

Set-2

1. Choose True/False for the following statements.

$1 \times 13 = 13$

- (a) Given a heuristic function, IDA* expands a node deterministically whereas Limited-Horizon search expands a node probabilistically. **True**
- (b) In the Minimax algorithm, the opponent may not play its best strategy. **False**
- (c) Depth-limited search guarantees optimal solution. **False**
- (d) Iterative deepening search, which calls DFS with increasing depth limits until a goal is found, *is complete*. **True**
- (e) For a given graph, the number of expanded nodes is higher in BFS compared to Depth-limited search. **True**
- (f) An Approximate search may not always lead to a goal state. **True**
- (g) If the consistency condition on h is satisfied, then if A^* expands a node n , it has already found an optimal path to n . **True**
- (h) In a Bayes network, if Z is an effect of the causes X and Y , then X and Y are dependent. **False**
- (i) At a given level of Alpha-beta pruning, we update either the upper or lower bound of reward. **False**
- (j) Probabilistic inference extends *Modus tollens* to probabilistic setting. **True**
- (k) If A and B are propositional variables, then $p(\neg A, B) + p(A, B) = 1$. **False**
- (l) If two random variables are unconditionally independent then they are conditionally independent. **False**
- (m) A drawback of Bayes network is it cannot reason the occurrence of a cause given an effect. **False**

2. Consider a two-player sequential game as follows:

- Player-1 is a maximizing player, Player-2 is minimizing.
- Possible actions of player-1: $\{a_i^1\}_{i=1}^2$ (superscript: player, subscript: action),
- Possible actions of player-2: $\{a_j^2\}_{j=1}^3$,
- Player 1 needs to choose its strategy by analyzing one move of herself followed by a move by player-2.
- Reward of player-1 after one move of player-1 herself followed by one move of player-2, (a_i^1, a_j^2) , is defined as $R_{i,j}^1 := 10 - i - j$.

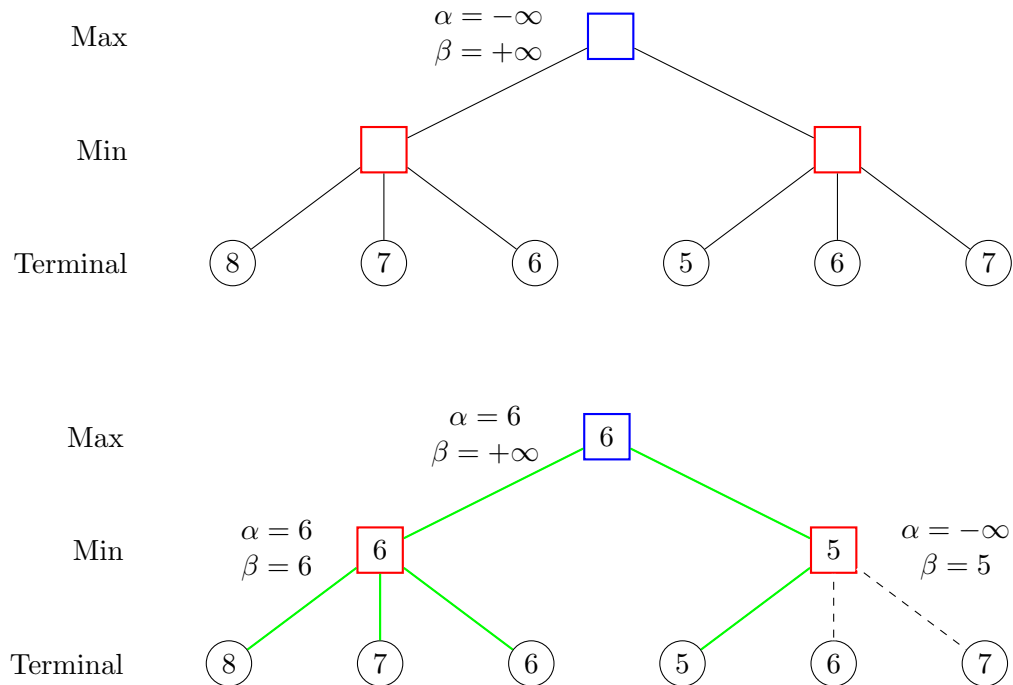
Draw the game tree. Show the steps in Alpha-beta pruning to find the best strategy for player-1. 1+4

Solution:

Game Tree

The initial game tree is as follows. The root node is Player 1's decision, then Player 2 moves, leading to terminal payoffs.

Step 1: Initial Tree (Initialise $\alpha = -\infty, \beta = +\infty$)



Explanation of terminal payoffs:

$$R_{1,1}^1 = 10 - 1 - 1 = 8, \quad R_{1,2}^1 = 7, \quad R_{1,3}^1 = 6,$$

$$R_{2,1}^1 = 7, \quad R_{2,2}^1 = 6, \quad R_{2,3}^1 = 5.$$

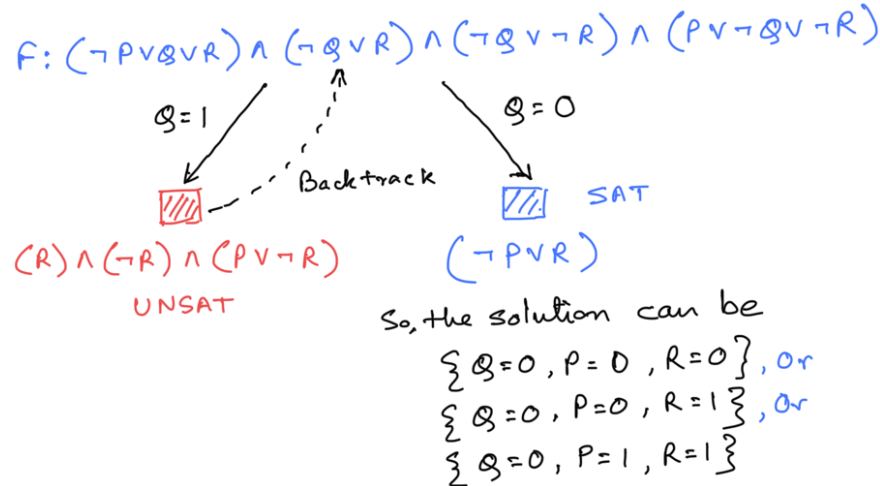
The optimal strategy for Player 1 is to choose action a_1^1 , which guarantees her a payoff of 6 under optimal play by Player 2.

- Full marks if Game tree is correct, intermediate step is correct and final answer is correct.
- Full marks for any order of the nodes $\{6, 7, 8\}$ and $\{5, 6, 7\}$.
- In case Game tree is incorrect, but the intermediate steps are as per alpha-beta pruning, give partial marks.
- If final answer is wrong, deduct 2 marks.

3. (a) Show the steps in DPLL method for solving the formula:
 $F : (\neg P \vee Q \vee R) \wedge (\neg Q \vee R) \wedge (\neg Q \vee \neg R) \wedge (P \vee \neg Q \vee \neg R)$. You have to begin with $Q = \text{true}$. 3
- (b) Write a mathematical expression of an *admissible* heuristic function. 1

Solution:

- (a) The solution is as follows:



- This question tests whether the student knows the backtracking and clause elimination based on partial assignment of literal values of the DPLL algorithm.
 - One of the above answers is sufficient to get **full marks**.
 - If it does not begin with $Q = 0$, give him/her minimum marks.
- (b) **Admissible heuristic:** An admissible heuristic function is an estimation of the cost, from a current state to the goal state, *that never overestimates the actual cost*. Thus, $h(n) \leq h^*(n)$ where $h(n)$ is the estimated cost from state n to the goal, and $h^*(n)$ is the optimal cost from state n to the goal state.
- Full marks only if the mathematical expression is correct and the notations are clearly mentioned.
4. Suppose n vehicles occupy squares $(1,1)$ through $(n,1)$ (i.e. the bottom row) of an $n \times n$ grid. The vehicles must be moved in a way that vehicle i that starts in $(i,1)$ ends up in $(n-i+1,n)$. On each time step, each of the vehicles can move one square up, down, left, or right, or no-movement. Two vehicles cannot occupy the same square. Suppose for any vehicle i , h_i is the Euclidean distance.

Which of the following heuristics are admissible *for the problem of moving all n vehicles to their destinations*? Explain your answer *mathematically*:

- (i) $\sum_{i=1}^n h_i$ and (ii) $\max\{h_1, \dots, h_n\}$. ? 2.5 + 2.5

Solution: A heuristic function $h(m)$ is admissible iff $h(m) \leq h^*(m)$, where $h(m)$ is the estimated cost from state m to goal state, and $h^*(m)$ is the optimal cost from state m to goal state.

- (i) Let $H_1^n = \sum_{i=1}^n h_i$ be the heuristic. We have to prove the admissibility of H_1^n .
For any state, the following holds: $h_i \leq h_i^*$ for any $i = 1, \dots, n$ where h_i^* is the optimal cost from that state to the goal. This is because, from a given state, the Euclidean distance (h_i) is less than or equal to the actual distance (h_i^*) to reach the goal. This implies, $\sum_{i=1}^n h_i \leq \sum_{i=1}^n h_i^*$. That is, $H_1^n \leq H_1^{n*}$. Hence, admissibility of H_1^n is proved.
- (ii) From any state, $\max(h_1, \dots, h_n)$ is less than or equal to the corresponding optimal cost for reaching the goal state. That is, $\max(h_1, \dots, h_n) = h_j \leq h_j^*$ for any state. Hence, admissibility of the given heuristic is proved.

- For each question, 2 for reasoning and proof.
- For each question, 0.5 for concluding admissible.

5. Refer to Figure-1. Adam has installed an alarm that detects burglary and also responds to earthquakes. He has two humanoid robots, John and Mary, who are supposed to call Adam when they hear the alarm. But, they sometimes sense the alarm wrongly and miss to call Adam.

Given that earthquake happens and burglary does not happen, what is the probability of John not calling Adam ? 4

Solution: Given that earthquake(E) happens and burglary(B) does not happen, we have to find the probability of John not calling Adam(J), i.e., $P(\neg J \mid E, \neg B)$.

- (a) Rewrite as *joint probabilities* of $\neg J$ and its parent A (alarm):

$$P(\neg J \mid E, \neg B) = P(\neg J, \neg A \mid E, \neg B) + P(\neg J, A \mid E, \neg B) \quad (1)$$

- (b) Expand Eq. (1) using *Chain rule*:

$$\begin{aligned} P(\neg J \mid E, \neg B) &= P(\neg J, \neg A \mid E, \neg B) + P(\neg J, A \mid E, \neg B) \\ &= P(\neg J \mid \neg A, E, \neg B) \cdot P(\neg A \mid E, \neg B) + P(\neg J \mid A, E, \neg B) \cdot P(A \mid E, \neg B) \end{aligned} \quad (2)$$

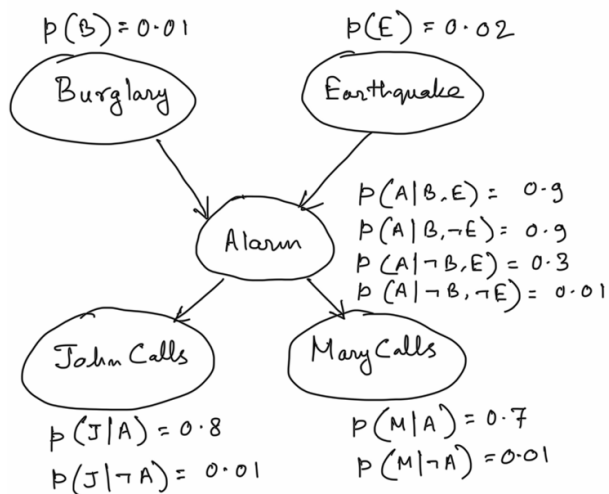


Figure 1

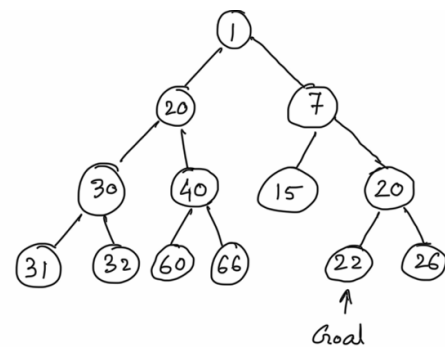


Figure 2

(c) Apply *conditional independence* to Eq. (2):

$$\begin{aligned}
 P(\neg J \mid E, \neg B) &= P(\neg J \mid \neg A, E, \neg B) \cdot P(\neg A \mid E, \neg B) + P(\neg J \mid A, E, \neg B) \cdot P(A \mid E, \neg B) \\
 &= P(\neg J \mid \neg A) \cdot P(\neg A \mid E, \neg B) + P(\neg J \mid A) \cdot P(A \mid E, \neg B) \\
 &= 0.99 \times 0.7 + 0.2 \times 0.3 \\
 &= 0.75 \text{ (approx.)}
 \end{aligned}$$

(3)

- 1 mark for writing Eq. (1) correctly. It tests concept of joint probability.
- 1 mark for writing Eq. (2) correctly. It tests concept of chain rule.
- 2 marks for writing Eq. (3) correctly. It tests concepts of conditional probability and Bayes net.
- If the above three equations are written correctly, give full marks even though the final value is not calculated correctly.

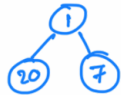
6. Refer to Figure-2. Consider that each node indicates the f -heuristic value for reaching the goal as indicated in the figure. Show the steps of IDA* to find the goal from the root node. Mention the set of nodes that are not expanded. 4

Solution:

① Start node: 1
Visited: $\{1\}$



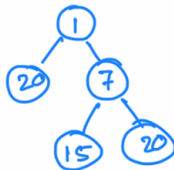
② Threshold: 1
Visited: $\{20, 7\}$



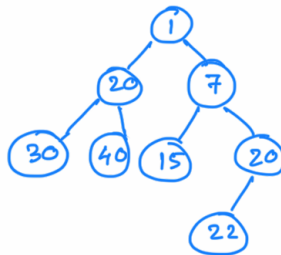
③ Threshold: 7
Visited: $\{15, 20\}$



④ Threshold: 15
Visited: $\{20\}$



⑤ Threshold: 20
Visited: $\{30, 40, 22\}$



* Set of unexpanded nodes: $\{30, 40, 22, 26, 31, 32, 60, 66\}$

- Full marks if the set of unexpanded nodes are correct and the steps are shown.
- Full marks even if unexpanded nodes are shown as $\{26, 31, 32, 60, 66, \}$.
- No marks if IDA* procedure is executed wrongly.
- Partial marks if both the nodes with 20 are not expanded.