Sol. 1 ⇒

$$x = \pm 1, \; z = \pm 1, \; y = 3 \text{ and } y = 5$$

$$I_x = \int_{-1}^{1}\int_{-1}^{1}\int_{3}^{5} y^2 + z^2 \, dy \, dz \, dx$$ — (1) mark for writing correct expression (limits must be correct)

$$= \frac{400}{3}$$ — (1) mark for getting answer after solving (with steps)

$$I_y = \int_{3}^{5}\int_{-1}^{1}\int_{-1}^{1} x^2 + z^2 \, dx \, dz \, dy$$ — (1) mark

$$= 16/3$$ — (1) mark

$$I_z = \int_{-1}^{1}\int_{3}^{5}\int_{-1}^{1} x^2 + y^2 \, dx \, dy \, dz$$ — (1) mark

$$= \frac{400}{3}$$ — (1) mark

Sol 2 ⇒

a) ⇒ $$(|u| \cos\theta)\hat{v} = |u| \left(\frac{u \cdot v}{|u||v|}\right) \frac{v}{|v|} = \frac{(u \cdot v)\, v}{|v|^2}$$ — (1) mark

b) ⇒ $(u+v) \times (u-v)$

$$= u \times (u-v) + v \times (u-v)$$

$$= u \times u - u \times v + v \times u - v \times v$$

$$= -u \times v + v \times u$$

$$= -2(u \times v) \text{ or } 2(v \times u)$$ — (1) mark

c) ⇒ Area of parallelogram = $|u \times w|$ — ① mark

d) ⇒ Volume of parallelopiped = $|u \cdot (v \times w)|$

— ① mark

give full mark if written any other correct form

Sol 3) ⇒

$$T(x,y) = 4x^2 - 4xy + y^2$$

$$\nabla T(x,y) = T_x \hat{i} + T_y \hat{j}$$
$$= (8x - 4y)\hat{i} + (2y - 4x)\hat{j}$$

$$g(x,y) = x^2 + y^2 - 25$$

$$\nabla g(x,y) = g_x \hat{i} + g_y \hat{j}$$
$$= 2x \hat{i} + 2y \hat{j}$$

Applying Lagrange Multipliers

$$\nabla T = \lambda \nabla g$$
$$(8x - 4y)\hat{i} + (2y - 4x)\hat{j} = \lambda(2x\hat{i} + 2y\hat{j})$$

$$\boxed{\begin{array}{ll} 8x - 4y = 2\lambda x & \text{— ①} \\ 2y - 4x = 2\lambda y & \text{— ②} \end{array}}$$

— ① mark for getting both equations

from ①

$$8x - 4y = 2\lambda x$$

$$\Rightarrow 8x - 2\lambda x = 4y$$

$$\Rightarrow y = 2x - \frac{\lambda x}{2}$$ — ① mark for getting relation b/w y & x. Give marks for any other correct expression

Put value of y in ②

$$2\left(2x - \frac{\lambda x}{2}\right) - 4x = 2\lambda\left(2x - \frac{\lambda x}{2}\right)$$

$$\Rightarrow 4x - \lambda x - 4x = 4\lambda x - \lambda^2 x$$

$$\Rightarrow \lambda^2 x - 5\lambda x = 0$$

$$\Rightarrow x(\lambda)(\lambda - 5) = 0$$

$$x = 0, \quad \lambda = 0, \quad \lambda = 5$$ — ① mark for getting values of x and λ

Case-1 - $x = 0$

$$\therefore y = 2(0) - \frac{\lambda(0)}{2} = 0$$

but $(0,0)$ is not on circle $\therefore x \neq 0$

— ① mark for this conclusion

Case-2 - $\lambda = 0$

$$\therefore y = 2x - 0 = 2x$$

$x^2+y^2=25$

$\therefore x^2+4x^2=25$

$5x^2=25$

$\Rightarrow x^2=5$

$\Rightarrow x=\pm\sqrt{5}$

$\therefore y=\pm 2\sqrt{5}$

$A(\sqrt{5}, 2\sqrt{5})$, $B(-\sqrt{5}, -2\sqrt{5})$ —— ① mark for getting points

Case-3 - $\lambda=5$

$\therefore y=2x-\frac{5x}{2}=-x/2$

$x^2+y^2=25$

$x^2+\frac{x^2}{4}=25$

$\Rightarrow 5x^2=100$

$\Rightarrow x^2=20$

$\Rightarrow x=\pm 2\sqrt{5}$

$\therefore y=\mp\sqrt{5}$

$C(2\sqrt{5}, -\sqrt{5})$, $D(-2\sqrt{5}, \sqrt{5})$ —— ① mark for getting points

$T(A) = 4x^2 - 4xy + y^2$

$= 4(\sqrt{5})^2 - 4(\sqrt{5})(2\sqrt{5}) + (2\sqrt{5})^2$

$= 0°$

$T(B) = 0°$

$T(C) = 125°$

$T(D) = 125°$

Min. temp = $0°$ unit —— ① mark for it

Max. temp = $125°$ unit —— ① mark for it

Sol 4) =>

$r(t) = \cos t\ \hat{i} + \sin t\ \hat{j} + (1-\cos t)\hat{k}, \quad 0 \leq t \leq 2\pi$

a) =>

$r(x,y,z) = x\hat{i} + y\hat{j} + z\hat{k}$

$x = \cos t$

$y = \sin t$

$z = 1 - \cos t$

$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$

$\therefore \boxed{x^2 + y^2 = 1}$ —— ① mark for eqn of cylinder

Its a right circular cylinder in three dimension with radius 1 and z-axis as its axis

Any point p lies on cylinder, p (cost, sint, 1-cost)

i) at $t=0$
$P(1,0,0)$

ii) at $t=\pi/2$
$Q(0,1,1)$

iii) at $t=\pi$
$R(-1,0,2)$

$$\vec{PQ} = -\hat{i}+\hat{j}+\hat{k}$$

$$\vec{QR} = -\hat{i}-\hat{j}+\hat{k}$$

$$\vec{PQ}\times\vec{QR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{vmatrix} = 2\hat{i}+2\hat{k}$$

— ① mark for finding normal vector

$2\hat{i}+2\hat{k}$ is a vector normal to plane P, Q, R

Eqⁿ of plane $\Rightarrow$ $2x+2z=c$

Put P in the eq^n of plane

$2(1) + 2(0) = C$

$2 = C$

$\therefore$ Eq^n of plane $\Rightarrow$ $2x + 2z = 2$

$\boxed{x + z = 1}$ — 1 mark for eq^n of plane

$\therefore$ Any point on the curve lies on the intersection of cylinder $(x^2+y^2=1)$ and plane $(x+z=1)$ $\therefore$ the curve is an ellipse.

b) $\Rightarrow$

$$V = \frac{dr}{dt}$$

$$V = -\sin t\,\hat{i} + \cos t\,\hat{j} + \sin t\,\hat{k}$$ — 1 mark for finding V

$$|V| = \sqrt{\sin^2 t + \cos^2 t + \sin^2 t}$$
$$= \sqrt{1 + \sin^2 t}$$

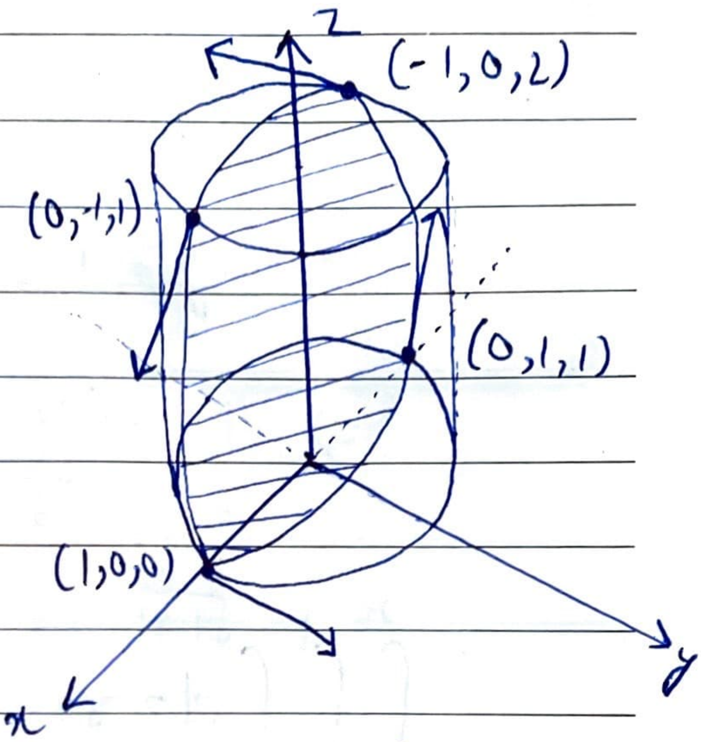
$$T = \frac{V}{|V|} = \frac{-\sin t\,\hat{i} + \cos t\,\hat{j} + \sin t\,\hat{k}}{\sqrt{1+\sin^2 t}}$$ — 1 mark for finding T

$$T(0) = \frac{-0\hat{i} + \hat{j} + 0\hat{k}}{\sqrt{1+0}} = \hat{j}$$

$$T(\pi/2) = \frac{-\hat{i} + \hat{k}}{\sqrt{2}}$$

$$T(\pi) = -\hat{j}$$

$$T(3\pi/2) = \frac{\hat{i} - \hat{k}}{\sqrt{2}}$$



c) ⇒

— ① mark for figure

$$a = \frac{dv}{dt}$$

$a = -\cos t\ \hat{i} - \sin t\ \hat{j} + \cos t\ \hat{k}$ — ① mark for finding a

vector normal to plane (n) $= \hat{i} + \hat{k}$

$$a \cdot n = -\cos t + \cos t = 0$$

$\therefore$ a & n are orthogonal

$\therefore$ a is parallel to plane $x + z = 1$

— ① mark for showing a is parallel to plane

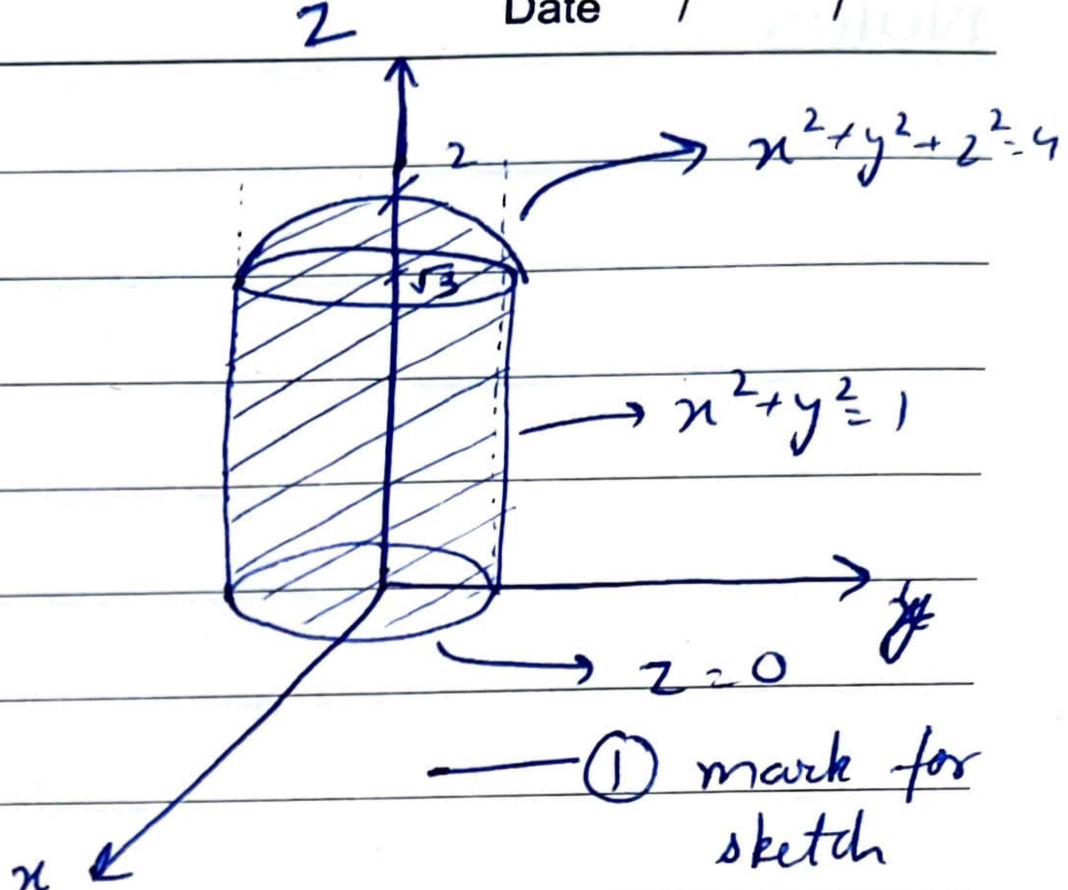
Sol 5) $\Rightarrow$

$$x^2+y^2+z^2=4$$

$$x^2+y^2=1$$

$$\Rightarrow 1+z^2=4$$

$$\Rightarrow z=\pm\sqrt{3}$$



— ① mark for sketch

$$\int_0^{2\pi}\int_0^{1}\int_0^{\sqrt{4-r^2}} dz\, r\,dr\, d\theta$$

— ③ marks

(1 mark for writing each limit)