# Game Theory: Final Exam

Total points: 25 | Date: 15/12/2022

Contribution to grade: 30% | Time: 10:00 - 12:00 AM

- Show all steps, as it can help you get partial credit.
- For Part I, do all questions.
- For Part I, do 3/4 questions. Each question is worth 5 marks.
- If you submit all 4 in Part II., the first 3 you did will be graded.

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**Part I.**

1. In a second price auction sealed bid, bidding one's own valuation is a strongly dominant strategy. True or False. Explain.

   *Answer.* False (0.75)

   It is a weakly dominant strategy (at least one line of example to illustrate why). (0.5)

2. Is cooperation possible in a finitely repeated Prisoner's Dilemma, as in the Folk Theorem? *Answer.* No, it is not. (0.5)

   This is because we solve such games by standard backward induction, and find that at each stage, the Nash Equilibrium is Confess, Confess at every stage. (0.75)

3. "Tacit collusion" is when firms collude/cooperate without explicitly talking to each other, i.e., they just collude because it is the SPNE. In a standard symmetric duopoly that mimics the prisoner's dilemma (same as what we have studied in class), when is tacit collusion possible?

*Answer.* The game has to be infinitely repeated. (0.5)

$\delta$ has to be high enough (0.75)

4. What does the term "bid shading" mean?

   *Answer.* In an auction, dinners bid an amount less than or equal to their true valuation (generally a little less). This is called bid shading (1.25)

5. What distinguishes a PBE from a BNE?

   *Answer.* Sequential rationality in terms of

   - Updating where one is in the game sequence, as in SPNE. (0.5).
   - Updating beliefs about priors/types (0.75)

6. We know that every SPNE is also an NE, and every PBE is also an NE and BNE. But is every BNE an SPNE?

   - No. (0.25).
   - SPNE requires sequential rationality but BNE does not. (0.75)

7. There is a concept called "partial cross ownership" in economics. The simplest interpretation of this is that firms have a share in each other's profits. However, firms cannot directly affect each other's decisions. Suppose we have a dupoly setting. Does partial cross ownership make cooperation more or less likely? Why?

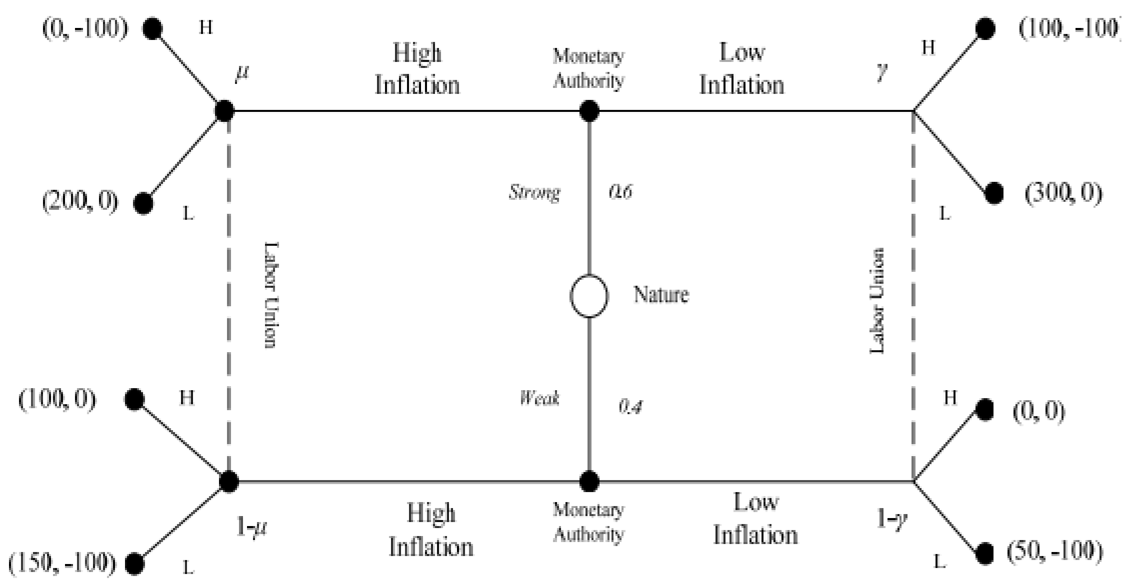
   - More likely. (0.50).
   - Since a firm gains some portion of their rival's profits, they see some value to their rival doing well as well. (0.75)

8. When finding PBE, we sometimes rely on costly signals. What do we mean? What if signals are not costly enough (or too costly)? *Answer.*

- A costly signal is one in which the player (typically the high type), by taking a costly action, tries to separate themselves from the low type. (0.25)
- If the signal is not costly enough, both types may use the signal, thus making it useless (extreme case is if the signal is free, like just saying you the high type). (0.5)
- If the signal is too costly, neither type will use it, making it useless again. (0.5)

## Part II.

1. A labor union, observing the message sent by the monetary authority (but not its real type), decides whether to ask for high increases in the wage level during their yearly increases (denoted as H), or to go for more moderate increases in their wages (represented by the letter L). Find all the sustainable PBEs. (5)




*Answer.* Separating with Strong type going low and weak type going high. Can be sustained., with the union going L and H respectively (1 mark)

Separating with Strong type going high and weak type going low. Cannot be sustained be sustained. (1 mark)

Pooling with both types going low. Cannot be sustained, but you have to show for both $\mu > \frac{1}{2}$ and $\mu < \frac{1}{2}$. In pooling, off the equilibrium beliefs have to be checked as well. If you do not check, you will lose a full mark (1.5 marks)

Pooling where both choose high. Yes, only if $\gamma > \frac{1}{2}$.(1.5 marks)

2. Suppose $n$ firms compete in quantities in an infinitely repeated game, $p = 1 - Q$, $Q$ is sum of all outputs. Discount factor is $\delta$ and marginal cost $c$ is constant and symmetric.

   (a) What is the quantity in a cooperative outcome? (1)

   *Answer.* This is same as a monopoly outcome in aggregate, but divided by $n$. It comes to $q^m = \frac{1}{n}\left(\frac{1-c}{2}\right)$ You just need to maximize combined profits. You must show the steps.

   (b) Indicate whether a larger number of firms in the industry facilitates or hinders the possibility of reaching a collusive outcome (assuming grim trigger strategy). (4)

   *Answer.* Optimal deviation comes from

$$\max_{q \geq 0} [1 - (n-1)q^m - q - c]q$$

   Solving yields $q^d = (n+1)\frac{(1-c)}{4n}$ (1 mark to get till here)

The profits that a firm obtains by deviating from the collusive output are hence

$$\pi^d = \left[1 - (n-1)q^m - q^d(q^m)\right] \times q^d(q^m) - c \times q^d(q^m)$$
$$= \left[1 - (n-1)\left(\frac{1}{n}\frac{1-c}{2}\right) - (n+1)\frac{1-c}{4n}\right](n+1)\frac{1-c}{4n} - c(n+1)\frac{1-c}{4n},$$

which simplifies to $\pi^d = \frac{(1-c)^2(n+1)^2}{16n^2}$.

***Incentives to collude***: Given the above profits from colluding and from deviating, every firm $i$ chooses to collude as long as

$$\frac{1}{1-\delta}\pi^m \geq \pi^d + \frac{\delta}{1-\delta}\pi^c$$

which, solving for $\delta$, yields

$$\delta \leq \frac{\pi^m - \pi^d}{\pi^c - \pi^d}$$

and multiplying both sides by $-1$, we obtain

$$\delta \geq \frac{\pi^d - \pi^m}{\pi^d - \pi^c}$$

2.5 marks for this part

Solving yields $\delta \geq \frac{(1+n)^2}{1+6n+n^2}$ (0.5 marks)

3. **Bayesian Nash Equilibrium.** Consider two candidates fighting an election, each with a privately known personal cost for entering the race. The probability of having a low entry cost, $f_L$, is $p$ and the probability of having a high entry cost, $f_H$, is $1-p$. A candidate's payoff depends on whether she enters the race and whether the other enters or not as well. Let $v_2$ be a candidate's payoff when she enters and the candidate does as well, $v_1$ be a candidate's payoff when she enters and the other candidate does not, and 0 be the payoff when she does not enter (regardless of the other candidate's

decision). Assume that

$$
\begin{aligned}
v_1 > v_2 > 0 \\
f_H > f_L > 0 \\
v_2 - f_L > 0 > v_2 - f_H \\
v_1 - f_H > 0
\end{aligned}
$$

(a) Show the conditions for when it is a symmetric BNE for a candidate to enter only when she has a low cost from doing so. (2.5)

*Answer.* It is a symmetric BNE for a candidate to enter if she has low personal cost iff, for a low type

$$p(v_2 - f_L) + (1 - p)(v_1 - f_L) \geq 0$$

Rearranging,

$$pv_2 + (1 - p)v_1 \geq f_L \tag{1}$$

Similarly, when a candidate has high cost, she will not enter if

$$0 \geq p(v_2 - f_H) + (1 - p)(v_1 - f_H)$$

which yields

$$f_H \geq pv_2 + (1 - p)v_1 \tag{2}$$

Combining (1) and (2), the condition for a candidate to enter only when she has a low personal cost is

$$f_H \geq pv_2 + (1 - p)v_1 \geq f_L$$

**Marks breakdown:**

- 1 marks for equation (1). 1 mark Partial credit for setup as well.

- 1 marks for equation (2). 1 mark Partial credit for setup as well.
- 0.5 mark for final combined result.

(b) Show the conditions for when it is a symmetric BNE for a candidate to enter for sure when she has a low cost (i.e., with probability 1) and to enter with some probability between 0 and 1 when she has a high cost (mixed strategy). *Hint: Assume that if she has high cost she enters with probability $q$. This problem is a bit difficult so all you need to do is set it up the type-wise conditions and find the expression for $q$ for full marks. You do not have to compare all conditions to get the final answer.* (2.5)

*Answer.* The conditions are that if she has low cost, she enters but if she has high cost, she enters with probability $q$. For a low type,

$$(p + (1 - p)q](v_2 - f_L) + (1 - p)(1 - q)(v_1 - f_L) \geq 0$$

The first expression counts the cases when the rival is a low type (probability $p$) or a high type who chooses to enter (probability $(1 - p)q$. The second case considers a situation in which the rival does not enter (probability $(1 - p)(1 - q)$. Rearranging gives

$$[p + (1 - p)q]v_2 + (1 - p)(1 - q)v_1 \geq f_L \tag{3}$$

And for a high type,

$$0 = [p + (1 - p)q](v_2 - f_H) + (1 - p)(1 - q)(v_1 - f_H)$$

Since the high type is randomizing, the condition has to be equal to 0 as he has to be indifferent between participating and not participating (and not participating yields 0).

$$q = \frac{(1-p)(v_1 - v_2) + v_2 - f_H}{(1-p)(v_1 - v_2)} \tag{4}$$

**Not graded after this point.**

For this to be an equilibrium we need, $0 < q < 1$. So

$$0 < \frac{(1-p)(v_1 - v_2) + v_2 - f_H}{(1-p)(v_1 - v_2)} < 1$$

$$pv_2 + (1-p)v_1 > f_H \tag{5}$$

Using (3), (4) and (5), you will get $f_H \geq f_L$ (true by definition), and the only binding condition here is the expression for $q$, i.e., (4)

**Marks breakdown:**

- 1 marks for equation (3).
- 1.5 marks for equation (4). 0.75 marks Partial credit for setup as well.