Ques 1

(a) TRUE

Since X_i is IID with $M_{X_i}(S) = e^{s^2/2}$

$$M_{X_1+X_2}(S) = E\left[e^{(X_1+X_2)s}\right]$$

$$= \prod_{i=1,2} E\left[e^{X_i}\right]$$ since X_i's are independent.

$$= \prod_{i=1,2} M_{X_i}(S)$$

$$= e^{s^2/2} \cdot e^{s^2/2}$$

$$= e^{s^2}$$

(b) n, independent shots

with p, probability of successful shots

this can be modelled as Bernoulli

& for K successful target out of n trials

$N_t \sim \text{Binomial}(n, p)$

$$= {}^{n}C_K (p)^K (1-p)^{n-K}$$

ca

therefore, the process can be modeled as discrete time and discrete state Random process

∴ TRUE

(c) TRUE

Sum of interarrival time

$$S_n = X_1 + X_2 + \cdots + X_n$$

OR

$$S_n = S_{(n-1)} + X_n$$

thus, this is not independent

OR

we can prove by

$$\mathrm{Cov}(S_n, S_{n-1}) \neq 0$$

∴ not independent

(d)

Given, the no. of arrivals follow poisson distribution

probability of exactly one arrival in $(0,1]$ and $(1,2]$

$$= P[N(1) = 1 \text{ and } \tilde{N}(1,2) = 1]$$

$$= P[N(1) = 1] \text{ and } P[\tilde{N}(1,2) = 1]$$

$$= P[N(1) = 1] \cdot P[\tilde{N}(1,2) = 1]$$

$\therefore$ independent

$$\frac{e^{-\lambda}\lambda}{1!} \times \frac{e^{-\lambda}\lambda}{1!} = e^{-2\lambda}\lambda^2 \quad \text{TRUE}$$

OR

$$\frac{e^{-\lambda}\lambda^{1-1}}{1!} \times \frac{e^{-\lambda}\lambda^{1-1}}{1!} = e^{-2\lambda} \quad \text{FALSE}$$

Q2.

Sol

$$E\left[(X_n - X_{n-1})^2\right] = E\left[X_n^2 + X_{n-1}^2 - 2X_nX_{n-1}\right]$$

Since the interarrival times X_i are IID, we have

$$E\left[(X_n - X_{n-1})^2\right] = E\left[X_n^2\right] + E\left[X_{n-1}^2\right] - 2E[X_n]E[X_{n-1}]$$

We know that

$$\mathrm{Var}(X_n) = E\left[X_n^2\right] - \left(E[X_n]\right)^2$$

$$\Rightarrow \mathrm{Var}(X_n) + \left(E[X_n]\right)^2 = E\left[X_n^2\right]$$

llly, $E\left[X_{n-1}^2\right] = \mathrm{Var}(X_{n-1}) + \left(E[X_{n-1}]\right)^2$

Now,

$$E\left[(X_n - X_{n-1})^2\right] = \mathrm{Var}(X_n) + \left(E[X_n]\right)^2 + \mathrm{Var}(X_{n-1}) + \left(E[X_{n-1}]\right)^2 - 2E[X_n]E[X_{n-1}]$$

Since, mean & variance of X_n & X_{n-1} are same

$$= 2\left[\mathrm{Var}(X_n) + \left(E[X_n]\right)^2\right] - 2\left(E[X_n]\right)^2$$

$$= 2\,\mathrm{Var}(X_n)$$

$$= \frac{2(1-p)}{p^2}$$

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Ques-3. (a) Spatial characteristic is Time (sec) which is <u>DISCRETE</u> (given in the question).

for state space,

$$Z_t = \begin{cases} 1 \ ; \text{ mosquito bites} & \text{w.p } (0.1) \\ 0 \ ; \text{ mosquito doesn't bites} & \text{w.p } (0.9) \end{cases}$$

p = Probability of mosquito biting $= 0.5 \times 0.2 = 0.1$
$q = 1 - p = 0.9$

$$\boxed{p = 0.1 \ ; \ q = 0.9}$$

$Z_t \Rightarrow$ At t_{th} time instant mosquito bites.

As, Z_t can be modeled in the above mentioned way thus the state space is <u>discrete</u>.

State space $\longrightarrow$ mosquito bites/ mosquito doesn't bites (Discrete RV)

- Trials are independent of each other. Mosquito biting at a time is independent of mosquito bite at any other time (given in qs.).
- There are only 2 possible outcomes either mosquito bites (or) doesn't bites.
- All trials have same probability (p or q).

Hence, we can say state space (Z_t) is a Bernoulli(p) IID RV.

- Further, we can assume the process starts at $t=0$ and arrival happens after $t>0$.
- At a time, only one arrival can occur.

Hence, we can model this process as a <u>BERNOULLI PROCESS</u>.

(b) Since, We modeled the process as Bernoulli. We know that, Interarrival times follow geometric (p) distribution in case of Bernoulli process.

$\{X_i\} \Rightarrow$ Interarrival times $\Rightarrow$ Time b/w successive arrivals

(time b/w successive mosquito Bites)

$$X_i \sim \text{Geometric}(p)$$

$$\boxed{E[X_i] = \frac{1}{p}}$$

$$E[X_i] = \frac{1}{0.1} = 10 \text{ seconds.}$$

$\therefore$, Expected time b/w successive bites is 10 sec.

Q3 c find the probability of getting 2 bites in 5 seconds.

Soln $N_t \sim \text{Binomial}(t, p)$

→ total no. of success / arrivals in time 't'

Now we know that,

$$P(N_t = k) = {}^{t}C_{k} (p)^{k} (q)^{t-k} \quad \text{— (1)}$$

In our case → $\boxed{\begin{array}{l} p = 0.1 \\ q = 0.9 \end{array}}$ → (probab. of getting mosquito bites)

→ from 3(a) part

putting values in (1)

$$P(N_5 = 2) = {}^{5}C_{2} (0.1)^{2} \times (0.9)^{3}$$

$$\Rightarrow 10 \times 0.01 \times (0.9)^{3}$$

$$\Rightarrow \underline{0.0729} \text{ Ans}$$

Q4 In the given ques, we can assume it is a Bernoulli process as Feed i

(1) Feed is giving out 1 sample at a time (1 arrival at a time)

(2) sample droping is independent to each other

(3) discrete time

(A) prob of door been answered $= \frac{3}{4}$

(B) prob that any houshold has a dog is $= \frac{2}{3}$

As ~~they~~ event A and B are independent

$P(E) = P(A \cap B)$ = Prob that feed gives away the sample $= \frac{\cancel{3}}{4} \times \frac{\cancel{2}}{\cancel{3}} = \frac{1}{2}$

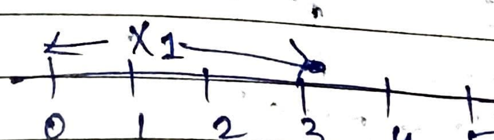
$\Rightarrow P(A) \times P(B)$

prob that feed doesn't give away the sample $= P(\bar{E}) = 1 - P(E)$

$= 1 - \frac{1}{2}$

$= \frac{1}{2} = q$

(a)



P(gives away first sample at 3rd call) $= P(X_1 = 3)$

in Bernoulli process X_i, ith interarrival times are iid

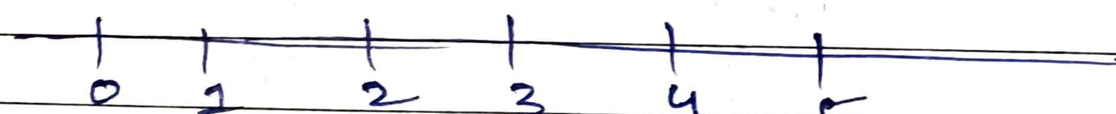
$$X_i \overset{iid}{\sim} \text{geometric}(p)$$

$$\therefore P(X_1 = 3) = q^{3-1} \times p$$

$$= q^2 \times p = \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{1}{8}$$

(b) P(2nd sample at 5th call) =

↳ means 2nd arrival on 5th call.



2nd arrival on 5th call.

~~This can hap~~

which means → $P(X_1 + X_2 = 5)$

$P(S_2 = 5)$ → this shows 2nd arrival happens at 5th call as then only the sum of X_1 and X_2 is 5

S_2 → sum of interarrival time $(X_1 + X_2)$

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This means first arrival can happen at any time bet first call and ~~fifth call~~ fourth call.

$\therefore$ ~~$P(S_2=5) = P(S_1)\, P(S_1=4)\times p$~~

=

$\therefore P(S_k=n) = P(S_{k-1}=n-1)\times p$

$P(S_2=5) = P(S_1=4)\times p$

$= {}^4C_1 \times (p)^1 \times (q)^3 \times p$

$= \dfrac{4!}{1!\times 3!} \times p^2 \times q^3$

$= \dfrac{4!}{1!\times 3!} \times \left(\dfrac{1}{2}\right)^5$

$= 4\times\dfrac{1}{32} = (2)^2\times\dfrac{1}{(2)^5}$

$= \dfrac{1}{(2)^3} = \dfrac{1}{8}$

~~we can diredly do this~~

we can directly do this by ~~convolution formula~~ using PMF of pascal distribution.

$P(S_K=n) = \binom{n-1}{k-1} p^{k-1} \times q^{(n-1)-(k-1)} \times p$

$= \binom{5-1}{2-1} p^2 \times q^3 \times p = \dfrac{1}{8}$

We can also do this with mgf of pascal.

S_2 = sum of 2 inter arrival times $= X_1 + X_2$

$X_i \overset{iid}{\sim}$ geometric (p)

$$M_{S_2}(s) = E[e^{S_2 s}]$$

$$= E[e^{(X_1+X_2)s}]$$

$$= E[e^{X_1 s} e^{X_2 s}]$$

$$= \prod_{i=1}^{n} E[e^{X_i s}]$$

$$= \prod_{i=1}^{n} M_{X_i}(s)$$

$$= \left(\frac{pe^s}{1-qe^s}\right)^2$$

This is mgf of Pascal with parameter p & order 2

Thus $S_2 \sim$ pascal $(k=2, p)$

$$P(S_2 = s) = \binom{s-1}{2-1} p^2 (1-p)^{s-2}$$

$s = 2, 2+1$

$$P(S_2 = 5) = \binom{4}{1} p^2 (1-p)^3 = 4 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^3$$

$$= 4 \times \left(\frac{1}{2}\right)^5 = \frac{1}{8}$$