Solutions

Short Answers

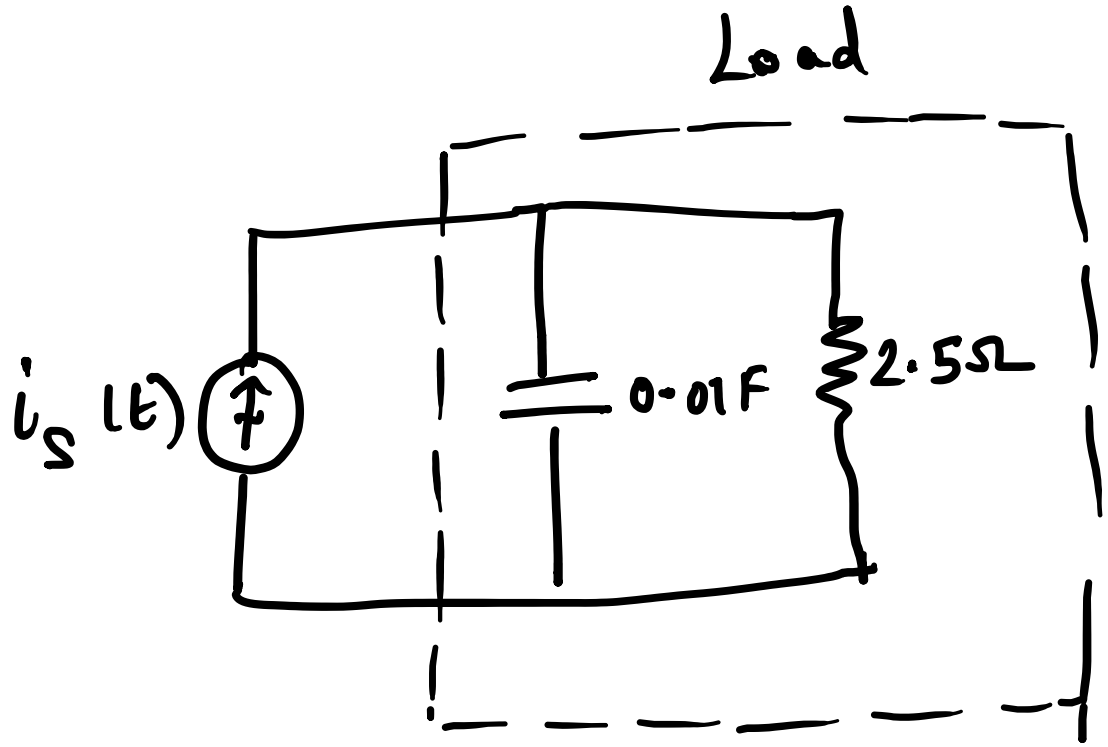
① (d) $i_s(t) = 3\cos(40t + 50^\circ)\ A$

same frequency (LTI ckt) ①

same phase (resistive ckt)

$$P_{avg} = \frac{1}{2} Re\{5e^{j50^\circ} \times 3e^{-j50^\circ}\} = \frac{15}{2} = 7.5\ W$$ ①

(2) $i_s(t) = 5\cos(30t)$ rms $\omega = 30$ rad/s



$$Y = j\omega C + \frac{1}{R}$$

$$= j \times 30 \times 0.01 + \frac{1}{2.5}$$

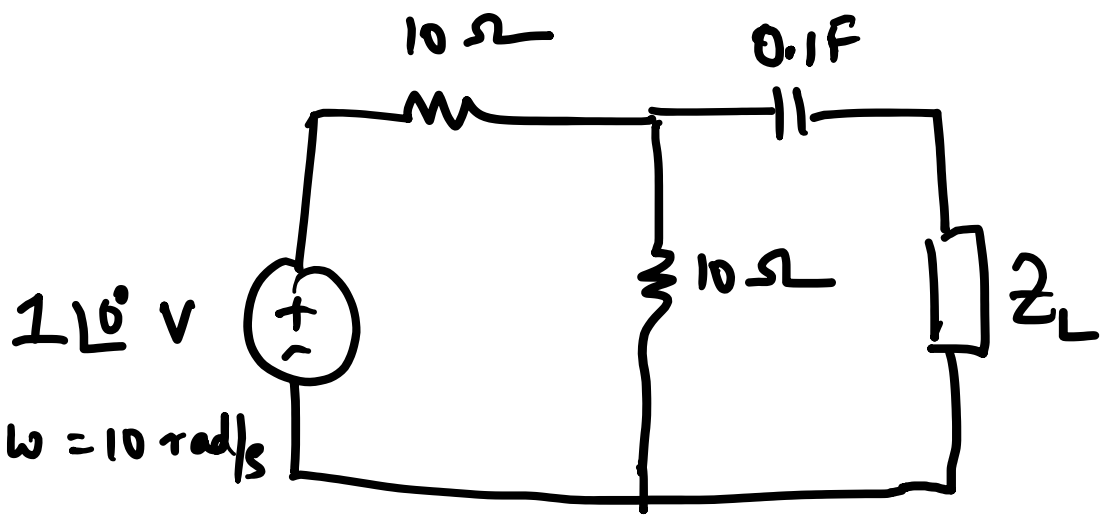
$$= j0.3 + 0.4$$

$$Z = 1/Y$$ (1)

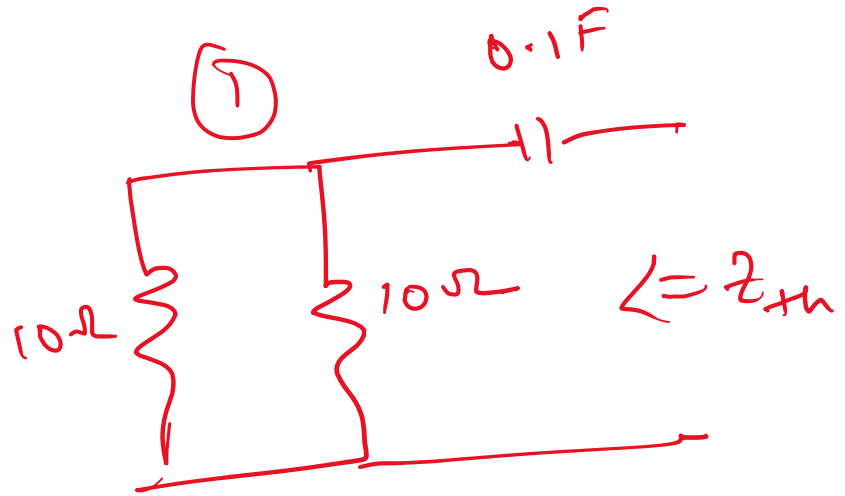
$$CP = \left\{ 5 \times \left(\frac{1}{0.4 + j0.3} \right) \times 5 \right\}$$ (No 1/2 because of rms)

$$= 40 - j30 \quad \therefore P_{avg} = 40\,W, \text{ Reactive power} = -30\,VaR$$ (1)

(3) $Z_{th} = [10\Omega \,||\, 10\Omega] + \frac{1}{j\omega C} = 5 + \frac{1}{j\,10\times 0.1} = 5 + \frac{1}{j} = 5 - j$



⇒



⇒ $Z_L = Z_{th}^{*}$ for maximum power transfer

$\therefore Z_L = (5 + j)\,\Omega$ (1)

(4) $V = 30\,e^{+j20^\circ}$, $S = -4+j5$

$v(t) = 30\,e^{-4t}\cos(5t+20^\circ)\ V$ (1)

(a) $t = 1\ ms$

$v(t) = 30\,e^{-4\times10^{-3}}\cos\left(5\times10^{-3}+20^\circ\times\frac{\pi}{180^\circ}\right] = 28.027\ V$ (1)

(b) $t = 500\ ms$

$v(t) = e^{-4\times500\times10^{-3}}\cos\left(5\times500\times10^{-3}+20\times\frac{\pi}{180}\right) = -3.88\ V$

(5) pf = 0.5 lagging

This means current lags behind voltage. So inductive circuit

$$I = \frac{V}{j\omega L}$$

(1)

$$CP = |CP| \cos\phi - j|CP| \sin\phi$$

$$= \underbrace{|CP| \times 0.5}_{5W} - j|CP| \sin\phi$$

$$\Rightarrow |CP| = 5 \times 2 = 10W$$

(1)

$\cos\phi = 0.5$ lagging

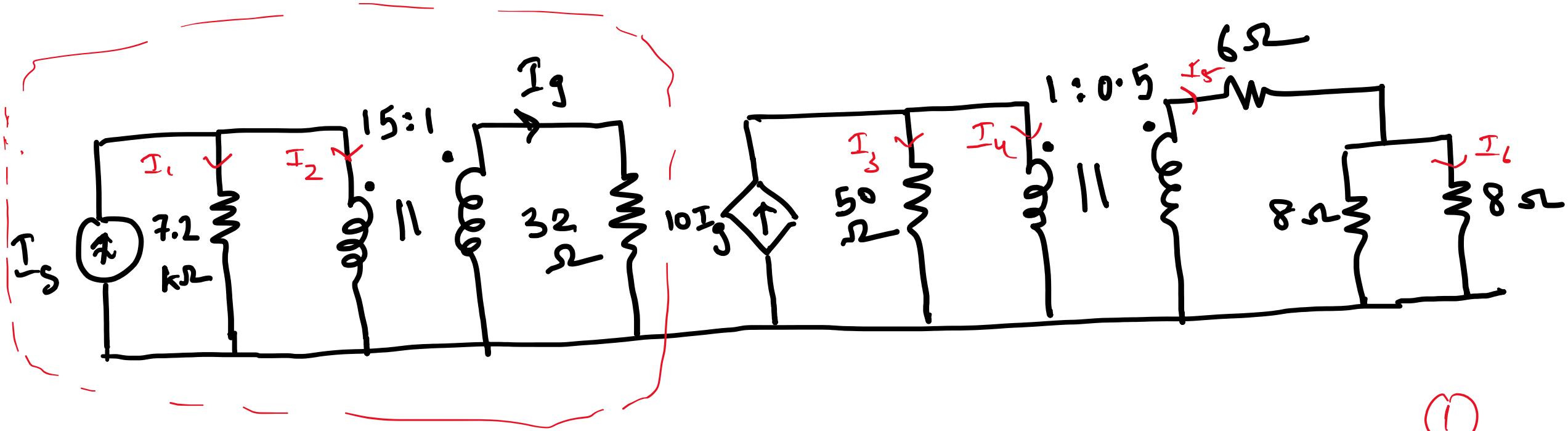
$\sin\phi = 0.866$

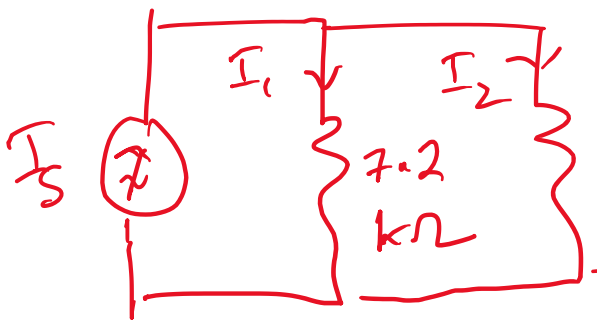
Reactive power =

$-10 \times 0.866 = -8.66W$

Long questions

(1)





$$\frac{Z_L}{a^2} = \frac{32 \times 15^2}{1} = 7200$$

$$I_1 = I_s \times \frac{7200}{7200+7200} = \frac{I_s}{2} = 16\,A$$ (1)

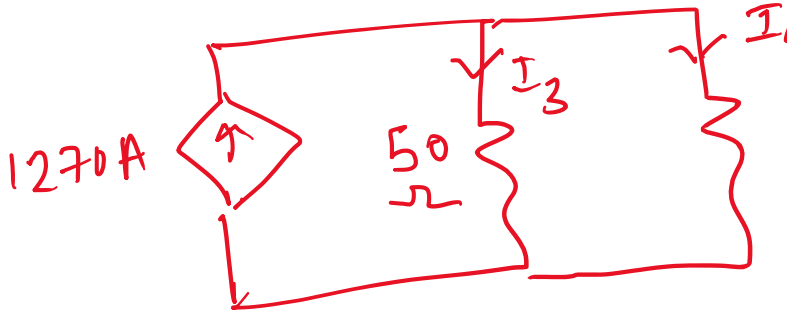
$$P_1 = \left\{\frac{I_s}{2} \times I_1^2 \times Z_1\right\} = 9.216 \times 10^5\,W$$ (1)

$$I_2 = \frac{I_s \times Z_2}{Z_1 + Z_2} = 32 \times \frac{1}{2} = 16\,A \quad (1)$$

$$\frac{I_g}{I_2} = \frac{1}{a} \Rightarrow I_g = \frac{1}{a} \times I_2 = 15 \times I_2 = 240\,A \quad (1)$$

$$P_2 = Re\left\{\frac{1}{2} |I_g|^2 \times Z_2\right\} = 9.216 \times 10^5\,W \quad (1)$$

$$10\,I_g = 2400\,A$$



$$\frac{(6+4)}{a^2} = \frac{10}{(0.5)^2} = 4 \times 10 = 40\,\Omega$$

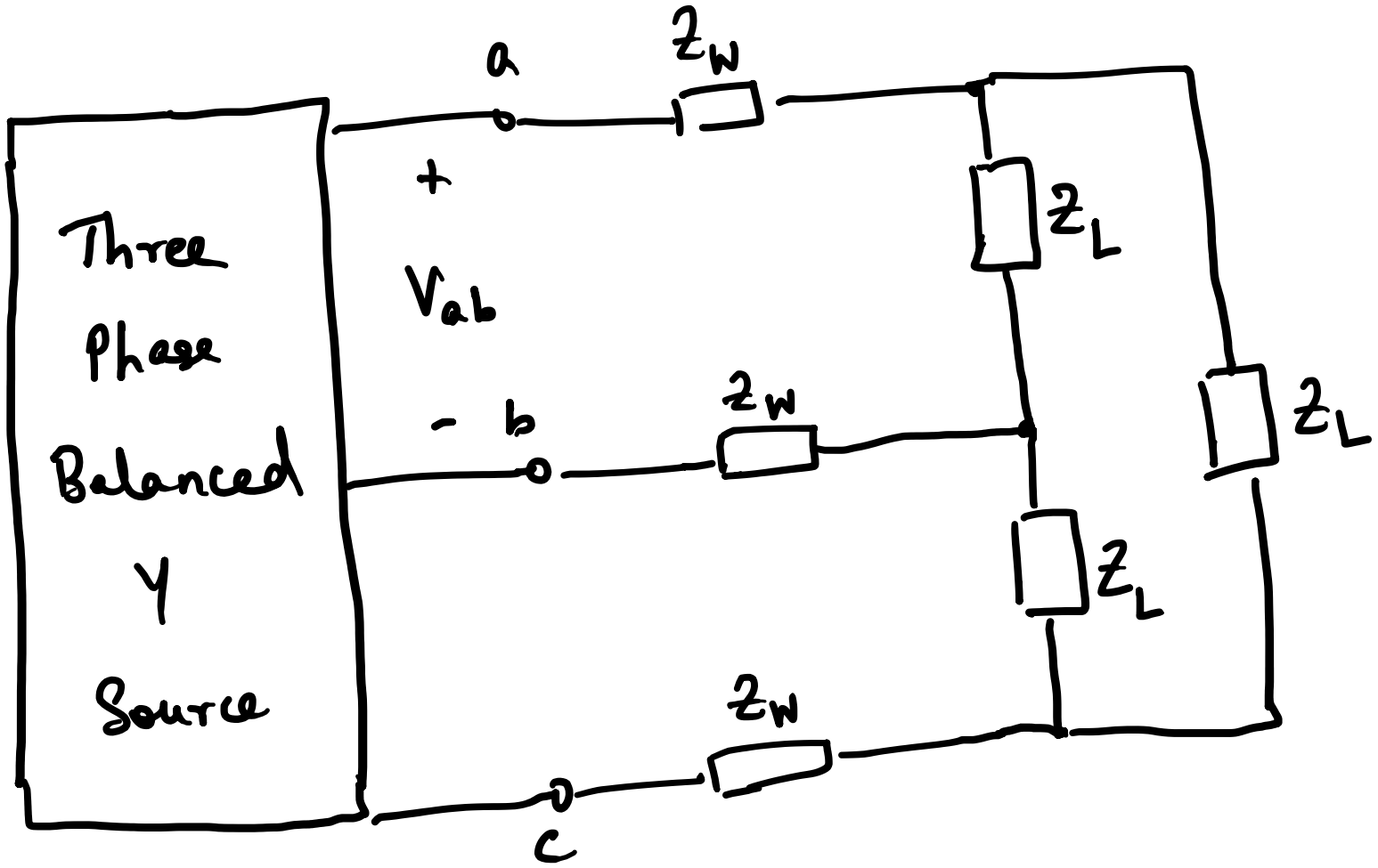
$$I_3 = \frac{10 \times I_g \times 40}{50+40} = 1066.67\ A \quad (1)$$

$$I_4 = \frac{10 \times I_g \times 50}{50+40} = 1333.33\ A \quad (1)$$

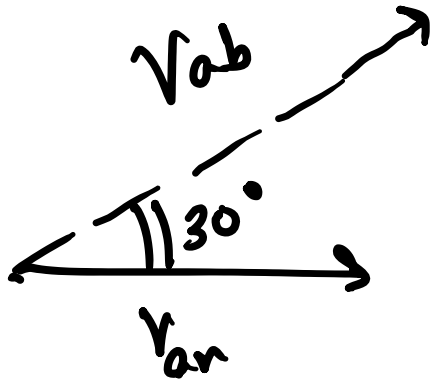
$$P_3 = \frac{1}{2} \times |I_3|^2 \times 50 = 28.4445 \times 10^5\ W \quad (1)$$

$$\frac{I_5}{I_4} = \frac{1}{a} \Rightarrow I_5 = \frac{I_4}{0.5}, \quad I_6 = \frac{I_5}{2} = 1333.33\ A \quad (1)$$

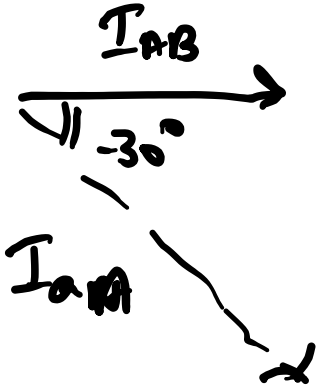
$$P_6 = \frac{1}{2} \times I_6 \times I_6 \times 8 = 7.11 \times 10^6\ W \quad (1)$$


a
Z_W
+
V_{ab}
- b
Z_W
Z_W
c
Three Phase Balanced Y Source
Z_L
Z_L
Z_L

$$V_{ab} = 380\, e^{-j15^\circ} \text{ V} \Rightarrow V_{an} = \frac{|V_{ab}|}{\sqrt{3}} \angle -15^\circ - 30^\circ$$



$$= 155 - j155 \text{ V} \quad (1)$$



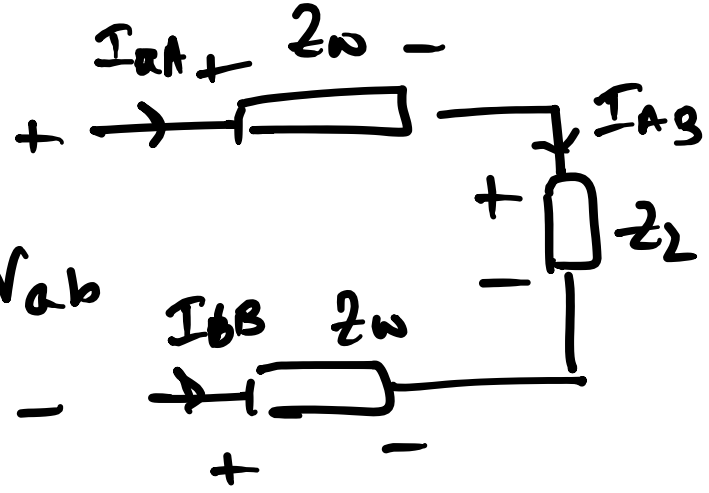
Let $I_{aA} = I_0$

$$I_{AB} = \frac{I_0}{\sqrt{3}} \angle I_0 + 30^\circ \quad (1)$$

$$= \frac{I_0}{\sqrt{3}} e^{+j30^\circ} \text{ A}$$

Through Loop

$$+ V_{AB} - I_{aA} Z_w - I_{AB} Z_L + I_{bB} Z_w = 0$$



$$\Rightarrow V_{AB} = I_{aA} Z_w + I_{AB} Z_L - I_{bB} Z_w$$

$$\Rightarrow V_{AB} = I_0 \left[Z_w + \frac{1}{\sqrt{3}} e^{+j30^\circ} Z_L - e^{-j120^\circ} Z_w \right]$$

$$\Rightarrow I_0 = \frac{V_{AB}}{Z_w + \frac{1}{\sqrt{3}} e^{+j30^\circ} Z_L - e^{-j120^\circ} Z_w} = 5.1616 - j\,41.866 \text{ A}$$

(1)

$$I_o = 42.1832 \angle -82.9716 \text{ A}$$

(1)

$$\Rightarrow i_{aA}(t) = 42.1832 \cos(120\pi t - 82.9716^\circ) \text{ A}$$

$$i_{bB}(t) = 42.1832 \cos(120\pi t - 202.9716^\circ) \text{ A}$$

$$i_{cC}(t) = 42.1832 \cos(120\pi t - 322.9716^\circ) \text{ A}$$

(1) (a)

$$I_{AB} = \frac{I_{aA}}{\sqrt{3}} e^{+j30^\circ} = \frac{42.1832 \angle -52.9716}{\sqrt{3}} = 24.3552 \angle -52.9716 \text{ A}$$

$$I_{BC} = 24.3552 \angle -172.9716 \text{ A}$$

(1)

$$I_{CA} = 24.3552 \angle -292.9716 \text{ A}$$

✓ Load currents (b)

$$P_{ms}(t) = v_{an}(t)\, i_{aA}(t) + v_{bn}(t)\, i_{bB}(t) + v_{cn}(t)\, i_{cn}(t)$$

(1)

$$= 9.2453\times10^{3} \cos(120\pi t - 45^{\circ}) \cos(120\pi t - 82.9716^{\circ}) +$$

$$9.2453\times10^{3} \cos(120\pi t - 45^{\circ} - 120^{\circ}) \cos(120\pi t - 82.9716 - 120^{\circ}) +$$

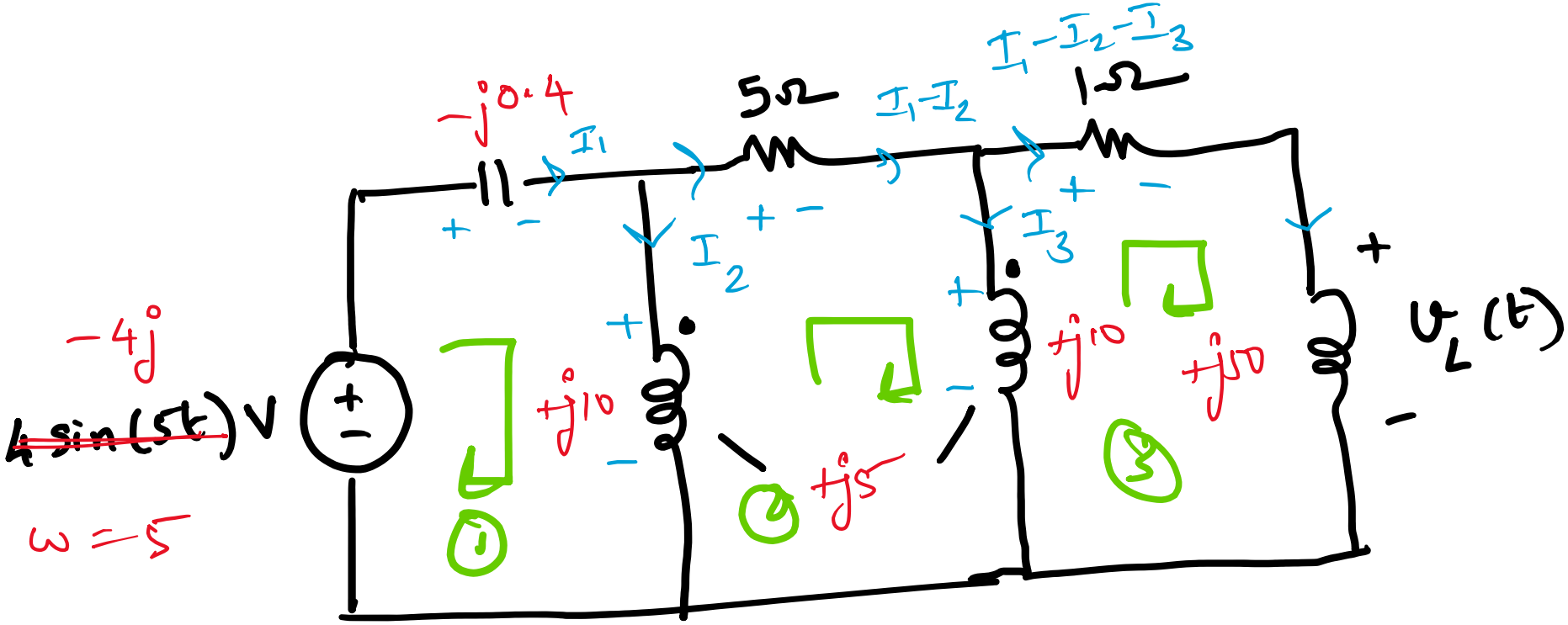
$$9.2453\times10^{3} \cos(120\pi t - 45^{\circ} - 240^{\circ}) \cos(120\pi t - 82.9716 - 240^{\circ})\ W$$

$$= 3 \times \frac{9.2453\times10^{3}}{2} \left[\cos(240\pi t - 127.9716^{\circ}) + \cos(37.9716^{\circ}) \right]$$

$$= 13.868 \cos(240\pi t - 127.9716^{\circ}) + 10.9323\ kW$$

(d) $P_L = \frac{3}{2} \times Re\left\{ I_{AB} \times Z_L \times I_{AB}^{*} \right\} = 10.677\ kW$ (1)

(e) $P_w = \frac{3}{2} \times Re\left\{ I_0 \times Z_w \times Z_w^{*} \right\} = 266.9131\ W$ (1)



$Z_C = -j0.4\,\Omega$

$Z_{L1} = +j10\,\Omega$

$Z_{L2} = +j10\,\Omega$

$M = k\sqrt{L_1 L_2} = 0.5 \times 2 = 1$

$Z_m = +j5\,\Omega$

$Z_{load} = j50\,\Omega$

(2)

Loop 1

$$+V_s - I_1(-j0.4) - I_2(+j10) - I_3(+j5) = 0 \quad (2)$$

$$\Rightarrow I_1(-j0.4) + I_2(+j10) + I_3(+j5) = V_s = -4j \quad \text{——} (1)$$

Loop-2

$$+I_2(+j10) + I_3(+j5) - (I_1 - I_2)(5) - I_3(+j10) - I_2(+j5) = 0$$

$$\Rightarrow I_1(-5) + I_2(+j10 + 5 - j5) + I_3(+j5 - j10) = 0 \quad (2)$$

$$\Rightarrow I_1(-5) + I_2(5 + j5) + I_3(-j5) = 0 \quad \text{——} (2)$$

Loop-3

$$+I_3(j10) + I_2(+j5) - (I_1 - I_2 - I_3)(1) - j50(I_1 - I_2 - I_3) = 0$$

$$I_1(-1 - j50) + I_2(+j5 + 1 + j50) + I_3(+j10 + 1 + j50) = 0 \quad (2)$$

$$\Rightarrow I_1(-1 - j50) + I_2(+j55 + 1) + I_3(+j60 + 1) = 0 \quad (3)$$

$$I_1 = -0.5699 - j0.1167 \text{ A}$$

$$I_2 = -0.3418 + j0.0732 \text{ A}$$

$$I_3 = -0.1620 - j0.1549 \text{ A}$$

$$I_L = I_1 - I_2 - I_3 = -0.0661 - j0.02501 \text{ A}$$

$$V_L = I_L \times Z_{load} = 1.2489 - j3.3037 \text{ V}$$

$$|V_L| = 3.5319 \text{ V}$$

$$\angle V_L = -69.2914^\circ$$

$$\Rightarrow v_L(t) = 3.35319 \cos(5t - 69.2914^\circ) \text{ V}$$

(2)