Electron Diffraction

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ABSTRACT

Diffraction occurs when a wave passes through a slit whose width is in the same order as the wavelength. In general, diffraction results in the redirection of the wave. The exact direction of the outgoing wave, θ, depends on the relationship of the slit width, d, the wavelength, λ, and the order of the reflection, n, according to Bragg's Law[1]. We used the diffraction of electron waves through graphite to find the spacing between the lattice layers of graphite and the corresponding order of reflection for the observed rings. We used an evacuated tube and a varying power supply to accelerate electrons through a graphite sample and captured the diffraction pattern on the phosphor coating inside the tube. We measured the diameter of the diffraction rings and found $d_{11} = 0.132$ and $d_{10} = 0.228$. Our results were consistent with the known values for the lattice spacing of graphite, which led us to a reasonable value of the order of reflection.

1 Introduction

De Broglie suggested that all particles with momentum p have wave-particle duality. He theorized that depending on how we measure these particles, they may instead act as waves according to the equation

$$\lambda = \frac{h}{mv}, \tag{1}$$

where λ is the de Broglie wavelength, m is the mass of the particle, and v is the velocity of the particle[2].

A slit experiment was required to observe the wave nature of electrons; however, man-made slits would not produce observable results due to the very small de Broglie wavelength of electrons and elementary particles. To overcome this experimental barrier, Max Von Laue suggested in 1912 using the grating pattern of raw materials such as graphite and some salts[3]. Lawrence Bragg demonstrated that the spacing between different layers of atoms in these materials works perfectly as a grating for an electron wave diffraction experiment. He formulated what came to be known as Bragg's Law based on his findings: the scattering, θ, can be described by

$$n\lambda = 2dsin(\theta), \tag{2}$$

where d is the lattice spacing and n is the order of reflection[1].

Electron diffraction was first experimentally shown in 1927 by physicists Clinton Davisson and Lester Germer[4] using a crystalline nickel target. The following year, G. P. Thompson performed a similar experiment with gold, aluminum, and silver targets[5]. Stem and Esterman did the same diffraction experiment in 1930 with Hydrogen molecules and the next year with helium molecules using a lithium fluoride crystal target. All of these experiments supported the de Broglie postulate of wave-particle duality.

In our experiment, we replicate the measurements done in previous experiments to observe electron diffraction[6]. We measured a diffraction pattern that indicated the wave nature of electrons. We then determined the specific order of reflection for the diffraction pattern and lattice spacing for our graphite target based on our collected data.

2 Experimental Setup

We used an evacuated tube along with a power supply shown in Fig. 1.

Fig. 2 shows a schematic of the evacuated tube. The cathode is heated from a 6.8 V ac voltage across pins 3 and 4 when the power supply is turned on, resulting in emitted electrons. These electrons are then accelerated via a high voltage across pins 5 and 7. Next, the electrons pass through the graphite lattice and hit the inside surface of the tube. This surface is coated with phosphor so that it will radiate a green light as electrons hit it and show the diffraction pattern.

We turned the power supply on and waited for the cathode temperature to stabilize before using a caliper to measure the diameters of the diffraction rings on the phosphor coating at 0.5 kV intervals between 2.5 kV and 4.5 kV. We used this measurement and the inverse square root of the voltage to create a graph. Performing a linear regression on our data produced a slope from which we could determine the order of reflection and the d-spacing of graphite for both the inner and outer diffraction rings.

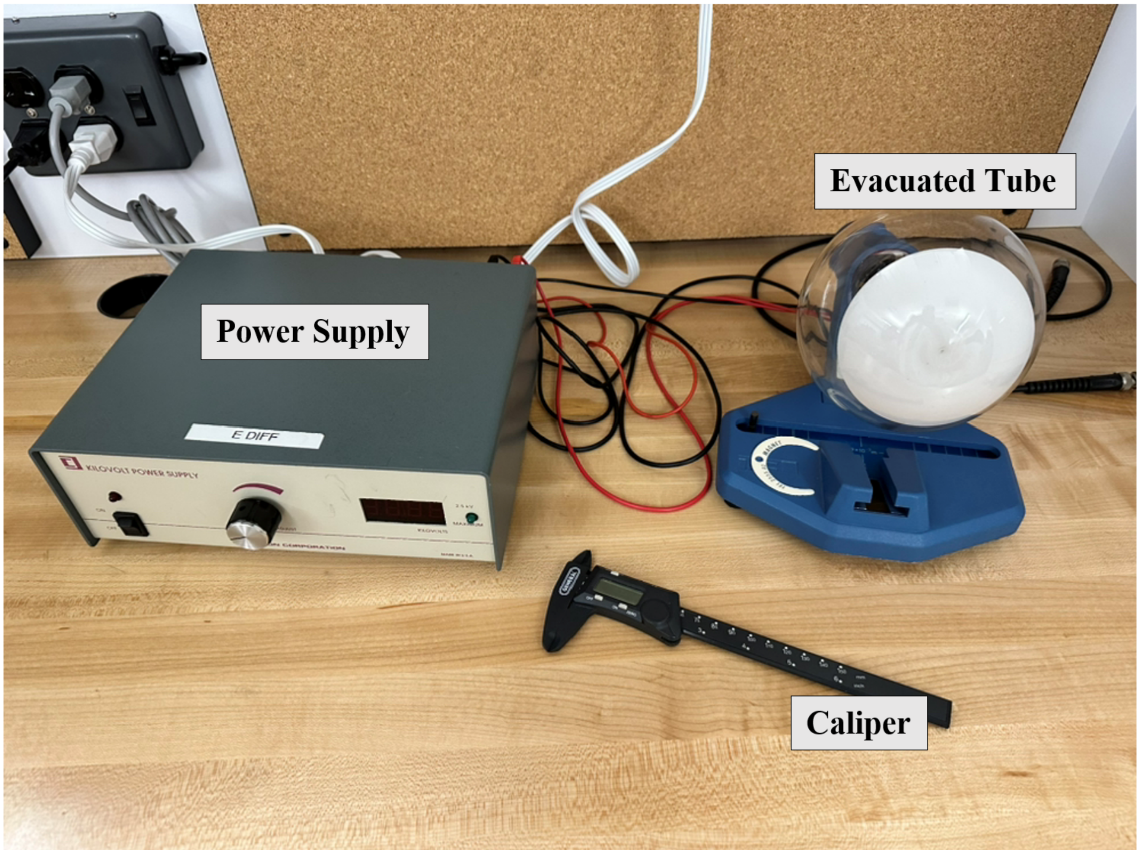

Figure 1. This is our experimental setup. You can see the power supply, being connected to the evacuated tube through the red and black wires. The caliper is also there because it helped us measure the diffraction rings.

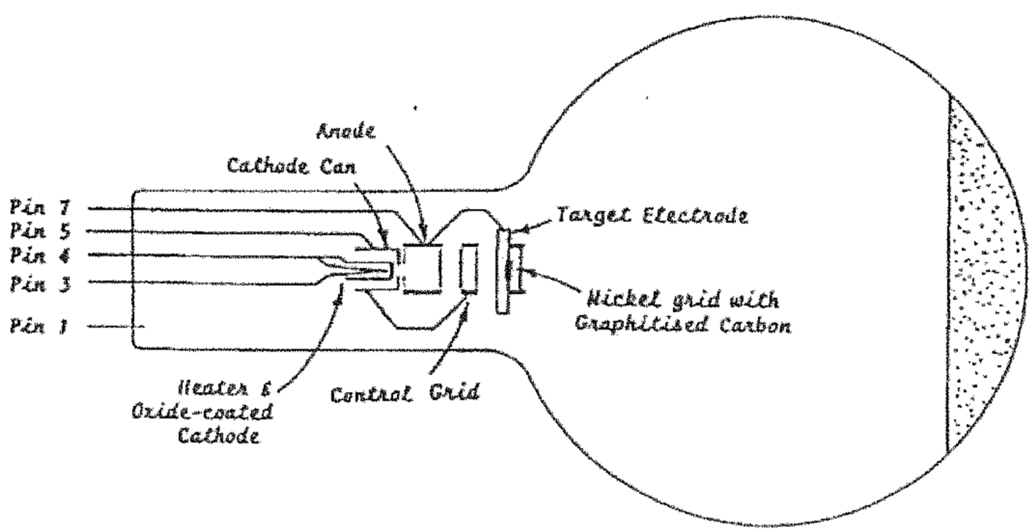

Figure 2. A schematic layout of the tube. It shows the connection pins behind the tube as well as the components of the tube responsible for the electron ray that creates the diffraction pattern.

3 Results

We apply the law of conservation of energy to an electron crossing the potential difference created by the anode:

$$\frac{1}{2}mv_1^2 + eV_1 = \frac{1}{2}mv_2^2 + eV_2, \tag{3}$$

where e is the charge of an electron and $V_2 - V_1$ is the anode voltage, V_a. In our case the initial velocity of electrons is 0, so the relationship between the velocity at which the electrons hit the graphite sample and the anode voltage is given by

$$eV_a = \frac{1}{2}mv^2. \tag{4}$$

[7] Combining Eq. 1 and Eq. 4 yields

$$\lambda = 1.23V_a^{-1/2}. \tag{5}$$

Consequently, increasing the anode voltage will accelerate the electrons more, decreasing the de Broglie wavelength. The directly proportional relationship of wavelength and diffraction angle in Bragg's Law reveals that shorter wavelengths (higher voltages) result in smaller diffraction angles (smaller ring diameters). Thus, an increase in the inverse square root of the anode voltage will result in an increase in ring diameter, as shown in Fig. 3.

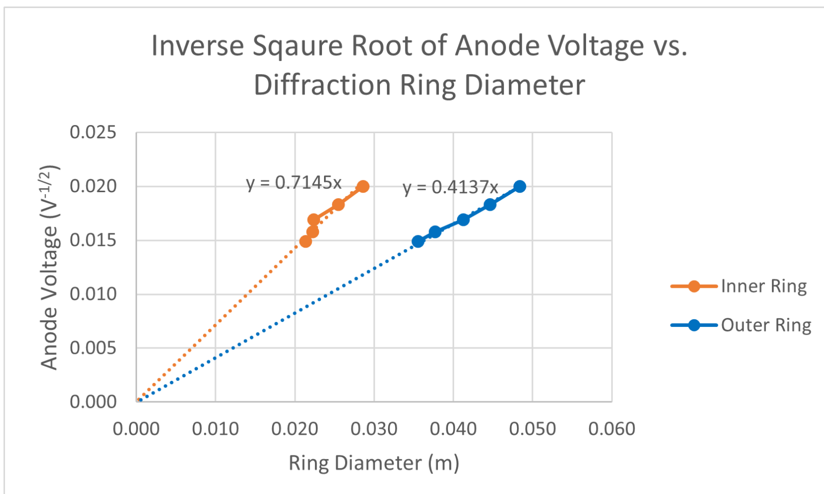

Figure 3. Graph of the inverse square root of the anode voltage vs diffraction ring diameter. The regression for each ring is plotted, and the equation for each is shown above each line.

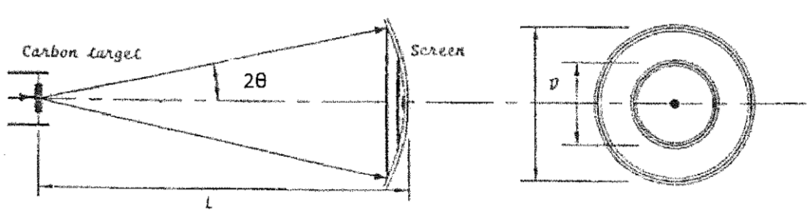

Figure 4. Diagram of the apparatus showing the relationship between the diffraction angle, the diameter of the rings, and the length of the apparatus.

Based on the diagram in Fig. 4 and the small angle approximation $\tan(x) \approx x$, the geometrical equivalent for θ is $\frac{D}{4L}$. By substituting this expression and Eq. 5 into Eq. 2, we arrive at the following relation for anode voltage and ring diameter:

$$V_a^{-1/2} = \frac{dD}{2.46nL}. \tag{6}$$

By comparing this equation to our regression, we can extract the relationship between the lattice spacing and the order of reflection. The regression for the inner diameter gives ${V_a}^{-1/2} = 0.7145D$ and comparing this to Eq. 6 we find that for $n = 1$, $d_{inner} = 0.2286nm$. The regression for the outer diameter ${V_a}^{-1/2} = 0.4137D$ gives $d_{outer} = 0.1323nm$.

A picture of the actual diffraction rings studied is shown below in Fig. 5.

Figure 5. Photo of the actual diffraction rings found during the experiment.

4 Conclusions

Our results for lattice spacing d are consistent with known values for graphite. Typical values are $d_{11} = 0.123nm$ and $d_{10} = 0.213nm$, meaning that $d_{inner} \approx d_{10}$ and $d_{outer} \approx d_{11}$[8]. Our results have some measurement error due to the calipers ($\pm 0.005m$), but it is negligible for our application since it only gives an error on the diameter measurement. The results led us to conclude that the order of reflection, n, was 1 because this n-value gave us the most consistent values of d-spacing to what was expected. It is possible for the n-value to be greater, but it would require values of voltage much greater than the maximum voltage on our power supply. In the end, the goal of this experiment was met as we observed diffraction rings and got reasonable values for the lattice spacing. This proves that electron diffraction is a viable method to find the d-spacing in graphite, and in the future, our experiment could be used as an accurate method to find the specific d-spacing for other materials and help in the classification of these materials.

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