Game Theory: Assignment 1

Total Points: 45 Due Date: 8/9/2021

Contribution to grade: 10% (3xx); 7.5%(5xx) Due Time: 11:59 PM

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1. From the following payoff matrix, show that when we delete weakly dominated strategies, the order affects the equilibrium outcome. **(6)**

		Player 2		
		a	b	c
Player 1	A	3,2	2,2	1,1
	B	2,3	4,2	3,2
	C	3,-1	2,0	0,0

Answer.

		Player 2		
		a	~~b~~	~~c~~
Player 1	A	3,2	~~2,2~~	~~1,1~~
	~~B~~	~~2,3~~	~~4,2~~	~~3,2~~
	~~C~~	~~3,-1~~	~~2,0~~	~~0,0~~

Order of elimination: Blue, Red Yellow Black.

		Player 2		
		a	b	~~c~~
Player 1	A	3,2	2,2	~~1,1~~
	B	2,3	4,2	~~3,2~~
	C	3,-1	2,0	~~0,0~~

Here, we can no longer eliminate anything more (note A and C do not weakly dominate each other as payoffs are identical). It is clear that we can no longer see the same equilibrium using IESDS/IEDS. Just to clarify, eliminating A or C is wrong.

2. **Bargaining game.** Consider a pizza with x slices. A parent wants to help 2 siblings share the pizza. The older sibling (the proposer) makes a take it or leave it offer to the younger sibling (the responder). She has 3 options: to offer $s = \left\{0, \frac{1}{2}x, x\right\}$. If the

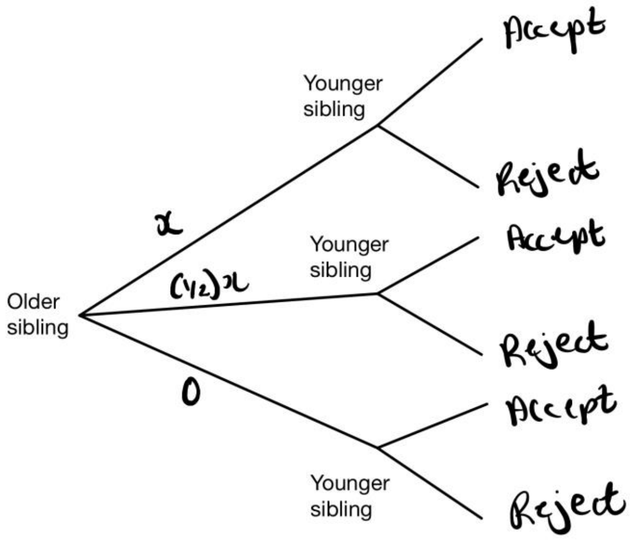
younger sibling accepts the offer, the older sibling gets $x - s$, and the younger sibling gets s. If the younger sibling rejects the offer, the parent confiscates the pizza to teach them a lesson in sharing. Remember that this is a sequential game.

a. Describe the strategy space for each player. **(2)**
Answer.

$$a_1 = \left\{0, \frac{1}{2}x, x\right\}, s_2 = \{AAA, AAR, ARR, RRR, RRA, RAA, ARA, RAR\}$$

Where A is accept and R is reject.

b. Draw the extensive form of the game. **(4)**
Answer.



c. Draw the normal form of the game. **(4)**
Answer.

		Younger Sibling							
		AAA	AAR	ARA	ARR	RAA	RAR	RRA	RRR
	x	$0, x$	$0, x$	$0, x$	$0, x$	0,0	0,0	0,0	0,0
Older Sibling	$\frac{1}{2}x$	$\frac{1}{2}x, \frac{1}{2}x$	$\frac{1}{2}x, \frac{1}{2}x$	0,0	0,0	$\frac{1}{2}x, \frac{1}{2}x$	$\frac{1}{2}x, \frac{1}{2}x$	0,0	0,0
	0	$x, 0$	0,0	$x, 0$	0,0	$x, 0$	0,0	$x, 0$	0,0

d. Does any player have strictly dominated pure strategies (answer in detail)? **(4)**
Answer.

The answer is no, but you have to explain why for each player. There are abundant weakly dominated strategies.

3. Consider the set of actions $x_i \in [0,1]$ available for player i where $i \in \{1,2\}$. Their payoff functions are given by v_i. Which of the following settings represent a game theoretic setting, and which are simply individual maximization problems? Explain.
   a. $v_1 = x_1^2 - x_2$ and $v_2 = x_2^2 - \frac{x_1}{2}$ **(2.5)**

This is a game theoretic setting, as the choices of each player affect the payoff of the other.

   b. $v_1 = x_1^3$ and $v_2 = \frac{x_2}{3000}$ **(2.5)**

   This is not a game theoretic setting, as the choices of each player do not affect the payoff of the other.

   c. $v_1 = x_1 - v_2$ and $v_2 = x_1$ **(2.5)**

Here, Player 1's payoff is not a function of player 2's choice, while player 2 does not have a choice at all to make. This is not game theoretic.

   d. $v_1 = x_2 + v_2$ and $v_2 = 31$ **(2.5)**

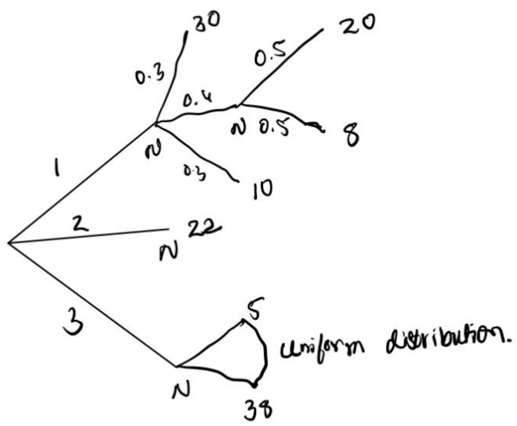
Here, player 1 does not have a choice to make, while player 2's choice does not affect his own payoff. Again, it is not game theoretic.

In cases c. and d., if you pointed out the confusion, it would be fine. But there is no game as one of the two players have no actions at all. This is a basic ingredient of a game.

4. Calculate the expected payoff of each lottery 1,2 and 3. Which one will you choose, assuming that you are risk neutral, i.e., your choices are based strictly on expectation (and not variance). **(10)**
   Note:
   a) The lottery 3 basically says that outcomes are distributed continuously and uniformly from 5 to 38.
   b) N means nature plays.
   c) In the first lottery, 0.3, 0.4 and 0.3 indicate probabilities.

1. Lottery one, expected payoff is $0.3 * 30 + 0.4[0.5 * 20 + 0.5 * 8] + 0.3 * 10$ =17.6.
2. Outcome of the second case is 22.
3. Expected payoff for third lottery is 21.5 (steps must be shown).

One of course, chooses 2.



5. Find the dominated strategies, if any. (next page). Specify which strategy dominates which, and for which player. **(5)**

Answer. This is a very simple question. Blue dominates orange weakly (because of 0,0) and green dominates red strongly. This is because, regardless of what j chooses, the payoff for i is higher in each respective case. We cannot say anything about player j's payoffs. The box brackets indicate that the choice is continuous on 0 to 1.

$S_j = [0,1]$
$S_i = \{Red, Blue, Green, Orange\}$