1. (a) Consider the weighted-A* algorithm which assigns a certain weightage to the heuristic cost in the conventional formula of A*.

   For which value of the weight A* does not guarantee optimal solution ? Which property is violated in this cas ? 1+1

   Solution: For weight > 1 weighted-A* does not guarantee optimal solution. The *admissibility* property is violated in this case.

   (b) Suppose there is an object A moving at a *variable speed* in a straight line. That is, due to certain factors A's speed is changing frequently. Suppose you have a mechanism to *track* A's position at any time point.

   You have a mobile robot B which *you want to program* in such a way that B wants to meet A as soon as possible. The final meeting point l and meeting time t of A and B is unknown a priori. Suggest an algorithm to program B in a way that it calculates the meeting point and time despite the variable speed of A. 4

   Solution: Suppose the current time is t and planning horizon length is l. B will iteratively run *limited horizon A** search until it meets A. In the $i+1$-th execution of the algorithm, i.e. execution triggered at time $t + i.l$, B uses A* algorithm to find the optimal path to B's current location and move accordingly.

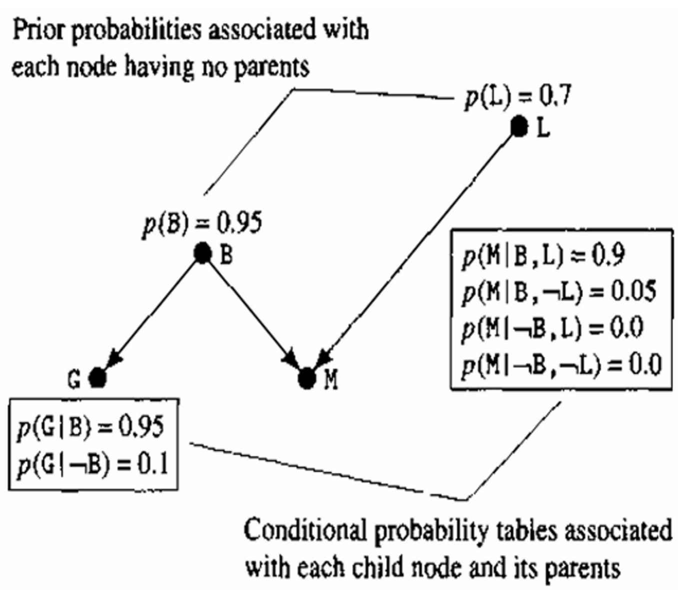

Figure 1: A Bayes network

2. (a) Consider four *boolean* random variables A, B, C, D. If any two of the variables are conditionally dependent, write the chain rule for calculating $p(A, B, C, D)$. How does the *value* change for calculating $p(B, A, D, C)$? (Answer without writing the chain rule again.) Here, p(.) refers to the probability function. 1+1

Solution:

- $p(A, B, C, D) = p(A \mid B, C, D) \cdot p(B \mid C, D) \cdot p(C \mid D) \cdot p(D)$
- The value remains *unchanged* when the order of the order of the variables is changed.

– No marks if anything else is written.

(b) Refer to Figure 1 for a battery-driven robot lifting an object. The four boolean variables are B (battery is charged), L (object is liftable), M (robot is moving) and G (Gauge indicates battery is charged).

Any probability value can be calculated using the probabilities shown in Figure 1. Mention the conditional independencies ? In this example, what is the *reduction* in probability specification due to the conditional independencies? What is the *probability of the object being liftable* given that the robot does not move ? 1+1+3

Solution: Following are the conditional independencies:

- G: conditionally independent of M given B.
- G: conditionally independent of L given $\emptyset$.
- B: conditionally independent of L given $\emptyset$.
- M: conditionally independent of G given B.
- L: conditionally independent of B given $\emptyset$.
- L: conditionally independent of G given $\emptyset$.

– Give full marks if any two of the above is written.

- Due to the conditional independencies, the reduction in probability specification is $2^4 - 8 = 8$ (there are 4 variables, and 8 probability specifications are given in Figure 1).

prob. of the object being liftable given that the robot does not move : $p(L|\neg M)$

$$p(L|\neg M) = \frac{p(\neg M|L)\, p(L)}{p(\neg M)} \quad \text{(1)}$$

$p(L) \rightarrow$ Known

$p(\neg M) \rightarrow$ Unknown

$p(\neg M|L)$: unknown

Let's find $p(\neg M|L)$.

$$p(\neg M|L) = p(\neg M, B|L) + p(\neg M, \neg B|L)$$
$$= p(\neg M|B, L)\, p(B|L) + p(\neg M|\neg B, L) \cdot p(\neg B|L)$$

We can see that $p(B|L) = p(B)$ and $p(\neg B|L) = p(\neg B)$ as B and L are independent.

So, $p(\neg M|L) = 0.1 \times 0.95 + 1 \times 0.05 = 0.1$

$$\therefore p(L|\neg M) = \frac{0.1 \times 0.7}{p(\neg M)} = \frac{0.07}{p(\neg M)} \rightarrow \text{(1a)}$$

$\rightarrow$ We still don't know $p(\neg M)$ value

We know $p(L|\neg M) + p(\neg L|\neg M) = 1$

Similarly we find $p(\neg L|\neg M)$.

$$p(\neg L|\neg M) = \frac{p(\neg M|\neg L) \cdot p(\neg L)}{p(\neg M)} \rightarrow \text{(2)}$$

$$p(\neg M|\neg L) = p(\neg M, B|\neg L) + p(\neg M, \neg B|\neg L)$$
$$= p(\neg M|B, \neg L) \cdot p(B|\neg L) + p(\neg M|\neg B, \neg L) \cdot p(\neg B|\neg L)$$

Same independence applies as before,

$$= 0.95 \times 0.95 + 1 \times 0.05 = 0.95$$

From (2), $p(\neg L|\neg M) = \dfrac{0.95 \times 0.3}{p(\neg M)} = \dfrac{0.285}{p(\neg M)}$ — (2a)

From (1a) and (2a), $p(\neg M) = 0.355$

Therefore, $p(L|\neg M) = \dfrac{0.07}{p(\neg M)}$ (from 1a) $= 0.2$ (approx)

– 1.5 marks if solved up to **1a**.

3. (a) In an MDP, write the expression for value function with time horizon T. 1

Solution:

The value function for time horizon T is as follows:

$$V_T(x) = \gamma \max_u \left[r(x,u) + \int V_{T-1}(x')p(x' \mid x,u) \right]$$

where x, x', u, $r(.)$ and *gamma* are the current state, next state, control action payoff function and discount factor respectively.

(b) Consider a partially observable system with two states (x_1, x_2) and three actions u_1, u_2, u_3. The belief b is $b(x_1) = p_1$ and $b(x_2) = p_2$. Suppose the reward is as follows: $r(b,u_1) = -10p_1 + 10p_2$, $r(b,u_2) = 10p_1 - 5p_2$ and $r(b,u_3) = -100$. Calculate $V_1(b)$ to choose the best action before getting any measurement (or observation).

Also, consider there are two observations and the observation model is: $p(z_1 \mid x_1) = 0.9$, $p(z_1 \mid x_2) = 0.3$. What is the expected value of $V_1(b \mid z)$ before getting any observation ? 4

Solution: See next page.

3.(b) We need to calculate the optimal value function $V_1(b)$ for $T=1$.

$$V_1(b) = \max_u r(b,u)$$

$$= \max \{ r(b,u_1), r(b,u_2), r(b,u_3) \}$$

$$= \max \begin{Bmatrix} -10p_1 + 10p_2 \\ -10p_1 - 5p_2 \\ -100 \end{Bmatrix}$$

for any value of p_1 and p_2, -100 will be lesser than the other two values. So,

$$V_1(b) = \max \begin{Bmatrix} -10p_1 + 10p_2 \\ -10p_1 - 5p_2 \end{Bmatrix}$$

- So far, V_1 is calculated without considering the observation.
- Observation will surely provide more information about the state, which we see next.

After observation, the belief should change. But, we don't know what was the observation actually.

So, if it was z_1, we have

new belief
$$\left\{ \begin{aligned} p(x_1|z_1) &= \frac{p(z_1|x_1) \cdot p(x_1)}{p(z_1)} \\ &= \frac{0.9\,p_1}{p(z_1)} \\ p(x_2|z_1) &= \frac{0.3\,(1-p_1)}{p(z_1)} \end{aligned} \right.$$

Now, we have to find out the V_1 value if observation for z_1.

Refer to (1) and find the following

$$V_1(b \mid z_1) = \max \begin{cases} -10 \dfrac{0.9 p_1}{P(z_1)} + 10 \dfrac{0.3(1-p_1)}{P(z_1)} \\ -10 \dfrac{0.9 p_1}{P(z_1)} - 5 \dfrac{0.3(1-p_1)}{P(z_1)} \end{cases}$$

$$= \frac{1}{P(z_1)} \max \begin{cases} -9 p_1 + 3(1-p_1) \\ -9 p_1 - 1.5(1-p_1) \end{cases}$$

If observation was z_2, similarly we get

$$V_1(b \mid z_2) = \frac{1}{P(z_2)} \max \begin{cases} -1 \cdot p_1 + 7(1-p_1) \\ -1 \cdot p_1 - 3.5(1-p_1) \end{cases}.$$

$$\therefore E_z\left[V_1(b \mid z)\right] = P(z_1) \cdot V_1(b \mid z_1) + P(z_2) \cdot V_1(b \mid z_2)$$

$$= \max \begin{Bmatrix} -9 p_1 + 3(1-p_1) \\ -9 p_1 - 1.5(1-p_1) \end{Bmatrix} + \max \begin{Bmatrix} -1 \cdot p_1 + 7(1-p_1) \\ -1 \cdot p_1 - 3.5(1-p_1) \end{Bmatrix}$$

– Deduct 2 marks for correct calculation of $V_1(b)$ [i.e. observation factor is not considered].

4. Suppose you are given a finite set of n data points. You are asked to design a k-NN regressor. Suggest a way to find the optimal value of k. 2

   Solution:

   - Plot error rates for various k values.
   - Look for the k value where the performance improves drastically before leveling off.

   – Give marks for other answers which are logically correct.