Game Theory: Assignment 1 (Solutions)

Total points: 35 | Due Date: 23/09/2022

Contribution to grade: 10% (3xx); 7.5%(5xx) | Due time: 11:59 PM

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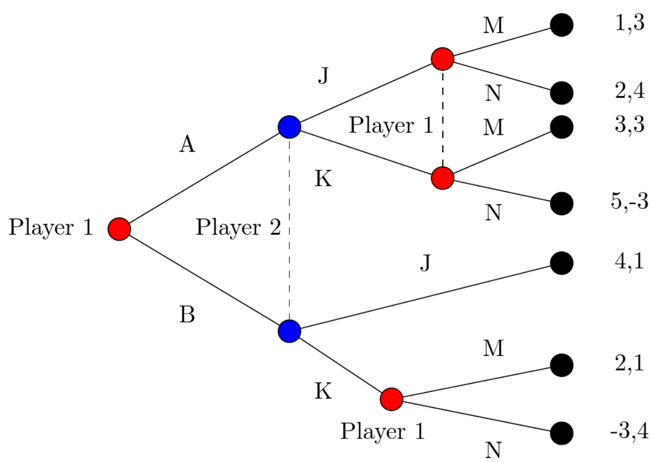
1. Represent the following game in normal form: (10)



Answer.

Player 1 (rows), Player 2 (columns)

	J	K
AMM	1,3	3,3
AMN	1,3	3,3
ANM	2,4	5,-3
ANN	2,4	5,-3
BMM	4,1	2,1
BMN	4,1	-3,4
BNM	4,1	2,1
BNN	4,1	-3,4

2. Use the following payoff matrix to demonstrate that when applying IEWDS (Iterated Elimination of Weakly Dominated Strategies), the outcomes can change when order of elimination changes. (5)

		Player 2		
		A	B	C
Player 1	X	2,1	1,1	0,0
	Y	1,2	3,1	2,1
	Z	2,-2	1,-1	-1,-1

Answer. Order of elimination: Black, Blue, Green, Orange. You can see that different orders lead to different outcomes. One leads to a specific strategy, while in the other case six strategies survive IEWDS.

		Player 2		
		A	~~B~~	~~C~~
Player 1	X	2,1	1,1	0,0
	~~Y~~	~~1,2~~	3,1	2,1
	~~Z~~	~~2,-2~~	~~1,-1~~	~~-1,-1~~

		Player 2		
		A	B	~~C~~
Player 1	X	2,1	1,1	0,0
	Y	1,2	3,1	2,1
	Z	2,-2	1,-1	-1,-1

3. This is a variation of the famous Bertrand Game, more specifically, a price matching game (more on this in lecture 7!). There are two firms $i \in \{1,2\}$, and firm i's objective (in this case profit) function is given by

$$\max_{p_i} \pi_i = \frac{1}{2}\left[\min\{p_i, p_{-i}\}\left(\frac{a - \min\{p_i, p_{-i}\}}{b}\right) - c\left(\frac{a - \min\{p_i, p_{-i}\}}{b}\right)\right] \text{ where } a, b > 0$$

Both firms choose their respective prices p_i simultaneously and independently. Without worrying about the details of the game, write down the game in terms of

(a) Players. (2)

Answer. Players are the two firms where firm $i \in \{1, 2\}$.

(b) Actions. (2)

Answer. There are infinite actions here. In general, action for each player i is price, where $p_i \in [0, \infty)$.

(c) Strategies. (2)

Answer. In this game, actions are same as strategies (as long as we do not allow for mixed strategies).

(d) Payoff Functions. (2)

The payoff function for player i is

$$\pi_i = \frac{1}{2}\left[\min\{p_i, p_{-i}\}\left(\frac{a - \min\{p_i, p_{-i}\}}{b}\right) - c\left(\frac{a - \min\{p_i, p_{-i}\}}{b}\right)\right]$$

Note: This game often plays out during the famous Black Friday sales in USA.

4. Suppose you have 4 actions available to you, $A_i = w, x, y, z$, while your rival's actions set is simply $A_j = [0, 1]$, and his action is given by a_j where $a_j \in A_j$. Now upon selecting a specific action, your payoff is a function of your rival's action. Suppose

(a) When you select w, your payoff $v_i(w, a_j) = a_j^2 - 1$.

(b) When you select x, your payoff $v_i(x, a_j) = 2a_j - 1$.

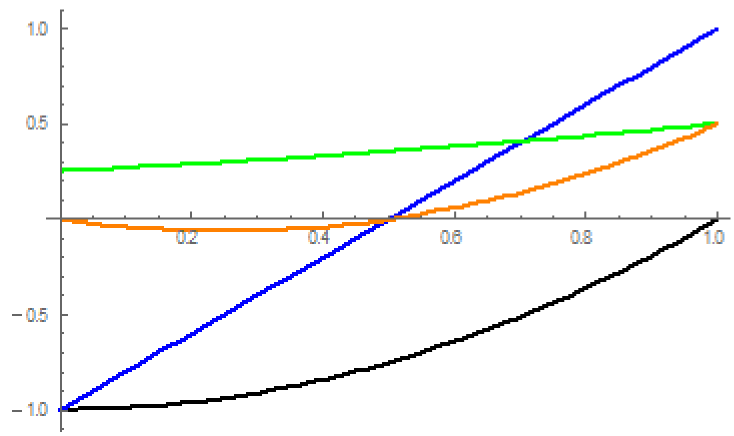
(c) When you select y, your payoff $v_i(y, a_j) = 0.5^{2-a_j}$.

(d) When you select z, your payoff $v_i(z, a_j) = a_j^2 - 0.5a_j$.

Which of the above strategies survive

(a) IESDS (3)

Answer. Let us first plot the payoff functions for all values of a_j. For convenience, w is black, x is blue, y is green and z is orange.



It is clear that w is eliminated by IESDS (dominated by y and z), while all others survive.

(b) IEWDS (3)

Answer. In addition to the above, y weakly dominates z. When $a_j = 1$, the payoffs from y and z are the same

Does your answer change is $A_j = (0, 1)$. If yes, what is the new answer? (1)

Answer. If the boundaries 0 and 1 are not included in A_j, y strictly dominates z, and z fails to survive IESDS. w which was anyway strongly dominated by y and z, but was only weakly dominated by x, is now strongly dominated by all 3.

5. Explain the difference between the uncertainty exhibited by a rival in a game theoretic setting and uncertainties exhibited by "Nature". Use the terms discussed in class while answering this question. (3)

*Answer.*The primary difference is that Nature's action is a random draw over a distribution. "Nature" does not optimize. A rival however, has an objective function, and though we do not know the strategy employed by a rival, we do know this objective function. This also allows us to anticipate how a rival may act, as opposed to nature.

6. What is the role of the "payoff function"? What is one property of preferences over outcomes that it must retain? (2)

   *Answer.*The main role of the payoff function is mapping preferences over outcomes to real numbers. This gives preference both ordinal and cardinal characteristics. The main property that it must retain is for any outcomes x and y, if v is a payoff function, $x \succsim y$, then $v(x) \geq v(y)$