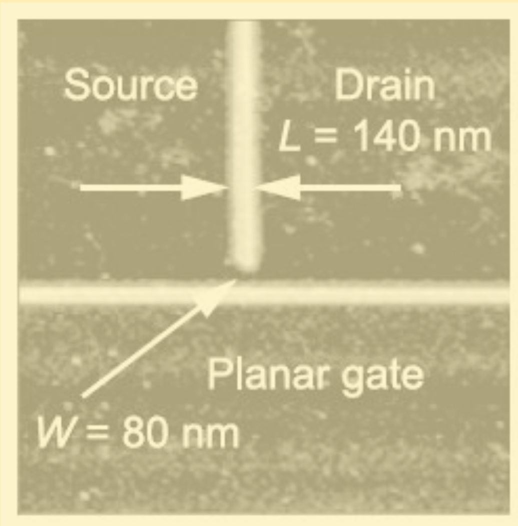
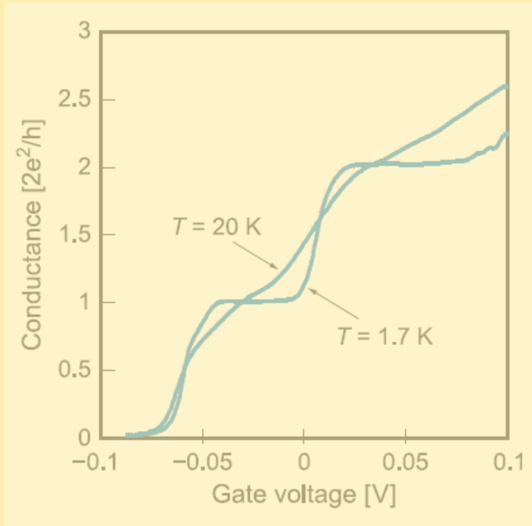
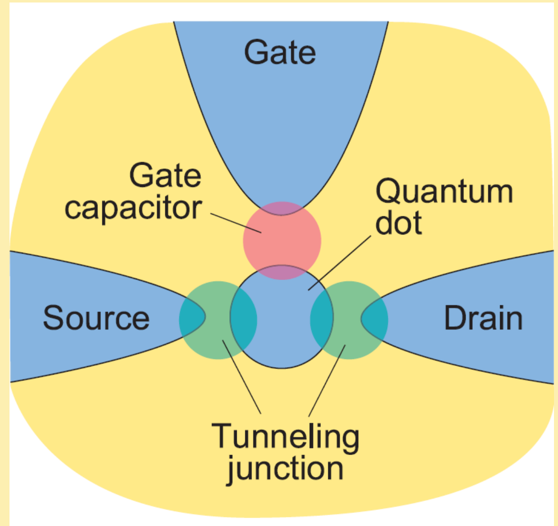
1 Mesoscopic electron transport







Transport in low dimensional structures

When is a system low dimensional?
When do we observe size effects?

Conductance quantization in ballistic 1D structures

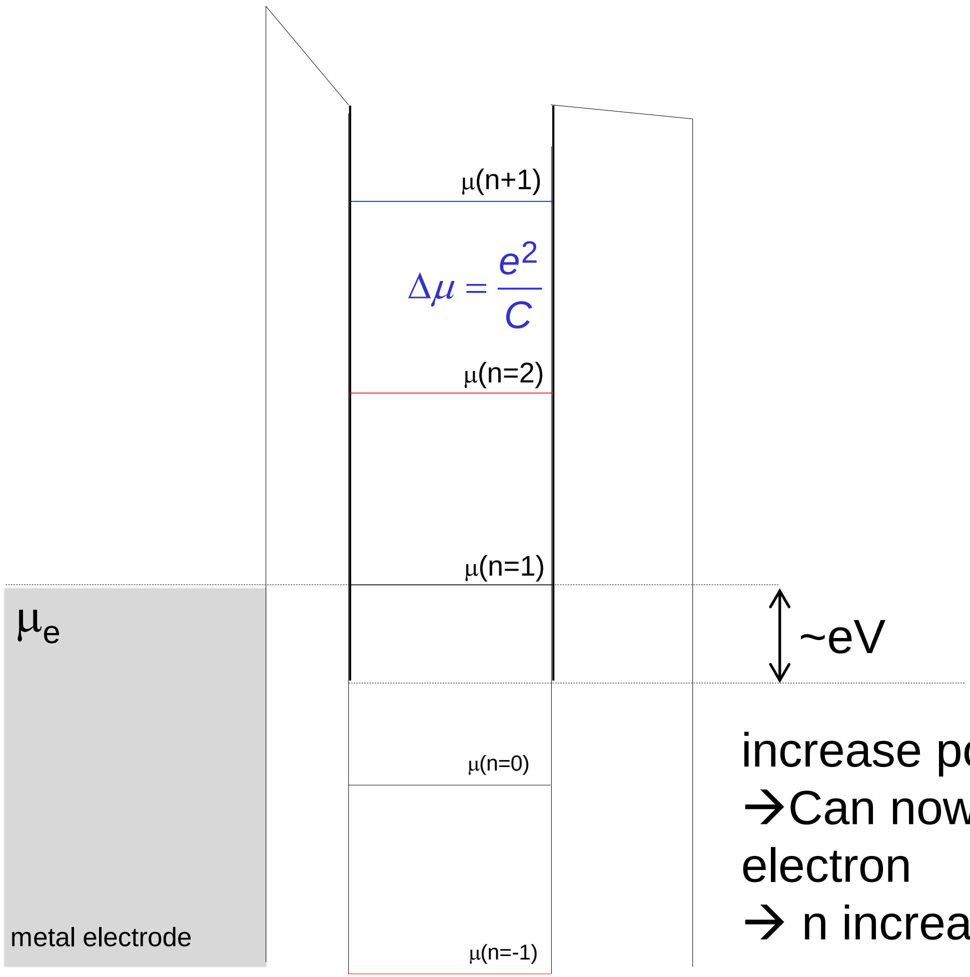
Does the conductance diverge when there is no scattering?

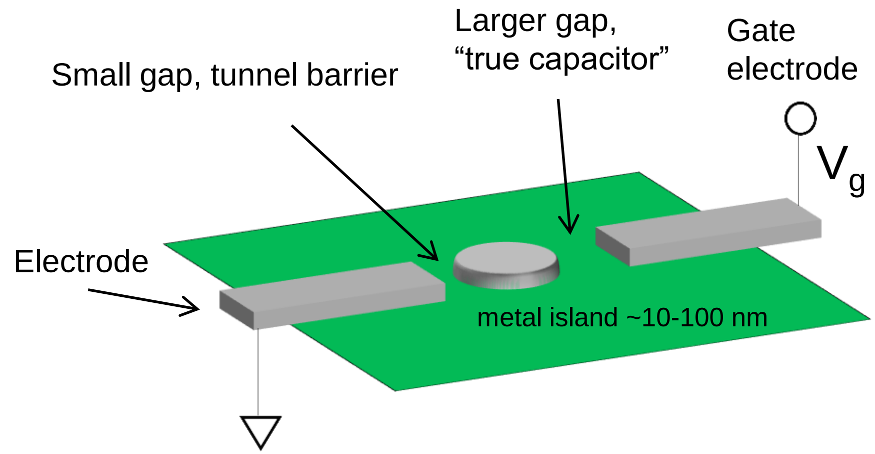
Coulomb blockade in quantum dot structures

Transferring single electrons across a quantum dot

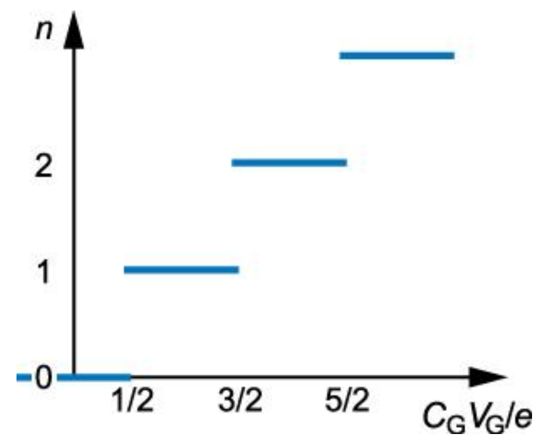
Recap: Single electron box

How can we charge the quantum dot?



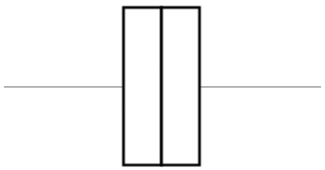


increase positive bias
→Can now tunnel electron
→ n increases



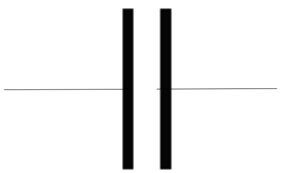
Single electron box – quantitative model

Equivalent circuit: QD is *coupled* to the surrounding via



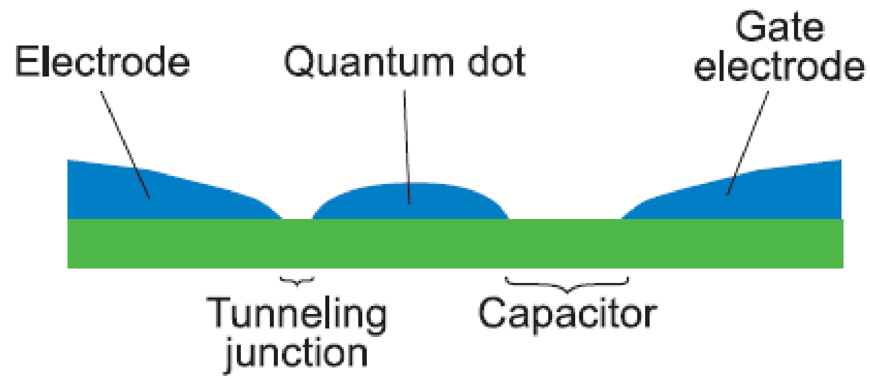
tunnel resistance

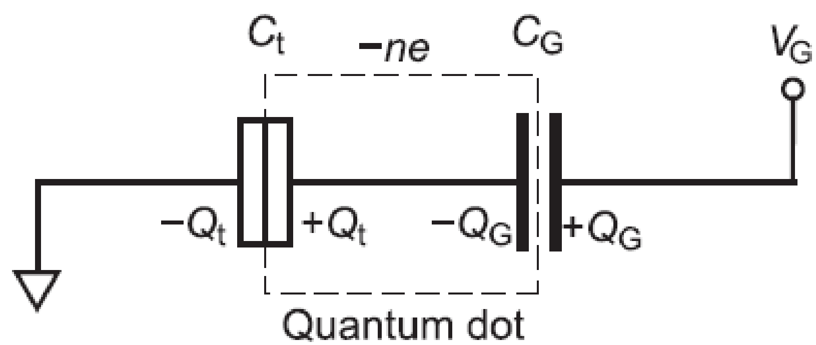
C_t, R_t



true capacitor

C_G, R_G=inf





- applying a gate voltage will produce charge Q_G
- system will try to form counter charge
- Charge on QD is *quantized,* Q_{dot}=-ne, while charge on capacitors is continuous

→ *A second (continuous) charge is formed at the tunnel resistance,* Q_t

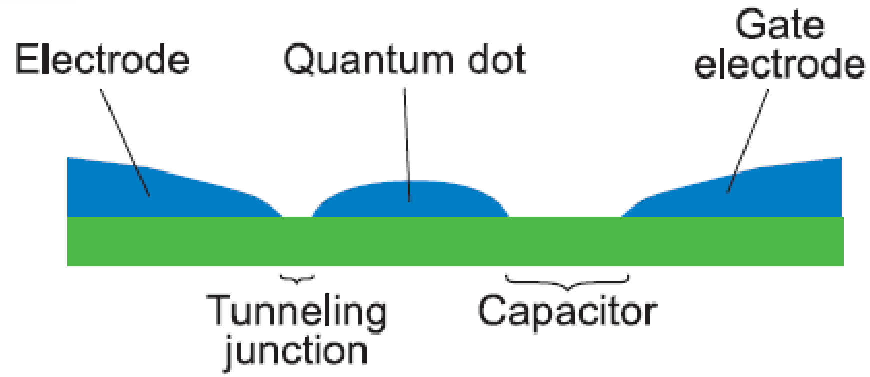
Single electron box – quantitative model

What is the free energy of the system when we apply a gate voltage?

$$F(n) = W_Q(n) - A(n)$$

electrostatic energy stored in the two capacitors

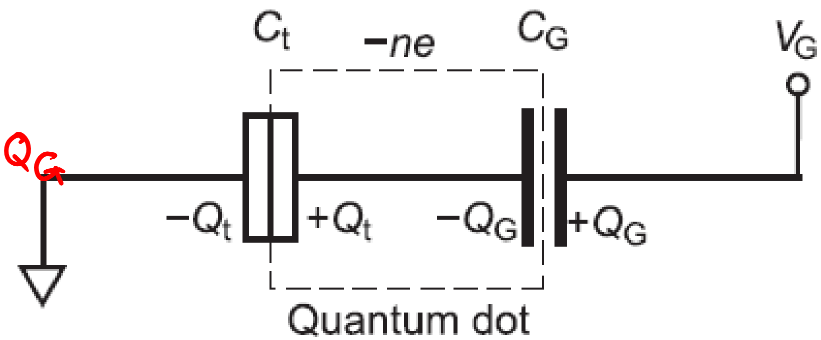
Work done by external voltage source to get C_G charged



$$W_Q(n) = \frac{Q_t^2}{2C_t} + \frac{Q_G^2}{2C_G}$$

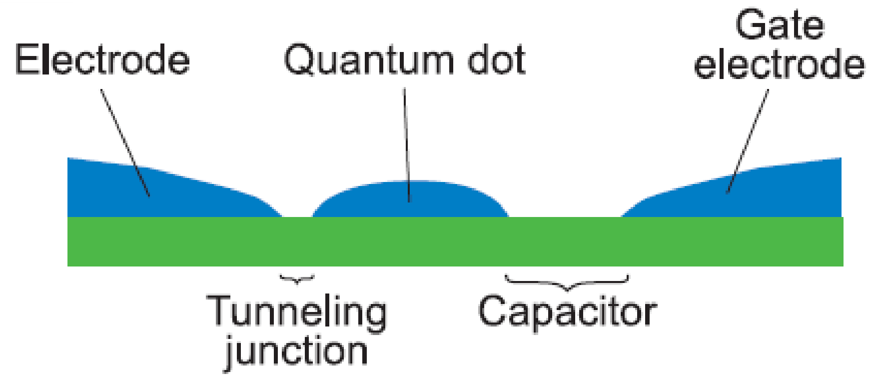
use $I = \frac{dQ}{dt}$

$$A(n) = \int_0^\infty dt\, I(t) \cdot V_G = V_G \int_0^{Q_G} dQ = V_G \cdot Q_G$$



Single electron box – quantitative model

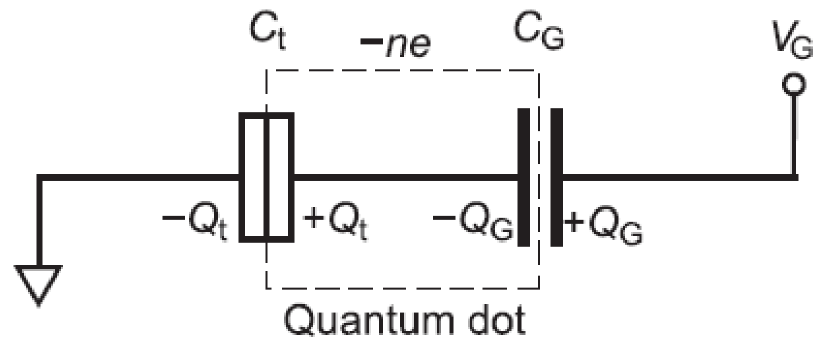
What is the free energy of the system when we apply a gate voltage?



Boundary conditions

Charge balance

$$Q_t - Q_G = -en$$

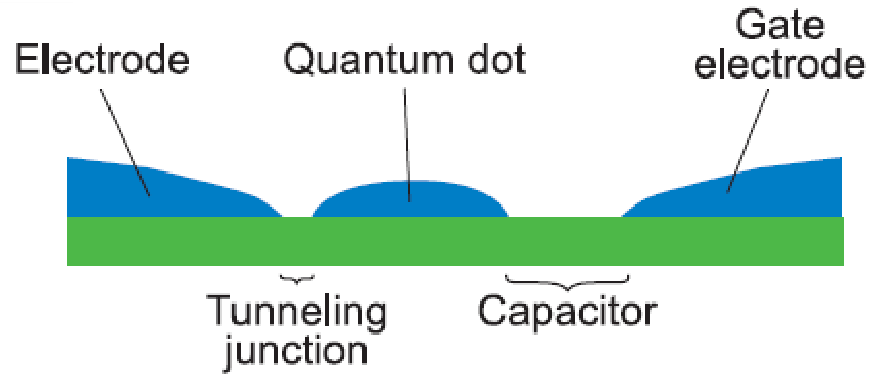


Voltage drop

$$V_G = \frac{Q_t}{C_t} + \frac{Q_G}{C_G}$$

Single electron box – quantitative model

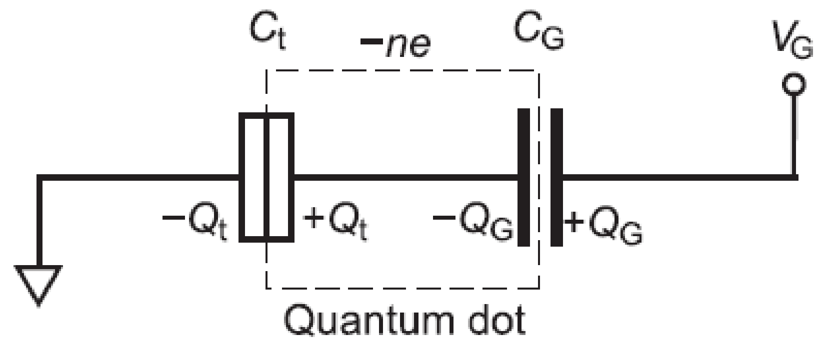
What is the free energy of the system when we apply a gate voltage?



From boundary conditions it follows

$$Q_t = \frac{C_t C_G}{C_\Sigma}\left(V_G - \frac{en}{C_G}\right)$$

$$Q_G = \frac{C_t C_G}{C_\Sigma}\left(V_G + \frac{en}{C_t}\right)$$

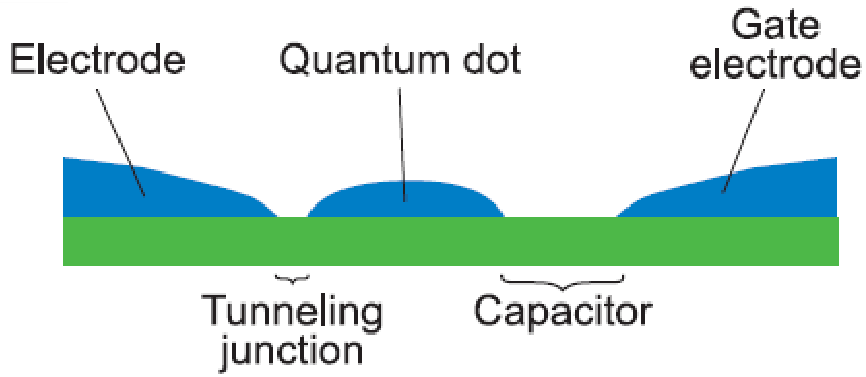


with $C_\Sigma = C_t + C_G$

take Q_t and Q_G and insert into $F(n)$

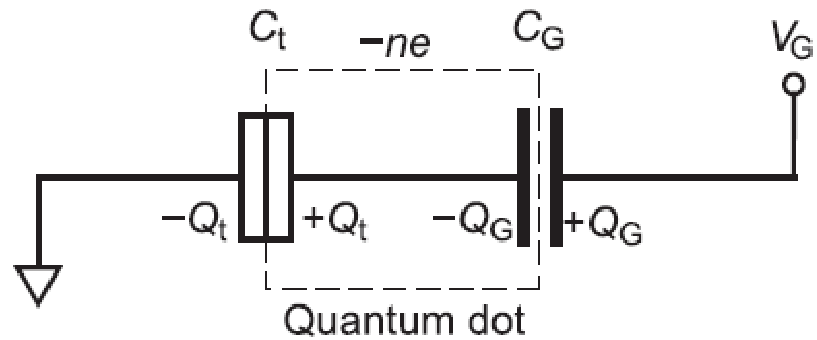
What is the free energy of the system when we apply a gate voltage?

Plugging in Q_t and Q_G, it follows



$$W_Q(n) = \frac{1}{2C_\Sigma}(en)^2 + \frac{C_t C_G V_G^2}{2C_\Sigma}$$

$$= \frac{Q_t^2}{2C_t} + \frac{Q_G^2}{2C_G}$$



$$A(n) = \frac{C_t C_G}{C_\Sigma} V_G^2 + \frac{C_G}{C_\Sigma} V_G (en)$$

$$= V_G \cdot Q_G$$

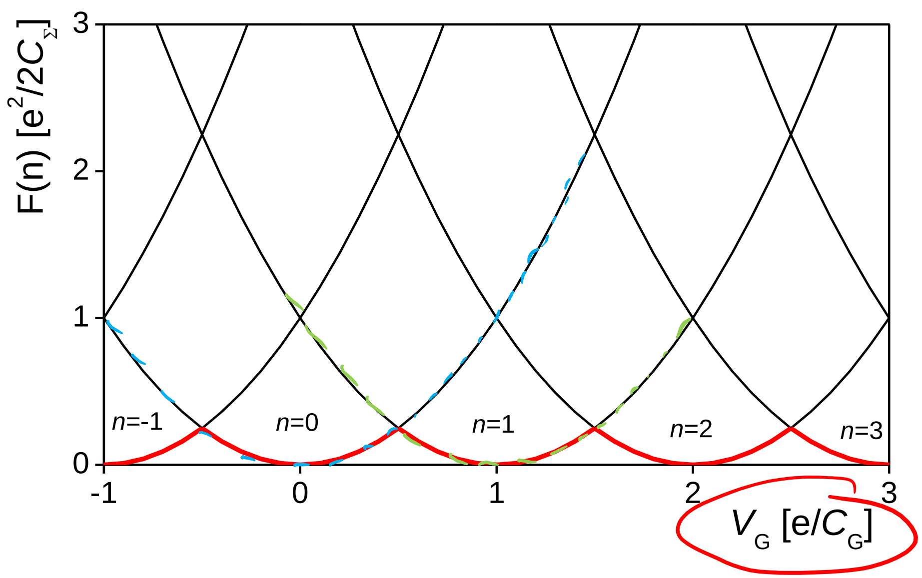
$$\Rightarrow \quad F(n) = W_Q(n) - A(n) = \text{const.}\,(n - \text{const.})^2 + \text{const.}\,2$$

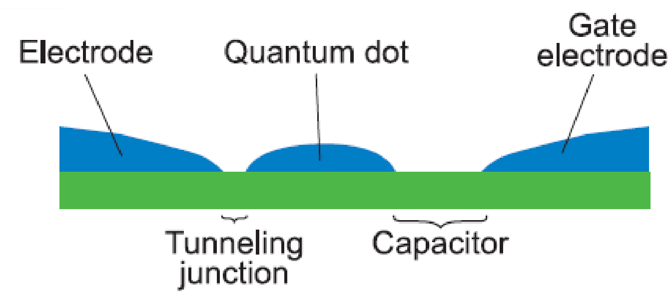
Single electron box – quantitative model

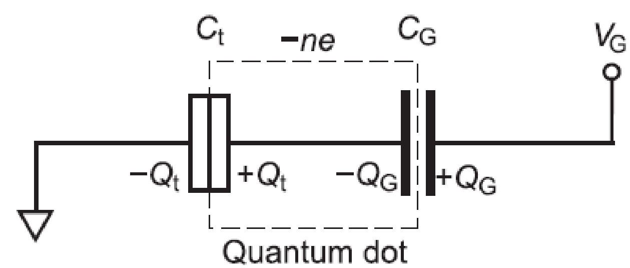
How many electrons will sit on the QD *when we apply a gate voltage?*

$$F(n) = \underbrace{\frac{e^2}{2C_\Sigma}\left(n - \frac{C_G V_G}{e}\right)^2}_{\text{n-dependent}} + \underbrace{f(V_G)}_{\text{n-independent}}$$

graphical solution







With increasing bias voltage V_G the free energy of the system is minimum for a different charge number n!
→ Electrons can be transferred into/out of the dot ***one by one***!

Weak and strong coupling

Requirements for the observation of *Coulomb blockade phenomena*

***n* should be a well-defined number, QD has to be reasonable decoupled from surrounding**

1. thermal fluctuation of *n* has to suppressed $$\frac{e^2}{C} \gg k_B T$$

2. Quantum fluctuations have to be suppressed: the electrons have to be reasonably localized on the QD,
   i.e spend enough time on the quantum dot before fluctuating back and forth

$$\Delta W \cdot \Delta t = \frac{e^2}{C_t} \cdot R_t \cdot C_t > h$$

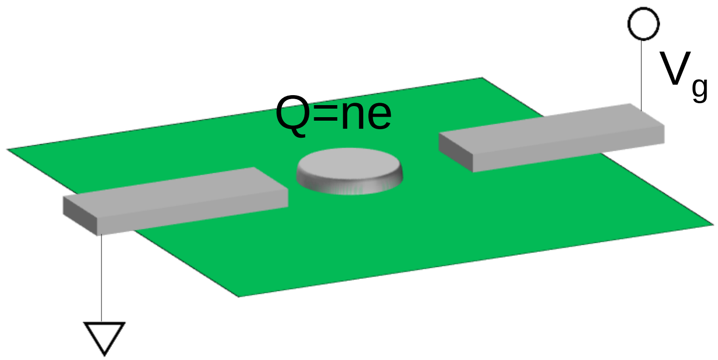
This timescale Δt is given by the $R_t C_\Sigma$ value of the coupling tunnel contact. Heisenberg´s uncertainty relation gives

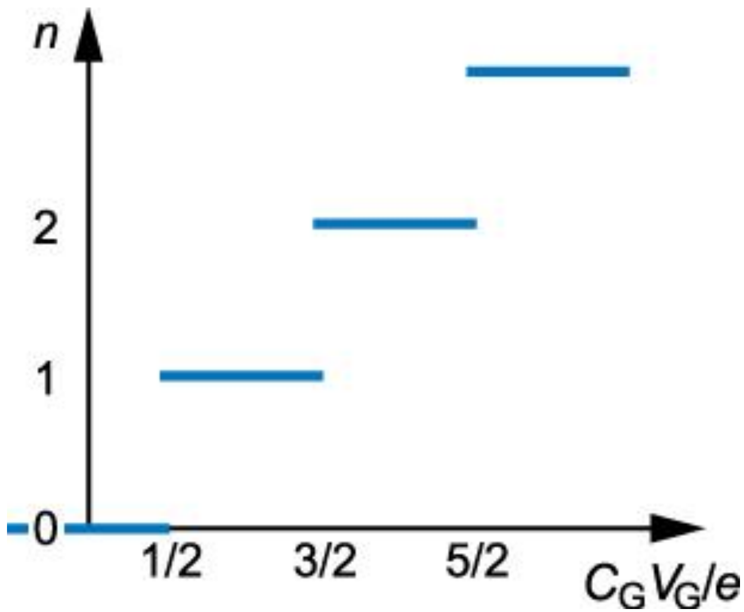
$$R_t > \frac{h}{e^2} \approx 25.8\ \mathrm{k}\Omega$$

$R_t > \frac{h}{e^2}$ is called **weak coupling**

$R_t < \frac{h}{e^2}$ is called **strong coupling**

IWE





The effect of an electron not being able to tunnel into a quantum dot due to an energy barrier is called *Coulomb blockade*

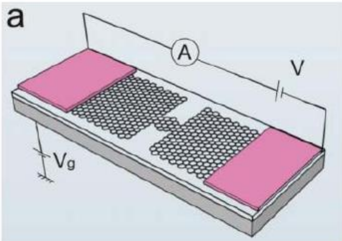
The Coulomb blockade occurs due to
- low capacitance of quantum dots (metals) # more important
- size quantization (semiconductors) # additional effect

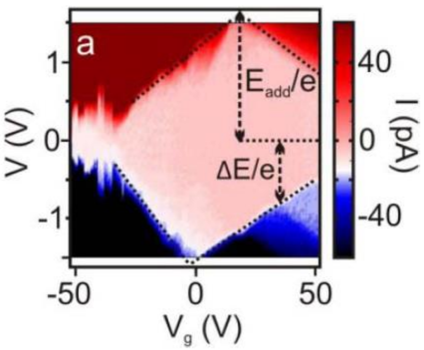
Using the Coulomb blockade effect single electron charges can be transferred and controlled on a nanoscale quantum dot

Requirements

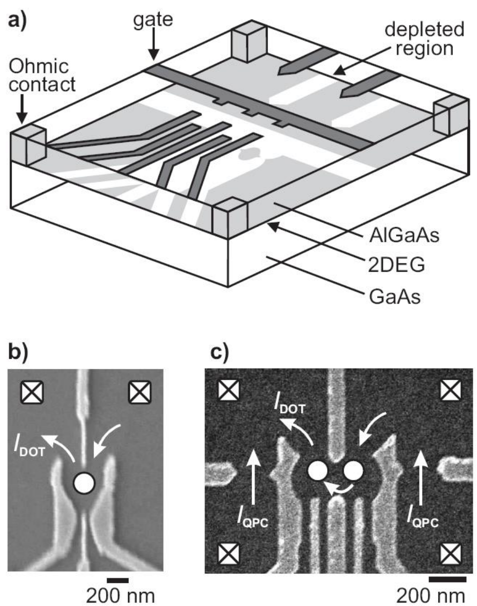
$$R_t > \frac{h}{e^2} \qquad \frac{e^2}{C} \gg k_B T$$

Coulomb blockade - examples

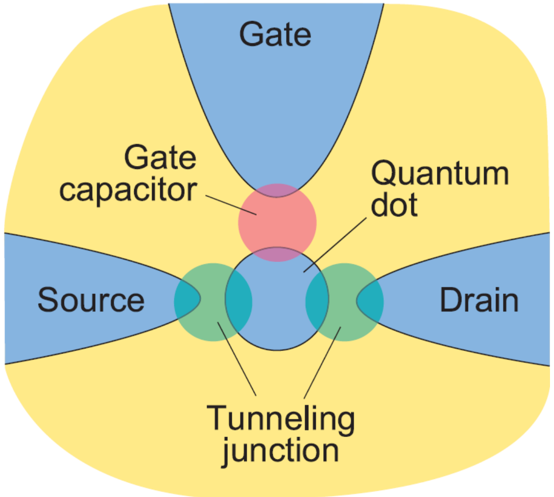


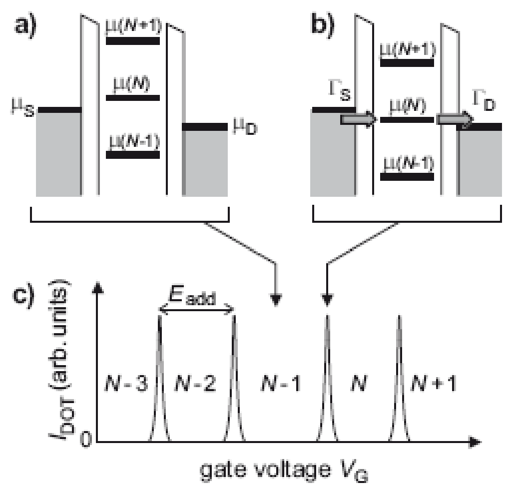


A. Barreiro, H.S.J. van der Zant, L.M.K. Vandersypen, "Quantum Dots at Room Temperature carved out from Few-Layer Graphene", arXiv:1211.4551 (21 Nov. 2012)



Hanson et al., Rev. Mod. Phys. 79, 1217 (2007)





Hanson et al., Rev. Mod. Phys. 79, 1217 (2007)

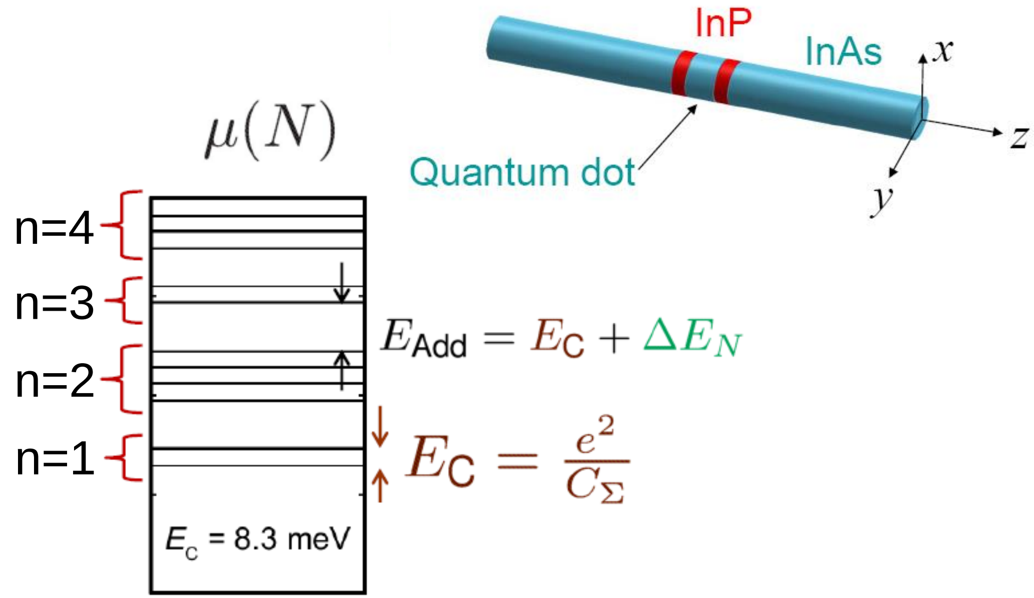
Artificial atoms: Semiconductor quantum dots

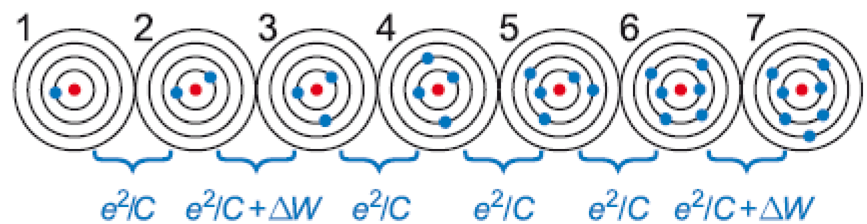
Charging energy

$$\mu(n) = W_C n + W_{k_n}$$

Size quantization energy

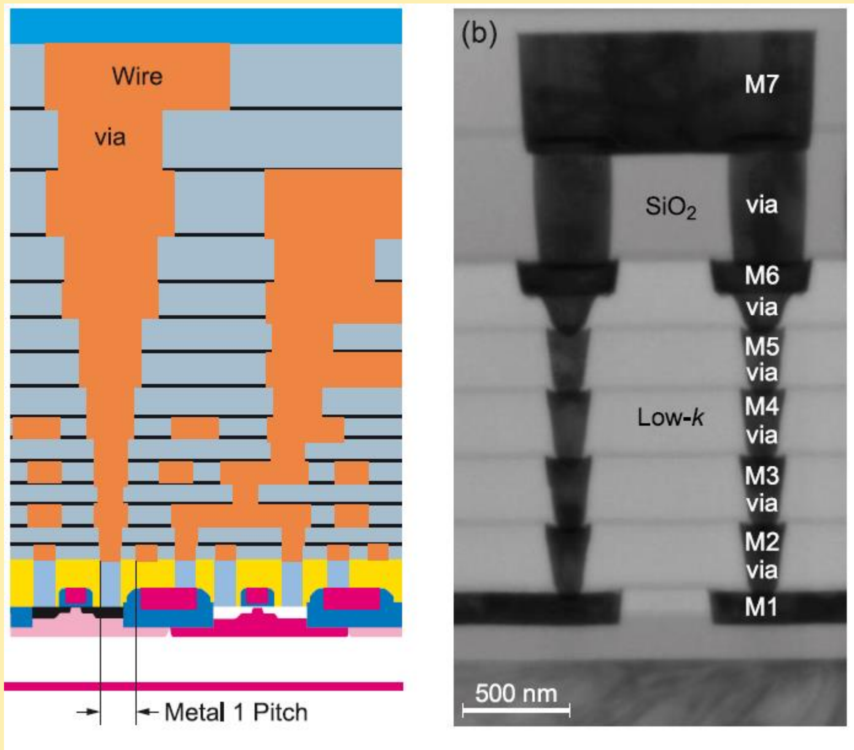
$$W_{k_n} = \frac{h^2}{2m^* L^2}(n_x^2 + n_y^2 + n_z^2)$$

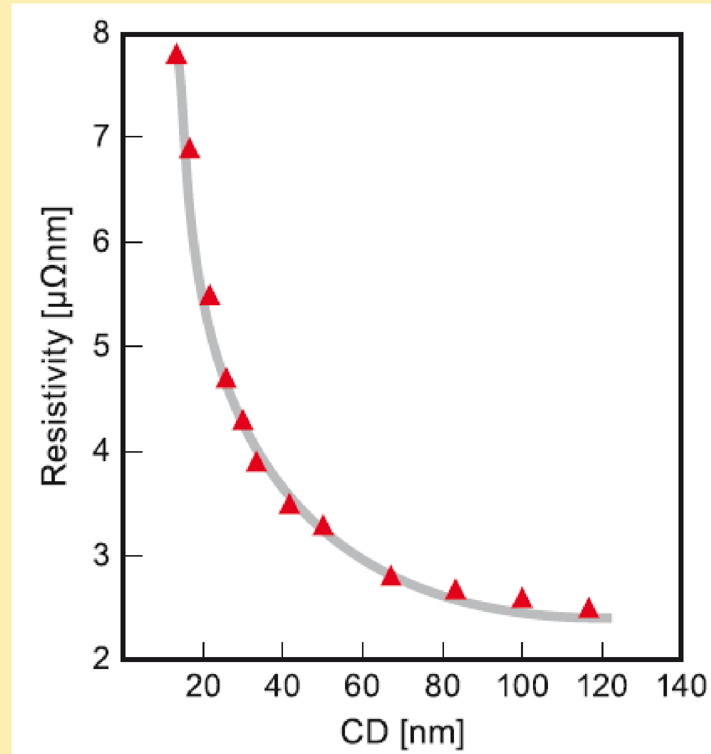




Band-like clustering of states in semiconductor QD

2 Metal interconnects





Interconnects – basic principles

Delay time
Power consumption

Size-dependent resistivity of metal wires
Furchs-Sondheimer formalism

Resistivity of metal point contacts
Sharvin-Conductance

Metal interconnects - basics

Interconnects *interconnect* transistors and functional entities on chips

- provide power
- provide ground
- usually a stacked-layer wiring system/network

global

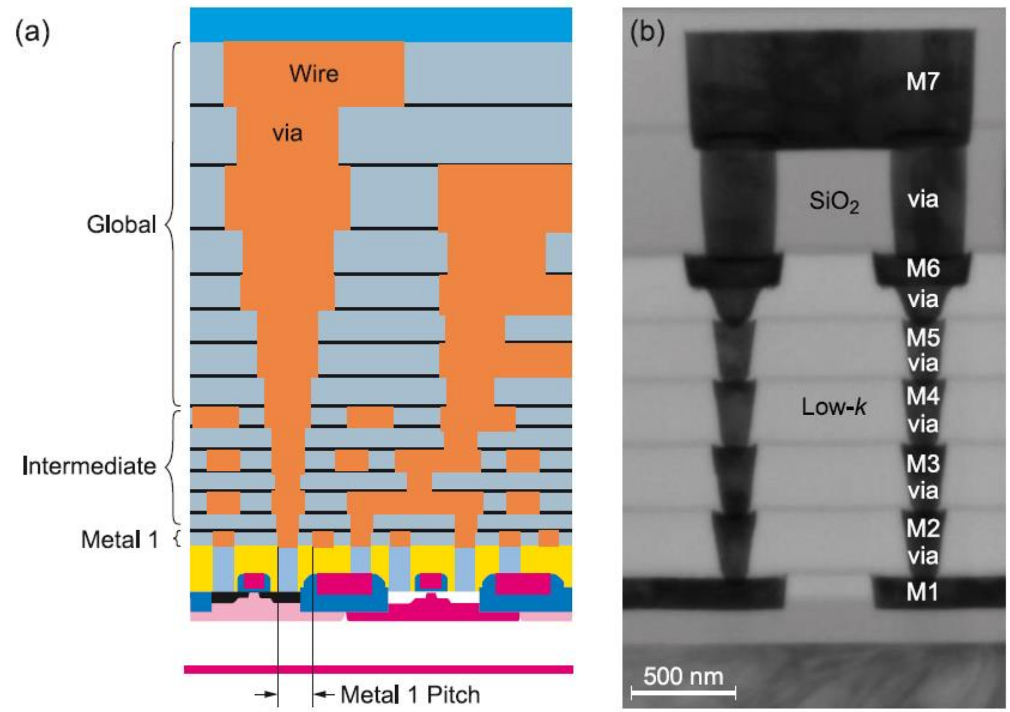
- ground, power supply, long distance communications
- 500 nm – 20 μm

intermediate

- interconnections between cells
- 100 nm – 500 nm

local

- Connecting single transistors
- < 100 nm

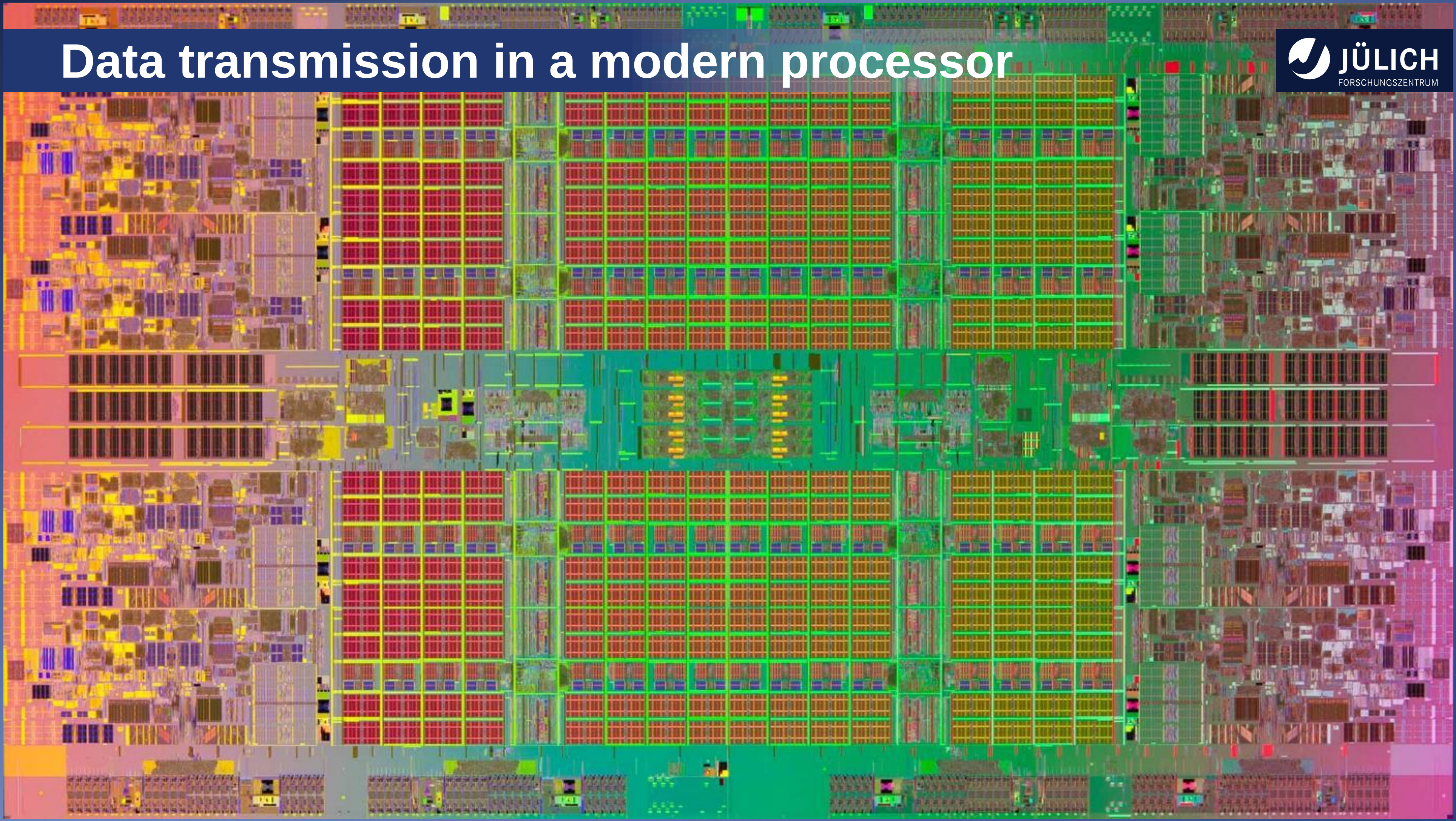


interconnects can cause >50% power consumption of a system

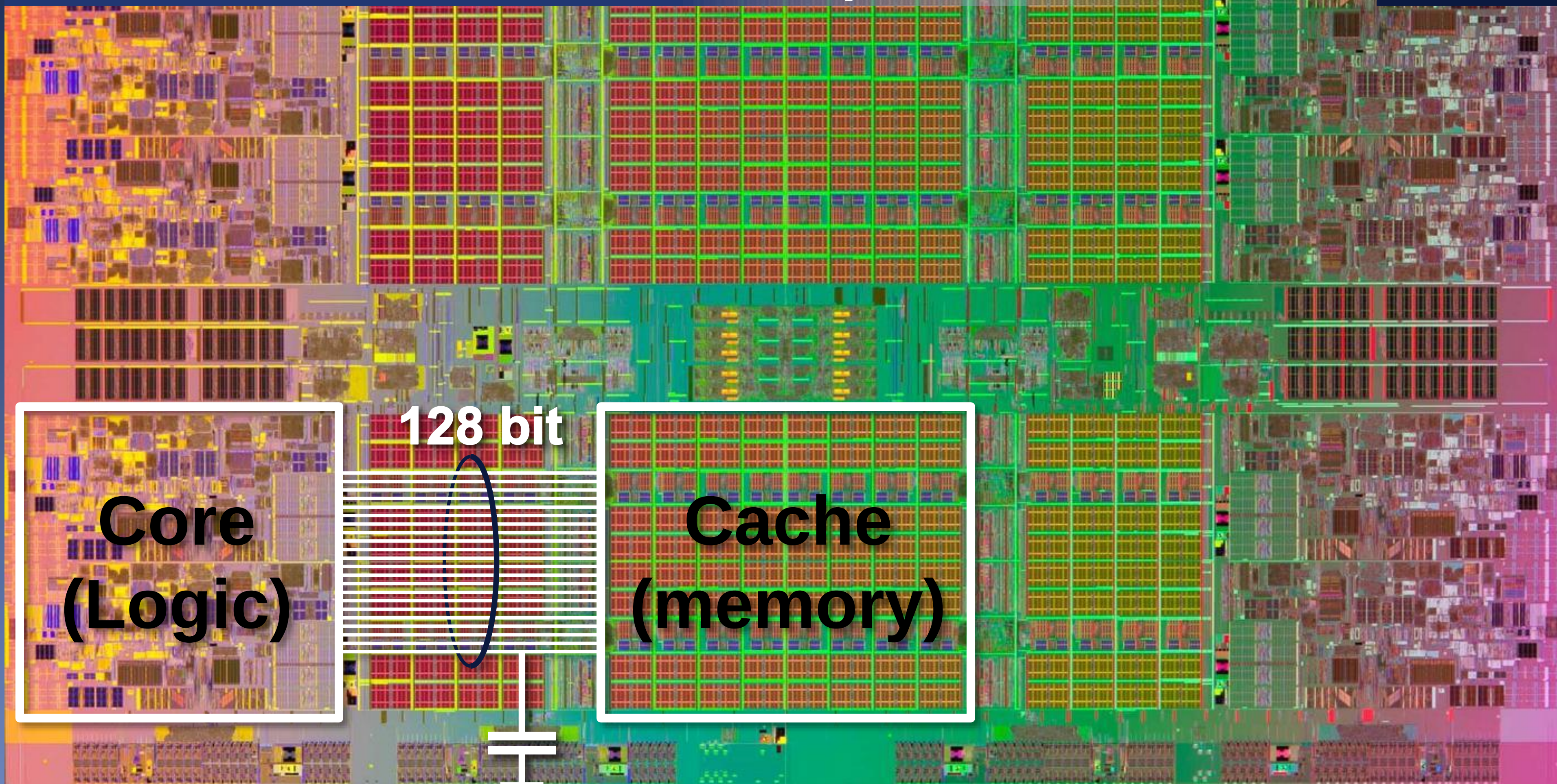
→resistance of *thin* metal wires

→ capacitive coupling to surrounding

Data transmission in a modern processor



Data transmission in a modern processor



2.1 Delay time and power consumption

Delay time and power consumption

Delay time of signal propagation through wire system

$\tau = R \cdot C$

plate capacitors

$C = \varepsilon\varepsilon_0 \frac{A}{d}$

total capacitance of the system

$C = 2(C_l + C_v) = 2\varepsilon\varepsilon_0 Ltw\left(\frac{1}{w^2} + \frac{1}{t^2}\right)$

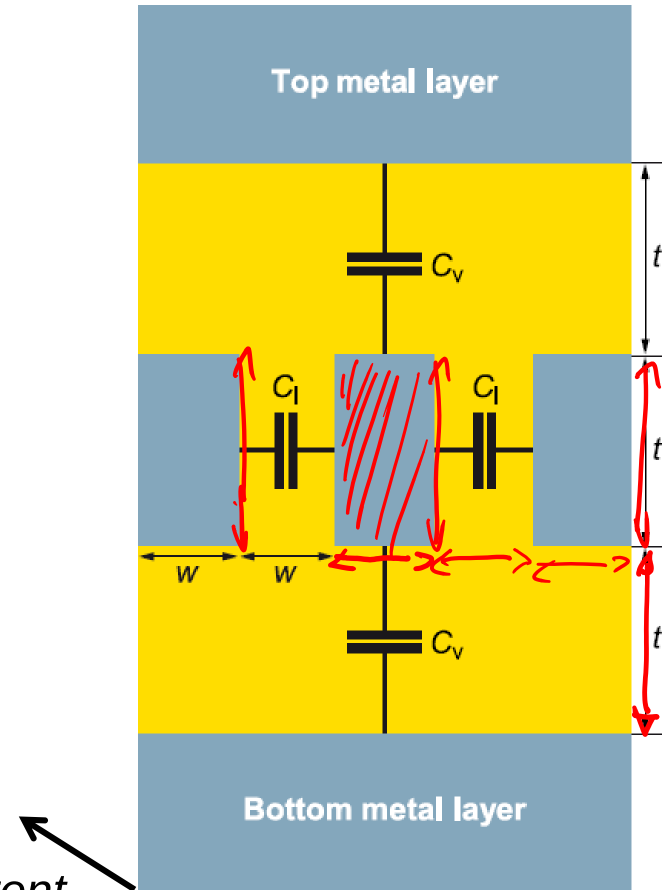
resistance of central wire

$R = \rho \cdot \frac{L}{wt}$

$$\tau_{delay} = 2\rho\varepsilon_0\varepsilon L^2\left(\frac{1}{w^2} + \frac{1}{t^2}\right)$$

dynamic power dissipation

$$P = \frac{1}{2} C \cdot V^2 \cdot f$$

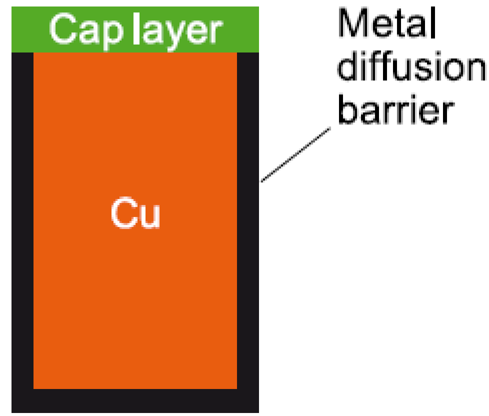


L, current

Simple equivalent circuit of interconnect network

Strategies to reduce delay time and/or power consumption

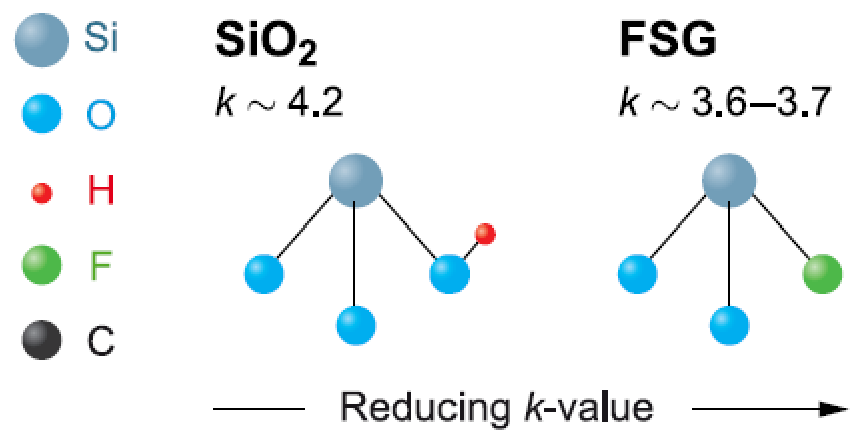
Reducing ρ
from W (5.6 $\mu\Omega$cm)
to Al alloys (3.0 $\mu\Omega$cm)
to Cu (1.67 $\mu\Omega$cm)
challenge: electromigration of Cu



Reducing ε ***(or k)***
chemical manipulation of SiO_2
→reduces diplole moment

e.g. Si-O-bond replaced by Si-F-bond
(fluorosilicate glasses, FSG)

challenge: too low ε increases leakage



$k \equiv \varepsilon$

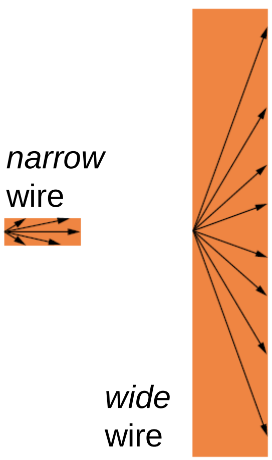
2.2 Size-dependent resistivity of metal wires

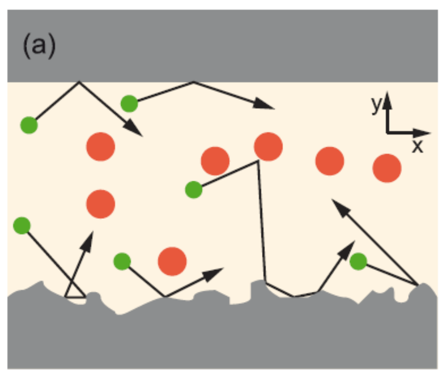
Size-dependent resistivity of metal wires

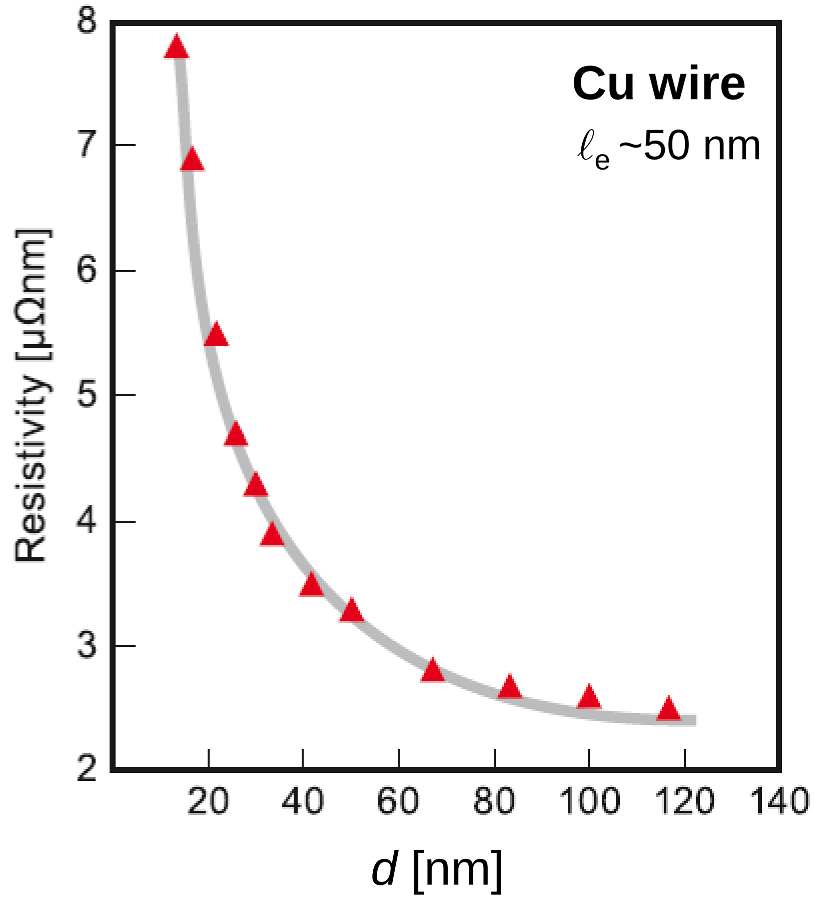
bulk resistivity: $\rho_{bulk} \neq \rho_{bulk}(L,w,t)$

→independent of geometry

However, ***experiments*** show that ρ increases with decreasing feature size







Reminder: Boundary scattering!

For metals (λ_F~1 nm), the boundary of the wire appear *rough* and limits ℓ_e

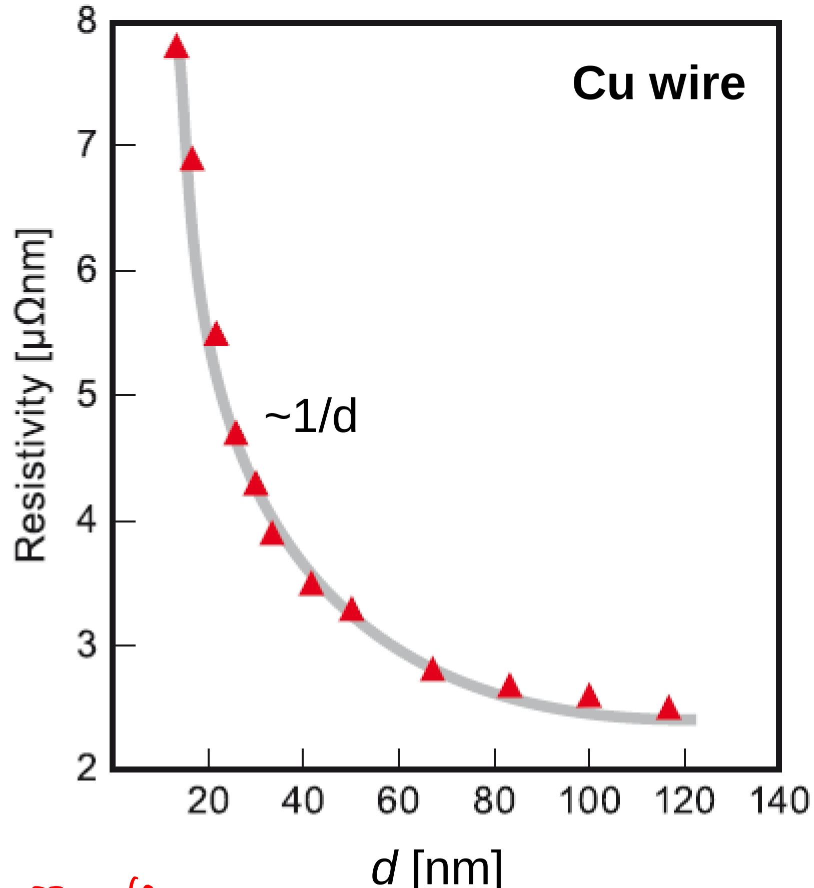
Fuchs-Sondheimer formalism

*First quantitative model by Fuchs and Sondheimer considering **surface scattering***

$$\boxed{\rho_{FS}} = \frac{\rho_{bulk}}{1-\frac{3}{2k}(1-p)\int_1^{\infty}\left(\frac{1}{t^3}-\frac{1}{t^5}\right)\frac{1-e^{-kt}}{1-pe^{-kt}}\,dt}$$

p = surface reflectivity

$k = \frac{d}{\ell_e}$ $\quad$ *ℓ_e means "bulk" mean free path here*



for d >> ℓ_e $\quad \frac{3}{4}\frac{\ell_e}{d}$ rectangular cross section

$$\rho_{FS} = \rho_{bulk}\left[1+\frac{3\ell_e}{8d}(1-p)\right]$$

Fuchs-Sondheimer formula ← circular crosssection of the wire