[redacted]

1. Given: sample size $n = 10$; sample mean $\bar{x} = 27$

Pop^n is normally distributed (one of the assumptions).

pop^n variance $\sigma^2 = 20$.

level of significance $\alpha = 0.05$.

Claim (to be tested). is pop^n mean different from 30.

So the hypothesis will be,

$H_0: \mu = 30$ v/s $H_1: \mu \neq 30$.

Assumptions
i. pop^n is normally distributed.
ii. Pop^n variance is known.
iii. the sample is a Simple random sample (as a random sampling was done).

Test. As we have to test for mean with known variance and pop^n is assumed to be normally distributed. The test statistic used will ~~depend on~~ be of the form 'Z' i.e. z score.

Test statistic
$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$
$$= \frac{27 - 30}{\sqrt{20/10}}$$
$$= -3/\sqrt{2}$$
$$= -2.121$$

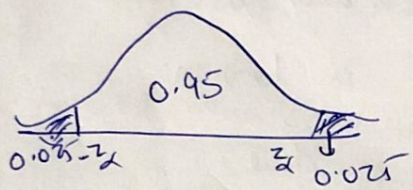
The alternative hypothesis is $H_1: \mu \neq 30 \Rightarrow$ it is a 2-sided.

As the test statistic is $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$.

i.e., the test statistic follows std. normal distribution.

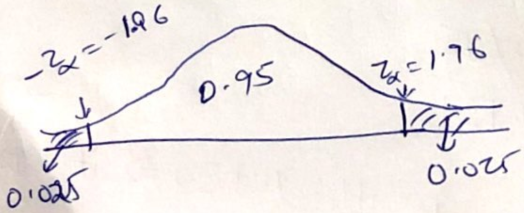
So, to get the rejection region of $\alpha = 0.05$ we need to consider both the tails.

The size of the rejection region should be $\alpha = 0.05$. Recall, std. normal is symmetric about 0. Thus, on both the tails we will have $\alpha/2 = 0.025$ of area i.e,



We have to find critical values (the point at which acceptance region & rejection regions are divided). By using the Z-table (given on Google classroom), $Z_\alpha = 1.96$ (area under the curve will be $0.95 + 0.025 = 0.975$). ~~By symmetric~~ As std. normal is symmetric about zero $\Rightarrow -Z_\alpha = -1.96$.

So the critical value is $|Z| = |1.96|$



Now, if the value of Z statistic lies in the acceptance region then we accept H_0 otherwise we will reject H_0. From previous calculations, $Z_{stat} =$ ~~-2.121~~ -2.121. This will lie to the left of $-Z_\alpha$ (-1.96) so the value of Z_{stat} lies in the rejection region. Thus we will reject H_0.

Scanned with CamScanner

Inference $|Z_{stat}| > |Z_{C.V.}|$ (C.V. = Critical Value)

Thus Z_{stat} lies in the rejection region and at $\alpha = 0.05$ ~~the pop^n mean is equal to 30.~~

~~we reject the claim that the pop^n mean is equal to 30.~~
~~We don't have sufficient evidence to accept the hypothesis.~~

The sampled data supports the claim that the mean is different from 30 years.

(2). Given : $n = 30$; $\bar{x} = \$43,260$; $\sigma^2 = \$(5230)^2$
$\alpha = 0.05$.

$H_1 : \mu > \$42,000$

Claim $H_0 : \mu \le \$42,000$

Assumptions. i sample size $n = 30$, so we can assume pop^n is normally distributed

ii. sample is a simple random sample (SRS).

iii σ is known.

Test. It will be solved in a similar way as question 1. The only difference here is that the alternative hypothesis is one-sided. This will reflect in rejection region but the test statistic will remain same as Q1. Thus

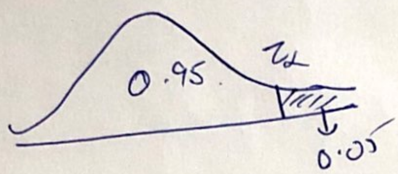
test statistic $Z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

$$= \frac{43,260 - 42,000}{5230/\sqrt{30}}$$

$$= 1.3196$$

As $H_1: \mu > \mu_0$
So the Rejection region will lie on the right side of the Curve. (Right-tailed test).

As $\alpha = 0.05$



We have to find C.V. $Z_\alpha = ??$.
using Z-table $Z_\alpha = 1.645$.
$Z_{stat} = 1.3196$, the Z_{stat} lies in the Acceptance region. Thus
we don't reject H_0.

$< Z_{C.V.}$

<u>Inference</u>. Z_{stat}
Thus Z_{stat} lies in the acceptance region and at $\alpha = 0.05$
we fail to reject H_0. We don't have enough evidence to
accept the claim that the assistant prof.s earn more than
\$42,000 per year.

Edit: normal popⁿ is also given.

③. Given $n = 10$; $\bar{x} = 17.7$; $s = 1.8$

$\alpha = 0.05$.

Claim $H_0: \mu = 16.3$ V/S $H_1: \mu \neq 16.3$.

Assumptions i. Popⁿ is normally distributed (here $n = 10 < 30$, so we need this assumption o/w we will not be able to apply the test).

ii. popⁿ variance is unknown.

iii. Sample is a SRS.

Test. Here as popⁿ variance (or popⁿ. std. deviation) is unknown. Thus we can't use Z-score here.

When σ or σ^2 is unknown but s or s^2 (i.e. sample std. deviation or sample variance) is known then we use t-statistic which is given by,

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{17.7 - 16.3}{1.8/\sqrt{10}}$$

$$= 2.4596$$

The test statistic 't' follows t-distribution with (n-1) degrees of freedom, i.e.,

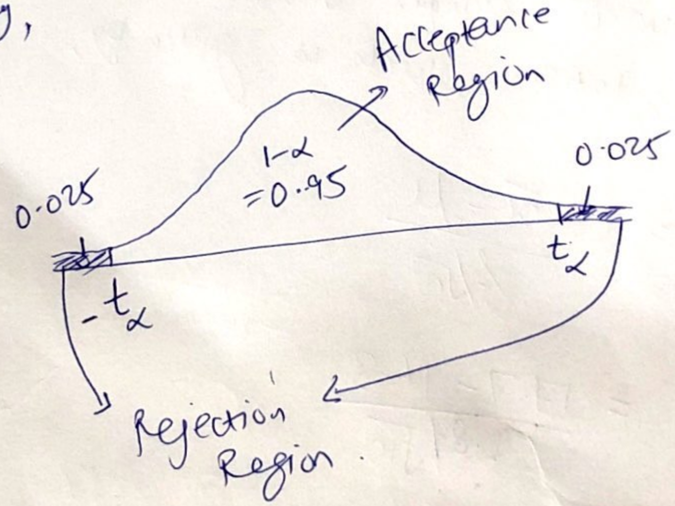
$$t \sim t_{(n-1)}$$

So here $t \sim t_9$.

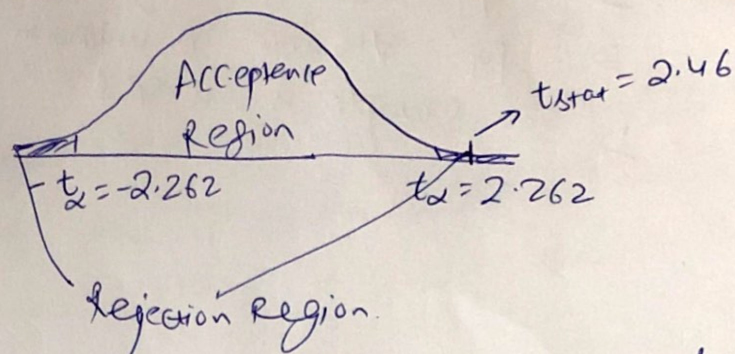
Here like question 1 the test is 2-tailed. The rejection region will be formed in a similar way as shown in Ques. 1. but remember here the dist^n is changed. (In Ques 1 it was std. normal but here it is t-dist^n.)

A t-dist^n is similar to std. normal. It is also symmetric about zero. The main difference is that the tails of t-distribution are heavier

Thus, the rejection region of t-dist^n will be graphically,



We have to find C.V. Here because the dist^n is t so we will use t-tables (~~attached~~ provided on google classroom). From the t-table, $|t_\alpha| = 2.2621$. (look under the column degrees of freedom $=9$, 2-tailed test of 5%). Thus, the critical values are $-t_\alpha = -2.262$ & $t_\alpha = 2.262$. The graphical representation is



$t_{stat} = 2.4586 \simeq 2.46$, clearly lies in the ~~acceptance~~ rejection region. Thus we will ~~not~~ reject H_0.

Inference: $|t_{stat}| > |t_{cv}|$. Thus $|t_{stat}|$ lies in the ~~acceptance~~ rejection region and at $\alpha = 0.05$ we ~~fail to~~ will reject H_0. At $\alpha = 0.05$, we have enough evidence to reject the claim that avg. # of infections per week at the hospital is 16.3

④. $n = 15$; $\bar{x} = 40.6$ (ml/kg) ; $s = 6$ ml/kg

$\alpha = \cancel{0.05}\ 0.10$

$H_0: \mu = 36.7$ v/s $H_1: \mu > 36.7$

(Note, here the units ml/kg are used everywhere. In such questions pay attention, sometimes you might have to convert the units so that the units are same throughout the analysis).

Edit: pop^n is normal.

Assumptions. i Pop^n is normally distributed
ii Pop^n variance is unknown
iii Sample is a SRS.

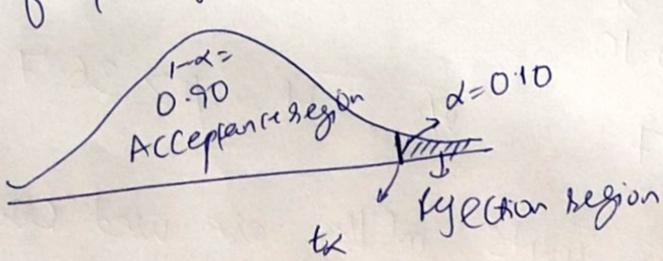
Test. This Ques is similar to Ques 3, but this is a One-Sided test. The t-statistic will be,

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

$$= \frac{40.6 - 36.7}{6/\sqrt{15}}$$

$$= 2.5174 \simeq 2.517$$

Here, the t-statistic will follow t_{14}. The graphical representation of finding C.V is



(Rejection Region will be on the right side as it is a right-tailed test.)

From the table $t_{0.10}$ for 1-tailed test will be less than 1.761. (Infact it is 1.345 using software, you can also refer to other t-tables).
So $t_{stat} > t_\alpha$ and thus t_{stat} will lie in the rejection region. Thus we will reject H_0 at $\alpha = 0.10$.

Inference $t_{stat} > t_{\alpha}$ (or $t_{stat} > t_{c.v.}$). Thus t_{stat} lies in rejection region and at $\alpha = 0.10$ we reject H_0.
The sample supports the physician's claim that joggers' maximal volume oxygen uptake is greater than 36.7 ml/kg.

[redacted] 5 Given Here we have 2 sample information.

Sample 1: College A

$n_1 = 11$

$\bar{x}_1 = 4$

$s_1 = 1.5$

Sample 2: College B.

$n_2 = 9$

$\bar{x}_2 = 3.5$

$s_2 = 1$

$\alpha = 0.01$

$$\boxed{H_0: \mu_1 = \mu_2 \quad V/s \quad H_1: \mu_1 > \mu_2}$$

Assumptions i There is no information on pop^n variances (or pop^n std. deviation so we can assume that σ_1, σ_2 are unknown and it is safe to assume that they are not equal.

ii The colleges are 2 different colleges in study, and the sample is taken from each individually so we can assume they are independent of each other.

iii further, we can assume that the samples (both of them) are SRS.

iv. it is given that the samples came from normal pop^n.

Test. Based on the information provided and assumptions it is clear that is a 2-sample test about mean. The samples are independent of each other and the pop^n std. dev.s (or pop^n. variances) are unknown. The test statistic used in this case will be

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t_{d.f.}$$

$d.f. =$ smaller of $(n_1 - 1), (n_2 - 1)$

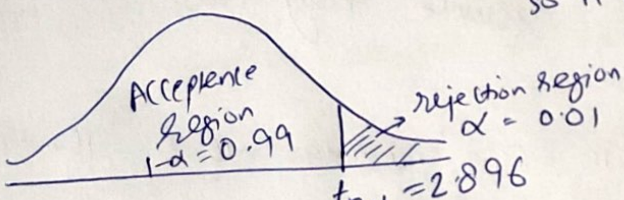
$$t = \frac{(4-3.5)-0}{\sqrt{\frac{(1.5)^2}{11}+\frac{(1)^2}{9}}} \qquad (\because (\mu_1 - \mu_2) = 0)$$

$$= 0.8899$$

$(n_1 - 1) = 10$; $(n_2 - 1) = 8$ Thus we have to look for $t_{8, \alpha = 0.01}$. Recall, $H_1: \mu_1 > \mu_2$ thus it is a 1-sided test, infact it is a right-tailed test.

Graph will be,

(As $H_0: \mu_1 = \mu_2 \Rightarrow H_0: \mu_1 - \mu_2 = 0$
& $H_1: \mu_1 > \mu_2 \Rightarrow H_1: \mu_1 - \mu_2 > 0$
So it is a right-tailed test).



t_{stat} lies in the acceptence region. Thus at $\alpha = 0.01$ we do not reject H_0.

Inference. At $\alpha = 0.01$ the $t_{stat} < t_{c.v}$ and lies in the acceptence region thus H_0 is not rejected. There is not enough evidence to accept the claim that the students who graduates from college A, on average attend more classes than that of students of college B.

[redacted]. 6 This question is similar to Ques 6. The difference here is that popⁿ std. deviation is given.

<u>Given</u>

Sample 1 : wax 1

$n_1 = 20$

$\bar{x}_1 = 3$

$\sigma_1 = 0.33$

Sample 2 : wax 2

$n_2 = 20$

$\bar{x}_2 = 2.9$

$\sigma_2 = 0.36$

$H_0: \mu_1 \leq \mu_2$ v/s $H_1: \mu_1 > \mu_2$

<u>Assumptions</u>. ~~Given~~

i. The pop^n std. deviations ~~are~~ of 2 ~~sample~~ pop^n are known and they are unequal.

ii) the 2 samples come from 2 different pop^{n}s and we can assume independency.

iii) They are random sample so we ~~can~~ can assume that we have SRS.

iv. it is given that the pop^{n}s follow normal distribution.

<u>Test</u>. This a test of 2 independent samples and known but different pop^n std. deviations.

The test statistic will be.

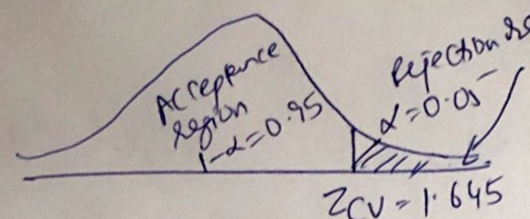
$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

$$= \frac{(3 - 2.9) - 0}{\sqrt{\frac{(0.33)^2}{20} + \frac{(0.36)^2}{20}}}$$

$$= 0.9157$$

$\Rightarrow Z_{stat} = 0.9157$

The graph will be.



(it is a right. tailed test like Ques 6)

$Z_{stat} < Z_{CV}$ lies in the acceptance region. Thus at $\alpha = 0.05$ we don't reject H_0.

<u>Inference</u> At 5% level of significance (or at $\alpha = 0.05$) the $Z_{stat} < Z_{C.V.}$ and lies in the acceptance region thus H_0 is not rejected. Based on the data, we do not have enough evidence to accept the claim that mean lasting time of wax 1 is more than that of wax 2.

[redacted]. 7 Here, the sample is taken ~~of~~ the same subjects. This type of data is called paired data. We are interested in studying before and after impact of hypotism. The sample is paired data so the within subject independency doesnot hold true but between subjects independency is required.

Given $n = 8$; $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i = \frac{1}{n}\sum_{i=1}^{n}(\text{after}_i - \text{before}_i)$

(Can be calculated from the data available).

S_d : Can be calculated

$\alpha = 0.05$ ~~$H_0: \mu_1 = \mu_2$ v/s $H_1: \mu_1 \neq \mu_2$~~

Assumptions i Random sample so we can assume SRS.

ii Pop^n comes from normal dist^n. (it is given)

iii Sample data are dependent.

Test Here we have to test if there is impact of hypotism which is captured by μ_d.

So $H_0: \mu_d \geq 0$ v/s $H_1: \mu_d < 0$

The test statistic will be

$$t = \frac{\bar{d} - \mu_d}{S_d/\sqrt{n}} \sim t_{(n-1)}$$

Now,
from the data.

d_i = after$_i$ - before$_i$

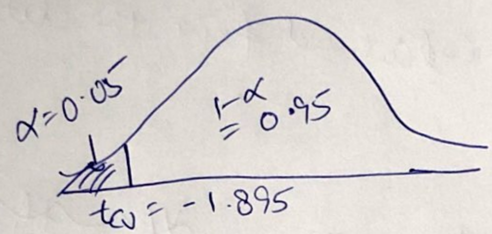
i	1	2	3	4	5	6	7	8
d_i	0.2	-4.1	-1.6	-1.8	-3.2	-2	-2.9	-9.6

$\Rightarrow \bar{d} = -3.13$; $s_d = 2.91$

$$\Rightarrow t = \frac{-3.13 - 0}{2.91/\sqrt{8}} = -3.0423$$

$t_{stat} = -3.0423$

By graph



$t_{C.V.} = -1.894$ (look in t-table 1-sided test for $\alpha = 0.05$ & $df = 7$).

Thus at $\alpha = 0.05$ we will reject Ho as t_{stat} lies in the rejection region.

Inference. At 5% level of significance, $t_{stat} < t_{cv}$ and lies in the rejection region thus Ho is rejected. Based on the data we can conclude that ~~hypnt~~ hypnotism results in lowering the pain (on avg.), thus is effective.