

- Continuity check:  $\lim_{t \rightarrow t_0} \vec{r}(t) = \vec{r}(t_0)$   
 $\rightarrow$  Continuous in each component
- Differentiate:  $\vec{r}(t) = (f(t), g(t), h(t))$   
 $\vec{r}'(t) = (f'(t), g'(t), h'(t))$
- Dot product:  $\frac{d(\vec{u} \cdot \vec{v})}{dt} = \vec{u}' \cdot \vec{v} + \vec{v}' \cdot \vec{u}$
- Cross product:  $\frac{d(\vec{u} \times \vec{v})}{dt} = (\vec{u}' \times \vec{v}) + (\vec{u} \times \vec{v}')$
- Chain rule:  $\frac{d[u(f(t))]}{dt} = \frac{du}{df} \frac{df}{dt}$   
 $\frac{du}{df}$  is scalar,  $\frac{df}{dt}$  is vector
- Parametric tangent =  $\frac{d\vec{r}}{dt} = \vec{r}'(t)$  at  $t = t_0$   
 (for every component)
- Arc length =  $\int_{t_A}^{t_B} \text{speed } dt$   
 $= \int_{t_A}^{t_B} \sqrt{x'^2 + y'^2 + z'^2} dt$  (if given)
- Tangent to arc =  $\frac{\vec{r}'}{|\vec{r}'|} = \frac{d\vec{r}/dt}{ds/dt} = \frac{d\vec{r}}{ds}$
- Curvature  $\kappa = \left| \frac{d\vec{T}}{ds} \right| = \frac{1}{|\vec{v}|} \left| \frac{d\vec{T}}{dt} \right| = \frac{1}{s^2} \left| \frac{d\vec{T}}{dt} \right|$
- Unit Normal  $\vec{N} = \frac{1}{\kappa} \frac{d\vec{T}}{ds} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|}$
- Binomial vector  $\vec{B} = \vec{T} \times \vec{N}$

$$\frac{d\vec{r}}{ds} = -\vec{r} \cdot \vec{N}$$

$$\begin{vmatrix} x' & y' & z' \\ x'' & y'' & z'' \\ x''' & y''' & z''' \end{vmatrix} / |\vec{v} \times \vec{a}|$$

$$\vec{a} = \frac{d^2 \vec{r}}{dt^2} = \kappa \vec{N} a_c + a_T \vec{T}$$

$$= \frac{d}{dt} \left( \vec{T} \frac{ds}{dt} \right)$$

$$\vec{v} \times \vec{a} = \vec{v} \times \left( \frac{ds}{dt} \vec{T} \right)$$

$$\vec{v} \times \vec{a} = \left( \frac{ds}{dt} \right)^3 \kappa \vec{B}$$

$$\vec{B} = \vec{v} \times \vec{a}$$

$$\kappa \left( \frac{ds}{dt} \right)^3$$

$$\kappa = |\vec{v} \times \vec{a}|$$

$$\left( ds/dt \right)^3$$

$$\text{To express } a_T \text{ in an}$$

$$\frac{d|\vec{v}|}{dt} = a_T$$

$$\kappa = \frac{a_n}{|\vec{v}|^2}$$

$$\kappa = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$$

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