

# AI Assignment 1

## Question 1

First assigning the propositional variables to the sentences in the given statements.

1.  $A \rightarrow$  The universe either will simply exist as it is.
2.  $B \rightarrow$  The universe either will end in a heat death.
3.  $C \rightarrow$  There was a big bang.
4.  $D \rightarrow$  Universe is expanding
5.  $E \rightarrow$  Universe is accelerated.

The propositional logical statements are as follows.

1.  $A \vee B$
2.  $\neg C \rightarrow A$
3.  $C \leftrightarrow D = (\neg C \vee D) \wedge (C \vee \neg D)$
4.  $(D \wedge E) \rightarrow B$

Contrapositives of the above statements are

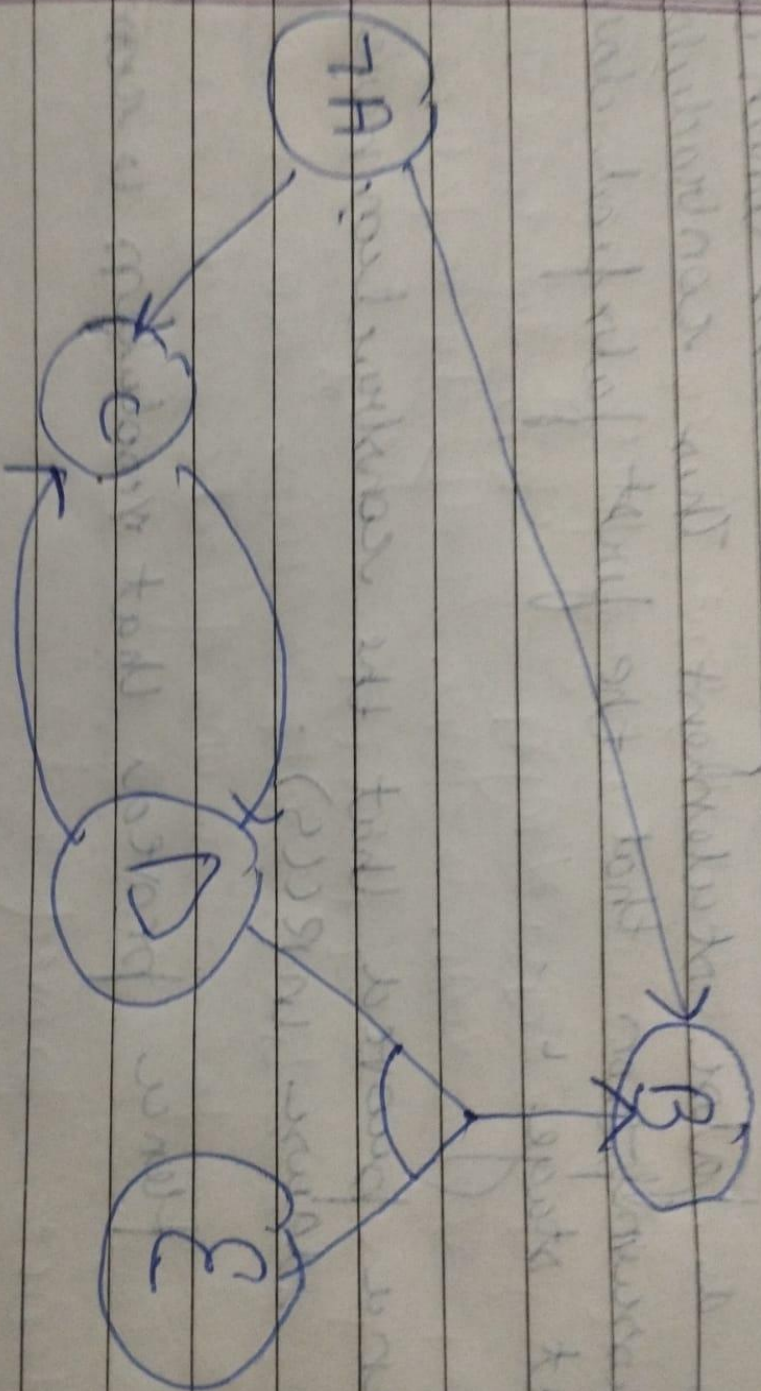
1.  $\neg B \rightarrow A$
2.  $\neg A \rightarrow C$
3.  $\neg C \leftrightarrow \neg D$
4.  $\neg B \rightarrow (\neg D \vee \neg E)$

Inferred:

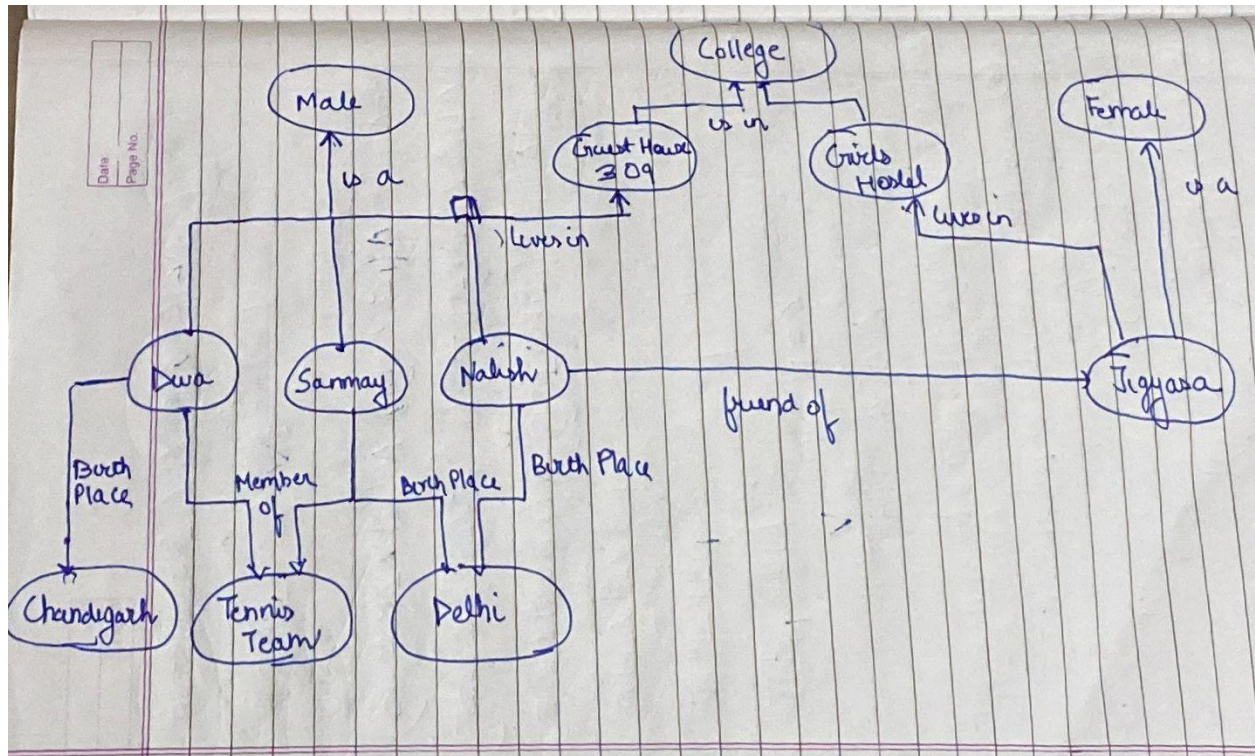
1. If the universe will not end in a heat death ( $\neg B$ ), then it will not simply exist as it is ( $\neg A$ ) (from  $\neg B \rightarrow \neg A$ ).
2. If the universe did not simply exist as it is ( $\neg A$ ), then there was a big bang ( $C$ ) (from  $\neg A \rightarrow C$ ).
3. If the universe is expanding ( $D$ ), then there was a big bang ( $C$ ), and if there was no big bang ( $\neg C$ ), then the universe is not expanding ( $\neg D$ ) (from  $D \rightarrow C$  and  $C \rightarrow D$ ).
4. If the universe will not end in a heat death ( $\neg B$ ), then it is not both expanding and accelerated ( $\neg(D \wedge E)$ ).

Not Inferred:

1. The original statements do not provide enough information to directly infer whether the universe is currently expanding ( $D$ ) or not ( $\neg D$ ). They mainly discuss the implications of expansion in relation to the big bang.
2. The original statements do not provide direct information about whether there was a big bang ( $C$ ) or not ( $\neg C$ ) if the universe is not currently expanding ( $\neg D$ ). They mainly discuss the conditions under which there was a big bang in relation to expansion.



## Question 2



Semantic Network

Single inheritance:

1. In the given semantic graph Guest House 309 and girls hostel show single inheritance as they inherit only from the parent entity college.

Multiple inheritance:

1. Dua and Sanmay show multiple inheritance as they inherit from 'Male' and 'Tennis Team'.

Location:

1. Nalish, Dua and Sanmay live in Guest House 309 whereas Jigyasa lives in Girls Hostel.

Gender:

1. Gender of Nalish, Dua and Sanmay is male as they inherit from 'Male' whereas the gender of 'Jigyasa' is female as she inherits from 'Female'.

### Question 3

Ans 3 Soundness

Consider two given ~~valid~~ statements

$$P: l_1 \vee l_2 \vee l_3 \quad \vee l_k$$

$$Q: m_1 \vee m_2 \vee m_3 \quad \vee m_n$$

Consider the literal  $l_i$  that is complementary to  $m_j$

If  $l_i$  is true then  $m_j$  is false & hence

$$m_1 \vee \dots \vee m_j = 1 \vee m_j = 1 \quad \vee m_n \text{ must be true as}$$

$$m_1 \vee \dots \vee m_n \text{ is given. If } l_i \text{ is false then}$$

$$l_i \vee \dots \vee l_i = 1 \vee l_i = 1 \quad \vee l_k \text{ must be true}$$

because  $l_i \vee \dots \vee l_k$  is given. Now  $l_i$  is either

true or false - so one of the conclusion holds

hence an 'or' of them will be true too.

Hence proved



Completeness of Resolution:

Let us define resolution closure ~~the~~  $RC(S)$  of a set of clauses  $S$ , which is the set of all clauses derivable by repeated application of the resolution rule to clauses in  $S$  or their derivatives.

Resolution closure has the property that if a set of clauses is unsatisfiable, then the resolution closure of these clauses contains the empty clause.

If we ~~prop~~ prove the above statement  
The above statement implies that resolution is complete. We need to ~~also~~ prove the statement.

We will prove its contrapositive.

If the closure  $RC(S)$  does not contain empty clause, then  $S$  is satisfiable.

Let us construct a model  $S$  with suitable truth values  $P_1, \dots, P_n$ .

From 1 to For 1 to  $n$

If a clause in  $RC(S)$  contains literal  $\neg P_i$  & all its other literals are false, then assign false to  $P_i$ .  
- otherwise assign true to  $P_i$ .

This assignment  $P_1, \dots, P_n$  is a model of  $S$ .

To prove this assume the opposite.

At some stage  $i$ , <sup>assigning</sup>  $P_i$  causes some clause  $C$  to be false.

$C$  must look like either  $(\text{false} \vee \text{false} \vee \dots \vee P_i)$  or  $(\text{false} \vee \text{false} \vee \dots \vee \neg P_i)$ .

If just one of them is present in  $RC(S)$  then the algo will assign truth value to  $P_i$  to make  $C$  true so  $C$  can only be falsified if both of these clauses are in  $RC(S)$ .

Since  $RC(S)$  is closed under resolution, it will contain resolvent of these clauses which will be a false statement. This contradicts our assumption that the first falsified clause appears at stage  $i$ .

Hence proved that the construction never falsified a clause in  $RC(S)$ .

Hence proved that resolution is complete.