Game Theory: Assignment 2

Total points: 35 — Due Date: 07/10/2022

Contribution to grade: 10% (3xx); 7.5%(5xx) — Due time: 11:59 PM

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1. Find the mixed strategy Nash Equilibria (msNE) in the following game (7)

			Player 2: q Left	Player 2: $1-q$ Right
Player 1	p_1	Up	3,2	1,4
	p_2	Middle	1,3	2,1
	$1-p_1-p_2$	Down	2,2	2,0

Hint: Just calculate expected payoffs like you would in a 2×2 game. It will work out.

Answer. Step #1 is always to calculate expected payoffs for each action. There are 5 actions here.

(a) $EU_1(Up) = 3q + 1(1-q) = 1 + 2q$.

(b) $EU_1(Middle) = 1q + 2(1-q) = 2 - q$.

(c) $EU_1(Down) = 2q + 2(1-q) = 2$.

(d) $EU_2(Left) = 2p_1 + 3p_2 + 2(1-p_1-p_2) = 2 + p_2$.

(e) $EU_2(Right) = 4p_1 + 1p_2 + 0(1-p_1-p_2) = 4p_1 + p_2$.

****2.5 marks for expected payoff calculation (0.5 each).**

If at all player 2 wants to mix, they will do so only if

$2 + p_2 = 4p_1 + p_2 \Rightarrow p_1 = \frac{1}{2}$ **(0.5 marks)**.

- Mixing between Up and Middle will make sense if

$1 + 2q = 2 - q \Rightarrow q = \frac{1}{3}$. In such a case, if player 1 had to mix between Up and Middle, he would mix such that he were indifferent between the two strategies, which happens when $p_1 = p_2 = \frac{1}{2}$. The expected payoff for player 1 in this case becomes

$$\frac{1}{2}\left[3 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3}\right] + \frac{1}{2}\left[1 \cdot \frac{1}{3} + 2 \cdot \frac{2}{3}\right] = \frac{5}{3}$$

Player 1's payoff from playing down is $2 > \frac{5}{3}$. Hence, this is not an MSNE. **(1 mark)**.

- Mixing between Up and Down will make sense if (do the same calculations) $q = \frac{1}{2}$. We know that Player 2 mixes if $p_1 = \frac{1}{2}$.

  There is no optimal deviation. Therefore, msNE is $\left\{\left(\frac{1}{2}Up, \frac{1}{2}Down\right), \left(\frac{1}{2}Left, \frac{1}{2}Right\right)\right\}$ **(1 mark).**

- Mixing between Middle and Down would be possible if $q = 0$, driving player 1 to best respond with Middle or Down as a pure strategy. Moreover, it requires a pure strategy from player 2 to even consider it. Hence, no msNE here **(1 mark)**.

- Mixing between all 3 would require (a)=(c) and (b)=(c) to simultaneously hold. This is not the case. Hence, no msNE here. **(1 mark)**.

2. Keeping in mind that you can also use mixed strategies to eliminate a pure strategy, eliminate at least one pure strategy from the following game. (3)

		Player 2		
		Left	Middle	Right
Player 1	Low	1,1	3,2	2,0
	Moderate	2,2	2,1.8	3,3.2
	High	3,1	1,1.2	2,0

Answer. Suppose player 2 has to mix between Middle and Right to dominate left. Assign probability q to Middle and $1 - q$ to Right. We need:

(a) $2q + 0(1-q) > 1 \Rightarrow q > \frac{1}{2}$ **(0.5 marks)**.

(b) $1.8q + 3.2(1-q) > 2 \Rightarrow q < \frac{6}{7} \approx 0.86$ **(1 mark)**.

(c) $1.2q + 0(1-q) >= 1 \Rightarrow q > \frac{5}{6} \approx 0.83$ **(1 mark)**.

Therefore, a mix of Middle and Right dominates Left for player 2 for $q \in \left(\frac{5}{6}, \frac{6}{7}\right)$ **(0.5 marks)**. You will get full marks even if you write in decimal approximations.

3. In Cournot duopoly, we showed that if our rival chooses "Cartel", it made sense to defect to the competitive outcome. However, was the defection to the competitive outcome the optimal deviation? Or can a firm do even better by defecting to some third quantity? If yes, what is this quantity? (Don't ask for hints). (10)

Answer. The answer is very simple, and not actually worth 10 marks. High weightage in marks is only to test if you understand best responses properly, as it is a key concept moving forward. Also, I am hoping this was a relatively easy 10 marks for everyone. Answer is as follows:

- Plug in q_{Cartel} in the best response function, to get

$$\frac{a-c}{2b} - \frac{1}{2}q_{\text{Cartel}} = \frac{a-c}{2b} - \frac{1}{2}\left(\frac{a-c}{4b}\right) = \frac{3(a-c)}{8b}$$

- In theory, you do not even need to do anything else. Since this is the best response, and the Cournot deviation is $\frac{a-c}{3b}$, it is clear that the Cournot deviation is not the best response, and this is. **This is all you need to do for 10 marks**. However, just to double check, we can find the profits as well, to compare. Plugging this into the payoff function (and putting rival's quantity as the cartel amount) yields a profit greater than the Cournot deviation. Cross check! .

- Caveat: Though it did not happen, in the event that the answer we were getting was negative, it would be more complicated, since quantity cannot be negative and is lower bounded at 0. We would have to make sure that there does not exist a better q than the Cournot outcome to deviate to, because the actual BRF is a piece function.

4. In Bertrand, if payoff from not selling was $-c$, what would be the pure strategy NE (psNE)? (5)

   Answer. This is actually a little tricky if your concepts are unclear.

   - We still have $p_1 = p_2 = c$ as a PSNE of course **(1 mark for this PSNE)**. However, there is a possibility that firms can get caught in a price war below the cost of production as well.
   - However, there is a possibility that firms can get caught in a price war below the cost of production as well. Imagine if firm i prices a good at $p_i[0, c)$. In this range, i is suffering a loss. If j undercuts i, they will get an even higher loss than i. If they go for $p_j > p_i$, their payoff will be will be $-c$. To minimize losses, they must match the price p_i has chosen and take a negative payoff of of $\frac{1}{2}(p_i - c) < 0$. Here, the set of all PSNE are $p_i = p_j \in [0, c]$ **(4 marks for this)**.
   - This is probably an even better example of how cut throat price competition can be, and sometimes a reason why there are industries in which firms persist even though they are making a loss.

5. An empty room is available in the hostel. There are two interest groups among the students, $i \in \{S, E\}$. S students want to fill the room up with sports equipment like TT tables, carrom boards etc., while E students want to fill it up with entertainment equipment like a TV, speakers, relaxing chairs etc. On a unit line, think of S's preferred policy as 0 and E's preferred policy as 1. The decision will be taken by the hostel student body, and will be based on monetary contributions made by each group. Groups simultaneously and independently choose a contribution to the student body

$s_i \in [0,1]$. The student body will eventually choose a policy

$$r(s_S, s_E) = \frac{1}{2} - s_S + s_E$$

Student groups have the following utility functions:

$$v_S(s_S, s_E) = -\left[r(s_S, s_E)\right]^2 - s_S$$

$$v_E(s_S, s_E) = -\left[1 - r(s_S, s_E)\right]^2 - s_E$$

Find the Nash equilibrium of this simultaneous-move game. (10)

Answer. This is the easiest of the 4 questions. Just follow the routine steps. The answer is as follows:

- Take FOC for student group S. You get

  $$2\left(\frac{1}{2} - s_S + s_E\right) - 1 = 0 \Rightarrow s_S(s_E) = s_E$$

  ****3 Marks**

- Take FOC for student group E. You get

  $$2\left(1 - \frac{1}{2} - s_S + s_E\right) - 1 = 0 \Rightarrow s_E(s_S) = s_S$$

  ****3 Marks**

- Solve to get PSNE is $s_S = s_E \in [0,1]$.

  ****4 Marks**