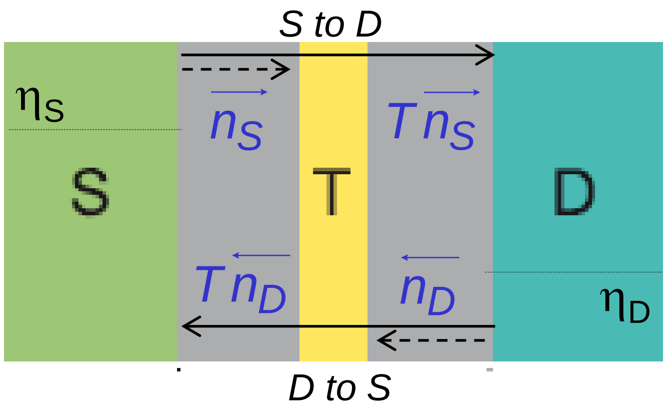


$$\overleftrightarrow{n_{S,D}}(W)\,v_{S,D} = \tfrac{1}{2} g_{1D}(W)\cdot f_{Fermi}(W-\eta_{S,D})\cdot v_{S,D}$$

$$= \sum_j \frac{1}{\cancel{2}}\sqrt{\frac{\cancel{2}\cancel{m}^*}{\pi^2\hbar^2}}\cdot \cancel{(W-W_j)^{-1/2}}\cdot\sqrt{\frac{\cancel{2}\cancel{(W-W_j)}}{\cancel{m}^*}}\cdot f_{Fermi}(W-\eta_{S,D})$$

$$= \sum_j \frac{1}{\pi\hbar}\cdot f_{Fermi}(W-\eta_{S,D})$$

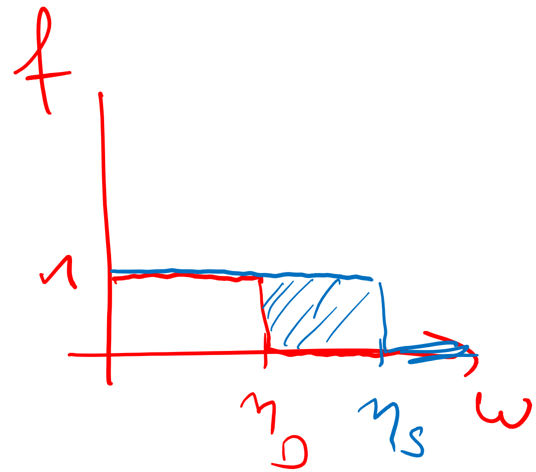
Electrical transport in ballistic wires (point contacts)



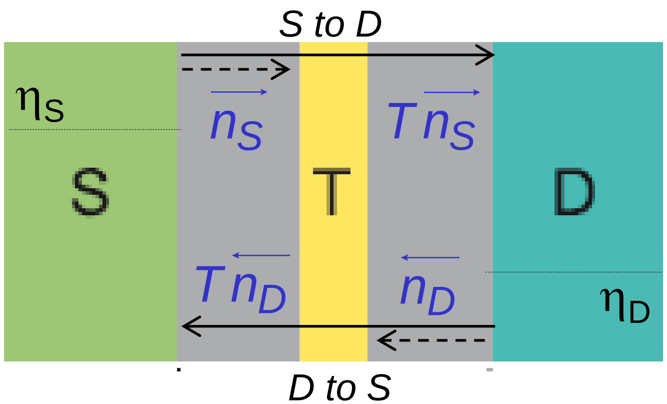
$$I(W) = \frac{2eT}{h}\sum_{\underline{j}} f_{Fermi}(W-\eta_S) - f_{Fermi}(W-\eta_D)$$

$$= \frac{2eT}{h}\underline{M}\cdot\left[f_{Fermi}(W-\eta_S) - f_{Fermi}(W-\eta_D)\right]$$

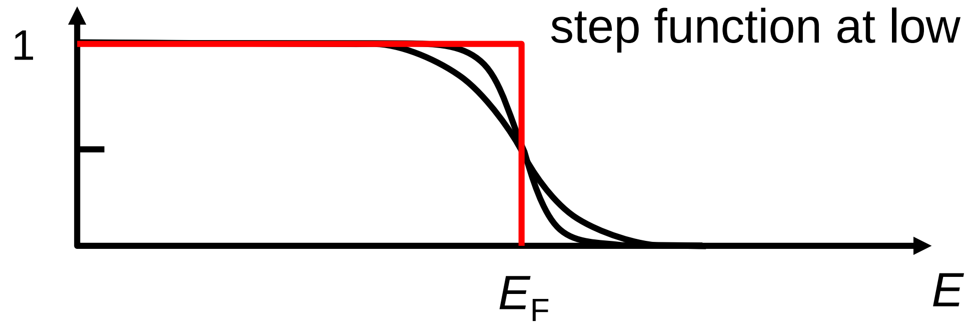
integer No



Electrical transport in ballistic wires (point contacts)



Fermi-function is a step function at low T

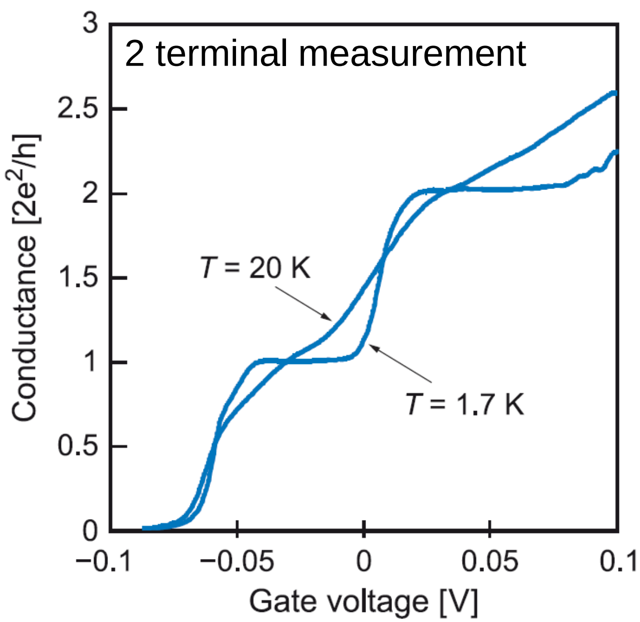


$$I = \int_0^\infty I(\omega)\, d\omega$$

$$= \frac{2e}{h} T M \overbrace{(\eta_S - \eta_D)}^{eV} = \frac{2e^2}{h} T M \cdot V$$

$$\Rightarrow G = \frac{I}{V} = \frac{2e^2}{h} T \cdot M$$

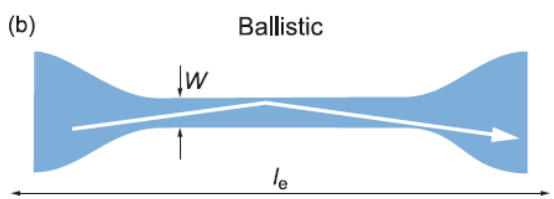
Electrical transport in ballistic wires (point contacts)



G is quantized because the energy dependence of the one-dimensional density of states and that of the electron velocity cancel out

- No of modes (M) can be tuned by gate voltage
→it controls how many 1D sub-bands are filled

- Increased temperature smears out step-like distribution function
→ Steps smear out

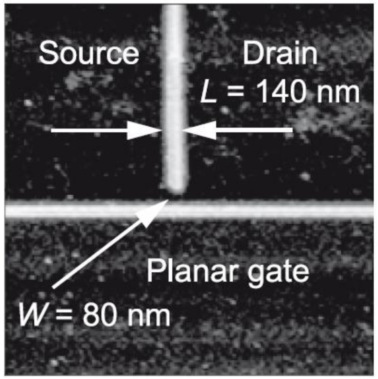


$$G = \frac{I}{V} = \frac{2e^2}{h} TM$$

Landauer Formula

no barrier means T→1
M is number of activated modes

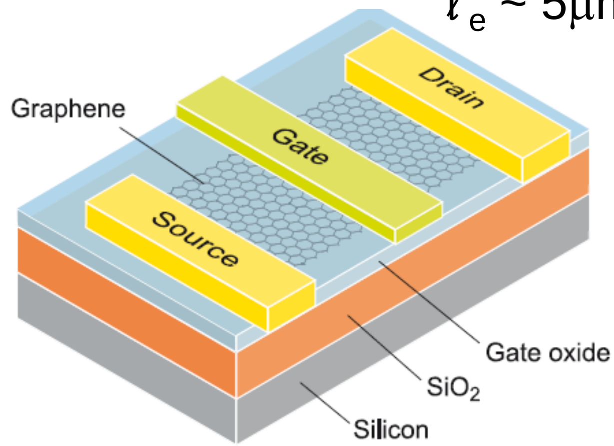
Other ballistic systems



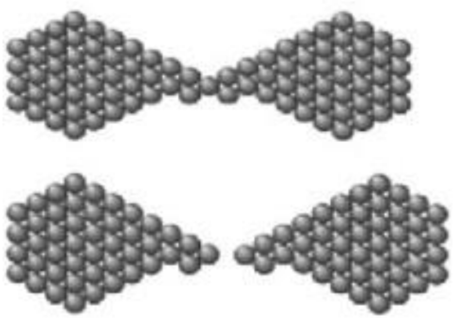
QPC in semiconductor devices

graphene

$\ell_e \sim 5\mu m$



Breaking metal contacts…

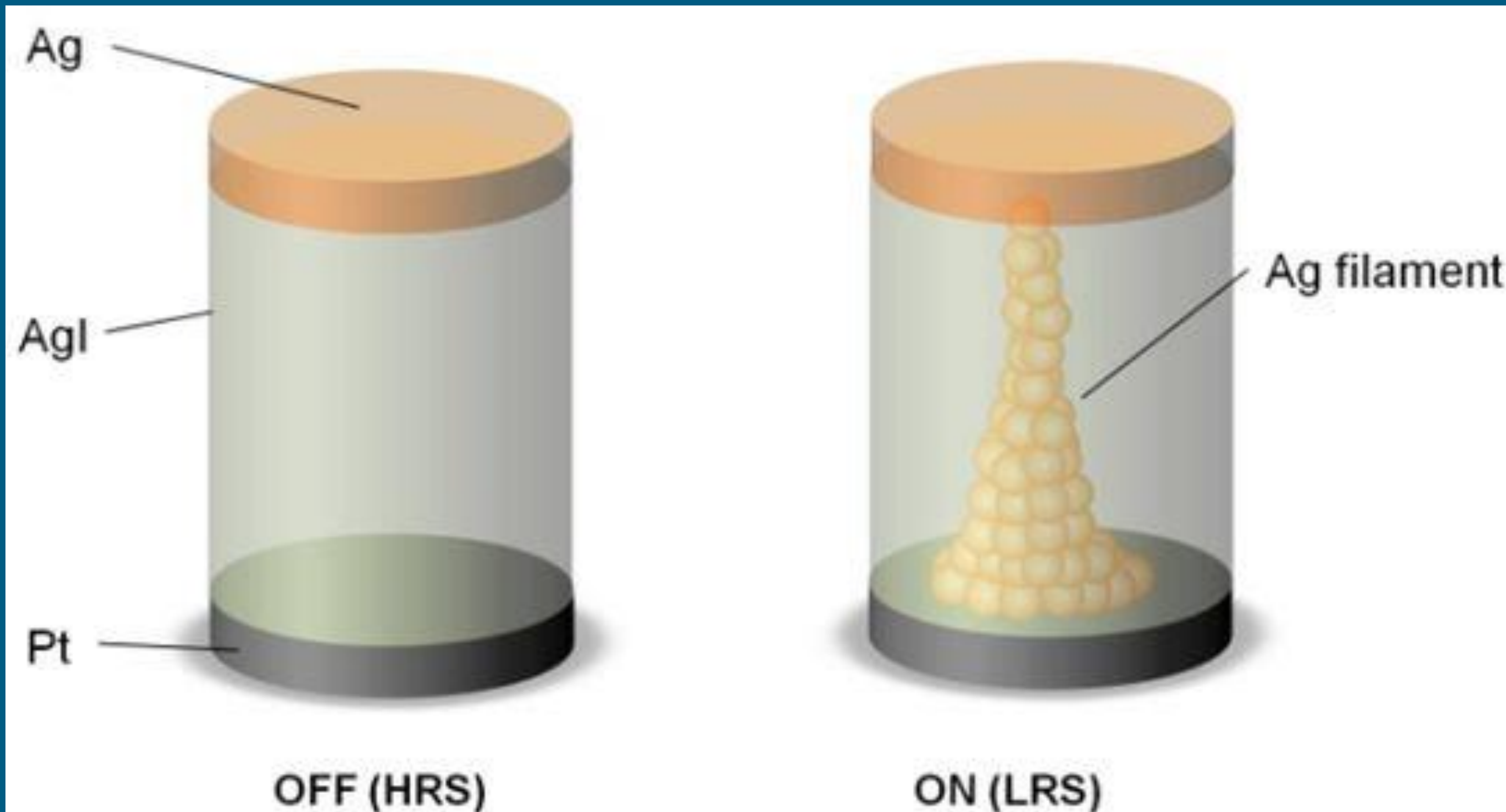


CNT FET

Carbon nanotubes

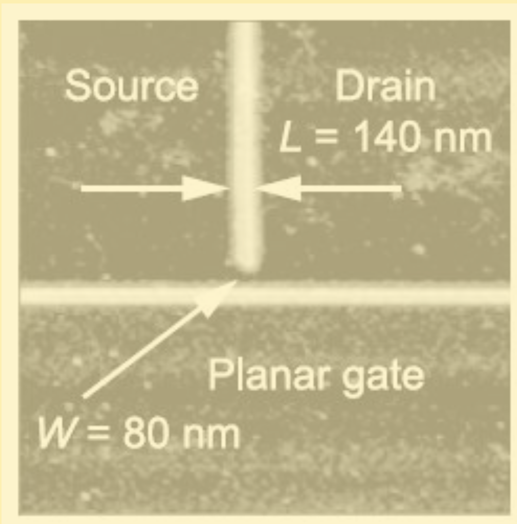
Atomic switch – a novel data memory device

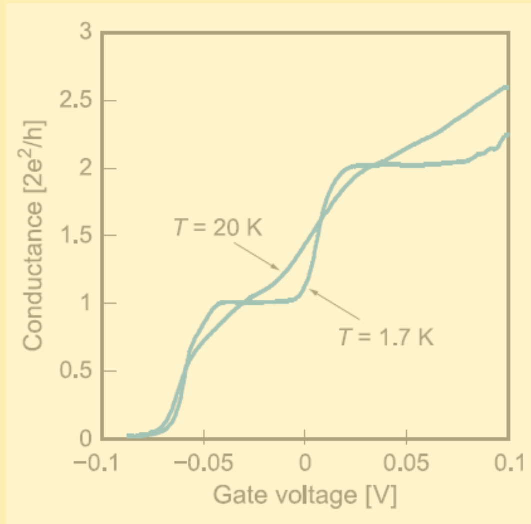
„0“ „1“, („2“, „3“, …)

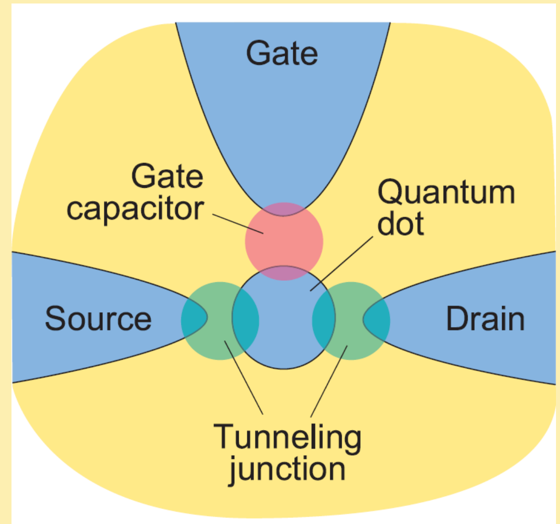


…addressed/controlled by external voltage…

1 Mesoscopic electron transport







Transport in low dimensional structures

When is a system low dimensional?
When do we observe size effects?

Conductance quantization in ballistic 1D structures

Does the conductance diverge when there is no scattering?

Coulomb blockade in quantum dot structures

Transferring single electrons across a quantum dot

1.3.5 Single electron charging – charge quantization

Conventional capacitors

So far, we considered free electrons,
however, if we confine electrons into a small volume there will be Coulomb repulsion

→Electrostatic energy can be expressed by the capacitance

$$W_Q = \frac{Q^2}{2C}$$

capacitance

charge

$$C = \frac{Q}{U}$$

voltage

C ~ 100pF

How much does charge change when voltage is varied? → Say we increase voltage 1mV

$$\Delta Q = C \cdot \Delta U = 100pF \cdot 1mV = 10^{-10} \cdot 10^{-3} C = 10^{-13} C = 10^{6} e$$

Charge is a continuous quantity in conventional capacitors

Capacitance of particles

Coulomb energy of a charged particle

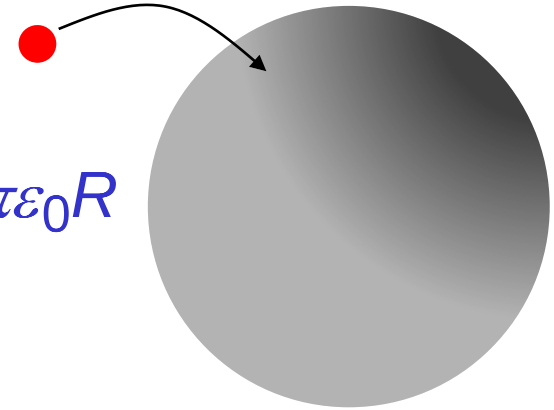
$$W_Q = \frac{Q^2}{2C}$$; if $C \searrow$ $W_Q \nearrow$

Generally W_Q increases with decreasing feature size

Radius of sphere [nm]	Capacitance $C=4\pi\varepsilon_0 R$ [F]	Energy W_Q of single electron on capacitor [meV]	Corresponding temperature $T=W_Q/k_B$ [K]
10000	1.1×10^{-15}	0.07	0.83
1000	1.1×10^{-16}	0.7	8.3 (LH2)
100	1.1×10^{-17}	7	83 (LN2)
10	1.1×10^{-18}	70	830
1	1.1×10^{-19}	700	8300

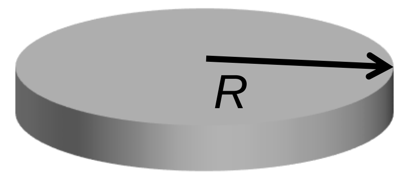
$$C_{sphere} = 4\pi\varepsilon_0 R$$

charged sphere



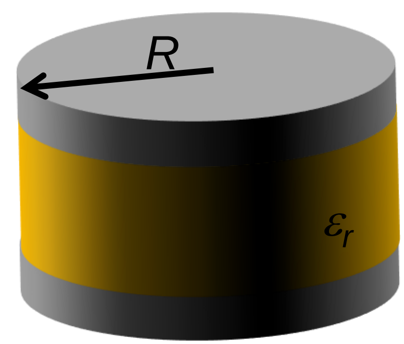
$$C_{disc} = 8\varepsilon_0 R$$

charged disc



$$C_{ppc} = \varepsilon_0 \varepsilon_r \frac{\pi R^2}{d}$$

parallel-plate capacitor



Relevant temperature range to observe single electron charging

if size becomes low, C becomes small and the *single electron Coulomb energy* may determine the *charge state of the particle* → **which then is quantized**

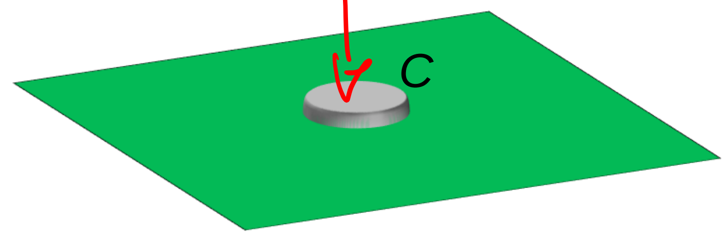
Charge quantization can be observed if the *relevant energy scale*

$$W_C = \frac{e^2}{C}$$

let us consider 10–100 nm metal island

exceeds the *thermal energy*

$$\frac{e^2}{C} \gg k_B T$$



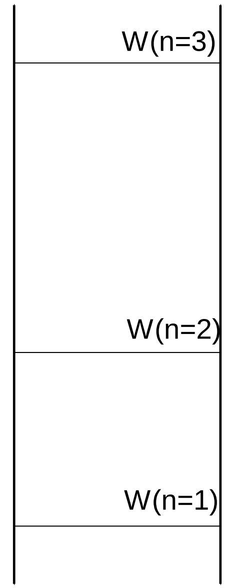
one usually talks about
Quantum dot
→ This does not necessarily mean that the DOS is discrete (size quantization)!

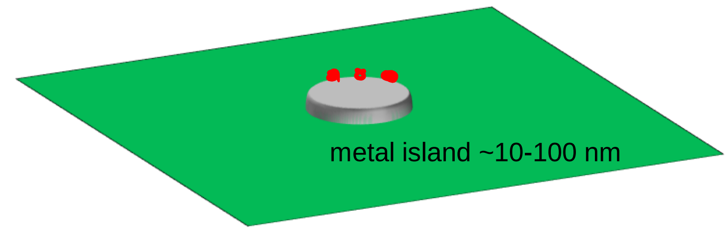
Thermal fluctuations of the charge on the particle are suppressed!

Coulomb energy of (small) charged particles

$$W_Q = W_Q(n) = \frac{e^2 n^2}{2C}, \quad n = \text{No of electrons on QD}$$

$$\Delta W_Q = W_Q(n) - W_Q(n-1) = \frac{e^2}{C} n - \frac{e^2}{2C}$$

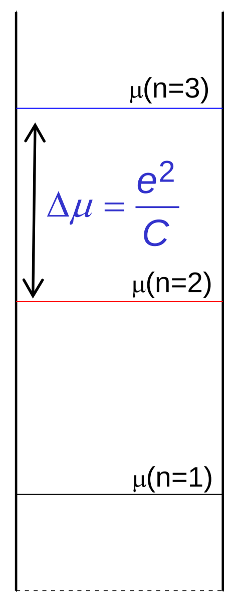




$$\mu := \frac{\partial W}{\partial n}$$

Chemical potential

$$\mu(n) = \frac{\Delta W_Q}{\Delta n} = \frac{e^2}{C} n - \frac{e^2}{2C}$$



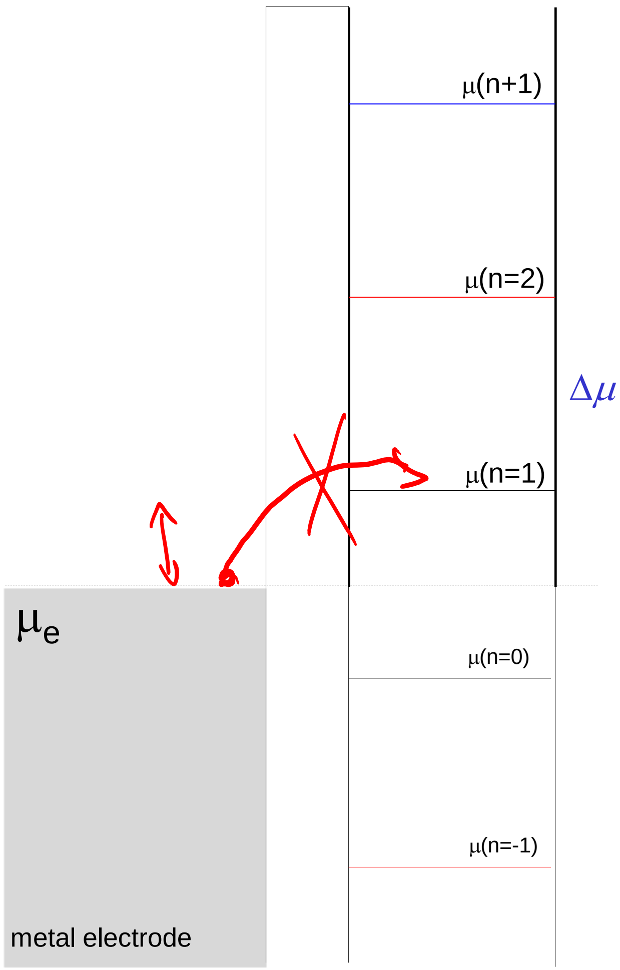
The chemical potential of the system takes
- discrete values
- equidistant values

$$W_C = \Delta\mu = \frac{e^2}{C}$$

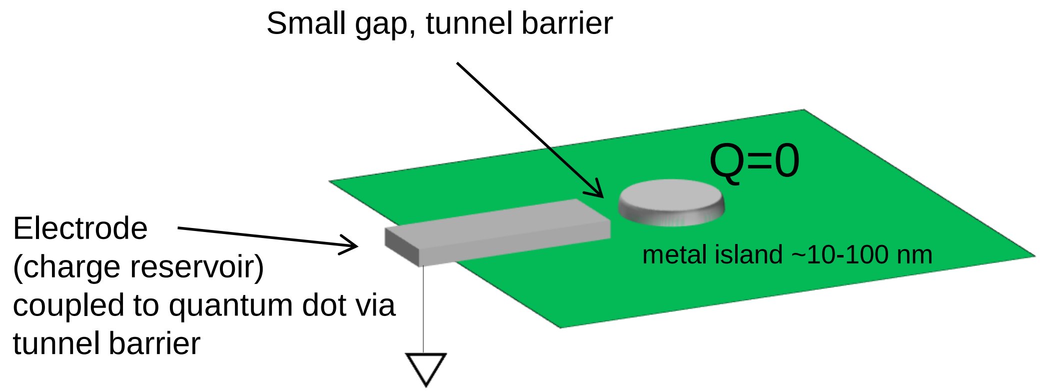
Charging energy

Single electron box

How can we charge the quantum dot?



$$\Delta\mu = \frac{e^2}{C}$$



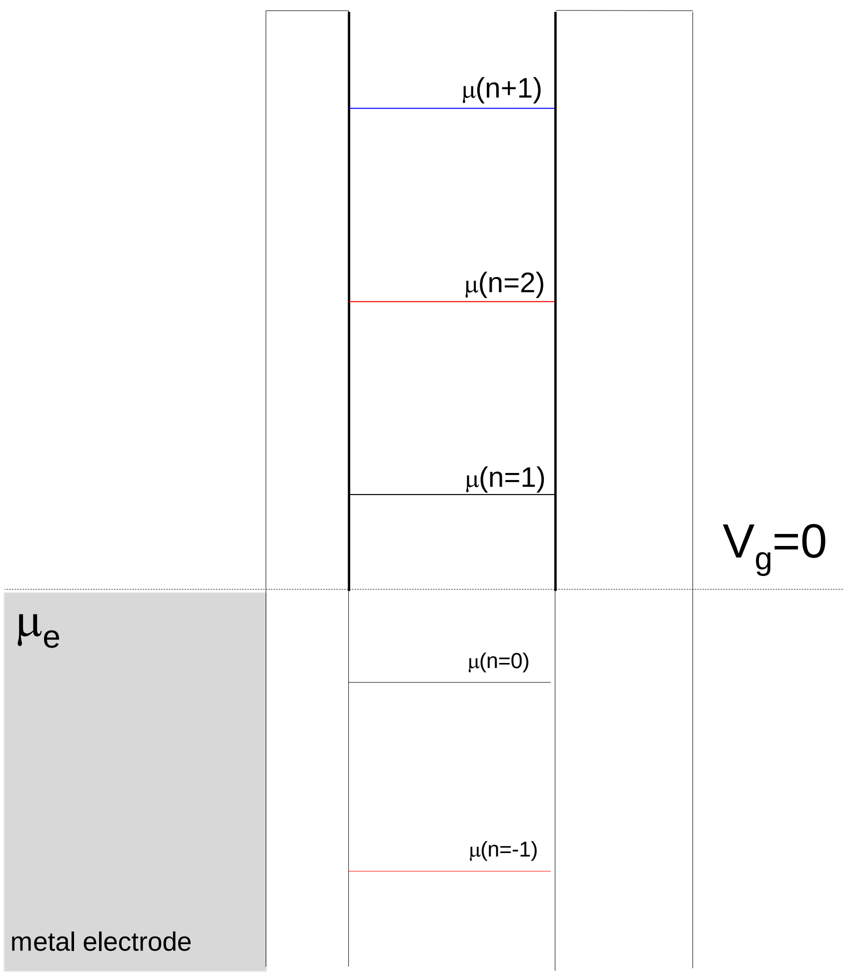
$$\mu(n) = \frac{e^2}{C}n - \frac{e^2}{2C}$$

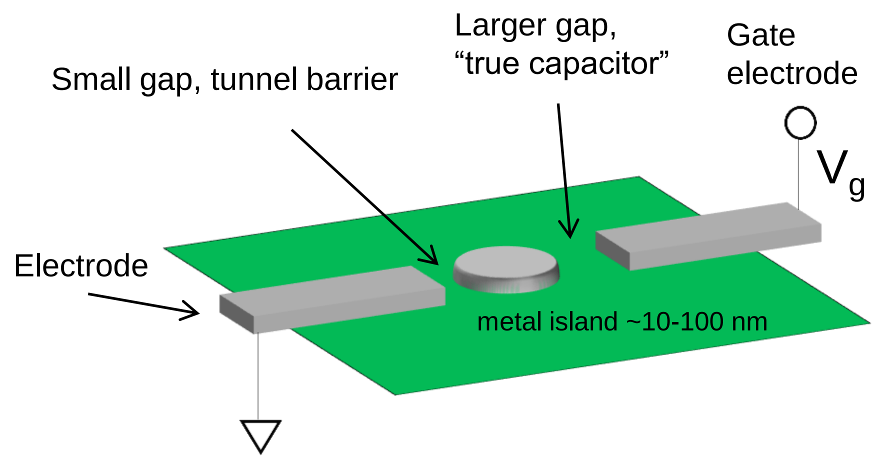
In equilibrium, *no tunneling into/out of* QD is allowed due to energetic offset of μ(n) in the QD

$$\frac{e^2}{C} \gg k_B T$$

Single electron box

How can we charge the quantum dot?





Single electron box

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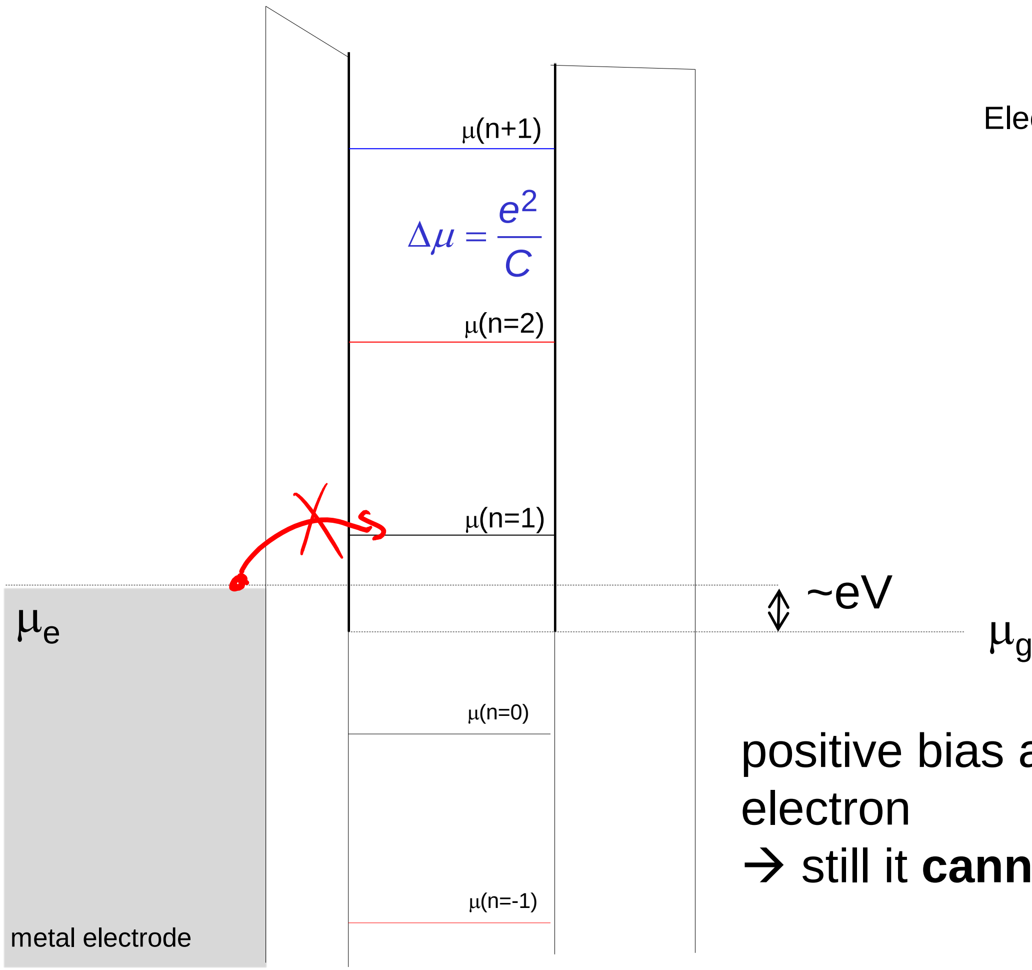
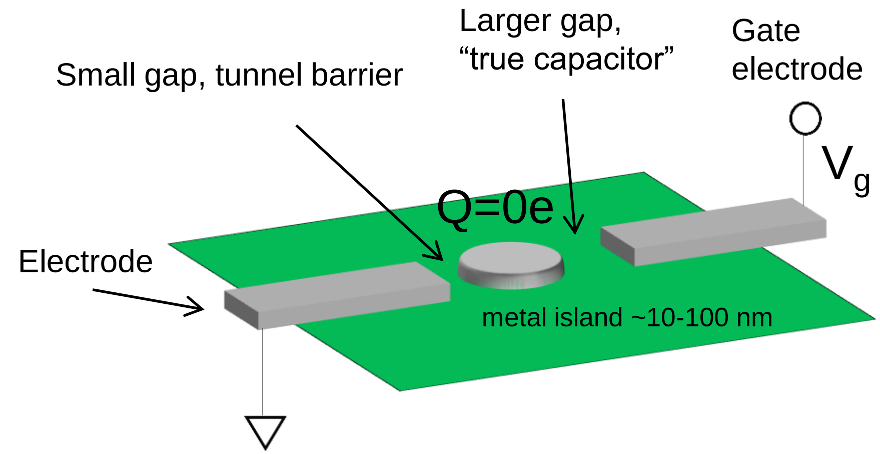
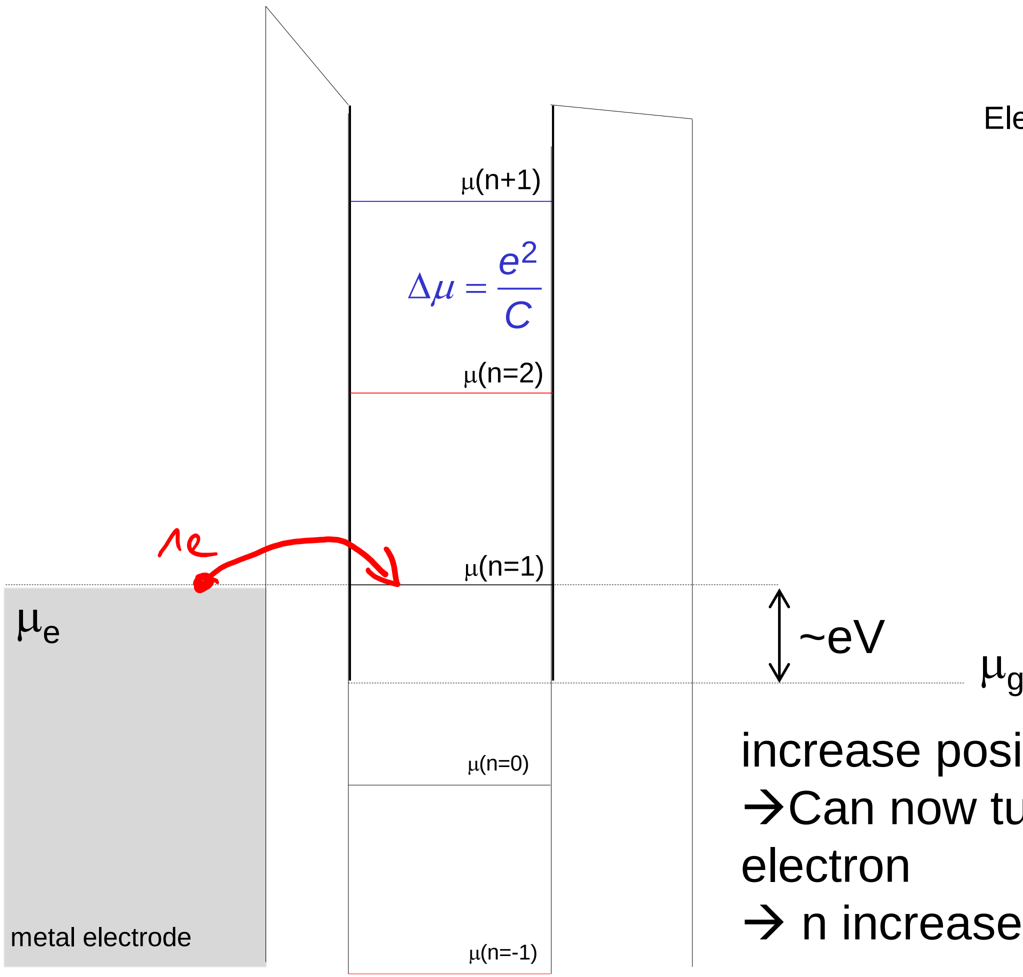


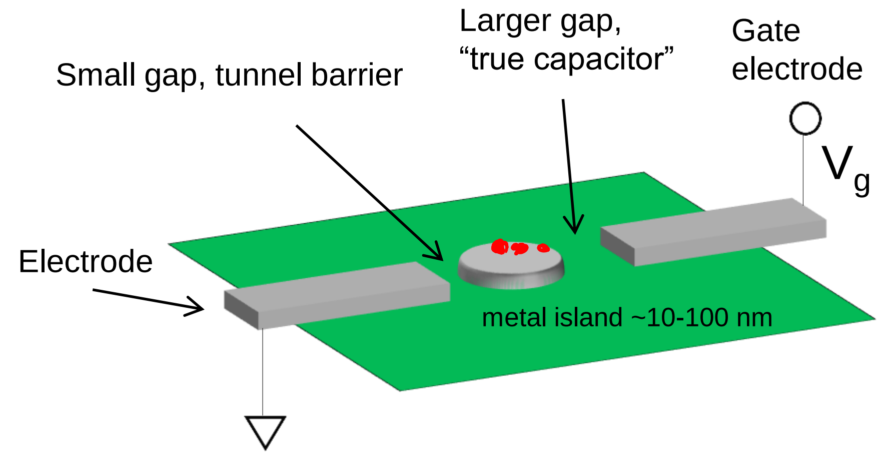


positive bias attracts electron
→ still it **cannot** tunnel

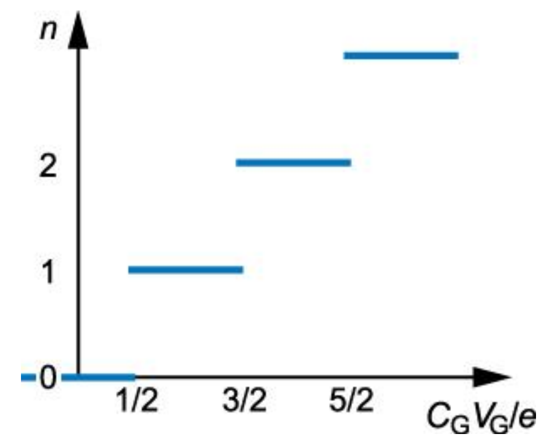
Single electron box

How can we charge the quantum dot?

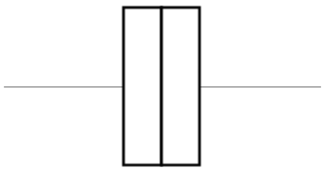




increase positive bias
→Can now tunnel electron
→ n increases

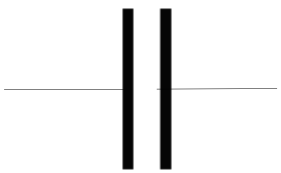


Single electron box – quantitative model



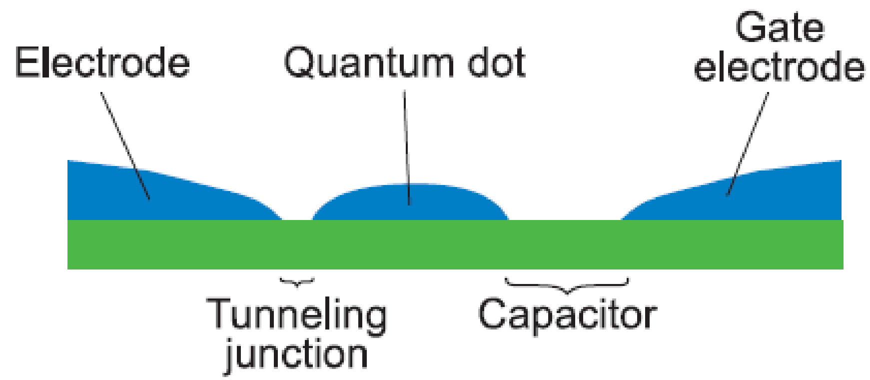
tunnel resistance

C_t, R_t



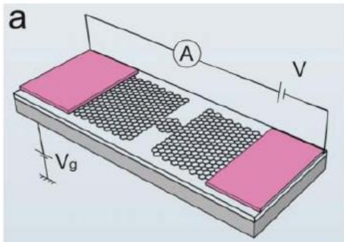
true capacitor

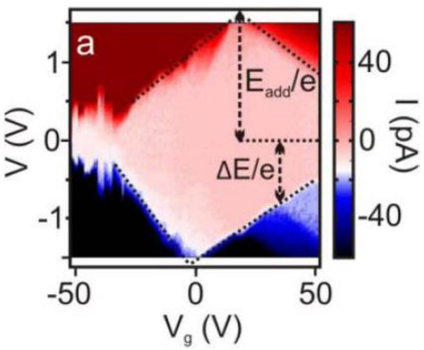
C_G, R_G=inf



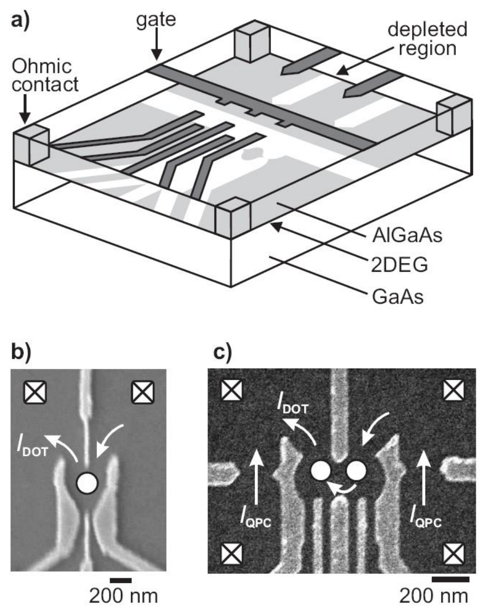
… to be continued

Coulomb blockade - examples

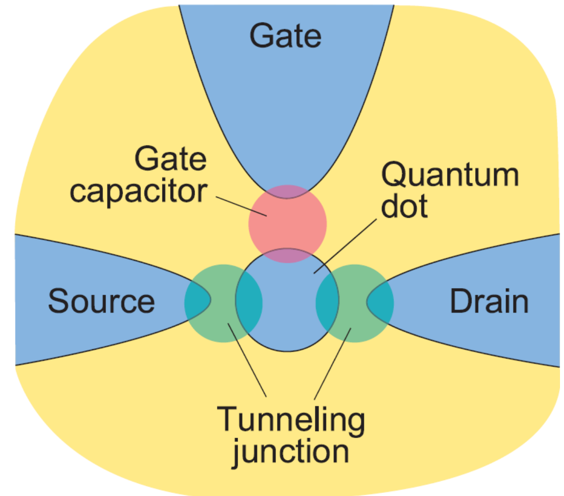


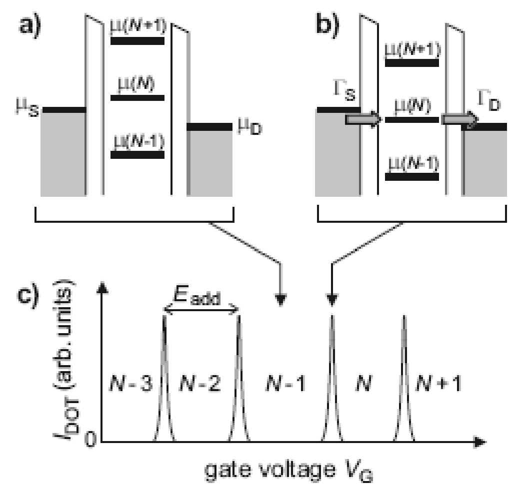


A. Barreiro, H.S.J. van der Zant, L.M.K. Vandersypen, “Quantum Dots at Room Temperature carved out from Few-Layer Graphene”, arXiv:1211.4551 (21 Nov. 2012)



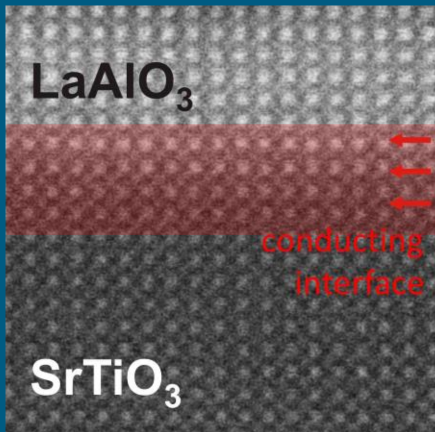
Hanson et al., Rev. Mod. Phys. 79, 1217 (2007)





Hanson et al., Rev. Mod. Phys. 79, 1217 (2007)

Single electron box and transistor realized in oxide heterostructures



2DEG in oxide heterostructure (cf. previous lecture)

Sketched oxide single-electron transistor
G. Cheng, P. F. Siles, F. Bi, C. Cen, D. F. Bogorin, C. Bark, C. M. Folkman, J.-W. Park, C.-B. Eom, G. Medeiros-Ribeiro, J. Levy, Nature Nanotechnology 6, 343–347 (2011)