Game Theory: Mid Semester Exam

Total points: 30 | Date: 16/10/2022

Contribution to grade: 30% (3xx); 20% (5xx) | Time: 10:00 - 11:00 AM

- Show all steps, as it can help you get partial credit.
- For Part I, do 4/5 questions. Each question is worth 2.5 marks. If you submit all 5, the first 4 you did will be graded.
- For Part I, do 2/3 questions. Each question is worth 10 marks. If you submit all 3, the first 2 you did will be graded.

Part I.

1. In a regular Bertrand Duopoly (2 firms and payoff 0 upon not selling), name a pure strategy, if any, that is strongly dominated.

   Answer. No pure strategy is strongly dominated. This is because no matter which two positive prices one chooses to compare, they can yield equal payoffs (of 0) the the rival prices lower. If we compare 0 to a positive price, both strategies can yield 0 as well, if the rival's price is 0 **(Explanation is must for marks, but less detail is okay.)**

   Note: However, there are plenty of weakly dominated strategies

2. In a simple 2×2 game (2 players, 2 actions each), if we allow for mixed strategies, what does the choice variable for each player become? Explain in one line. *Answer.* The problem then becomes the probability we want to assign to each action (or to simplify it further, any one action, since the other one will just be one minus that probability). **This answer only has one part, worth 2.5 marks.**

3. In strictly competitive games, the sum of payoffs for all players is always the same regardless of the outcome. True or false? Explain in 2 lines. *Answer.* This is not the case. That is what happens in constant sum games. It is true that constant sum games are always strictly competitive, but not vice-versa. **Any basic explanation will get 2.5 marks. Without explanation it is worth 1.5 marks.**

4. Two players i and j are playing a game in which their choice variables are a_i and a_j, where a_i and a_j have to be natural numbers. The best response functions are as follows:

   (a) $a_i(a_j) = 2 + \frac{a_j}{100}$.

   (b) $a_j(a_i) = 3 + 4a_i$.

   The above situation is a case of strategic __________. Fill in the blank and explain.

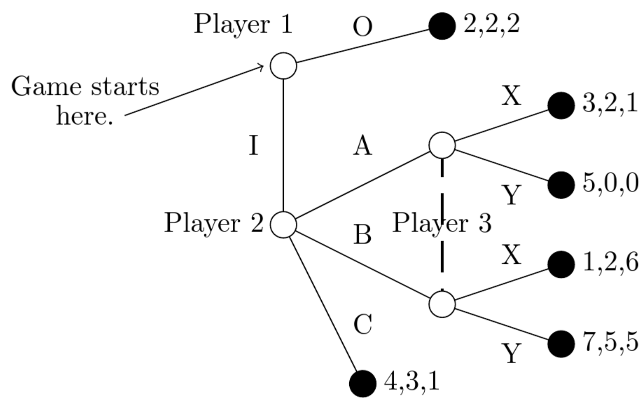
   Answer. The above is a case of strategic complements. **(1.5 marks)**.As one player's strategy increases/decreases, so does the others' best response **(1 mark)**.

5. SPNE have another property because of which we call it a "refinement" of Nash Equilibrium. What is this property and when is it important/valid?

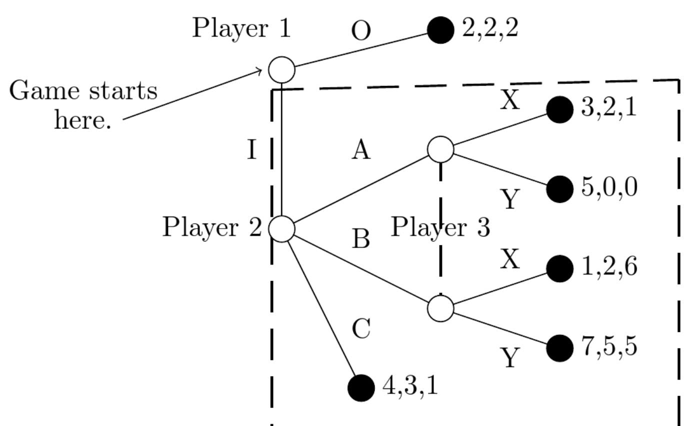
   Answer. The property is sequential rationality **(1.5 marks)**, and is only important if the game is a sequential game **(1 mark)**, at least in part.

Part II.

1. Find the SPNE of the following game.



Answer. Start with the smallest proper subgame. This is the game that starts at Player 2's node.



A simultaneous move game between Player 2 (actions A,B and C), and Player 3 (X and Y). A simple 3×2 table will yield the PSNE for that subgame to be (C,X).

			Player 3	
			X	Y
	A		2,1	0,0
Player 2	B		2,6	5,5
	C		3,1	3,1

This leads to payoff profile (4,3,1). Since Player 1 knows this by backward induction, he will opt for I (as O would give hi ma payoff of 2). Hence, SPNE is (I, C, X).

****Marking scheme: 2 marks for identifying the smallest proper subgame, 3 marks for table (can be written a bit differently, with all payoffs included, i.e., player 1's payoffs as well, as long as you do not involve player 1 in decision making), 3 marks for figuring out the NE of the subgame and 2 marks for the final SPNE. (This game is from Watson).**

****Alternately, if you find $\{A, X\}$ as NE in the simultaneous part of the game and then say player 2 prefer C, that is okay too. Arriving till here will fetch you 8 marks. 2 marks for the final SPNE.**

2. **Negative Advertising.** Two soft drink companies, P and CC can choose to run negative ads against each other during a national cricket league. Due to demand from other industries, the organizers have decided to allow a maximum of 2 hours of such advertising during the entire tournament. Given the pair of choices (a_P, a_{CC}), the payoff for firm $i \in \{P, CC\}$ is given by

$$v_i(a_i, a_j) = a_i - 2a_j + a_i a_j - (a_i)^2 \text{ where } j \neq i$$

Both firms make their choice simultaneously.

(a) Find the best response functions.

Answer. Each player maximizes $a_i - 2a_j + a_i a_j - (a_i)^2$ w.r.t. a_i, which gives 2 best response functions:

$$a_1(a_2) = \frac{1 + a_2}{2} \quad \text{and} \quad a_2(a_1) = \frac{1 + a_1}{2}$$

****1.5 marks for each best response function.**

(b) Find the optimal level of advertising for each firm.

Answer. Solving this simultaneously yields a Nash equilibrium of $a_1 = a_2 = 1$. This gives each party a payoff of $a_i - 2a_j + a_i a_j - (a_i)^2 = 1 - 2(1) + (1)(1) - (1)^2 = -1$. It is clear that negative advertising hurts both firms in Nash equilibrium.

****3 marks for correct plugging in of values, 1 mark for correct calculation.**

(c) If the companies could sign binding agreement on how much to campaign, what levels would they choose?

Answer. Since this is a symmetric game, a binding agreement would equally help both equally if $a_i = a_j = a$. This would yield an individual payoff of $a_i - 2a_j + a_i a_j - (a_i)^2 = a - 2a + (a)(a) - (a)^2 = -a$. Given that $a_i \geq 0$, this is minimized at $a = 0$. Thus, they should choose to not run negative ads. ****2 marks for finding $-a$ after plugging in, 1 mark for stating the optimal contract.**

3. Consider 10 firms, and each firm $i \in \{1, ..., 10\}$. $S_i = \{N, M\}$ $\forall i$. Suppose a total of n firms choose N and m firms choose M. Every firm chooses one of the two. Payoff from choosing N is $5n - n^2 + 50$. Payoff from choosing M is $48 - m$. Find the pure strategy Nash Equilibrium of the game. How many PSNE exist?

Answer. This is the easiest of all the problems. To know the NE, we need to find values for n and m such that there will be no incentive for any firm to deviate. This will happen when

$$5n - n^2 + 50 = 48 - m \ \textbf{(2.5 marks)} \Leftrightarrow 5n - n^2 + 50 = 48 - (10 - n) \ \textbf{(1 mark)}$$

$$\Leftrightarrow -n^2 + 4n + 12 = 0$$

$n = 6$ and $n = -2$ **(1.5 marks)**. Since -2 is not meaningful, we consider the positive result. Hence, NE is that 6 firms choose N and 4 choose M **(2 marks)**.

The total number of NE is given by ${}_{10}C_6 = {}_{10}C_4 = 210$ **(3 marks)**.