Game Theory: Assignment 3 (Solutions)

Total points: 25 | Due Date: 18/11/2022

Contribution to grade: 10% (3xx); 7.5%(5xx) | Due time: 11:59 PM

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1. Let us work with a very basic Bertrand model, but repeated infinitely. Suppose *delta* > 0.

   (a) Is there always a pricing strategy that can sustain cooperation, and if yes, how many such strategies exist? (Explanation is mandatory, it is not a yes/no question) (5)

   Answer. Let us only check for the symmetric case. We wil lfind that cooperation is not always sustainable, and though this may seem a bit counter-intuitive (there should always be a high enough p regardless of δ), we will see in detail why in the next part of the question.

   (b) Again, if yes, find the set of prices that can sustain cooperation in terms of δ. If no, show the minimum value of δ needed to ensure cooperation is possible. (5)

   Answer. Here, I show only the symmetric one. Consider a cooperative price $p^c > 0$. Cooperation yields a payoff of $\frac{1}{2}(p^c - c)$ in each stage. Defection yields $p^c - c$ (we can assume defection is almost 0, just some infinitesimally small amount), and 0 in all other periods. Thus coperation is possible if

$$\frac{\frac{1}{2}(p^c - c)}{1-\delta} \geq p^c - c \Rightarrow \delta \geq \frac{1}{2}$$

   Thus, symmetric cooperation is only sustainable if $\delta \geq \frac{1}{2}$, and it will work for any value of $p^c > c$.

2. The Folk Theorem says that there exists a participation constraint below which cooperation is not sustainable. However, we saw that players getting payoffs below that

constraint in some periods were still willing to cooperate under some conditions. What were those conditions? (5)

Answer. The Folk Theorem simply states that the average payoff must be greater than the participation constraint (not the payoff in every period). Hence, as long as the average payoff was greater than the participation/Individual Rationality Constraint, cooperation can be sustained if δ is high enough.

3. The discounting we discussed in class (using δ) is known as hyperbolic discounting. David Laibson in a famous paper in 1997 proposed a more sophisticated version of this: "quasi-hyperbolic discounting". Explain what this added to the current method, and comment on whether or not this new intervention makes cooperation more or less sustainable. (5)

   Answer. The new method has two components. The δ component continues to exist. However, we now have an addition weight β, which distinguishes the present from any future period. The idea behind this is while we add more value to periods closer to us in time, we make a binary distinction between the present and any future period. Mathematically, it is stated as

$$1 + \beta\delta + \beta\delta^2 .... = 1 + \beta \sum_{t=1}^{\infty} \delta^t$$

   $\beta \in [0, 1]$ is known as present bias. This further reduces the emphasis put on future payoffs by an individual, making cooperation less likely.

4. Suppose we have a symmetric 3 player prisoner's dilemma, instead of 2. Is cooperation more or less likely with the added player. Show your proof (5) *Answer.* This is very easy. The conditions for defection are identical to the 2 player prisoner's dilemma. Cooperation is equally likely. You do have to show your work.