Page No.

1a Expanded form equation

$$y_p = w_1 x + w_2 x_2 + w_3 x_3 + w_4 x_4 + w_5 x_5 + b$$

Loss for SGD $= L = (y_p^{(i)} - y^{(i)})^2$

To calculate $\frac{\partial L}{\partial w}$ we will have to take partial derivatives wrt $w_1, w_2, w_3, w_4, w_5, b$

$$\frac{\partial L}{\partial w_1} = \frac{\partial (y_p^{(i)} - y^{(i)})^2}{\partial w_1} = 2(y_p^{(i)} - y^{(i)}) \frac{\partial y_p}{\partial w_1}$$

$$= 2(y_p^{(i)} - y^{(i)}) \frac{\partial (w_1 x + w_2 x_2 + w_3 x_3 + w_4 x_4 + w_5 x_5 + b)}{\partial w_1}$$

$$= 2(y_p^{(i)} - y^{(i)}) x_1$$

By similarity

$$\frac{\partial L}{\partial w_2} = 2(y_p^{(i)} - y^{(i)}) x_2, \quad \frac{\partial L}{\partial w_3} = 2(y_p^{(i)} - y^{(i)}) x_3$$

$$\frac{\partial L}{\partial w_4} = 2(y_p^{(i)} - y^{(i)}) x_4, \quad \frac{\partial L}{\partial w_5} = 2(y_p^{(i)} - y^{(i)}) x_5$$

$$\frac{\partial L}{\partial b} = \frac{\partial (y_p - y^i)^2}{\partial b} = 2(y_p^{(i)} - y^{(i)}) \frac{\partial y_p}{\partial b}$$

$$= 2(y_p^{(i)} - y^{(i)}) 1$$

Now gradient descent update rule

$b = b - \eta(2)(yp^i - y^i)$

$w_1 = w_1 - \eta(2)(yp^i - y^i)x_1$

$w_2 = w_2 - \eta(2)(yp^i - y^i)x_2$

$w_3 = w_3 - 2\eta(yp^i - y^i)x_3$

$w_4 = w_4 - 2\eta(yp^i - y^i)x_4$

$w_5 = w_5 - 2\eta(yp^i - y^i)x_5$

b

Vector form

$y = w^T x + b$ where $w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$

~~$\frac{\partial L^{(i)}}{\partial w}$~~ =

$L^i = (w^T x^i + b - y^i)^2$

$\frac{\partial L^i}{\partial w} = 2(w^T x^i + b - y^i)x^i$, $\frac{\partial L^i}{\partial b} = 2(w^T x^i + b - y^i)$

Gradient descent update rule

$$w = w - 2\eta\,(w^T x^i + b - y^i)\,x^i$$
$$b = b - 2\eta\,(w^T x^i + b - y^i)$$