EE4150 Control Systems Lab-Experiment 2

Design of lead compensator using root locus and frequency response methods

Objective:

To design lead compensator using frequency response and root locus methods.

Software required: MATLAB editor.

Expectations:

- Before coming to the lab-session, read the material presented below and go through related textbooks.
- Write a brief report about your design with the diagrams/plots, results obtained/noted and inferences, if any. Indicate your answers in observation tables wherever necessary.
- Submit the report along with the simulation files. MATLAB uploaded files should be error free. There will be a penalty for erroneous code.
- Report should be written clearly and briefly.
- Ensure the following are presented in your report:
  - Name and Roll Number.
  - Appropriate graphical representations.
  - Obtained parameters.
  - Results and inferences.
- Avoid copying, it will attract zero marks. We encourage discussions, of course. In case you need any clarifications, please contact:

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Problem 1: Design a lead compensator using root locus method for the following system with open loop transfer function

$$G(s) = \frac{1}{s(s+4)(s+6)}$$

such that the closed loop unity negative feedback system with the compensator will satisfy the following specifications:

(A) Settling time (for 2% tolerance band), $t_s \leq 2s$ for unit step input

(B) Percentage peak overshoot for unit step input $\leq 30\%$

Draw the root-locus plots of uncompensated and compensated systems in MATLAB and also draw the unit step response plot of the closed loop compensated system indicating the settling time and peak overshoot in the step response plot.

- If only specification B is given, suggest a way to calculate the desired roots.
- Comment on the effect of lead compensator on transient response specifications and system stability.

Procedure:

Lead compensators are generally designed to enhance the transient response specifications. The root-locus of the uncompensated system with open loop gain will not pass through the desired root location, indicating the need of compensation. The lead compensator $G_c(s)$ has to be designed such that the root-locus of the compensated system will pass through the desired root location. Consider the following lead compensator:

$$G_c(s) = \frac{K(s+z)}{s+p}$$

The steps are

1. List the system specifications and translate them into a desired root location for the dominant roots.
2. Sketch the uncompensated root locus, and determine whether the desired root locations can be realized with an uncompensated system.
3. If a compensator is necessary, place the zero (z) of the compensator directly below the desired root location (and to left of the first two real poles of open loop transfer function of the uncompensated system).
4. Determine pole (p) of the compensator so that the total angle at the desired root location is $180°$ (angle criterion) and therefore is on the compensated root locus.
5. Evaluate the total system gain (K) at the desired root location using magnitude criterion and then calculate the error constant (if asked).
6. Repeat the steps if the error constant is not satisfactory. (if asked)

Results:

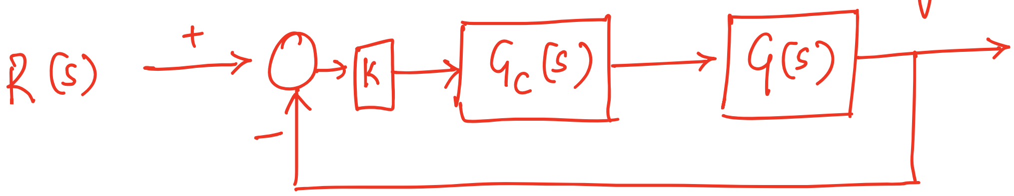
1. The compensator transfer function is ________________
2. The compensated system transfer function is ________________
3. For the compensated system subject to unit step input,

   The settling time is ________________

   The maximum overshoot is ________________

An example is provided in the following for reference. Try to automate the calculations as much as possible.

ex: (Phase lead compensator design using rootlocus)



Consider $G(s) = \frac{1}{s(s+2)}$, then design lead compensator

$G_c(s) = \frac{(s+z)}{s+p}$ such that

i) Settling time (with a 2% criterion), $t_s \leq 1$ sec

ii) Peak percentage overshoot for a step input $\leq 35\%$

Ans: Uncompensated system: $\frac{1}{s^2+2s+1}$

$\omega_n = 1, \quad \xi\omega_n = 1 \quad \Rightarrow \quad t_s = 4$ sec.

No oscillations

Consider lead compensator $G_c(s) = \frac{(s+z)}{s+p}$

First locate the dominant pole locations to satisfy the design requirements

$$t_s = \frac{4}{\xi\omega_n} = 1 \quad \Rightarrow \quad \xi\omega_n = 4 \quad \text{——} \quad (1)$$

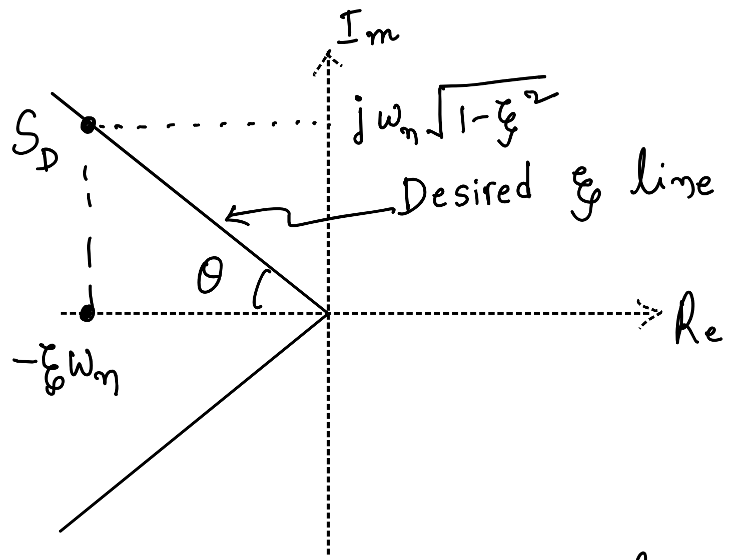
Peak overshoot $\quad e^{-\pi\xi/\sqrt{1-\xi^2}} \leq 0.35$

$$\Updownarrow$$

$$\Rightarrow \quad \xi \geq 0.3169$$

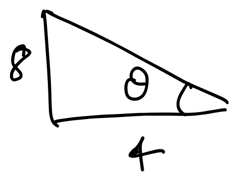
Let $S_D: -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$ be the desired dominant poles, then

$$\cos\theta = \xi \quad \text{—— (2)}$$



$\xi\omega_n = 4$ from (1), Let $\omega_n\sqrt{1-\xi^2}$ be 8, then

$$\cos\theta = \frac{4}{\sqrt{4^2+8^2}}$$



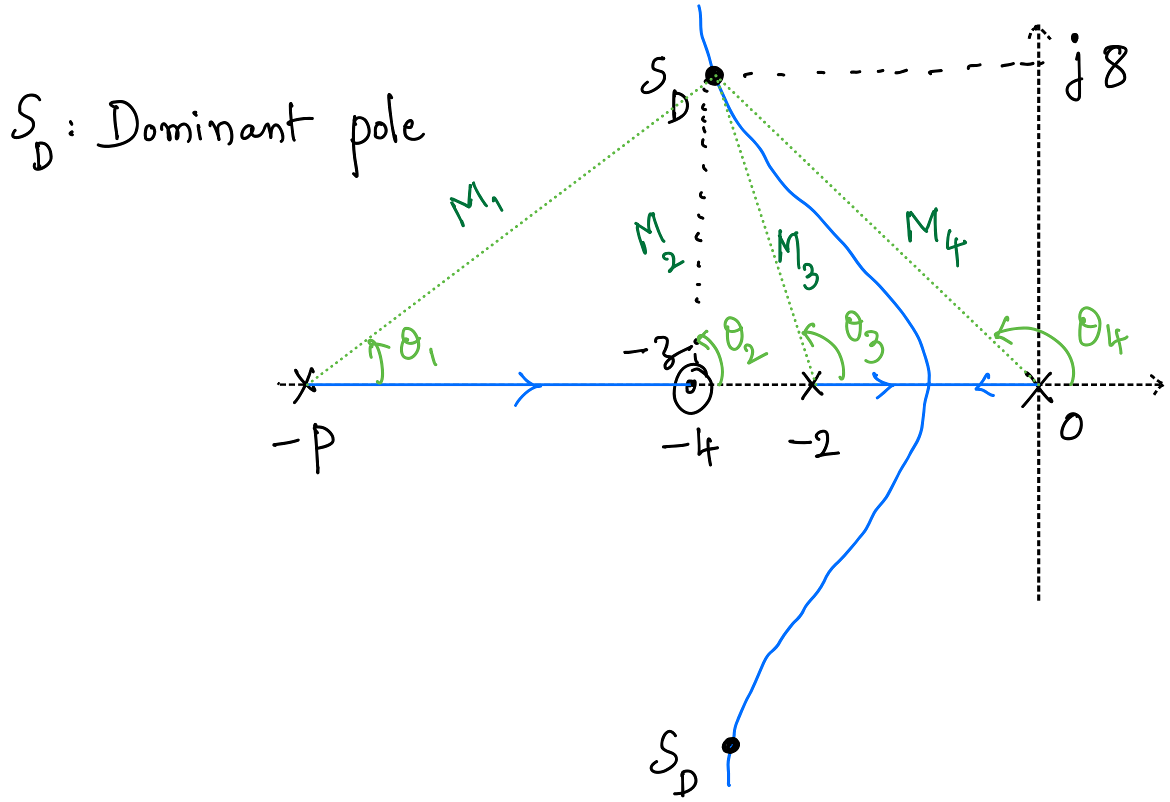
$$= 0.4472 = \xi$$

$\therefore$ This choice of dominant pole location is satisfying $t_s \leq 1\,s$ and $\xi \geq 0.3169$.

Now design z & p of the compensator.

(place the zero of the phase-lead compensator directly below the desired root location and to the left of the first two real poles of the loop transfer function of the uncompensated system)

So $z = 4$. Lets obtain the root locus with some $p > z$ (Unknown p)



How to obtain p ? (Use Angle criterion)

$$-\theta_1 + \theta_2 - \theta_3 - \theta_4 = 180^\circ$$

$$\Rightarrow \quad \theta_1 = 50^\circ \quad \Rightarrow \quad p = 10.6$$

How to obtain K ? (Use Magnitude Criterion)

$$\frac{K\, M_2}{M_1 M_3 M_4} = 1 \quad \Rightarrow \quad K = 96.5$$

Last step is to verify if the design specifications are met with the compensator.

Problem 2: Design a lead compensator using Bode plot for the following system with open loop transfer function

$$G(s) = \frac{1}{s(s+1)}$$

to meet the following specifications

(A) Phase margin $\Phi_m \geq 45°$

(B) Steady state error subjected to unit ramp input $\leq (1/15)$ units.

(C) Gain cross over frequency ≤ 7.5 rad/s.

Procedure:

1. Assume the following lead compensator:

$$\begin{aligned} G_c(s) &= \frac{K(s+z)}{s+p} \\ &= K\left(\frac{z}{p}\right)\frac{(s/z+1)}{s/p+1} \\ &= K_1\frac{(s/z+1)}{s/p+1} \end{aligned}$$

   with $\alpha = \frac{p}{z} > 1$. First determine K_1 to satisfy the requirement on the given steady state error specification.

2. Using the gain thus determined, draw a bode plot of $K_1 G(s) = K_1 \frac{1}{s(s+1)}$

3. Determine the necessary phase lead angle Φ_m to be added to the system. Consider adding 10% more to account for phase lags

4. Determine α from

$$\sin(\Phi_m) = \frac{\alpha - 1}{\alpha + 1}.$$

5. Determine the frequency where the magnitude of uncompensated system $K_1 G(jw)$ is equal to $-10\log(\alpha)$. Because the compensation network provides a gain of $10\log\alpha$ at w_m, this frequency is the new 0-dB crossover frequency and w_m simultaneously.

6. Calculate the pole $p = w_m\sqrt{\alpha}$ and $z = \frac{p}{\alpha}$

7. Value of $K = K_1(p/z) = K_1\alpha$

8. Check if the phase margin of the compensated system to be satisfactory. If not, repeat the design process by increasing the percentage in step 3 or raising the gain of the amplifier K.

Results:

1. The compensator transfer function is ________________

2. The compensated system transfer function is ________________

3. For the compensated system,

   The phase margin is ________________

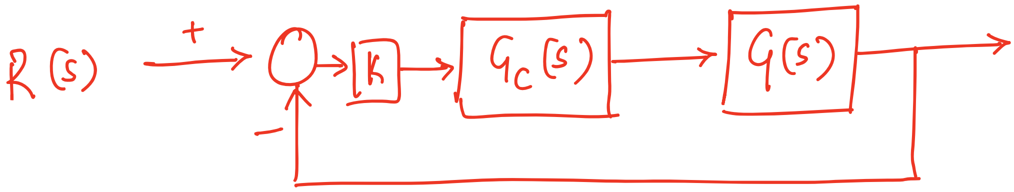
   The Velocity error constant K_v is ________________

   Steady state error is ________________

   Gain cross over frequency is ________________

An example is provided in the following for reference. Try to automate the calculations as much as possible.

ex 1: (lead compensator using Bode plots)



Consider $G(s) = \dfrac{1}{s(s+2)}$, then design lead compensator $G_c(s) = \dfrac{s+z}{s+p}$ such that

i) Phase margin $\geq 45^\circ$

ii) Steady state error for a ramp input is equal to 5% of the velocity of the ramp.

Ans :

Consider lead compensator

$$G_c(s) = \frac{(s+z)}{s+p} = \left(\frac{z}{p}\right)\left(\frac{\frac{s}{z}+1}{\frac{s}{p}+1}\right)$$

$$= \frac{z}{p}\left(\frac{\frac{s}{z}+1}{\frac{s}{p}+1}\right)$$

ii) $$E_{ss} = \lim_{s\to 0} \frac{s\,R(s)}{1+G(s)\,G_c(s)} = \lim_{s\to 0} \frac{s\,A/s^2}{1+\left[\frac{K'}{s(s+2)}\,\frac{(s/z+1)}{(s/p+1)}\right]} = \frac{2A}{K_1}$$

$$\frac{2A}{K_1} = 0.05A \Rightarrow K_1 = 40$$

where $K' = K\left(\frac{z}{p}\right)$.

i) PM should be atleast 45°

ⓐ Find PM without compensator

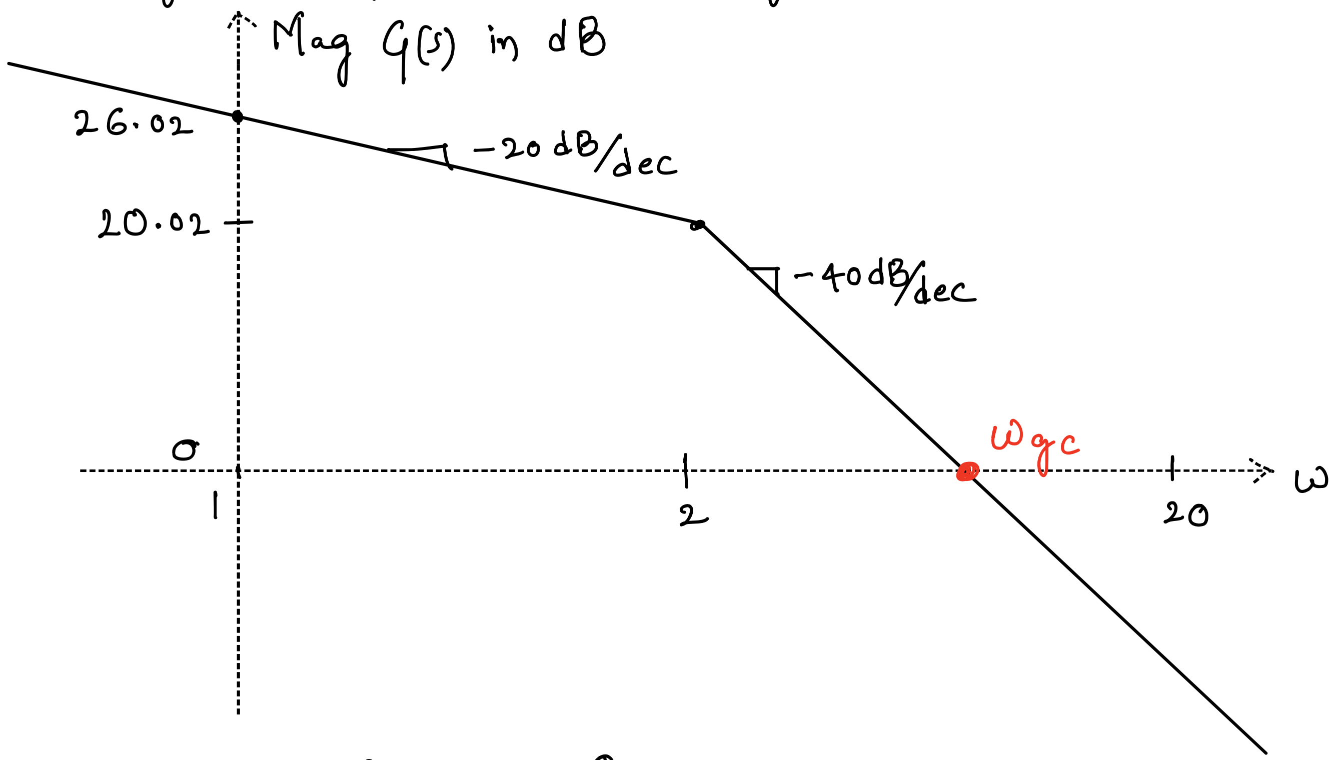
ⓑ Determine necessary additional phase lead.

(a) Uncompensated system (with K_1)

$$G(s) = \frac{K_1}{s(s+2)} = \frac{40}{s(s+2)} = \frac{20}{s(s/2+1)}$$

Obtain Bode plot and compute PM.

Magnitude plot $\quad 20 \log 20 = 26.02\,dB$



$$-40 = \frac{20.02 - 0}{\log 2 - \log \omega_{gc}} \Rightarrow \omega_{gc} = 6.2 \text{ rad/s}$$

$$PM = \angle G(j\omega)\Big|_{\omega_{gc}} + 180^\circ$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(\omega/2)$$

$$\angle G(j\omega)\Big|_{\omega_{gc}} = -90 - \tan^{-1}\left(\frac{6.2}{2}\right) = -162^\circ$$

$$PM = -162 + 180 = 18^\circ$$

(b) Required phase lead $= 27^\circ$ to make PM 45°

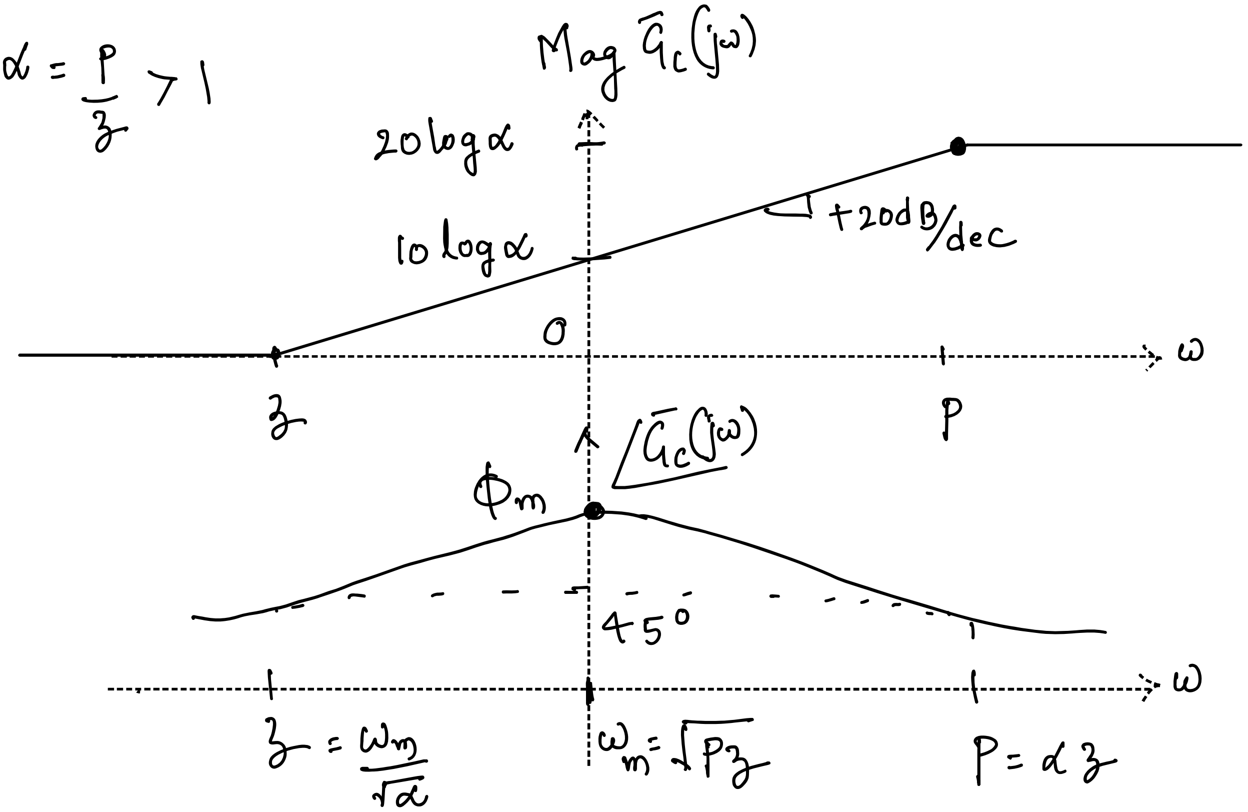
Usually phase lead = 27° + 10% of actual phase lead
$\approx 30^\circ$

when compensated cross over frequency ω_{gc} is greater than uncompensated crossover frequency 2 rad/s to account for any additional phase lags.

Q: Design a phase lead compensator that provides additional lead 30°

$$\bar{G}_c(j\omega) = \alpha G_c(j\omega) = \frac{\left(\frac{j\omega}{z}+1\right)}{\left(\frac{j\omega}{p}+1\right)}$$

$$\alpha = \frac{p}{z} > 1$$



What should be ω_m and ϕ_m ?

Let $\phi_m = 30^\circ$

$$\sin\phi_m = \frac{\alpha-1}{\alpha+1} \Rightarrow \sin 30^\circ = \frac{\alpha-1}{\alpha+1} = 0.5 \Rightarrow \alpha = 3$$

Let $\omega_m = \omega_{gc} = 6.2$ rad/s (Is it a good choice)

At ω_m, there is also a lead of magnitude $10 \log \alpha$

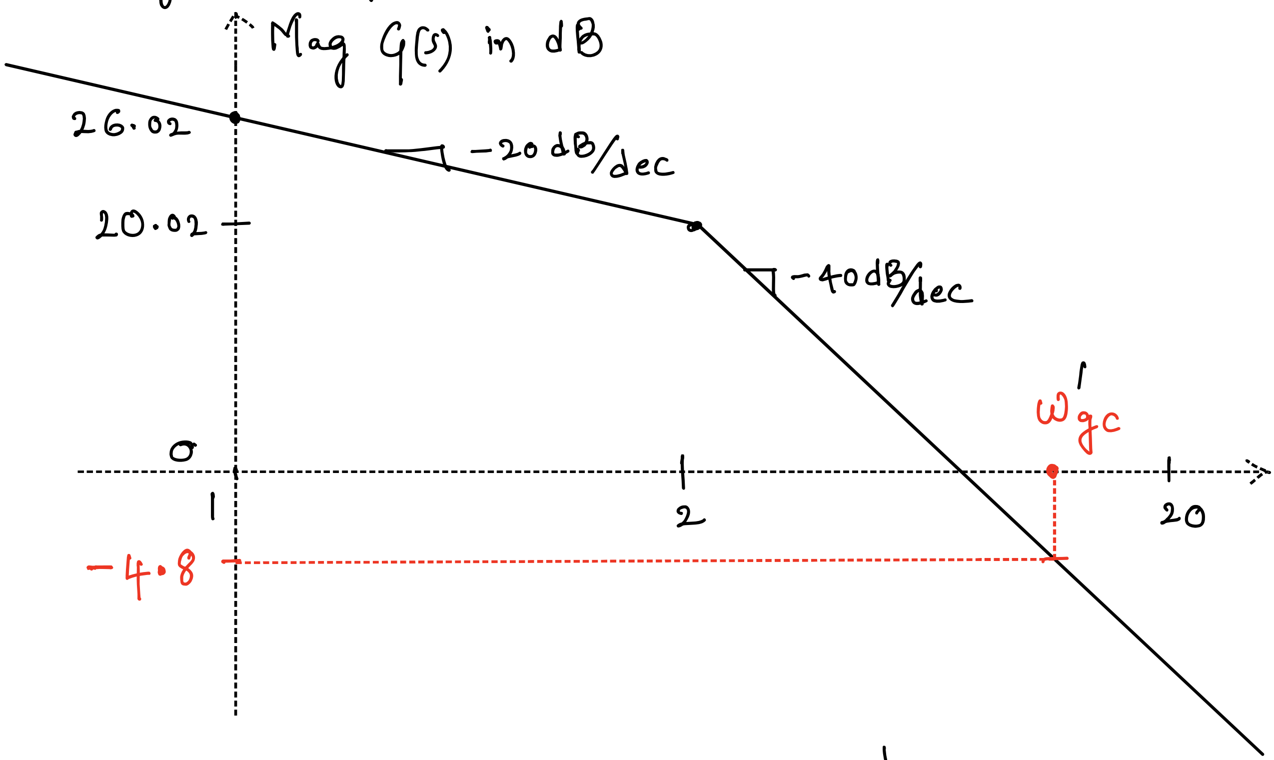
$\Rightarrow \quad 10 \log \alpha = 10 \log 3 = 4.8$ dB

ω_{gc} need to evaluated again!

$\Downarrow$

New ω_{gc} is where the magnitude of $G(s)$ is -4.8 dB

Magnitude plot



$$-40 = \frac{0 + 4.8}{\log 6.2 - \log \omega'_{gc}} \Rightarrow \omega'_{gc} = 8.1732.$$

place $\omega_m = \omega'_{gc} = 8.1732$

$$z = \omega_m / \sqrt{\alpha} = 4.7188$$

$$p = \alpha z = 14.1564$$

$$\therefore \quad G_c(s) = \frac{s+z}{s+p}$$

$$= \frac{s+4.7188}{s+14.1564}$$

Also $K' = K\left(\frac{z}{p}\right) \Rightarrow 40 = \frac{K}{\alpha} \Rightarrow K = 40\times 3 = 120.$

<u>Verification:</u>

$$\underline{\angle G_c(s)\,G(s)} = -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{4.7188}\right) - \tan^{-1}\left(\frac{\omega}{14.1564}\right)$$

$$\underline{\angle G_c(s)\,G(s)}\Big|_{\omega=\omega_{gc}=8.1732} = -136.25$$

$PM = -136.25 + 180 = 43.75^\circ$ & also check the steady state error

To the required phase lead 27° addition of 3° is not sufficient, add a bit more angle & repeat the procedure if exact PM of 45° is desired.