Devaj Rathore
2023190

MTH203 - Multivariate Calculus
Mid Semester Exam

Section - B
Total Marks - 30
11th October 2024

Problem - 1

A solid cube, 2 units on a side, is bounded by the planes $x = \pm 1$, $z = \pm 1$, $y = 3$, and $y = 5$. Find the moments of inertia about the coordinate axes.

[6 marks]

Problem - 2

Given nonzero vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$, use dot product and cross product notation, as appropriate, to describe the following:

a. The vector projection of $\mathbf{u}$ onto $\mathbf{v}$

b. A vector orthogonal to both $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$

c. The area of the parallelogram determined by $\mathbf{u}$ and $\mathbf{w}$

d. The volume of the parallelepiped determined by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$

[4 marks]

Problem - 3

The temperature at a point (x, y) on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant? **(Do it using Lagrange Multipliers)**

[8 marks]

Problem - 4

Solve the following questions:

a. Show that the curve $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + (1 - \cos t)\mathbf{k}$, $0 \leq t \leq 2\pi$, is an ellipse by showing that it is the intersection of a right circular cylinder and a plane. Find equations for the cylinder and plane.

[3 marks]

b. Sketch the ellipse on the cylinder. Add to your sketch the unit tangent vectors at $t = 0$, $\pi/2$, π, and $3\pi/2$.

[3 marks]

c. Show that the acceleration vector always lies parallel to the plane (orthogonal to a vector normal to the plane).

[2 marks]

Problem - 5

Let D be the region bounded below by the plane $z = 0$, above by the sphere $x^2 + y^2 + z^2 = 4$, and on the sides by the cylinder $x^2 + y^2 = 1$. Sketch the figure and Set up the triple integrals in cylindrical coordinates that give the volume of D using the following order of integration: $dz\, dr\, d\theta$

[4 marks]