Electron E/M Ratio

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ABSTRACT

Modern experiments have determined individually both the charge and mass of the electron. Previously, only the ratio of these quantities could be measured. In our experiment, we accelerate electrons into a uniform magnetic field created by Helmholtz coils. We use the accelerating voltage, the current through the coils, and the radius of the circular path of the electron beam to calculate the charge-to-mass ratio of the electron. We find the ratio to be 1.81×10^{11} C/kg, which is within 2.82% of the known value: 1.76×10^{11} C/kg.

1 Introduction

J.J. Thomson first discovered the electrons' charge-to-mass ratio through experiments with cathode ray tubes. Based on the behavior of cathode rays deflected by both a magnetic and electric field, he concluded that the rays were made up of charged particles (electrons) and was able to determine their mass-to-charge ratio[1].

In our experiment, we use an electron beam that is accelerated by an electron gun through a given potential V into a uniform magnetic field B created by Helmholtz coils. This magnetic field, perpendicular to the path of the beam, causes the electrons to form a circular path of radius r.

The charge-to-mass ratio can be determined by equating the magnetic and centripetal forces experienced by the electrons:

$$evB = \frac{mv^2}{r}, \tag{1}$$

where e is the electron charge, v is the velocity of the electrons, and m is the electron mass.

The kinetic energy of the electrons is determined by the potential through which they are accelerated. Thus the velocity of the electrons can be expressed in terms of this potential:

$$v = \sqrt{\frac{2eV}{m}}. \tag{2}$$

The magnetic field produced by Helmholtz coils is

$$B = \frac{N\mu_0 I}{(\frac{5}{4})^{\frac{3}{2}} a}, \tag{3}$$

where N is the number of turns in the coil, I is the current through the coils, and a is the radius and separation of the coils. Solving for $\frac{e}{m}$ and substituting in Eq. (2) and Eq. (3) yields:

$$\frac{e}{m} = \frac{2V(\frac{5}{4})^3 a^2}{(N\mu_0 I r)^2}. \tag{4}$$

During our experiment, we measured various radii for different values of coil current at a constant electrode voltage. Pairing these measurements with Eq. (4), we determined the value of the charge-to-mass ratio.

We can apply the principles of charge-to-mass ratio to differentiate between the isotopes of the same atom where the only difference is mass and its too small to measure individually (This is also known as Mass Spectrometry)[2]. In the following sections, we will discuss the experimental setup that allowed us to accomplish this and go over the results from our specific experiment.

2 Experimental Setup

Our setup includes two power supplies, one to create an accelerating voltage in the electron gun and one to pass a current through the Helmholtz coil to create a magnetic field. Our set-up is shown in Fig. 1.

The magnetic field from the coils is perpendicular to the motion of the beam which deflects the electron in a circular path. This beam is illuminated because our evacuated tube is filled with helium gas which interacts the beam causing it to glow and be visible for measurement. The apparatus also includes a reflective ruler behind the tube that allows the viewer to align the ring inside the tube with the one reflected. This setup allows us to eliminate parallax errors when measuring the radius. Since the strength of the magnetic field determines the path radius of the electron beam, we measured radii at ten different values of coil current varied at a singular voltage.

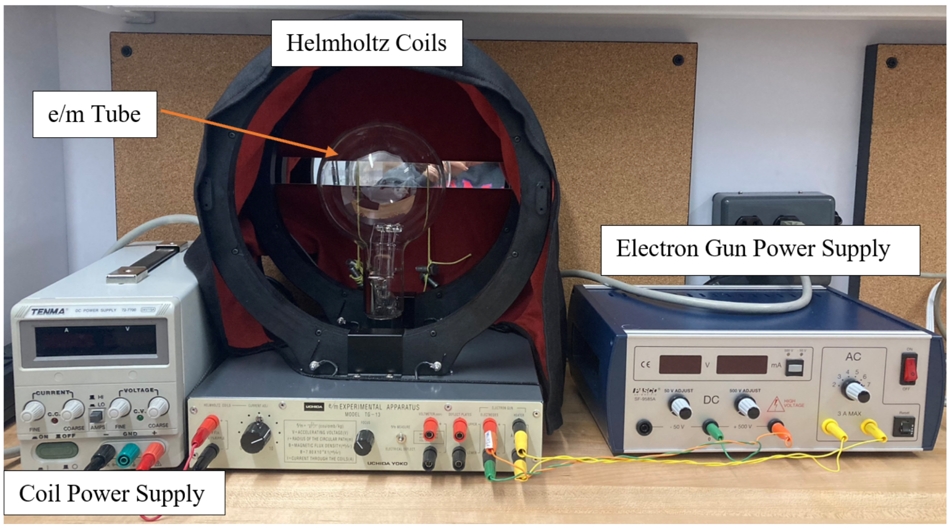

Figure 1. The experimental setup consisting of two power supplies, a Helmholtz coil apparatus and the E/M tube.

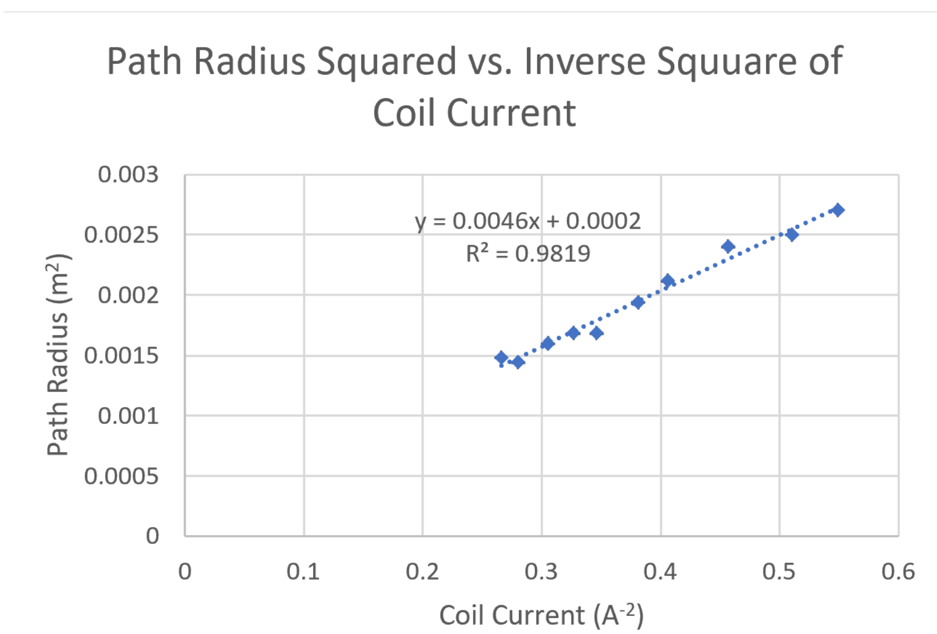

Figure 2. The graph of our data. We converted our raw measurements of radius and current into radius squared and inverse square of current to fit the model of Eq. 5.

3 Results

To compare our results to the expected value for charge-to-mass ratio, we need an appropriate mathematical model to which we can fit our data. Rearranging Eq. (4) and substituting $V = 259\text{V}$, $a = 0.15\text{m}$, $N = 130$, $\mu_0 = 4\pi \times 10^{-7}\text{N}\cdot\text{A}^{-2}$ produces a useful model:

$$r^2 = \frac{2V(\frac{5}{4})^3 a^2}{(N\mu_0 I)^2 \frac{e}{m}} = \frac{8.54 \times 10^8}{I^2 \frac{e}{m}}. \tag{5}$$

This equation indicates that since our voltage is constant, the only variable quantity that affects the radius of the electron beam is the current through the coils. Thus, we measured pairs of radius and current values. Linear regression of our data (shown in Fig 2) yields the following line of best fit:

$$r^2 = \frac{4.6 \times 10^{-3}}{I^2} + 2 \times 10^{-4}. \tag{6}$$

Comparing the slope of our line with Eq. 5 determines our measured value of $\frac{e}{m}$.

$$\frac{e}{m} = \frac{8.54 \times 10^8}{4.6 \times 10^{-3}} = 1.86 \times 10^{11}. \tag{7}$$

4 Conclusions

We calculate the error of our result using the expected value[3] $\frac{e}{m} = 1.76 \times 10^{11}$.

$$\frac{1.86 \times 10^{11} - 1.76 \times 10^{11}}{1.76 \times 10^{11}} \cdot 100 = 5.68\% \tag{8}$$

This percent error is low, indicating that our experiment is a success and our results are conclusive. One of our sources of error is the accuracy of our ruler measurements. Our result would have been more accurate if we had a better way to measure the radius of the beam path. The reflectivity of the ruler creates some visibility issues, and the accuracy of our readings depends on our ability to minimize parallax errors.

The largest error in this experiment comes from the velocity of the electrons. As shown in Fig. 3, the anode in the electron gun has a small hole in it, which causes the magnetic field to be non-uniform. As a result, the electron velocity is slightly slower than the theoretical value. The electron velocity is additionally slowed by collisions with the helium atoms inside the tube. This leads to a somewhat high error since $\frac{e}{m} \propto \frac{1}{r^2}$ (as shown in Eq. (4)) and $r \propto v$ (shown by rearranging Eq. (1)). The only practical way to reduce this error is by using the highest possible accelerating voltage. Too high of a voltage will cause the curvature of the tube to interfere with the radius measurements. We chose $V = 259\text{V}$, but this does not completely eliminate the error.

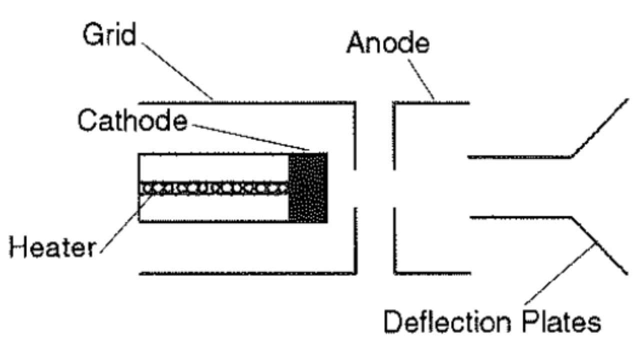

Figure 3. Diagram of the electron gun. The hole in the anode causes a somewhat high error due to it causing a non-uniform magnetic field.

As stated earlier, the principles of our experiment could be applied in mass spectrometry[2]. For particles too small to measure the mass individually, we charge them to a specified charge and launch them in a magnetic field. We, then, use the curvature of the path in the magnetic field to find the E/M ratio. Since we know the charge (we specified it earlier) we can measure the mass.

References

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