

```
In [ ]: # import necessary packages
import numpy as np
import scipy.special as ss
import matplotlib.pyplot as plt
from copy import deepcopy
from scipy.constants import *
from matplotlib.patches import Circle
from scipy.integrate import quad, dblquad
from ipywidgets import interactive
from scipy.special import fresnel
from mpl_toolkits.mplot3d import axes3d
```

1. np.where(): How to deal with singular points?

Plot

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Explicitly handle the case where $x=0$.

```
In [ ]: def sinc(x):
    arg=np.pi*np.where(x==0,1e-15,x)
    # arg=np.pi*x
    y=np.sin(arg)/arg
    return y
```

```
In [ ]: def interactive_plot(a=11):
    x=np.linspace(-5,5,a)
    y=sinc(x)
    plt.plot(x,y)

    interactive(interactive_plot,a=(11,101,10))
```

You can see that there was no divide by zero error.

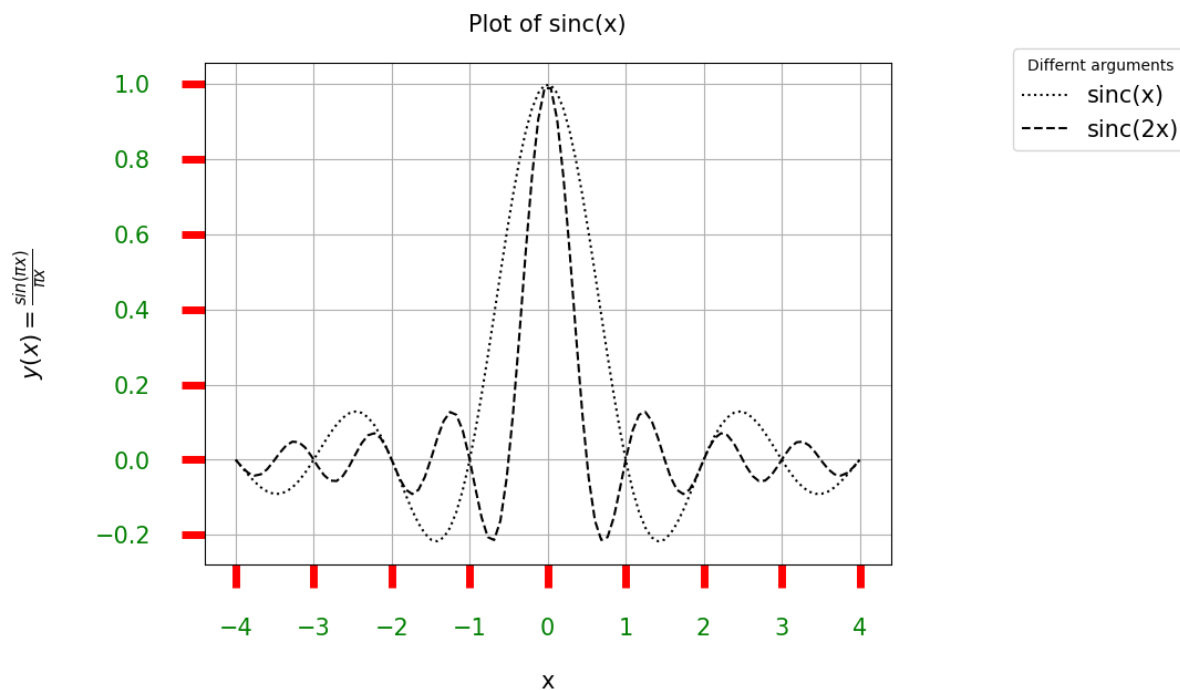
```

In [ ]: fig,ax=plt.subplots(figsize=(8,6),dpi=100)

ax.plot(x,sinc(x),label='sinc(x)',c='black',ls=':')
#ax.plot(x,sinc(x),label='sinc(x)',marker='^',c='black',mfc='red',mew=3.0,mec
='blue',ms=10,ls='-.')
ax.set_xlabel('x',fontsize=15,labelpad=20)
ax.set_ylabel('$y(x)=\frac{\sin(\pi x)}{\pi x}$',fontsize=15,labelpad=30)
# ax.set_title('Plot of sinc(x)',fontsize=15)

ax.plot(x,sinc(2*x),label='sinc(2x)',c='black',ls='--')
#ax.plot(x,sinc(2*x),label='sinc(2x)',marker='v',c='black')
# ax.set_xlabel('x',fontsize=15)
# ax.set_ylabel('$y(x)=\frac{\sin(\pi x)}{\pi x}$',fontsize=15)
ax.set_title('Plot of sinc(x)',fontsize=15,pad=20)
# ax.legend(fontsize=15)
ax.legend(bbox_to_anchor=(1.45,1.05),loc=1,fontsize=15,title='Differnt argumen
ts')
# ax.legend(loc=3,fontsize=15)
ax.tick_params(axis='both',size=15,pad=20,width=5,color='red',labelsize=15,lab
elcolor='green')
#ax.tick_params(axis='both',labelsize=15,labelcolor='blue',color='green',width
=4,pad=20,size=15)
ax.grid()

```



Make color-blind friendly plots. Use linestyle or ls whenever you can.

2. Spirals

Cornu's Spiral: Use packages as much as you can

This is created out of Fresnel integrals with usage in optics and are given by

$$C(l) = \int_0^l \cos\left(\frac{\pi t^2}{2}\right) dt$$

and

$$S(l) = \int_0^l \sin\left(\frac{\pi t^2}{2}\right) dt$$

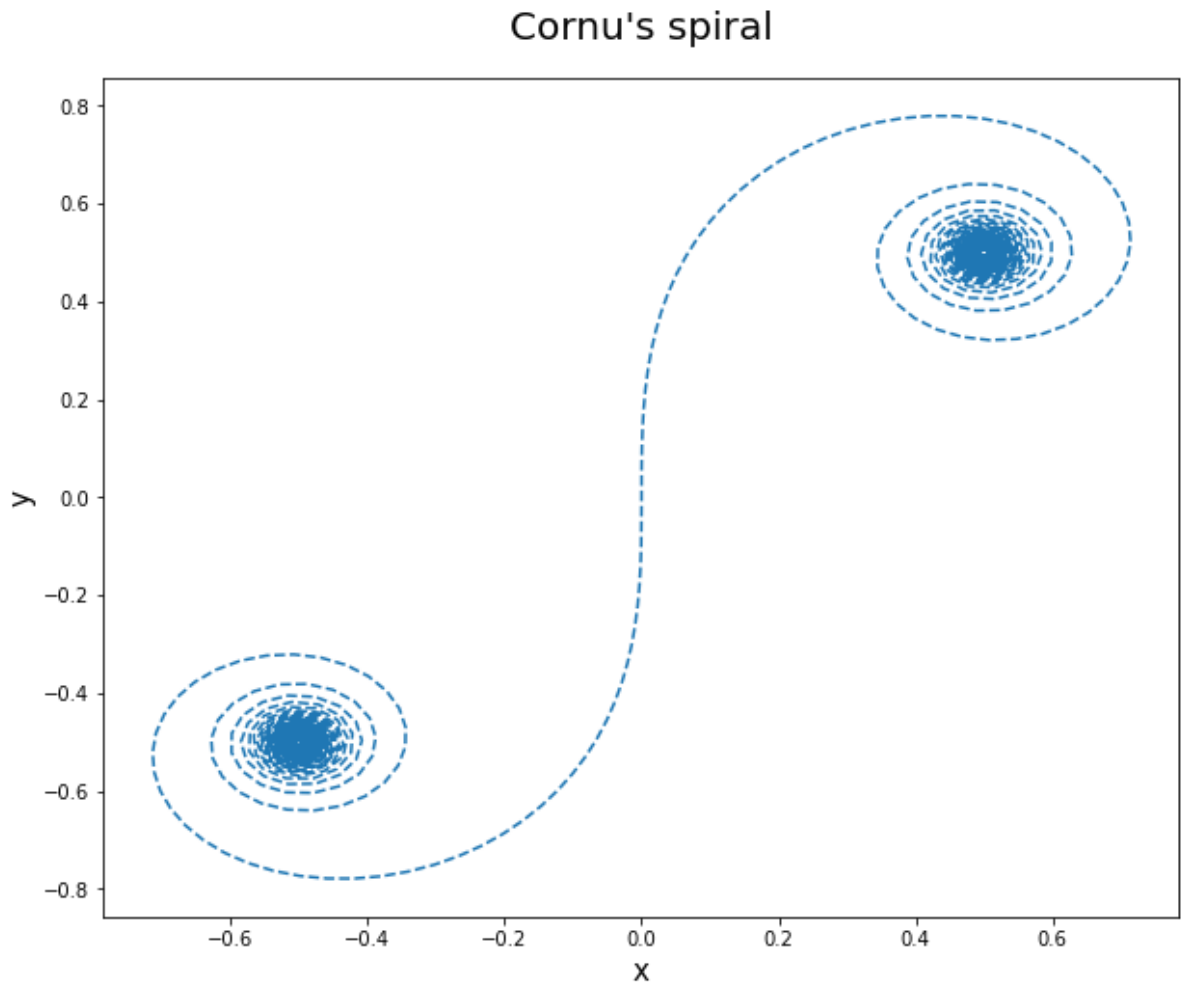
The parametric curve $(x(l), u(l)) = (C(l), S(l))$ is the Cornu's spiral. Use `scipy` package to plot this curve.

```
In [ ]: a=1e3  
a
```

```
Out[ ]: 1000.0
```

```
In [ ]: l=np.linspace(-20,20,int(1e3))
plt.figure(figsize=(10,8))
C,S=fresnel(l)
plt.plot(C,S,ls='--')
plt.title("Cornu's spiral",fontsize=20,pad=20)
plt.xlabel('x',fontsize=15)
plt.ylabel('y',fontsize=15)
```

Out[]: Text(0, 0.5, 'y')



H1: Prove that the angular acceleration on this curve is constant.

References:

1. CURVES: <https://faculty.sites.iastate.edu/jia/files/inline-files/curves.pdf>
(<https://faculty.sites.iastate.edu/jia/files/inline-files/curves.pdf>)
2. CURVATURE: <https://faculty.sites.iastate.edu/jia/files/inline-files/curvature.pdf>
(<https://faculty.sites.iastate.edu/jia/files/inline-files/curvature.pdf>)

Read these files in the order given.

Lituus Spiral: Interactive Polar Plot

```
In [ ]: theta=np.linspace(0.1,12*np.pi,100)
def radial(a):
    return a/np.sqrt(theta)
```

```
In [ ]: def interactive_plot(a=1):
    r=a/np.sqrt(theta)
    plt.figure(figsize=(10,8))
    plt.polar(theta,r,c='red')
    plt.title('Plot of a Lituus Spiral',fontsize=20,pad=20)

    interactive(interactive_plot,a=(1,10,1))
```

H2: Prove that the magnetic field at the centre of the Lituus Spiral is $\frac{\mu_0 I}{3a} \sqrt{2\pi}$. See Griffiths' Electrodynamics Book Question 5.51

3. Bowditch Curves/Lissajous Figures: Adding Subplots

Consider two sinusoidal motions

$$y_1(t) = \sin(t), y_2(t) = \sin(nt + \phi)$$

where n is an integer and ϕ is some offset. Make 4 subplots for various values of n and ϕ , but using only one for loop.

```
In [ ]: print(np.pi/4)
```

```
0.7853981633974483
```

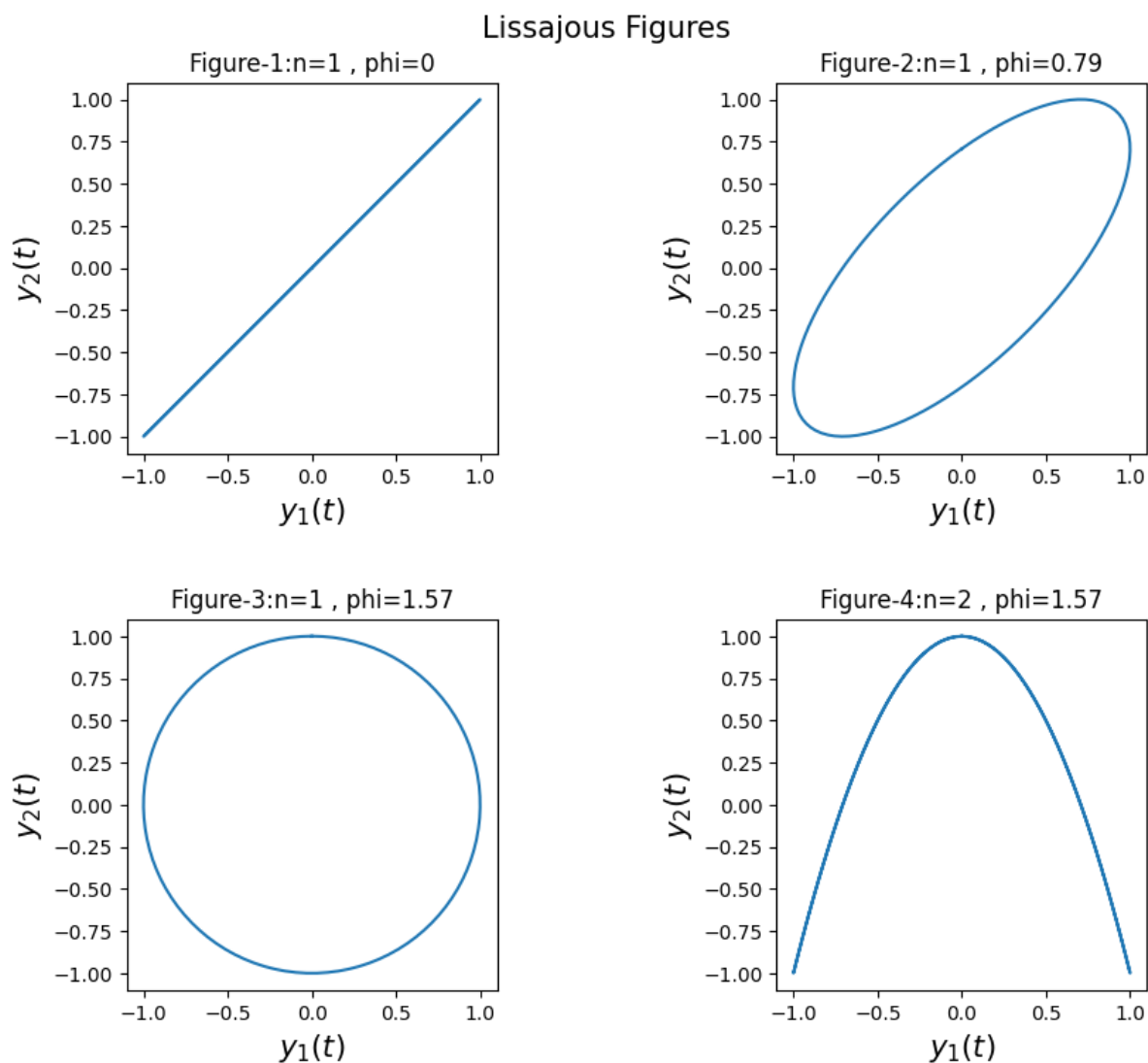
```

In [ ]: t=np.linspace(0,2*np.pi,100)
n=[1,1,1,2]
phi=[0,np.pi/4,np.pi/2,np.pi/2]
fig=plt.figure(figsize=(10,8),dpi=100)

for i in range(len(n)):
    ax=fig.add_subplot(2,2,i+1)
    ax.plot(np.sin(t),np.sin(n[i]*t+phi[i]))
    ax.set_xlabel('$y_1(t)$',fontsize=15)
    ax.set_ylabel('$y_2(t)$',fontsize=15)
    ax.set_title('Figure-'+str(i+1)+':'+n[0]+'n='+str(n[i])+', phi='+str(round(phi[i],2)))
    ax.set_aspect('equal')
fig.tight_layout(pad=3.0)
plt.suptitle('Lissajous Figures',fontsize=15)

```

Out[]: Text(0.5, 0.98, 'Lissajous Figures')



4. Fields in 2D

```

In [ ]: total=100
figure, ax = plt.subplots(figsize=(10,8),dpi=100)
x=np.linspace(-4,4,total)
y=np.linspace(-4,4,total)
X,Y=np.meshgrid(x,y)

k=1/(4*np.pi*epsilon_0)
Ex=[[0]*total]*total
Ey=deepcopy(Ex)
# create a dictionary of charges

qpos={(-2,0): 1, (2,0): 1, (0,-2): -5, (0,2): -1}

for charge_loc,charge in qpos.items():
    if(charge>0):
        ax.add_artist(Circle(charge_loc,abs(charge)/20,color='black'))
    else:
        ax.add_artist(Circle(charge_loc,abs(charge)/20,color='red'))
    ax.text(charge_loc[0]-0.25,charge_loc[1]+0.5,s=str(charge)+'q',fontsize=20)

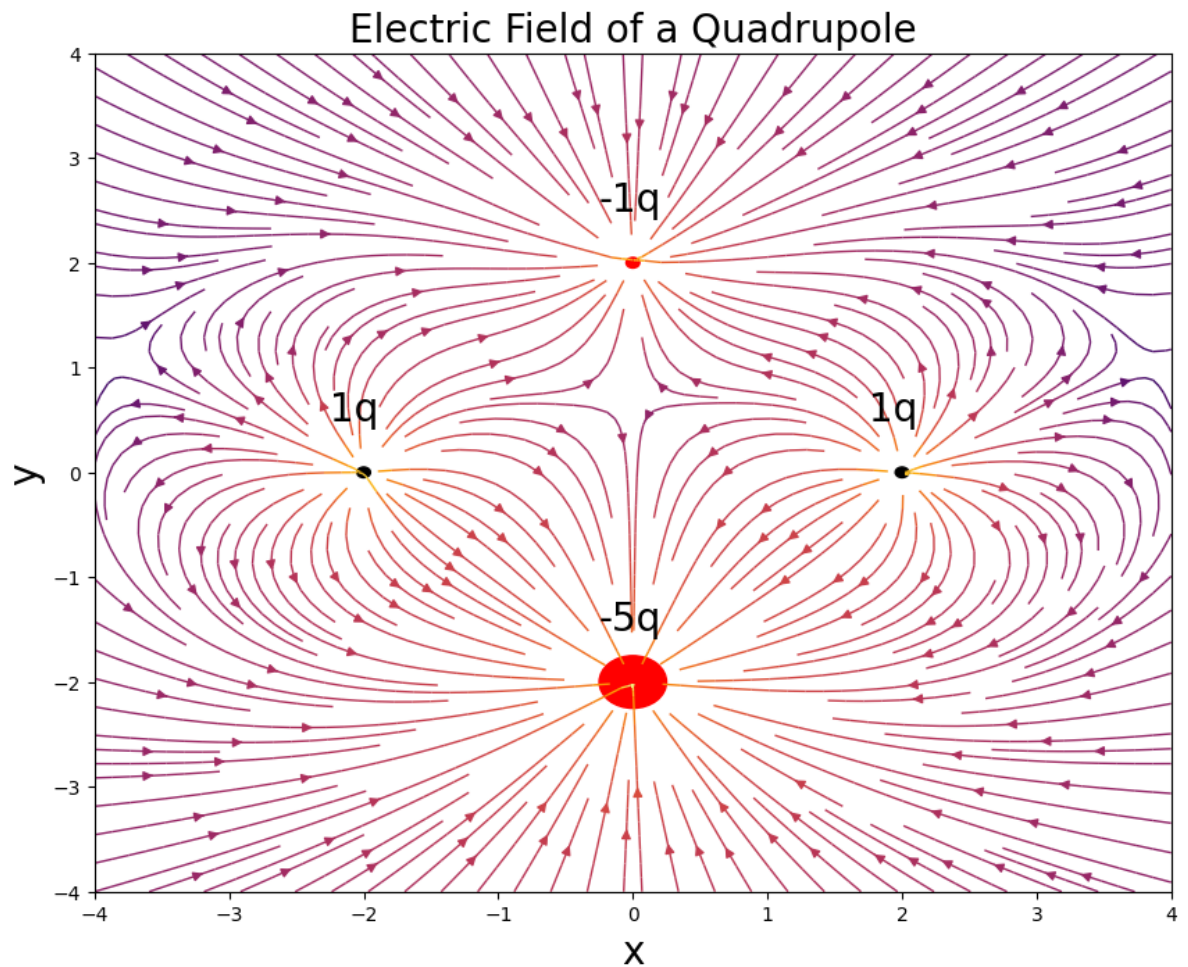
for charge_loc,charge in qpos.items():
    charge_xloc,charge_yloc=charge_loc
    R=np.sqrt((X-charge_xloc)**2+(Y-charge_yloc)**2)
    Ex+=charge*(X-charge_xloc)/R**3
    Ey+=charge*(Y-charge_yloc)/R**3

color=np.log((np.sqrt(Ex**2+Ey**2)))
print(color)

ax.streamplot(X, Y, Ex, Ey,color=color,linewidth=1,density=2, arrowstyle='->',
, arrowsize=1,cmap='inferno')
ax.set_xlabel('x',fontsize=20)
ax.set_ylabel('y',fontsize=20)
ax.set_title('Electric Field of a Quadrupole', fontsize=20)
print(len(color))

```

```
[ [-1.56605413 -1.52972443 -1.4929474 ... -1.4929474 -1.52972443
-1.56605413]
[-1.55152628 -1.51454332 -1.47708347 ... -1.47708347 -1.51454332
-1.55152628]
[-1.53754117 -1.49990528 -1.46176211 ... -1.46176211 -1.49990528
-1.53754117]
...
[-2.56143154 -2.53709413 -2.51203164 ... -2.51203164 -2.53709413
-2.56143154]
[-2.56913043 -2.54515929 -2.52051088 ... -2.52051088 -2.54515929
-2.56913043]
[-2.577501 -2.55390671 -2.52968015 ... -2.52968015 -2.55390671
-2.577501 ]]
100
```



References:

1. Hues, Lightness, Saturation: <https://vanseodesign.com/web-design/hue-saturation-and-lightness/>
(<https://vanseodesign.com/web-design/hue-saturation-and-lightness/>)
2. Colormaps: <https://matplotlib.org/stable/tutorials/colors/colormaps.html>
(<https://matplotlib.org/stable/tutorials/colors/colormaps.html>)

H3: Find out how the color scheme is working here.

H4: Plot the electric field lines due to an octopole distribution. Distribute the 8 charges with alternating signs on a circle. Do not hardcode the charge locations. Use a for loop.

Electric Field due to a Parallel Plate Capacitor

```

In [ ]: qpos={}
total_num_charges=40
int_dis=1e-2
for i in range(total_num_charges):
    qpos[(-2,4*i/(total_num_charges-1)-2)]=1
    qpos[(2,4*i/(total_num_charges-1)-2)]=-1

total=100
figure, ax = plt.subplots(figsize=(10,8),dpi=100)
x=np.linspace(-8,8,total)
y=np.linspace(-8,8,total)
X,Y=np.meshgrid(x,y)

k=1/(4*np.pi*epsilon_0)
Ex=[[0]*total]*total
Ey=deepcopy(Ex)

# qpos={1: (-2,0), 5: (2,0), -1: (0,-2), -5: (0,2)}
for charge_loc,charge in qpos.items():
    if(charge>0):
        ax.add_artist(Circle(charge_loc,abs(charge)/20,color='black'))
    else:
        ax.add_artist(Circle(charge_loc,abs(charge)/20,color='red'))

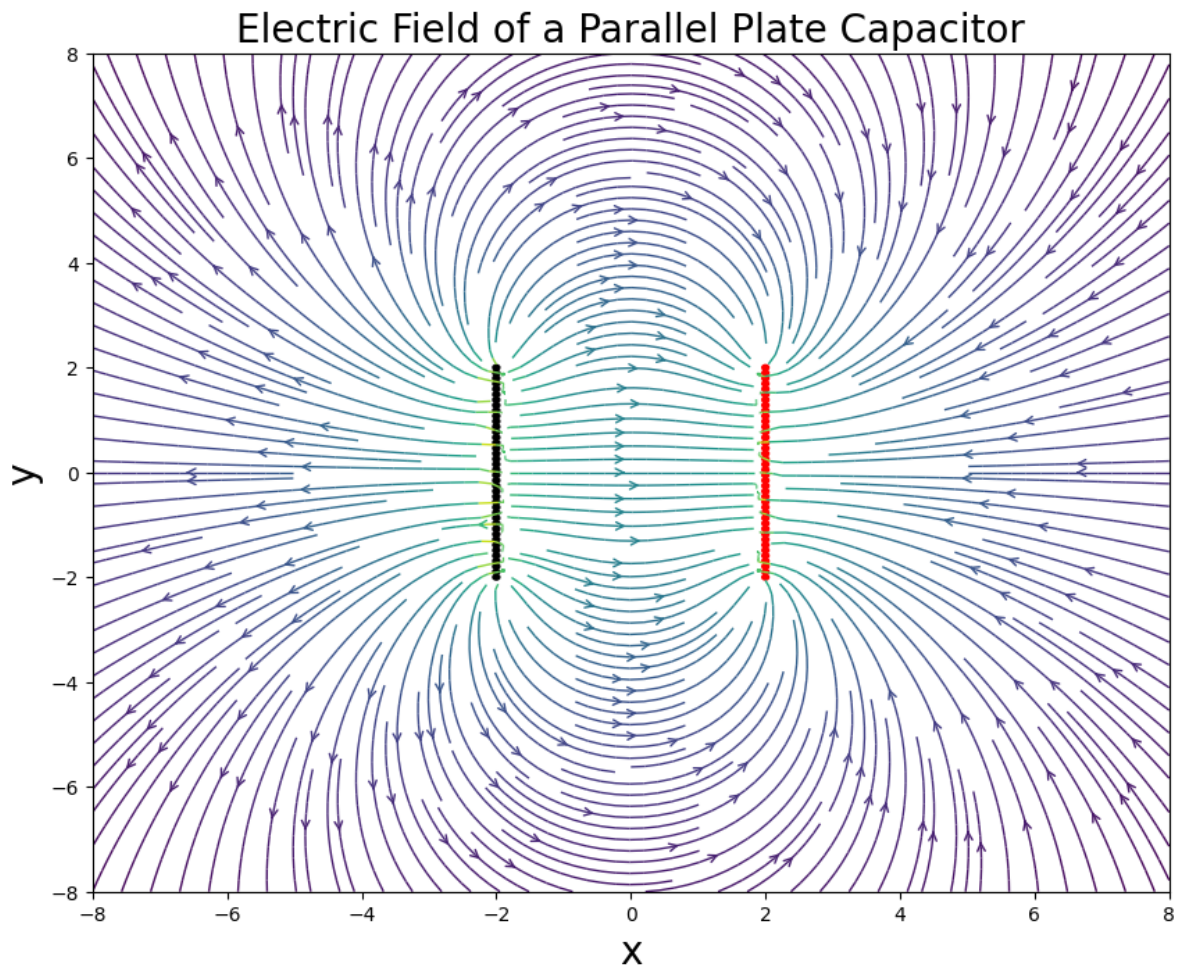
for charge_loc,charge in qpos.items():
    charge_xloc,charge_yloc=charge_loc
    R=np.sqrt((X-charge_xloc)**2+(Y-charge_yloc)**2)
    Ex+=charge*(X-charge_xloc)/R**3
    Ey+=charge*(Y-charge_yloc)/R**3

color=np.log((np.sqrt(Ex**2+Ey**2)))

ax.streamplot(X, Y, Ex, Ey, color=color,linewidth=1,cmap=plt.cm.viridis,densit
y=2.5, arrowstyle='->', arrowsize=1)
ax.set_xlabel('x',fontsize=20)
ax.set_ylabel('y',fontsize=20)
ax.set_title('Electric Field of a Parallel Plate Capacitor', fontsize=20)

```

```
Out[ ]: Text(0.5, 1.0, 'Electric Field of a Parallel Plate Capacitor')
```



5. Fields in 3D

Numerically integrate

$$I = \int_{-\infty}^{\infty} e^{-ax^2} dx$$

Pass a as an argument. The analytical value of I is

$$I = \sqrt{\frac{\pi}{a}}$$

Verify your numerical result against various values of a.

```
In [ ]: def integrate(x,a):
        return np.exp(-a*x**2)
```

```
In [ ]: a=2
ans=quad(integrate,-np.inf,np.inf,args=(a))[0]
print(ans,np.sqrt(np.pi/a))
#help(quad)
```

1.2533141373155017 1.2533141373155001

Let's now perform a double integral. We use dblquad. It returns the double (definite) integral of func(y, x) from x = a..b and y = gfun(x)..hfun(x). Perform the integration

$$I = \int_0^1 \int_{y=0}^{y=x} ay \, dy \, dx$$

```
In [ ]: def integrate(y,x,a):
        return a*y
a=2
ans=dblquad(integrate,0,1,lambdax:0, lambdax:x,args=[a])[0]
print(ans,a/6)
```

0.3333333333333337 0.3333333333333333

Reference: <https://problemsolvingwithpython.com/06-Plotting-with-Matplotlib/06.15-Quiver-and-Stream-Plots/>
(<https://problemsolvingwithpython.com/06-Plotting-with-Matplotlib/06.15-Quiver-and-Stream-Plots/>)

Magnetic Field of a Cylindrical Bar Magnet

Question: Plot the magnetic field due to a small cylindrical bar magnet with magnetization $1000kA/m$.

Solution: Consider a cylinder with its center at the origin of the coordinate system. Consider a disk of infinitesimally small thickness dz' . The current in this disc flows in the azimuthal direction with value Mdz' since surface density $\vec{K}_{bound} = M\hat{\phi}$. The line element in the azimuthal direction (in cylindrical coordinate system (ρ, ϕ, z)) is $\vec{dl}' = (-\rho' \sin(\phi')\hat{x} + \rho' \cos(\phi')\hat{y})d\phi'$, where the primed coordinates represent the source point. Let $\vec{r} = (x, y, z)$ represent the coordinates of a field point where these coordinates can take values from $(-100, 100)$ cm. Let the coordinates of the source point be $\vec{r}' = (\rho' \cos(\phi'), \rho' \sin(\phi'), z')$. Therefore, the magnetic field \vec{B} is given by

$$\vec{B} = \frac{\mu_0 M}{4\pi} \int_{-z_{min}}^{z_{max}} \int_0^{2\pi} \frac{\vec{dl}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dz' d\phi'$$

Using the above formula, we can calculate the components of the magnetic field B_x, B_y, B_z . We first create a meshgrid and then using quiver plots, plot the field components.

H5: Plot the magnetic field with the magnet in 3d.


```

In [ ]: cyl_rad=1
        cyl_len=2

        total=8
        l_lim=-100
        u_lim=100
        x=np.linspace(l_lim,u_lim,total)
        y=deepcopy(x)
        z=deepcopy(x)
        x,y,z=np.meshgrid(x,y,z)

        # Functions to calculate the integrands in the three directions
        def x_integrand(phi_prime,z_prime,x,y,z):
            R=np.linalg.norm([x-cyl_rad*np.cos(phi_prime),y-cyl_rad*np.sin(phi_prime),z-
            z_prime])
            return cyl_rad*np.cos(phi_prime)*(z-z_prime)/R**3

        def y_integrand(phi_prime,z_prime,x,y,z):
            R=np.linalg.norm([x-cyl_rad*np.cos(phi_prime),y-cyl_rad*np.sin(phi_prime),z-
            z_prime])
            return cyl_rad*np.sin(phi_prime)*(z-z_prime)/R**3

        def z_integrand(phi_prime,z_prime,x,y,z):
            R=np.linalg.norm([x-cyl_rad*np.cos(phi_prime),y-cyl_rad*np.sin(phi_prime),z-
            z_prime])
            return (cyl_rad*np.sin(phi_prime)*(y-cyl_rad*np.sin(phi_prime))+cyl_rad*np.c
            os(phi_prime)*(x-cyl_rad*np.cos(phi_prime)))/R**3

        # Function to calculate magnetic field
        def magnetic_field(x,y,z):
            M=1e6
            mu0_4pi=mu_0/(4*np.pi)
            print(mu0_4pi)
            cons=M*mu0_4pi
            x=x.reshape(-1)
            y=y.reshape(-1)
            z=z.reshape(-1)
            mag_x=[]
            mag_y=[]
            mag_z=[]

            for i in range(0,len(x)):
                mag_x.append(cons*dblquad(x_integrand,-cyl_len/2,cyl_len/2,lambda inner:0,
                lambda inner:2*np.pi,args=(x[i],y[i],z[i]))[0])
                mag_y.append(cons*dblquad(y_integrand,-cyl_len/2,cyl_len/2,lambda inner:0,
                lambda inner:2*np.pi,args=(x[i],y[i],z[i]))[0])
                mag_z.append(-cons*dblquad(z_integrand,-cyl_len/2,cyl_len/2,lambda inner:0
                ,lambda inner:2*np.pi,args=(x[i],y[i],z[i]))[0])

            mag_x=np.array(mag_x).reshape(total,total,total)
            mag_y=np.array(mag_y).reshape(total,total,total)
            mag_z=np.array(mag_z).reshape(total,total,total)

            return mag_x,mag_y,mag_z

```

```
Bx,By,Bz=magnetic_field(x,y,z)
```

```
1.0000000000000001e-07
```

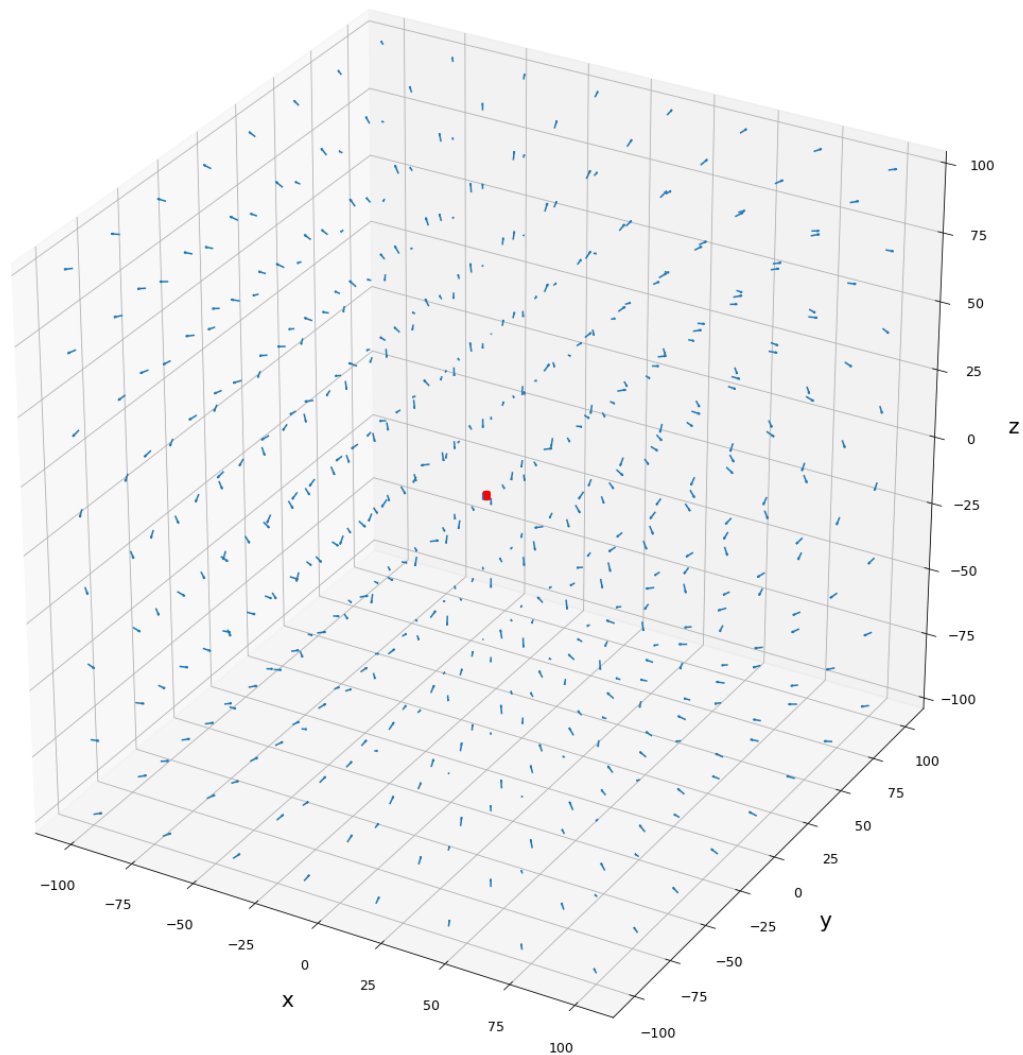
```

In [ ]: def cylinder(cyl_rad):
        phi = np.linspace(0,2*np.pi,100)
        return cyl_rad*np.cos(phi),cyl_rad*np.sin(phi)

fig=plt.figure(figsize=(20,20))
ax=fig.gca(projection='3d')
c_x,c_y=cylinder(cyl_rad)
for i in np.linspace(-int(cyl_len/2),int(cyl_len/2),100):
    ax.plot(c_x,c_y,i,color='r')
ax.set_xlabel('x',fontsize=20,labelpad=30)
ax.set_ylabel('y',fontsize=20,labelpad=30)
ax.set_zlabel('z',fontsize=20,labelpad=30)
ax.tick_params(labelsize=12.5,pad=15)
color=np.linalg.norm([Bx,By,Bz])
ax.quiver(x,y,z,Bx,By,Bz,length=3,normalize=True,label='Magnetic Field of a Cy
lindrical Magnet')

```

Out[]: <mpl_toolkits.mplot3d.art3d.Line3DCollection at 0x7fcb62f58250>



H5: Figure out how numbers are mapped to colors. How are the lowest and highest numeric values allocated the extremities of the colormaps?

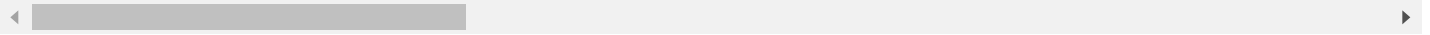
Flow Past a Sphere at Low Reynolds Numbers

Theory:

https://geo.libretexts.org/Bookshelves/Sedimentology/Book%3A_Introduction_to_Fluid_Motions_and_Sediment_Transport/Stokes'_Law%2C_The_Bernoulli_Equation%2C_Turbulence%2C_Boundary_Layers%2C_Flow_Separation/3.02%3A_Flow_Past_a_Sphere_at_Low_Reynolds_Numbers

(https://geo.libretexts.org/Bookshelves/Sedimentology/Book%3A_Introduction_to_Fluid_Motions_and_Sediment_Transport/Stokes'_Law%2C_The_Bernoulli_Equation%2C_Turbulence%2C_Boundary_Layers%2C_Flow_Separation/3.02%3A_Flow_Past_a_Sphere_at_Low_Reynolds_Numbers)

Check equation 3.2.1



```

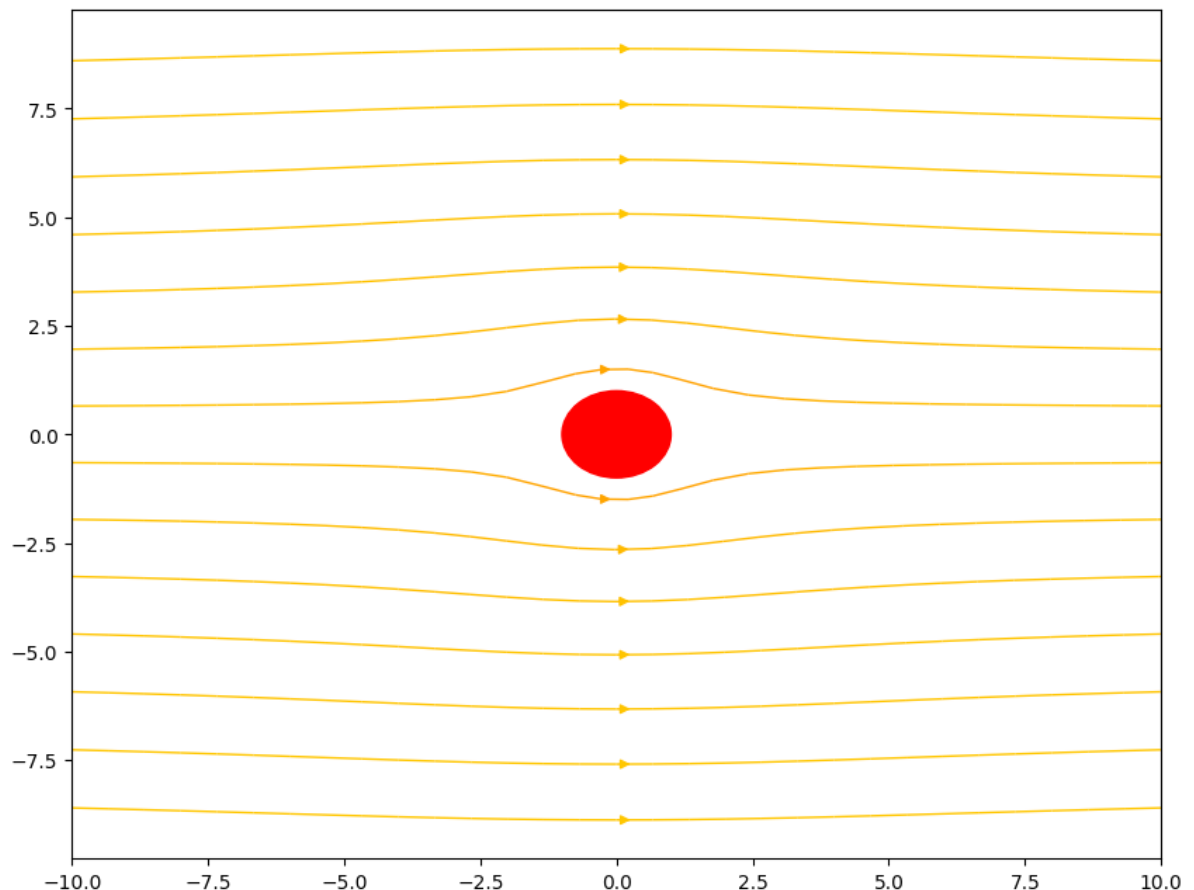
In [ ]: x=np.linspace(-10,10,100)
y=np.linspace(-10,10,100)
xg,yg=np.meshgrid(x,y)
rad=1
vel=1
def velocity(x,y,R,U):
    r=np.sqrt(x**2+y**2)
    cos_th=x/r
    sin_th=y/r
    u_r=U*cos_th*(1-3*R/(2*r)+R**3/(2*r**3))
    u_theta=-U*sin_th*(1-3*R/(4*r)-R**3/(4*r**3))
    u_x=u_r*cos_th-u_theta*sin_th
    u_y=u_r*sin_th+u_theta*cos_th
    return u_x,u_y

ux,uy=velocity(xg,yg,rad,vel)

color=np.log((np.sqrt(ux**2+uy**2)))
y_aux=np.linspace(-10,10,16)
x_aux=[-8]*len(y_aux)
figure, ax = plt.subplots(figsize=(10,8),dpi=100)
ax.streamplot(xg, yg, ux, uy, color=color,linewidth=1,cmap='hot',density=1, arrowstyle='->', arrowsize=1,minlength=0.4,start_points=np.array([x_aux,y_aux]).T)
ax.add_artist(Circle((0,0),1,color='red'))

```

Out[]: <matplotlib.patches.Circle at 0x7fcb633033d0>



In [281]: `!jupyter nbconvert --execute --to html Plots.ipynb`



```
[NbConvertApp] WARNING | pattern u'Plots:' matched no files
[NbConvertApp] WARNING | pattern u'Singularities,Lissajous' matched no files
[NbConvertApp] WARNING | pattern u'Figures,' matched no files
[NbConvertApp] WARNING | pattern u'Spirals,' matched no files
[NbConvertApp] WARNING | pattern u'Fields.ipynb' matched no files
This application is used to convert notebook files (*.ipynb) to various other
formats.
```

WARNING: THE COMMANDLINE INTERFACE MAY CHANGE IN FUTURE RELEASES.

Options

Arguments that take values are actually convenience aliases to full Configurables, whose aliases are listed on the help line. For more information on full configurables, see '--help-all'.

--execute

Execute the notebook prior to export.

--allow-errors

Continue notebook execution even if one of the cells throws an error and include the error message in the cell output (the default behaviour is to abort conversion). This flag is only relevant if '--execute' was specified, too.

--no-input

Exclude input cells and output prompts from converted document.

This mode is ideal for generating code-free reports.

--stdout

Write notebook output to stdout instead of files.

--stdin

read a single notebook file from stdin. Write the resulting notebook with default basename 'notebook.*'

--inplace

Run nbconvert in place, overwriting the existing notebook (only relevant when converting to notebook format)

-y

Answer yes to any questions instead of prompting.

--clear-output

Clear output of current file and save in place, overwriting the existing notebook.

--debug

set log level to logging.DEBUG (maximize logging output)

--no-prompt

Exclude input and output prompts from converted document.

--generate-config

generate default config file

--nbformat=<Enum> (NotebookExporter.nbformat_version)

Default: 4

Choices: [1, 2, 3, 4]

The nbformat version to write. Use this to downgrade notebooks.

--output-dir=<Unicode> (FilesWriter.build_directory)

Default: ''

Directory to write output(s) to. Defaults to output to the directory of each

notebook. To recover previous default behaviour (outputting to the current

working directory) use . as the flag value.

```

--writer=<DottedObjectName> (NbConvertApp.writer_class)
    Default: 'FilesWriter'
    Writer class used to write the results of the conversion
--log-level=<Enum> (Application.log_level)
    Default: 30
    Choices: (0, 10, 20, 30, 40, 50, 'DEBUG', 'INFO', 'WARN', 'ERROR', 'CRITICAL')
    Set the log level by value or name.
--reveal-prefix=<Unicode> (SlidesExporter.reveal_url_prefix)
    Default: u''
    The URL prefix for reveal.js (version 3.x). This defaults to the reveal CDN,
    but can be any url pointing to a copy of reveal.js.
    For speaker notes to work, this must be a relative path to a local copy
    of
    reveal.js: e.g., "reveal.js".
    If a relative path is given, it must be a subdirectory of the current
    directory (from which the server is run).
    See the usage documentation
    (https://nbconvert.readthedocs.io/en/latest/usage.html#reveal-js-html-slideshow)
    for more details.
--to=<Unicode> (NbConvertApp.export_format)
    Default: 'html'
    The export format to be used, either one of the built-in formats
    ['asciidoc', 'custom', 'html', 'latex', 'markdown', 'notebook', 'pdf',
    'python', 'rst', 'script', 'slides'] or a dotted object name that represents
    the import path for an `Exporter` class
--template=<Unicode> (TemplateExporter.template_file)
    Default: u''
    Name of the template file to use
--output=<Unicode> (NbConvertApp.output_base)
    Default: ''
    overwrite base name use for output files. can only be used when converting
    one notebook at a time.
--post=<DottedOrNone> (NbConvertApp.postprocessor_class)
    Default: u''
    PostProcessor class used to write the results of the conversion
--config=<Unicode> (JupyterApp.config_file)
    Default: u''
    Full path of a config file.

```

To see all available configurables, use `--help-all`

Examples

The simplest way to use nbconvert is

```
> jupyter nbconvert mynotebook.ipynb
```

which will convert mynotebook.ipynb to the default format (probably HTML).

You can specify the export format with `--to`.

Options include ['asciidoc', 'custom', 'html', 'latex', 'markdown', 'notebook', 'pdf', 'python', 'rst', 'script', 'slides']

```
book', 'pdf', 'python', 'rst', 'script', 'slides'].
```

```
> jupyter nbconvert --to latex mynotebook.ipynb
```

Both HTML and LaTeX support multiple output templates. LaTeX includes 'base', 'article' and 'report'. HTML includes 'basic' and 'full'. You can specify the flavor of the format used.

```
> jupyter nbconvert --to html --template basic mynotebook.ipynb
```

You can also pipe the output to stdout, rather than a file

```
> jupyter nbconvert mynotebook.ipynb --stdout
```

PDF is generated via latex

```
> jupyter nbconvert mynotebook.ipynb --to pdf
```

You can get (and serve) a Reveal.js-powered slideshow

```
> jupyter nbconvert myslides.ipynb --to slides --post serve
```

Multiple notebooks can be given at the command line in a couple of different ways:

```
> jupyter nbconvert notebook*.ipynb
```

```
> jupyter nbconvert notebook1.ipynb notebook2.ipynb
```

or you can specify the notebooks list in a config file, containing::

```
c.NbConvertApp.notebooks = ["my_notebook.ipynb"]
```

```
> jupyter nbconvert --config mycfg.py
```

In []: