Game Theory: Assignment 4 (Solutions)

Total points: 25 | Due Date: 5/12/2022

Contribution to grade: 10% (3xx); 7.5%(5xx) | Due time: 11:59 PM

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1. *Market for Lemons.* In the framework of PBE and BNE, we talked about "types" of players.

   (a) Explain using the "Market for Lemons" example discussed in class, how private information favours the "low type" and hurts the "high type". (2.5)

   Answer. This is simple. In the market for lemons problem, the lack of information means that low type is always traded. However, high type cars are only traded if the probability of high type cars are high enough (following the example from Watson). In that example, at times, it is even theoretically possible that a low type gets traded for a high price if q is high enough.

   (b) Can you modify one of the parameters in the Market for Lemons game discussed in class to ensure that it does not lead to market failure? (2.5)

   Answer. This is very easy. All we need is that the valuation of the seller of the high type exceeds how much the buyer values the high type. Then, trade for high type should not happen anyway (even in full information cases) for high type cars, and there is no market failure.

2. *Perfect Bayesian Equilibrium.* Answer the following:

   (a) What is a separating equilibrium, and how can it be useful for policy makers? (2.5)

   Answer. A separating equilibrium is a PBE in which different types take different actions (thus revealing their type by virtue of their action). This can be very useful for policy makers in cases where the policy maker wants the each type to take a specific decision, eg., they may want an environmentally friendly firm to produce, but a polluting firm to not produce, or efficient mergers to exist, but inefficient mergers to not exist.

(b) Generally speaking, if there is one uninformed player and one informed player (two possible types types), which type does the pooling equilibrium tend to favour? Why? (2.5)

Answer. A pooling equilibrium is likely to favour the low type from the informed player. (Just a short explanation will do).

(c) We saw that the updated belief q was either the same as the prior p, or 0, or 1. Obviously, this is not always the case. When will this not be the case? (2.5)

Answer. When we have a mixed strategy equilibrium in PBE, also known as semi separating PBE. (short explanation needed)

(d) In addition to sequential rationality in SPNE, PBE has another layer of sequential rationality. What is it? (2.5)

(e) Players also update their beliefs about the opponent's type, or any private information based on the actions they observe.

3. *Cournot competition.* Consider a setting in which two firms compete in quantities. They have symmetric marginal costs, $c = 1$, but they have asymmetric information about market demand. Firm 2 does not know the state of market demand, but think is it is $p(Q) = 10 - Q$ with probability 0.5 and $p(Q) = 5 - Q$ with probability 0.5. Firm 1 has full information. Find the BNE. (5)

Note: Here, by high type we always mean the type that is favourable for the uninformed player, for example, friend, owner of a good car, a skilled worker etc.

Answer to Question 3:

Let us consider an oligopoly game where two firms compete in quantities. Both firms have the same marginal costs, MC = \$1, but they are asymmetrically informed about the actual state of market demand. In particular, Firm 2 does not know what is the actual state of demand, but knows that it is distributed with the following probability distribution:

$p(Q) = (10 - Q)\ with\ prob. 1/2\ \&\ (5 - Q)\ with\ prob. 1/2$

On the other hand, firm 1 knows the actual state of market demand, and firm 2 knows that firm 1 knows this information (i.e., it is common knowledge among the players)

Firm 1.

First, let us focus on Firm 1, the informed player in this game, as we usually do when solving for the BNE of games of incomplete information. When firm 1 observes a high demand market its profits are:

$$Profits_1 = 10q_1^H - {q_1^H}^2 - q_2 q_1^H - 1.q_1^H$$

Differentiating with respect to q_1^H, we can obtain firm 1's best response function when experiencing low costs:

$$10 - 2q_1^H - q_2 - 1 = 0 \Longrightarrow q_1^H(q_2) = 4.5 - q_2/2$$

On the other hand, when firm 1 observes a low demand market, its profits are:

$$Profits_1 = (5 - q_1^L - q_2)q_1^L - q_1^L$$

Differentiating with respect to q_1^L, we can obtain firm 1's best response function when experiencing high costs:

$$5 - 2q_1^L - q_2 - 1 = 0 \Longrightarrow q_1^L(q_2) = 2 - q_2/2$$

Firm 2.

Let us now analyze Firm 2 (the uninformed player in this game). First note that its profits must be expressed in expected terms, since firm 2 does not know whether market demand is high or low.

$$Profits_2 = 1/2[(10 - q_1^H - q_2)q_2 - 1.q_2] + 1/2[(5 - q_1^L - q_2)q_2 - 1.q_2]$$

Rearranging and differentiating with respect to q_2 helps us obtain firm 2's best response function:

$$1/2[(10 - q_1^H - 2q_2) - 1] + 1/2[(5 - q_1^L - 2q_2) - 1] \Longrightarrow\ q_2(q_1^H, q_1^L) = \frac{13 - q_1^H - q_1^L}{4}$$

After finding the best response functions for both types of Firm 1 and for the unique type of Firm 2 we are ready to plug the first two BRFs into the latter:

$$q_2 = 3.25 - 1.625 + 0.25q_2 \Rightarrow q_2 = 2.167$$

With this information, it is easy to find the particular level of production for firm 1 when experiencing low market demand:

$$q_1^L(q_2) = 2 - \left(\frac{2.167}{2}\right) = 0.916$$

And, high market demand:

$$q_1^H(q_2) = 4.5 - \left(\frac{2.167}{2}\right) = 3.417$$

Therefore, the Bayesian Nash equilibrium (BNE) of this oligopoly game with incomplete information about market demand prescribes the following production levels:

(q_1^H, q_1^L, q_2)=(3.42, 0.92, 2.17)