

# Game Theory: Assignment 3

Total points: 50

Due Date: 25/10/2021

Contribution to grade: 10% (3xx); 7.5%(5xx)

Due time: 11:59 PM

---

1. Consider two players in a game, where player one has 10 rupees, and player 2 has 0. Player 1 can give any amount  $x$  to player 2, and once he does, the money gets tripled to  $3x$ . Then, player 2 can return any amount he chooses to out of this  $3x$  to player 1 in stage 2.

(a) What is the SPNE? (3)

(b) Is there an NE where players receive higher payoffs? (3)

(c) Now suppose player 2 does care at least a little about his rival's payoff (often known as altruism in behavioural economics). Can you think of a good way to represent this in the payoff function? Hint: Assume player 2 cares about player 1 by a coefficient  $\delta \in [0, 1]$ . Under what condition on  $\delta$  will this affect the SPNE? (4).

2. Suppose there are two firms in a market which sell differentiated products. The inverse demand function is given by

$$p_i = a - bq_i - dq_j \quad 0 \leq d \leq b$$

Suppose we have a two stage game, where in stage one a firm can commit to either a quantity  $\bar{q}_i$  or a price  $\bar{p}_i$ . In the second stage, they can choose the variable they had not committed to in the first stage, and then they go on to compete. Show that commitment to quantities in stage 1 is the dominant strategy for each firm (15)

3. Consider two firms competing a la Cournot, and linear inverse demand  $p(Q) = 100 - Q$ , where  $Q = q_1 + q_2$ . Per unit cost of production is  $c = 10$ .

- (a) For which discount factors this grim-trigger strategy can a cooperative outcome be sustained as the SPNE of the infinitely-repeated game? Hint: Go back to Cournot notes to understand what we mean by cooperation. (10)
- (b) Consider now an altered grim-trigger strategy. Here, if a deviation occurs, every firm deviates to the Cournot equilibrium during only 1 period, and then every firm returns to cooperation (producing the collusive output). Thus, punishment of deviating from the collusive agreement is now temporary since it lasts only one period. For which discount factors this altered grim-trigger strategy can be sustained as the SPNE of this game? (5)
- (c) Let us now generalize part (b) regarding temporary punishment facilitating collusive behavior. Consider a more general setting where the temporary punishment lasts  $T \geq 1$  periods. Find under which conditions for firms' discount factor  $\delta$  collusion can be sustained as the SPNE of the infinitely repeated game. How are your results affected by an increase in  $T$ ? (10)