1. (a) Consider the bi-directional A* search in which two search processes run simultaneously – one in the forward direction from the source, and the other in the backward direction – from the destination. Write a pseudocode for bi-directional A*. Mention the parameters, steps and output. Consider that a function *A-star(.)* for one-directional A* has already been defined. 3

Solution:

1. (a)

Start vertex: start

destination vertex: dest

branching factor: b

Bidirectional-A-star (start, dest, b)

1. Fork two threads simultaneously
   - (a) Thread 1: A-star (start, dest, b)
   - (b) Thread 2: A-star (dest, start, b)
2. Stop when the paths generated by the threads intersect.
3. Return the optimal path (start to p^*) merged with the optimal path (dest to p^*), where p^* is the optimal point at which the threads meet.

- No optimal path: deduct 0.5
- Stopping criteria: 0.5
- parameters: 1
- Calling of A^*: 1

(b) Use DPLL method to solve the following propositional formula:
$F : (P) \ \wedge \ (\neg P \vee Q) \ \wedge \ (R \vee \neg Q \vee S)$. You have to begin with $Q =$ `true`. 4

Solution:

1.(b)

F: $(P) \wedge (\neg P \vee Q) \wedge (R \vee \neg Q \vee S)$

$(P) \wedge (\neg P \vee Q) \wedge (R \vee \neg Q \vee S)$

$Q=1$ ↙ literal elimination

$(P) \wedge (R \vee S)$

$P=1$ ↙ Unit propagation

$(R \vee S)$

$R=1 / S=1$ ↙ literal elimination

$\boxed{\text{SAT}}$

Therefore, F is satisfiable for the following interpretation
$P=1$, $Q=1$ and ($R=1$ or $S=1$)

- For each step: 1 (show reduced formula at each step)

- Solution: 1

(c) What is conditional independence ? Is it symmetric ? When does a conditional independence reduce to absolute independence

Solution:

- *Conditional independence:* Variable A is conditionally independent of B, given C if **any** of the following:

$$p(A \mid B, C) = p(A \mid C) \text{ where } A, B, C \text{ are random variables.} \tag{1}$$

$$p(A, B \mid C) = p(A \mid C) \cdot p(B \mid C) \text{ where } A, B, C \text{ are random variables.} \tag{2}$$

$$p(A \mid B, C) = p(A \mid C) \text{ where } A \text{ is a variable, and } B, C \text{ are set of variables.} \tag{3}$$

  Intuition is that if C is given/known then B does not give more information about A.

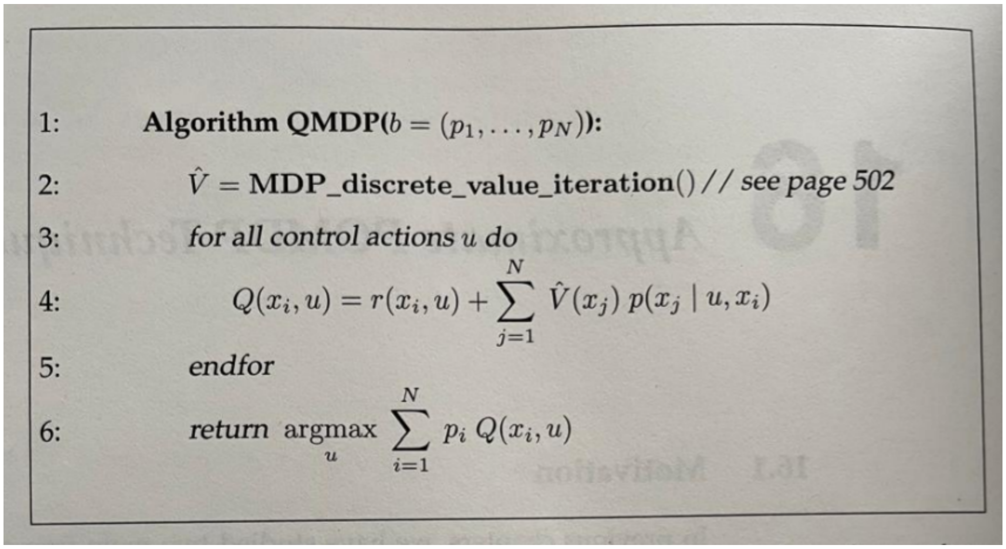
  – Full marks is any of 1,2 or 3 is written.
  – Give 1 if written in words, and not expressed mathematically.

- Yes, it is symmetric, because if A is conditionally independent of B, then B is conditionally independent of A.
- Conditional independence reduces to absolute independence when any of the following holds:
  i. C in 1,2 or 3 (above) is null.
  ii. Both A and B are independent of C.

  – Full marks is any of the above is written.

2+1+1

2. (a) Write the QMDP algorithm. 3



Algorithm QMDP($b = (p_1, \ldots, p_N)$):

$\hat{V}$ = **MDP_discrete_value_iteration**() // *see page 502*

for all control actions u *do*

$$Q(x_i, u) = r(x_i, u) + \sum_{j=1}^{N} \hat{V}(x_j)\, p(x_j \mid u, x_i)$$

endfor

$$\textit{return } \operatorname*{argmax}_u \sum_{i=1}^{N} p_i\, Q(x_i, u)$$

Figure 1: QMDP algolrithm

Here, MDP_discrete_value_iteration(.) is a general value iteration algorithm for discrete and finite state spaces.

– Deduct 0.5 marks if the purpose of MDP_discrete_value_iteration() is not written. More specifically, if "finite" state space is not mentioned.

(b) Consider a robot that operates in the triangular environment with three types of landmarks as shown in Figure 2:

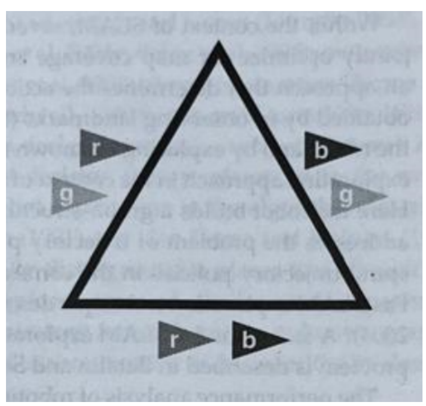


Figure 2:

Each location has two different landmarks, each with a different color. Assume that in every round the robot can only inquire about the presence of one landmark type: either the one labeled "r", the one labeled "g", or the one labeled "b". Suppose the robot first fires the detector for "b" and moves *clockwise* to the next arc. *What would be the optimal landmark detector to use next ?* 4

Solution:

2.(b)

Landmark detection

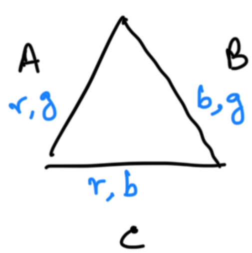
The goal is to reduce uncertainty about the location of the robot.

So, we have to maximize information gain.

$$I_b(u) = \underbrace{H(x)}_{\text{given}} - E_z\left[\underbrace{H_{b'}(x' \mid u, z)}_{\text{minimize this.}}\right]$$

Case 1:

- $z_b = 0$/false: In this case, we know that the robot is in A with probability 1. So, any of r, b or g can be fired next.



Case 2: Not sure whether the robot currently is in B or C. So, calculate $H_{b'}(x \mid z_g)$, $H_{b'}(x \mid z_r)$ and $H_{b'}(x \mid z_b)$.

$$H_{b'}(x' \mid z_g) = -(\underbrace{p_1 \log p_1}_{A} + \underbrace{p_2 \log p_2}_{B} + \underbrace{p_3 \log p_3}_{C})$$

$$= -(1 \cdot \log 1 + 0 + 0)$$

$$= 0$$

Similarly, $H_{b'}(x' | z_r) = 1$, $H_{b'}(x' | z_b) = 0$

Therefore, optimal landmark detector is b or g.

- case 1: 1.5
- case 2: 1.5
  - Decision with explanation: 1
  - Mathematical formulation of info gain: 0.5

3. (a) Suppose you have a dataset of n students. For each student, the dataset contains their marks in 10 different subjects and the probability of distinction of the student. Now, a student's marks in only 2 subjects are given to you. Write briefly how you will proceed to find the probability for her getting distinction (o need to define the functions you use). 3

Solution:

3.(a) Given

Data points: n

A data point: $(s_1, s_2, \ldots, s_{10}, p_{dist})$

Test data: (s_1^*, s_2^*)

To find: p_{dist}^*

1. Select (s_1^i, s_2^i) tuple for all $i = 1 \ldots n$
2. Set the 2-D range to select a relevant dataset

$$R = \begin{bmatrix} (a_1 - s_1^*, & s_1^* + b_1) \\ (a_2 - s_2^*, & s_2^* + b_2) \end{bmatrix}$$

3. Orthogonal range searching with range R, to find $n' < n$ data points.
4. Apply regression on n' data points to find p_{dist}^*

- Orthogonal range search: 1.5
- Regression: 1
- Setting range: 0.5
- Other approaches should also be fine.

(b) Given the root of a subtree of a kd-tree, and a range R, write the pseudocode for searching in a kd-tree. 5

```
Algorithm SearchKdTree(ν, R)
Input. The root of (a subtree of) a kd-tree, and a range R.
Output. All points at leaves below ν that lie in the range.
if ν is a leaf
   then Report the point stored at ν if it lies in R.
   else if region(lc(ν)) is fully contained in R
           then ReportSubtree(lc(ν))
           else if region(lc(ν)) intersects R
                   then SearchKdTree(lc(ν), R)
        if region(rc(ν)) is fully contained in R
           then ReportSubtree(rc(ν))
           else if region(rc(ν)) intersects R
                   then SearchKdTree(rc(ν), R)
```

Figure 3:

– Full marks if the pseudocode is written for any of left or right subtree, and the other subtree is handled similarly.
– Deduct 1.5 marks if the recursion terminating condition is not written.

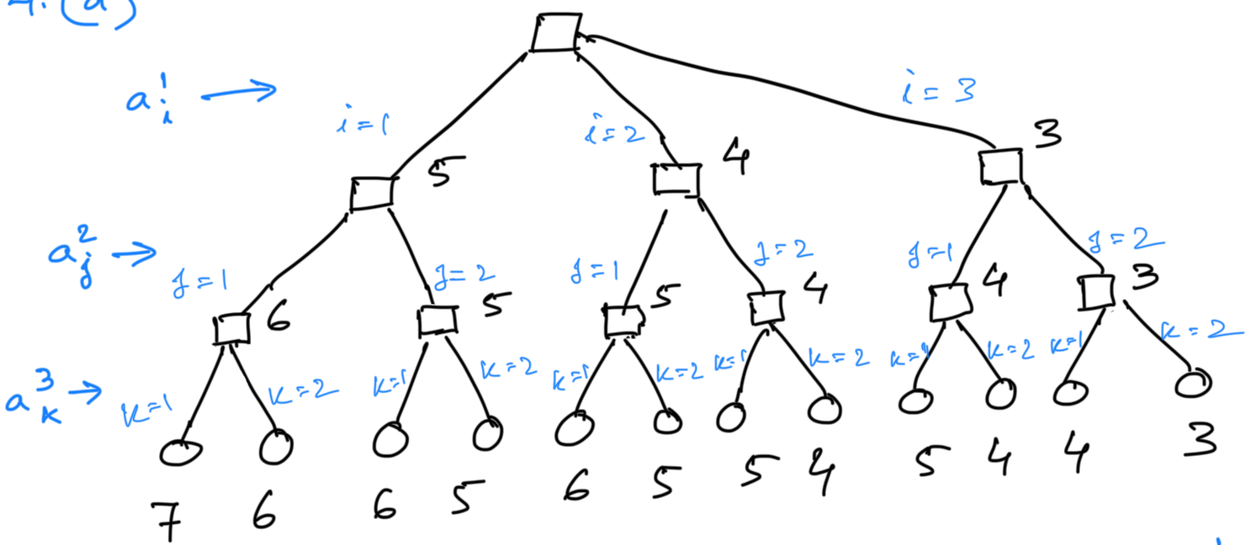
4. (a) Consider a THREE-player Alpha-Beta pruning game as follows:

- *Nature of play:*
  Player-1 - maximizing, Player-2 - minimizing. Player-3 -minimizing.
- *Actions: (superscript: player, subscript: action)*
  Player-1 - $\{a_i^1\}_{i=1}^3$; Player-2 - $\{a_j^1\}_{j=1}^2$; Player-3 - $\{a_k^1\}_{k=1}^2$.
- Player-1 plays first, followed by player-2 and player-3 in that order.
- *Reward of Player-1* after player-1, 2 and 3 play one move each, i.e. after action sequence (a_i^1, a_j^2, a_k^3), is defined as $R_{i,j,k}^1 := 10 - i - j - k$.

Draw the game tree. Find the best strategy for Player-1. 3+3

Solution:

4. (a)



- Player 3 is a minimizing player; The decision of player 1 after player 3 acts are given in level 2 of the game tree.

- Player 2 is a minimizing player; The decision of player 1 after player 2 acts are given in level 1.

- Player 1 chooses action a_1^1.

- Maximization/minimization order: 1
- Deduct 2 marks if any of the rewards is claimed to be of Player 2 or 3.

- Decision of action: 1.5

		The system suggests ...		
		tiger	leopard	cat
Actually the animal is a ...	tiger	0.7	0.2	0.1
	leopard	0.4	0.6	0.0
	cat	0.2	0.0	0.8

Table 1: Measurement model of the animal recognition system

(b) Consider a vision based pattern recognition system to identify some of the animals. Its measurements are governed by Table 1. Initially the system has uniform probability distribution as its belief. Then it identifies two animals sequentially and changes its belief accordingly.

i) Suppose, the first animal we ask it to identify is actually a tiger (we know it). Derive the belief of the system after its first observation. 2

ii) The system identifies the second animal to be a cat. What is the probability that the second animal is actually a tiger ? 3

iii) What is the probability of identifying both the animals correctly ? 2

Solution:

4. (b)

(i) Given that the animal is a tiger, the probabilities of the system observing it as a cat, leopard and tiger makes its belief.

When the animal is actually a tiger, the belief

$$bel_1(x) = [\underset{\text{tiger}}{\underset{\downarrow}{0.7}},\ \underset{\text{leo}}{\underset{\downarrow}{0.2}},\ \underset{\text{cat}}{\underset{\downarrow}{0.1}}] \quad \text{(Ans)}$$

(ii) System identifies the second animal as a cat.

$z_2 = \text{cat}$

$$\overline{bel}_2(x=\text{tiger}) = bel_1(x=\text{tiger}) = 0.7$$

$$\overline{bel}_2(x=\text{leo}) = bel_1(x=\text{leo}) = 0.2$$

$$\overline{bel}_2(x=\text{cat}) = bel_1(x=\text{cat}) = 0.1$$

$$bel_2(x=\text{tiger}) = \eta\, P(z=\text{cat} \mid x=\text{tiger})\, \overline{bel}_2(x=\text{tiger})$$

$$= \eta \times 0.1 \times 0.7 = 0.07\,\eta$$

$$bel_2(x=\text{leo}) = 0$$

$$bel_2(x=\text{cat}) = 0.08\,\eta$$

$$0.07\,\eta + 0 + 0.08\,\eta = 1 \Rightarrow \eta = 6.6$$

$0.07\,\eta + \ldots$

$$bel_2(x = tiger) = P(x = tiger \mid z = cat)$$

$$= 0.07\,\eta = 0.462 \text{ (Ans)}$$

(iii) Identifying both the animals correctly :

$$bel_1(x = tiger) \times bel_2(x = cat)$$

$$= 0.7 \times 0.54 = 0.378 \text{ (Ans)}$$

- 4.b.(i) correct belief : 1 ; explanation : 1
- 4.b.(ii) $\overline{bel}_i = bel_{i-1}$: 1 ; belief calc approach : 2
- 4.b.(iii) correct formula : 2

5. (a) For an MDP, write mathematical expression for the *value function* with time horizon T. Describe the terms in it. 2+1

Solution:

The value function for time horizon T is as follows:

$$V_T(x) = \gamma \max_u \left[r(x,u) + \int V_{T-1}(x') p(x' \mid x, u) \right]$$

where x, x', u, $r(.)$ and *gamma* are the current state, next state, control action payoff function and discount factor respectively.

– 2 marks for the mathematical expression.
– 1 mark for description of the terms.

(b) Answer the following in a *generalized* way. Consider a partially observable system with n states $\{x_i\}_1^n$ and m actions $\{u_j\}_{j=1}^m$. Current belief $b(x_i) = p_i$ is given for all the states. The reward functions $\{r(b, u_j)\}_{j=1}^m$ are also given. Write mathematical expressions for the following:

i) $V_1(b)$ for choosing the optimal control action, with no observation. 2

ii) Change of belief after taking an observation. 2

iii) V_1 with the changed belief, i.e. after taking observation. 2

iv) Expected V_1 over all possible observations. 2

Solution:

5.(b)

(i) $V_1(b) = \max\limits_{u_i} \left\{ r(b, u_i) \right\}$

(ii) New belief after observation z_k

$b' = p(x_i \mid z_k)$ for all x_i

(iii) V_1 with changed belief $\left[V_1(b \mid z_k) \text{ or } V_1(b') \right]$

$V_1(b') = \max\limits_{u_i} \left\{ r(b', u_i) \right\}$

(iv) Expected V_1 over possible observations

$$E_z\left[V_1(b \mid z_k) \right] = \sum_{z_k} p(z_k) \cdot V_1(b \mid z_k)$$

- Focus on generalization
- If an example is given instead of generalization, deduct 2 marks.
- If state is considered instead of belief, deduct 2 marks.
- Give full marks if observation space is assumed to be finite.