

Modern Algorithm Design : Quiz 4

Full Marks : 15

Time : 40 minutes

2/12/2024

Problem 1. (5 points) Recall that we had analyzed an algorithm MAJORITY in class for the $\{0, 1\}$ experts problem. We showed that the number of mistakes made by this algorithm is at most $\mathcal{O}(\log n)$, where n is the number of experts. Give an example to show that the analysis is tight - that is there is an instance such that MAJORITY will indeed make $\Omega(\log n)$ mistakes. Note that you need to be the adversary to fool the algorithm.

Problem 2. (5 points) Consider the bin packing problem done in lecture. Show that if all items are of size less than ε , then the First-Fit algorithm uses at most $(1 + \varepsilon)OPT$ many bins where OPT is the number of bins used by an optimal solution. (Hint: how much is each bin of First-Fit occupied?)

Problem 3. (5 points) Consider the $\{0, 1\}$ experts game again with n experts. Let us forget about all the smart algorithms we studied and instead just consider the following greedy algorithm. On day t , just choose the expert that has made the least number of cumulative mistakes so far and follow him/her. Show an example where this algorithm can make at least $n - 1$ times as many mistakes as the best expert.