Module 2

AC-to-DC Converters
Diode and thyristor converters

Content

- Diode Rectifiers
  - Three-Pulse Diode Rectifier
  - Six-Pulse Diode Rectifier
  - Continuous and discontinuous mode of operation
  - Line-current harmonics
- Phase-Controlled Rectifiers (Thyristor rectifiers)
  - Phase-Controlled Six-Pulse Rectifier
  - Line-current harmonics
  - Reactive power
  - Commutation and effect of line inductance
- Four-quadrant operation

Learning outcomes of the module

After the module, you can:

- describe the operation of three-phase diode and thyristor converters more detailed
- analyze line current harmonics and reactive power consumption of diode and thyristor converters
- make a difference between continuous and discontinuous operating mode of the converters
- analyze commutation and the effect of line impedance and how it affects the dc-voltage value
- understand the four operating quadrants (voltage and current polarities)

Three-pulse diode rectifier

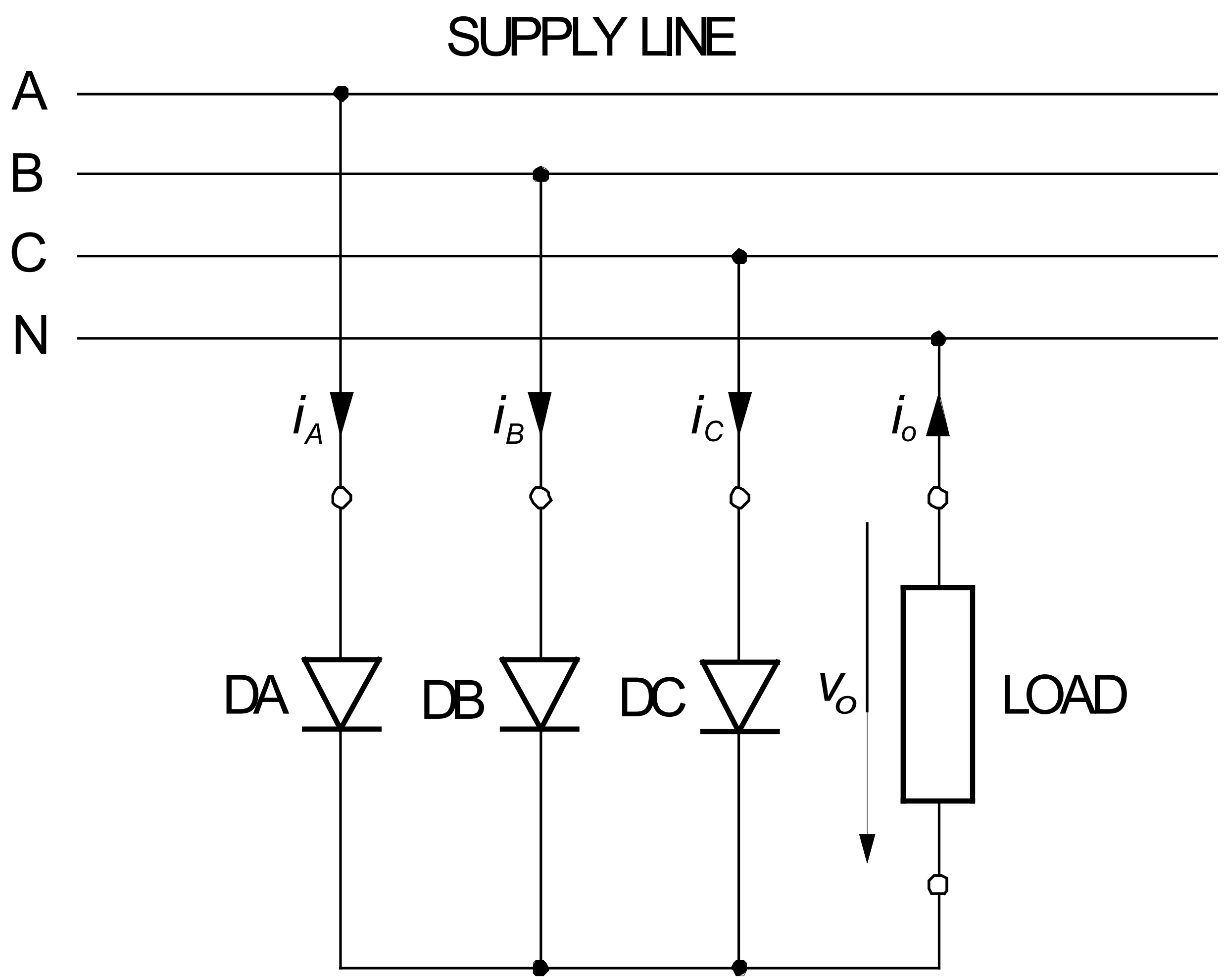
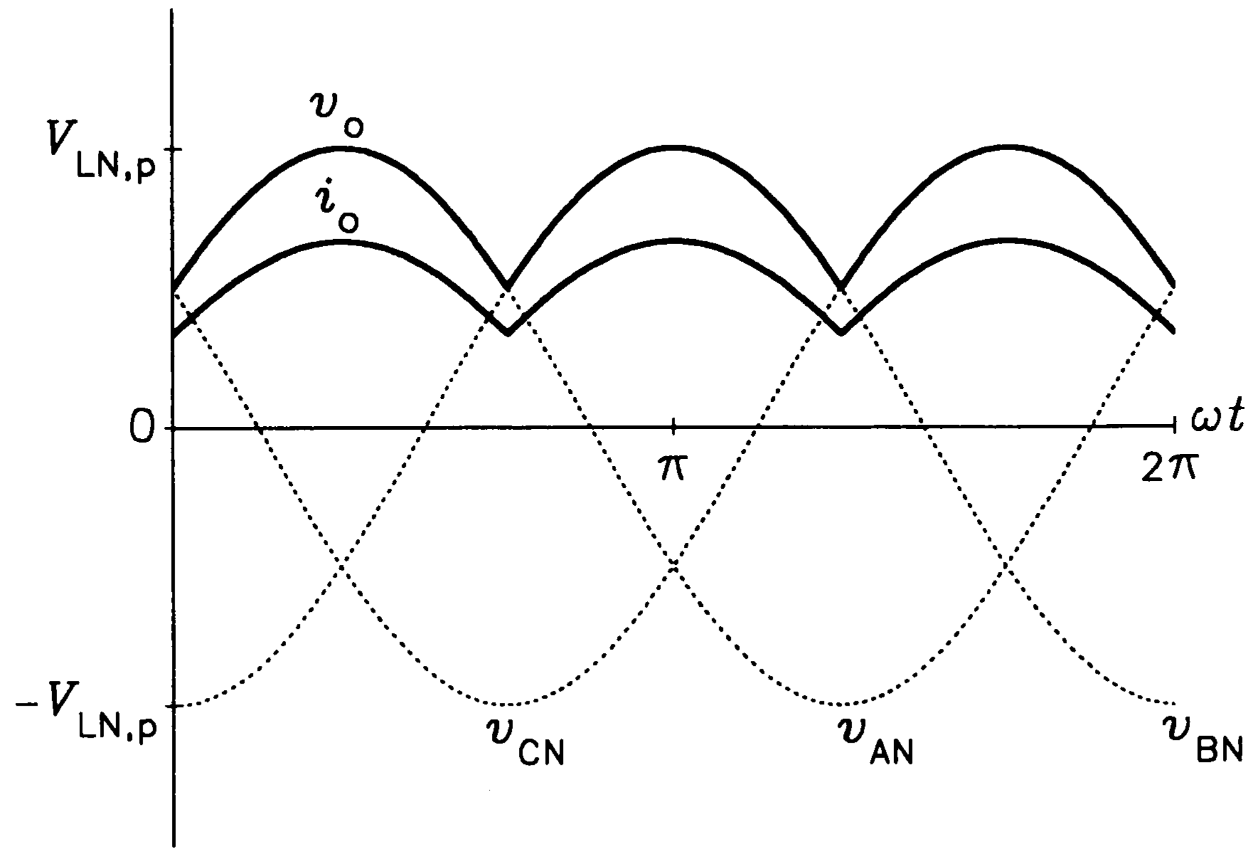


Fig. 4.1

Waveforms of output voltage and current in a three-pulse diode rectifier (R load)

Fig. 4.3



$$V_{o,dc}(C) = \frac{1}{\frac{2}{3}\pi} \int_0^{\frac{2}{3}\pi} V_{LN,p} \sin\left(\omega t + \frac{\pi}{6}\right) d\omega t = \frac{3}{2\pi} V_{LN,p} \left[\cos\left(\omega t + \frac{\pi}{6}\right)\right]_{\frac{2}{3}\pi}^{0}$$

$$= \frac{3\sqrt{3}}{2\pi} V_{LN,p} \approx 0.827 V_{LN,p}.$$

Six-pulse diode rectifier

• Example current path and voltage distribution in a six-pulse diode rectifier

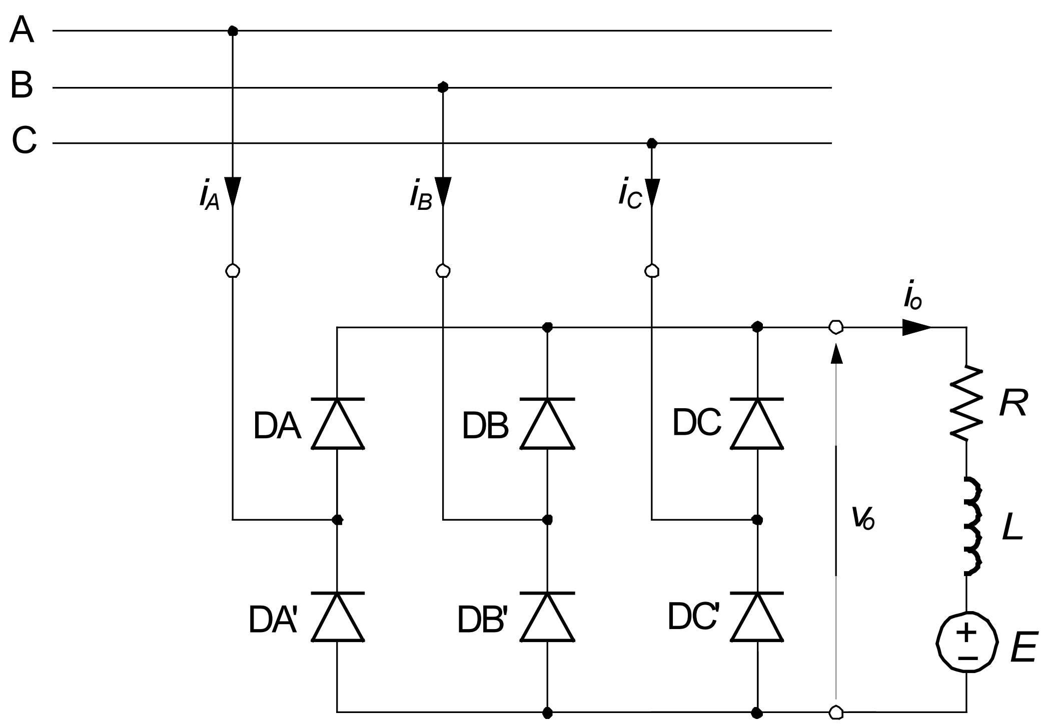


Fig. 4.5

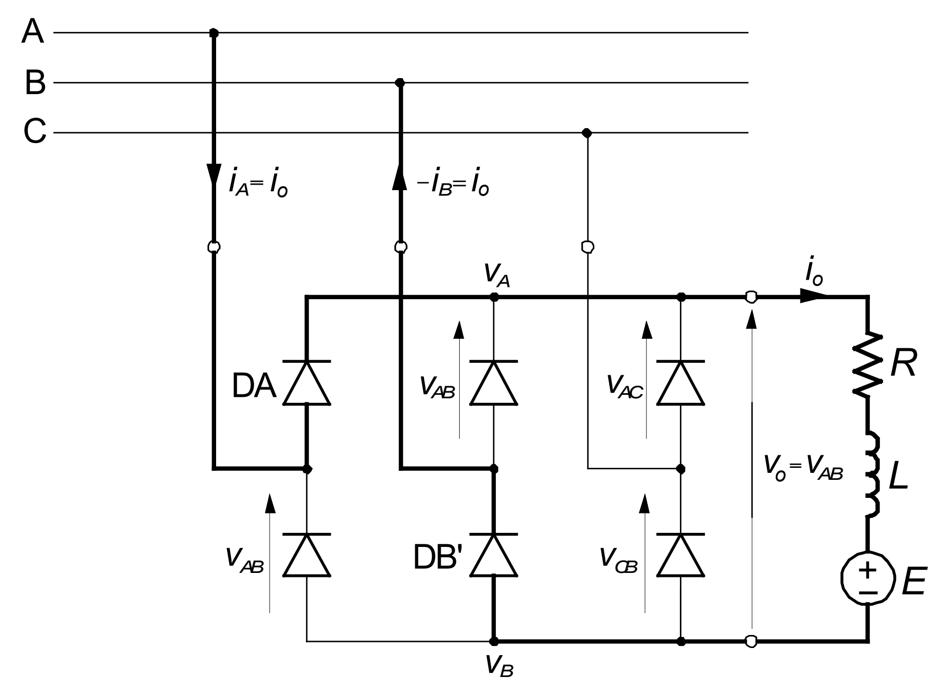


Fig. 4.6

Waveforms of output voltage and current in a six-pulse diode rectifier in the continuous conduction mode (RLE load)

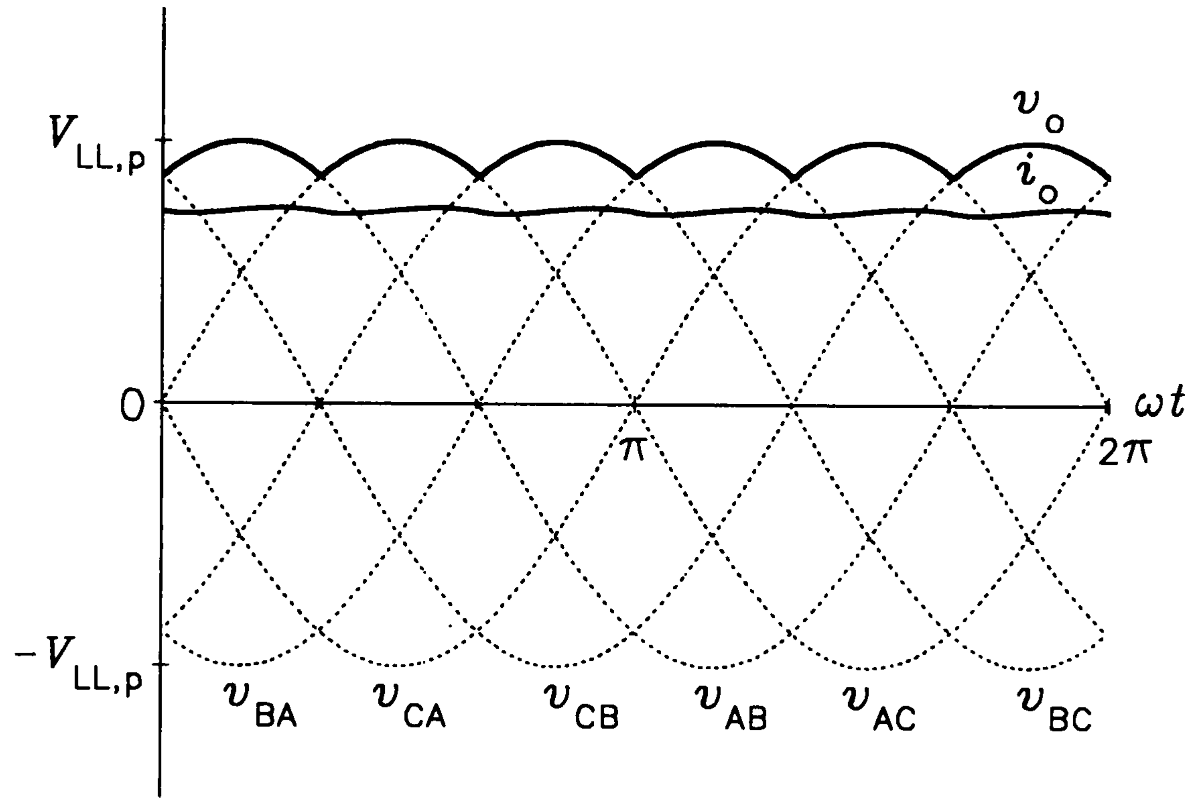


Fig. 4.8

$$V_{o,dc(C)} = \frac{1}{\frac{\pi}{3}} \int_{0}^{\frac{\pi}{3}} V_{LL,p} \sin\left(\omega t + \frac{\pi}{3}\right) d\omega t = \frac{3}{\pi} V_{LL,p} \left[\cos\left(\omega t + \frac{\pi}{3}\right)\right]_{\frac{\pi}{3}}^{0}$$
$$= \frac{3}{\pi} V_{LL,p} \approx 0.955 V_{LL,p}.$$

Continuous conduction mode

- Dc-current is always more than zero

$$V_{AB} = L\frac{di_o}{dt} + Ri_o + E,$$

- Forced component of current

$$i_{o(F)}(\omega t) = \frac{V_{LL,p}}{Z}\sin\left(\omega t + \frac{\pi}{3} - \varphi\right) - \frac{E}{R}$$

- Natural component of current

$$i_{o(N)}(\omega t) = A_{(C)}e^{-\frac{R}{L}t} = A_{(C)}e^{-\frac{R}{\omega L}\omega t} = A_{(C)}e^{-\frac{\omega t}{\tan(\varphi)}},$$

- Current in continuos conduction mode is the sum of these two

$$\begin{aligned} i_{o(C)}(\omega t) &= i_{o(F)}(\omega t) + i_{o(N)}(\omega t) \\ &= \frac{V_{LL,p}}{Z}\left[\sin\left(\omega t + \frac{\pi}{3} - \varphi\right) - \frac{\varepsilon}{\cos(\varphi)}\right] + A_{(C)}e^{-\frac{\omega t}{\tan(\varphi)}} \end{aligned}$$

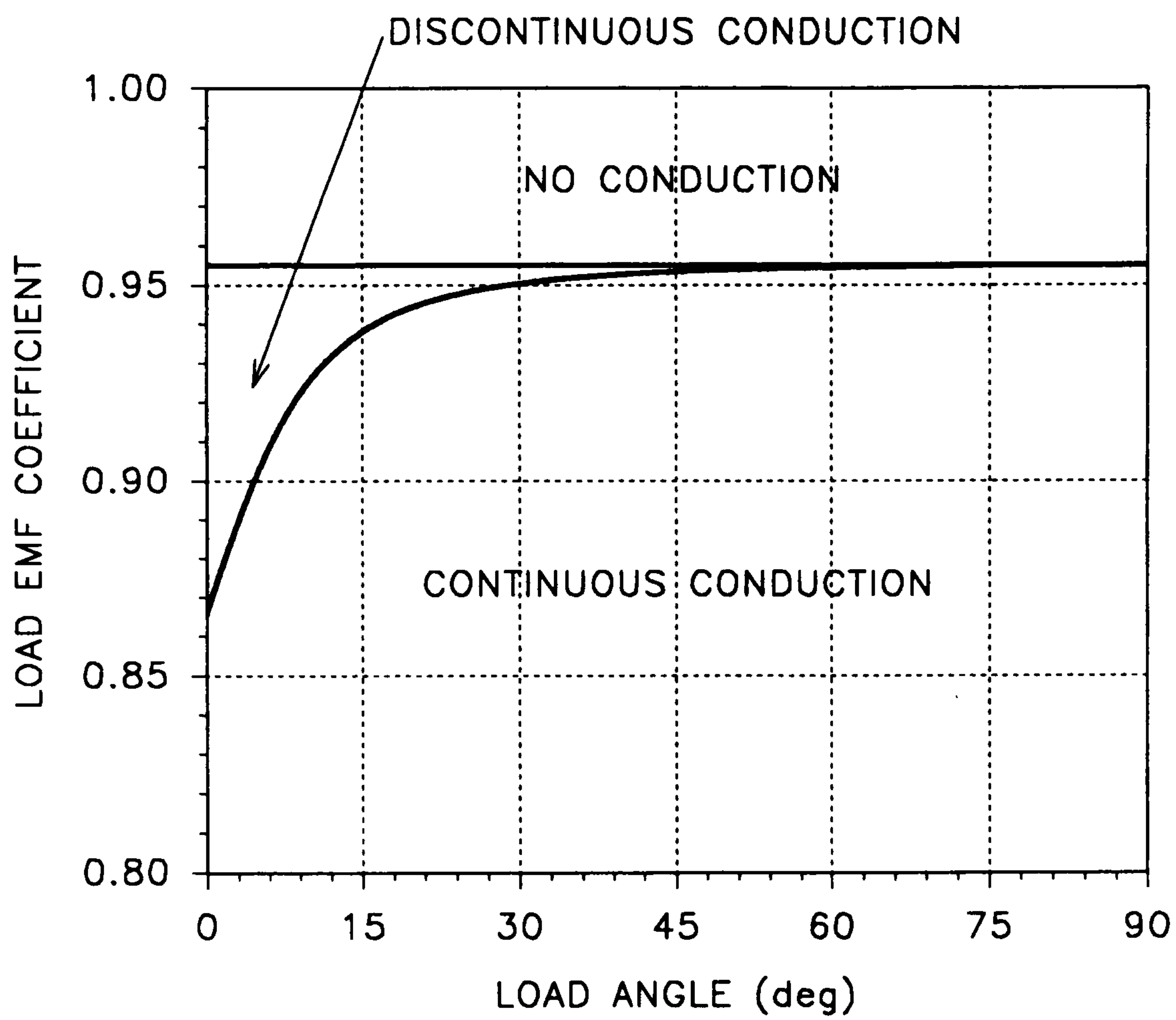
- And finally

$$i_{o(C)}(\omega t) = \frac{V_{LL,p}}{Z}\left[\sin\left(\omega t + \frac{\pi}{3} - \varphi\right) - \frac{\varepsilon}{\cos(\varphi)} + \frac{\sin(\varphi)}{1 - e^{-\frac{\pi}{3\tan(\varphi)}}}e^{-\frac{\omega t}{\tan(\varphi)}}\right]$$

- Where $\varepsilon \equiv E/V_{LL,p}$

Areas of conduction modes of a six-pulse diode rectifier

$$\varepsilon \equiv E/V_{\mathrm{LL,p}}$$



Load angle

$$\varphi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \cos^{-1}\left(\frac{R}{Z}\right)$$

Fig. 4.10

Waveforms of output voltage and current in a six-pulse diode rectifier in the **discontinuous conduction** mode (RLE load)

- Discontinuous conduction takes place when the load EMF, E exceeds the lowest instantaneous value of output voltage vo

$$i_{o(D)}(\omega t) = \frac{V_{LL,p}}{Z}\left\{\sin\left(\omega t + \frac{\pi}{3} - \varphi\right) - \frac{\varepsilon}{\cos(\varphi)} - \left[\sin\left(\alpha_c + \frac{\pi}{3} - \varphi\right) - \frac{\varepsilon}{\cos(\varphi)}\right] e^{-\frac{\omega t - \alpha_c}{\tan(\varphi)}}\right\}$$

$$V_{o,dc(D)} = \frac{1}{\frac{\pi}{3}}\left[\int_0^{\alpha_c} E d\omega t + \int_{\alpha_c}^{\alpha_e} V_{LL,p}\sin\left(\omega t + \frac{\pi}{3}\right) d\omega t + \int_{\alpha_e}^{\frac{\pi}{3}} E d\omega t\right]$$
$$= \frac{3}{\pi} V_{LL,p}\left[2\sin\left(\alpha_c + \frac{\beta}{2} + \frac{\pi}{3}\right)\sin\left(\frac{\beta}{2}\right) + \varepsilon\left(\frac{\pi}{3} - \beta\right)\right]$$

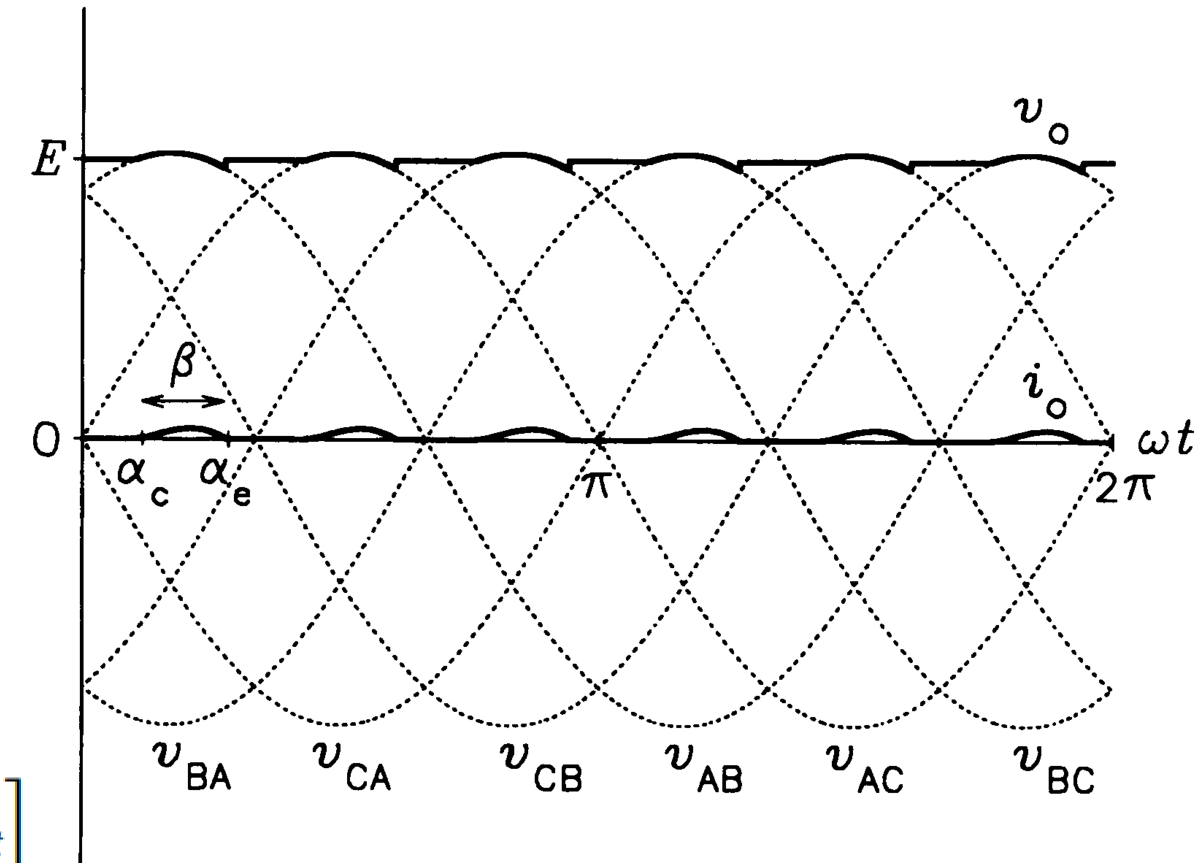
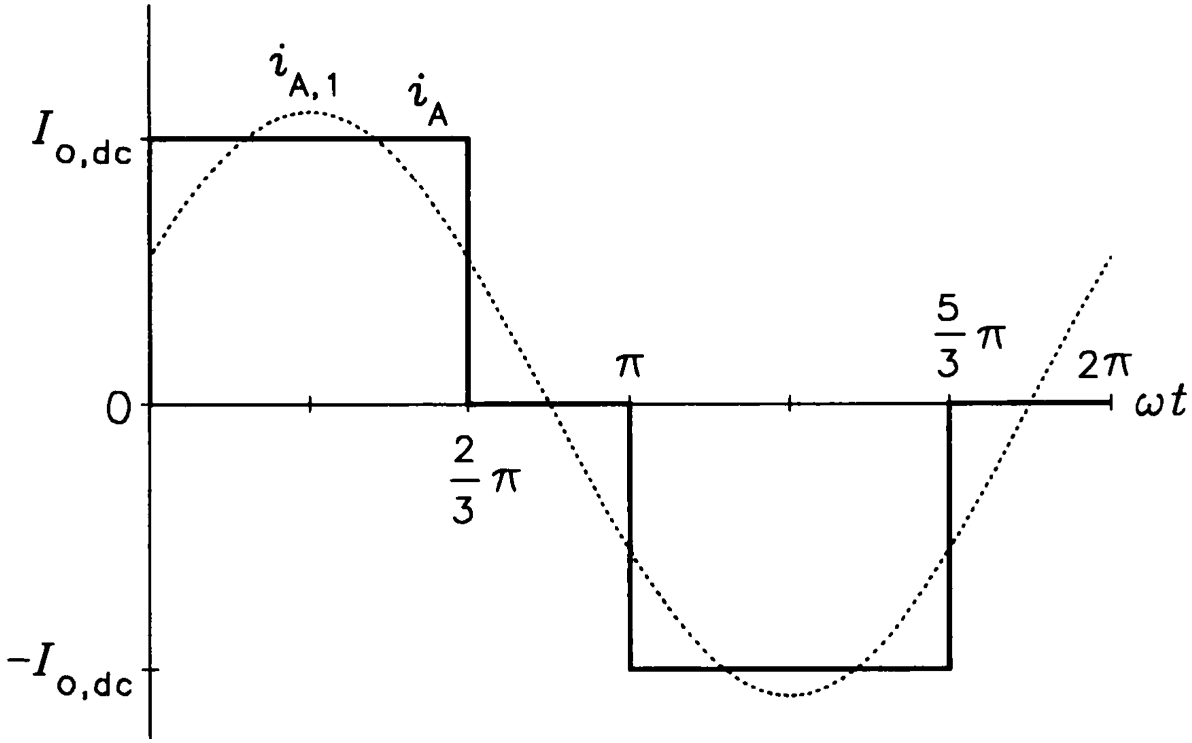


Fig. 4.11

Waveform of input current in a six-pulse diode rectifier (assuming an ideal dc output current)

- The rms value

$$I_A = \sqrt{\frac{1}{2\pi}\left[\int_0^{\frac{2}{3}\pi} I_{o,dc}^2 d\omega t + \int_{\pi}^{\frac{5}{3}\pi} I_{o,dc}^2 d\omega t +\right]} = \sqrt{\frac{2}{3}} I_{o,dc} = 0.82\, I_{o,dc}$$



Note: For Fourier-series derivation this selection of y-axis position is not the wisest because of no symmetry => both cos- and sin-series components (see next slide)

Fig. 4.12

Spectrum of input current in a six-pulse diode rectifier

$$I_{A,1c} = \frac{1}{\pi}\left[\int_0^{\frac{2}{3}\pi} I_{o,dc}\cos(\omega t)\,d\omega t + \int_{\pi}^{\frac{5}{3}\pi} -I_{o,dc}\cos(\omega t)\,d\omega t\right] = \frac{3}{\pi}I_{o,dc}$$

$$I_{A,1s} = \frac{1}{\pi}\left[\int_0^{\frac{2}{3}\pi} I_{o,dc}\sin(\omega t)\,d\omega t + \int_{\pi}^{\frac{5}{3}\pi} -I_{o,dc}\sin(\omega t)\,d\omega t\right] = \frac{\sqrt{3}}{\pi}I_{o,dc}$$

- The fundamental component

$$I_{A,1} = \frac{1}{\sqrt{2}}\sqrt{\left(\frac{3}{\pi}I_{o,dc}\right)^2 + \left(\frac{\sqrt{3}}{\pi}I_{o,dc}\right)^2} = \frac{\sqrt{6}}{\pi}I_{o,dc} \approx 0.78\,I_{o,dc}$$

- Harmonics are at $n = k6 \pm 1$, $k = 1,2,\ldots$ and the amplitude of an individual harmonic

$$I_{A,n} = \frac{I_{A,1}}{n}$$

- Harmonic content

$$I_{A,h} = \sqrt{I_A^2 - I_{A,1}^2} = \sqrt{\left(\sqrt{\frac{2}{3}}I_{o,dc}\right)^2 - \left(\frac{\sqrt{6}}{\pi}I_{o,dc}\right)^2}$$

$$= \sqrt{\frac{2}{3} - \frac{6}{\pi^2}}I_{o,dc} \approx 0.24\,I_{o,dc},$$

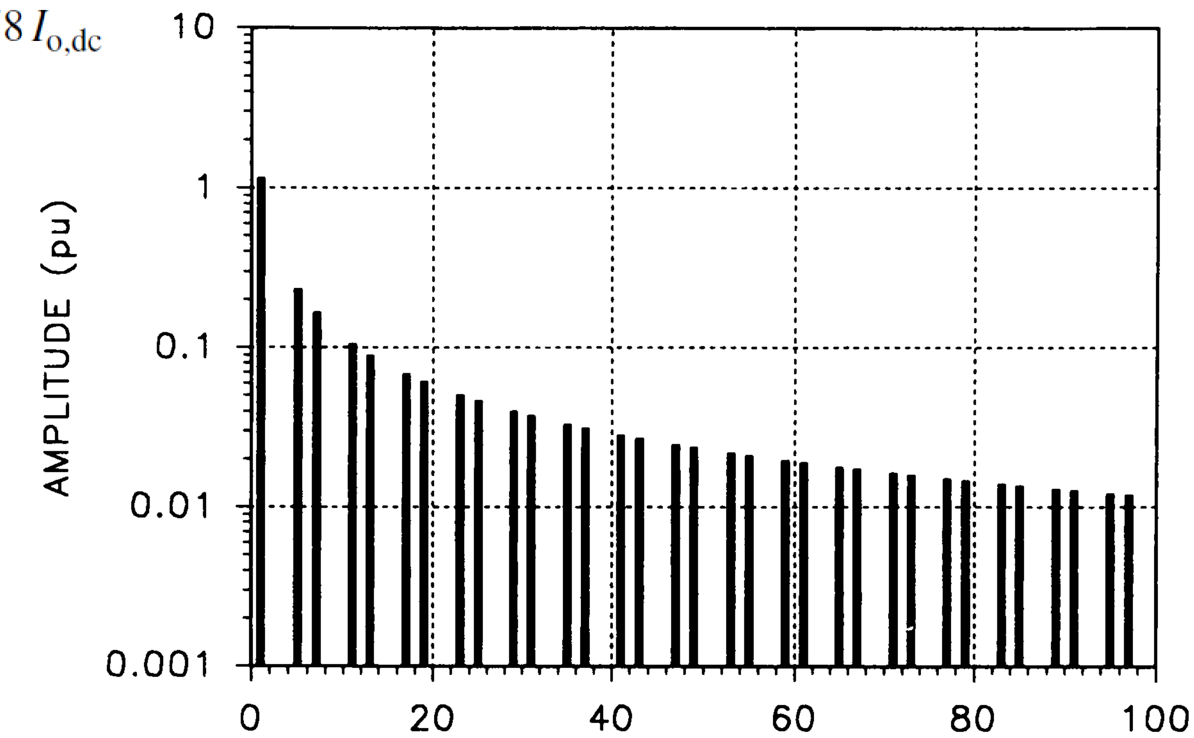


Fig. 4.13

- Total harmonic distortion

$$\mathrm{THD} = \frac{I_{A,h}}{I_{A,1}} = \frac{\sqrt{\frac{2}{3} - \frac{6}{\pi^2}}I_{o,dc}}{\frac{\sqrt{6}}{\pi}I_{o,dc}} = \sqrt{\frac{\pi^2}{9} - 1} \approx 0.31$$

- Power factor

$$\mathrm{PF} = \frac{P_i}{S_i} = \frac{P_o}{S_i} = \frac{V_{o,dc}I_{o,dc}}{\sqrt{3}V_{LL}I_L} = \frac{\frac{3}{\pi}V_{LL,p}I_{o,dc}}{\sqrt{3}\frac{V_{LL,p}}{\sqrt{2}}\sqrt{\frac{2}{3}}I_{o,dc}} = \frac{3}{\pi} \approx 0.955$$

Phase-controlled six-pulse rectifier

- Diodes are replaced by thyristors
- Animation with resistive and inductive load

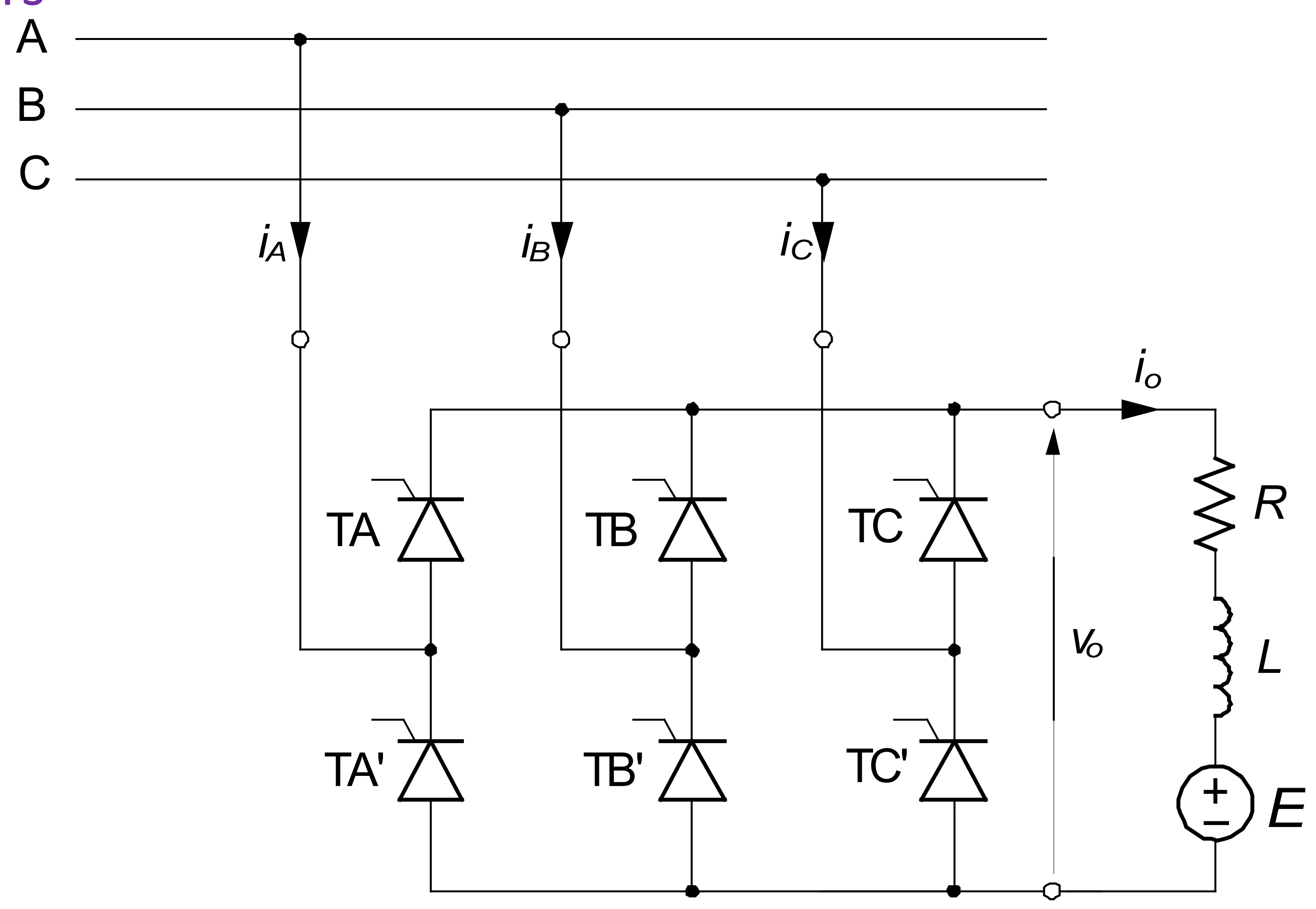


Fig. 4.15

Waveforms of output voltage and current in a phase-controlled six-pulse rectifier

- Average value of the dc-voltage in continuous conduction mode

$$V_{\mathrm{o,dc(C)}} = \frac{1}{\frac{\pi}{3}} \int_{\alpha_f}^{\alpha_f+\frac{\pi}{3}} V_{\mathrm{LL,p}} \sin\left(\omega t + \frac{\pi}{3}\right) d\omega t = \frac{3}{\pi} V_{\mathrm{LL,p}} \cos(\alpha_f)$$

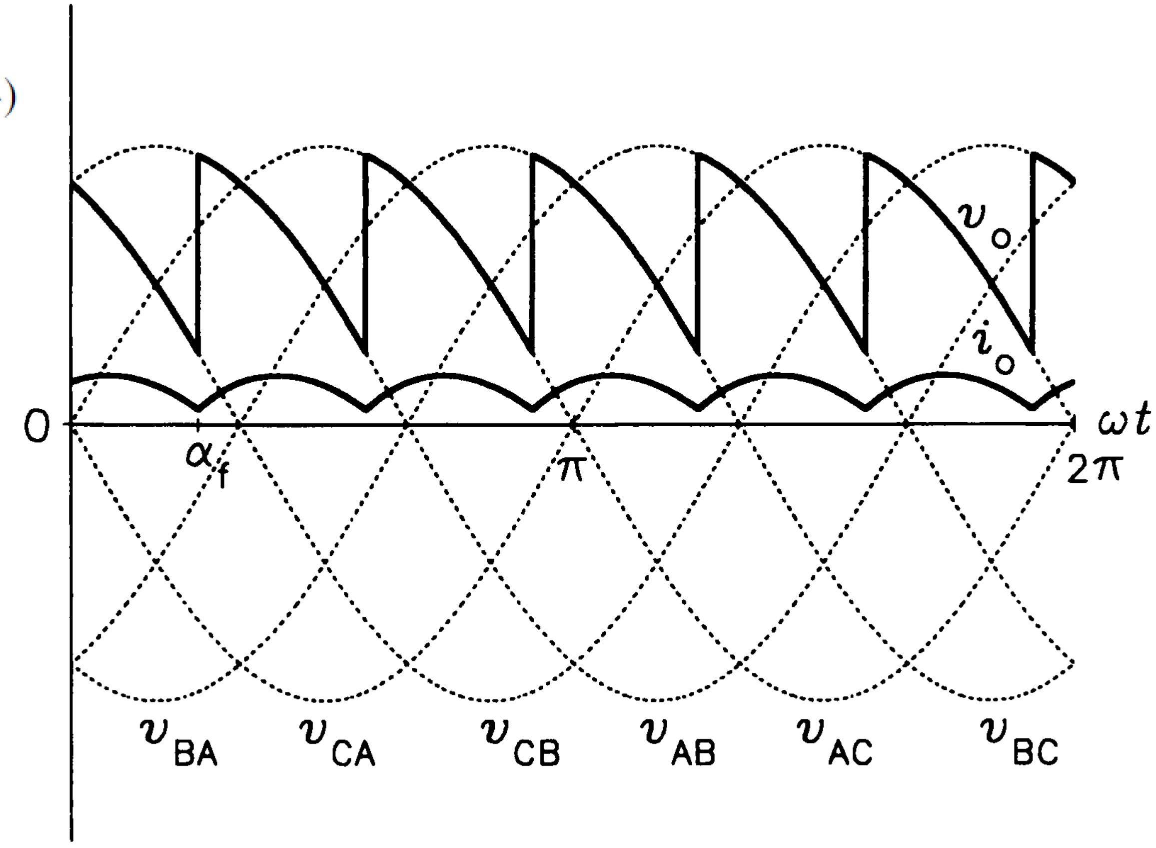


Fig. 4.17

Waveforms of output voltage and current in a phase-controlled six-pulse rectifier in the discontinuous conduction mode: (a) rectifier operation ($\alpha_f = 45^o$), (b) inverter operation ($\alpha_f = 135^o$)

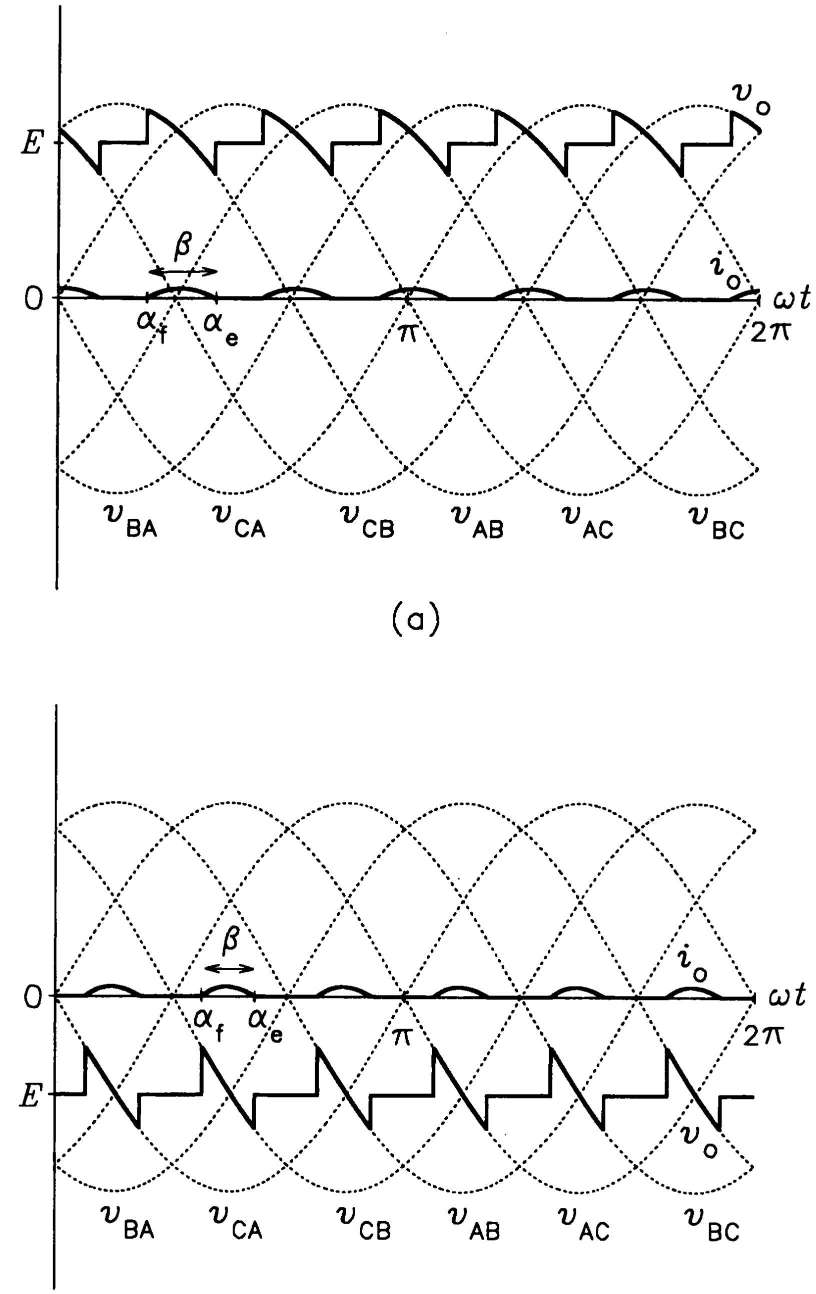


Fig. 4.23

Waveform of input current in a phase-controlled six-pulse rectifier (ideal dc output current assumed)

- Current conduction is delayed by the amount of the control angle
- This causes reactive power taken from the supply system
- Power factor of a diode bridge was

$$\mathrm{PF} = \frac{P_\mathrm{i}}{S_\mathrm{i}} = \frac{P_\mathrm{o}}{S_\mathrm{i}} = \frac{V_\mathrm{o,dc} I_\mathrm{o,dc}}{\sqrt{3} V_\mathrm{LL} I_\mathrm{L}} = \frac{\frac{3}{\pi} V_\mathrm{LL,p} I_\mathrm{o,dc}}{\sqrt{3} \frac{V_\mathrm{LL,p}}{\sqrt{2}} \sqrt{\frac{2}{3}} I_\mathrm{o,dc}} = \frac{3}{\pi} \approx 0.955$$

- Calculating from the ratio of active and apparent power and replacing voltage with the controlled dc voltage => Power Factor of a thyristor bridge is

$$\mathrm{PF} = \frac{V_\mathrm{o,dc}}{V_\mathrm{LL,p}} = \frac{3}{\pi} \cos(\alpha_\mathrm{f}) \approx 0.95 \cos(\alpha_\mathrm{f})$$

- PF of diode bridge is 3/π and there is an additional phase-shift, which is equal to the cosinus of control angle

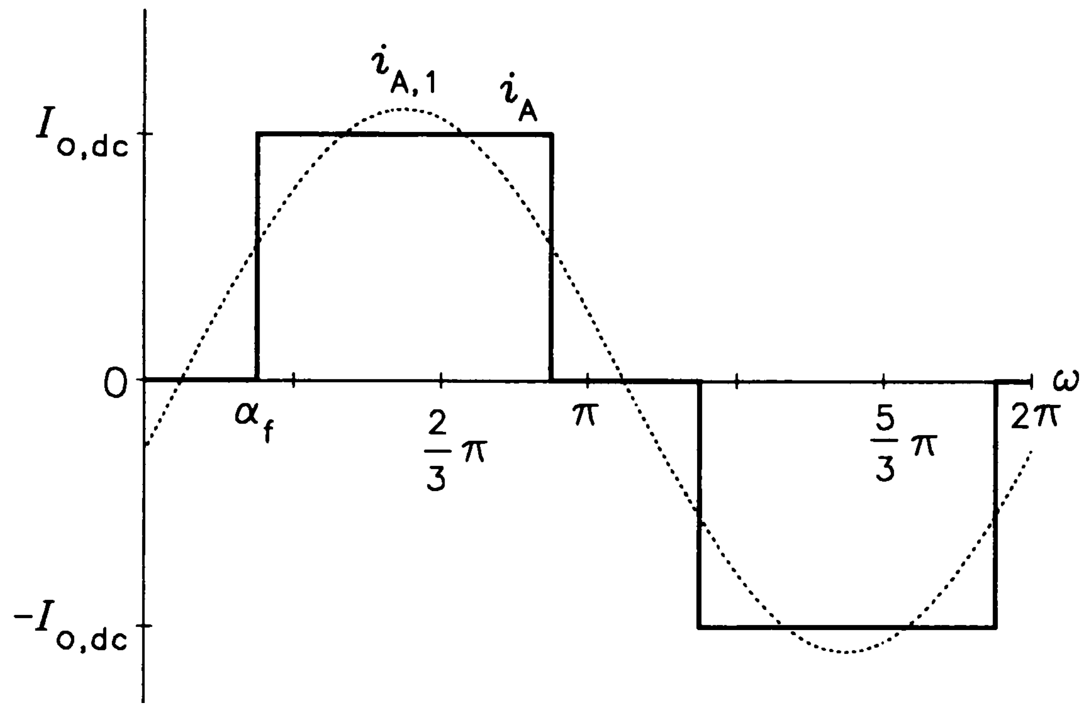


Fig. 4.24

DC voltage and line current of six-pulse rectifier with varying control angle

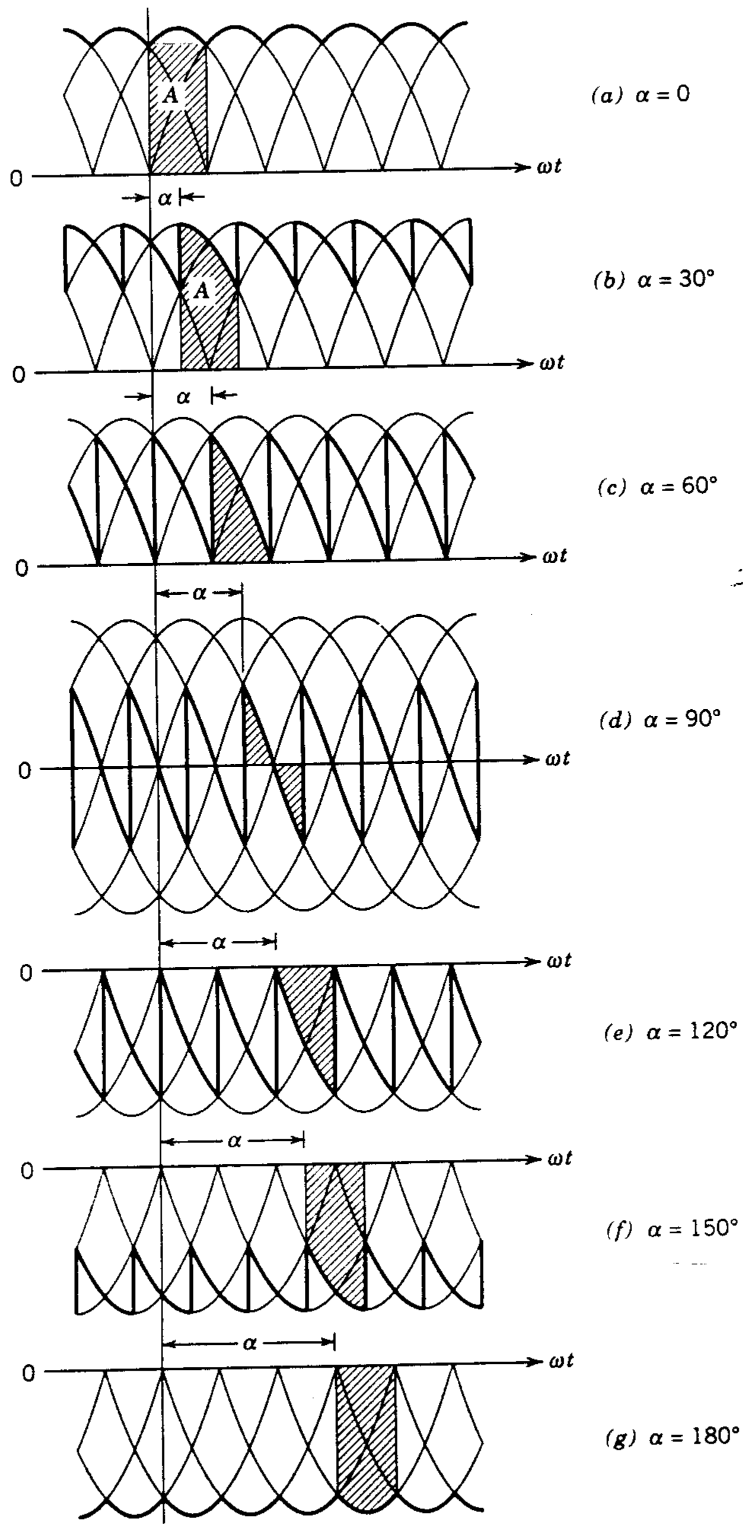


Figure 6-21 The dc-side voltage waveforms as a function of α where $V_{d\alpha} = A/(\pi/3)$. (From ref. 2 with permission.)

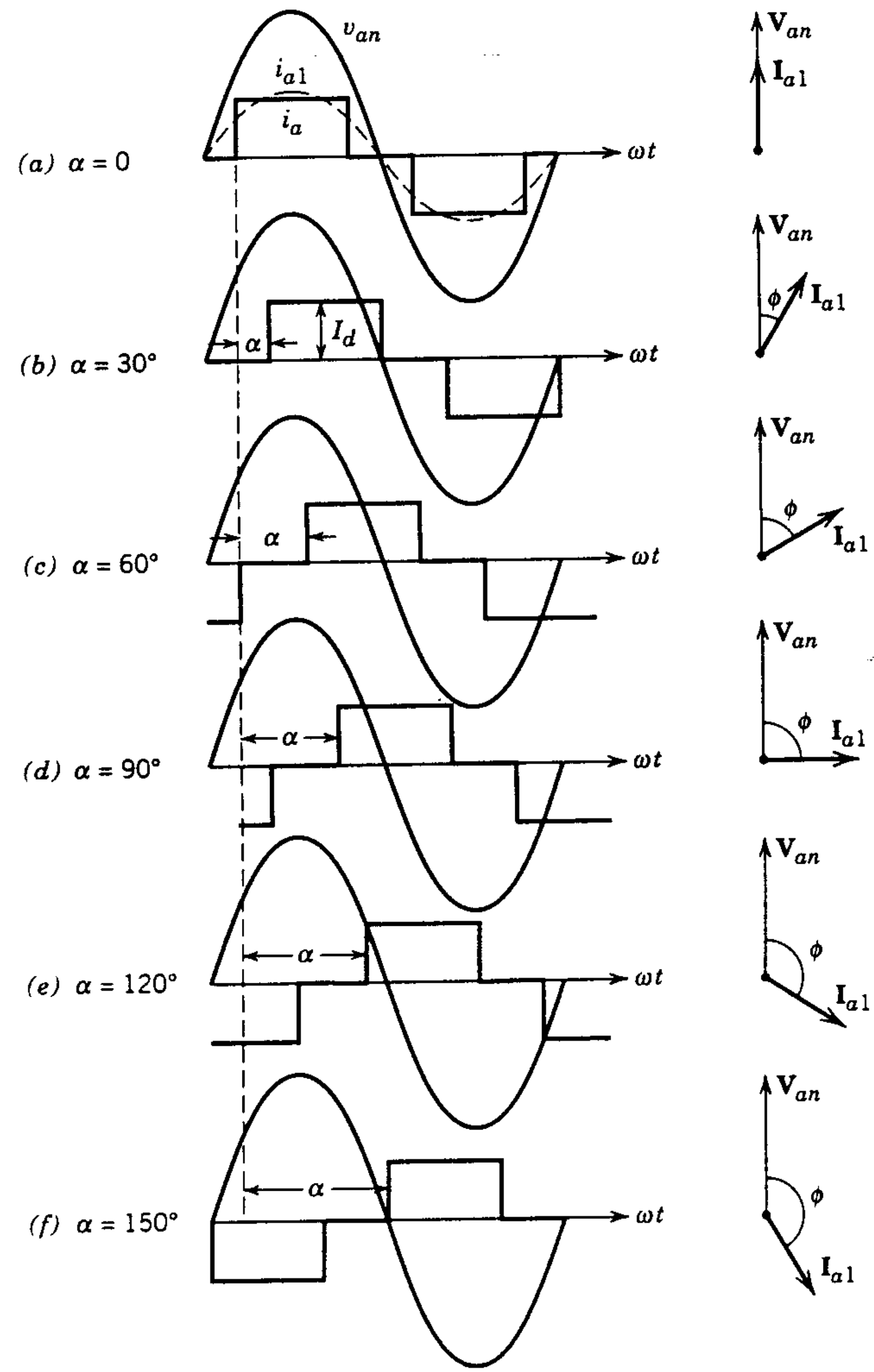


Figure 6-23 Line current as a function of α. (With permission from ref. 2.)

Active, reactive and apparent power

- Active three-phase power is with ac quantities $P = \sqrt{3} V_{LL} I_{A,1} \cos\phi_1$

- When we are using the previously derived results for the fundamental component of current and the phase shift being equal to control angle α we end up on

$$P = \sqrt{3} V_{LL} I_{A,1} \cos\phi_1 = V_{o,dc} I_{o,dc} \cos\alpha$$

- Which is equal to the dc side power (dc voltage multiplied with dc current) when the converter itself has no losses
- In the same way we get the reactive power

$$Q_1 = \sqrt{3} V_{LL} I_{A,1} \sin\phi_1 = V_{o,dc} I_{o,dc} \sin\alpha$$

- Thus, the apparent power at fundamental component remains constant if dc current is constant

$$S_1 = \sqrt{3} V_{LL} I_{A,1} = \sqrt{P^2 + Q_1^2} = V_{o,dc} I_{o,dc} \sqrt{\cos^2\alpha + \sin^2\alpha} = V_{o,dc} I_{o,dc} \approx 1{,}35 \cdot V_{LL} I_{o,dc}$$

Visualization of different power components versus control angle

- Apparent power is calculated with the rms value of the current and

$$S = \sqrt{3} V_{LL} I_A = \sqrt{6} V_{LL} I_{o,dc} \approx 2,44 \cdot V_{LL} I_{o,dc}$$

- It is higher than the previously calculated fundamental part

$$S_1 \approx 1,35 \cdot V_{LL} I_{o,dc}$$

- When the dc current is assumed constant the effect of the control angle on active, reactive and apparent power can be visualized as in the figure

- When control angle increases the dc voltage reduces and thus also the active power taken from the grid
- At the same time, the phase displacement of voltage and current increases and if dc current is constant, reactive power from the grid increases
- Apparent power remains constant

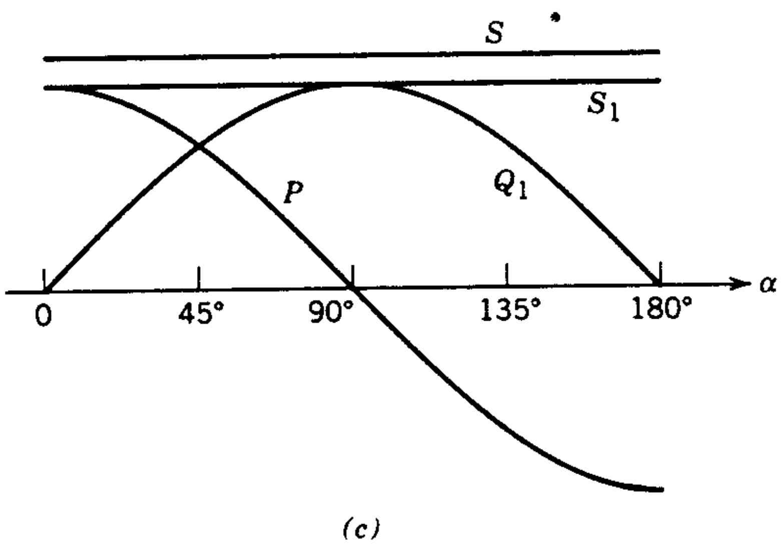


Figure 6-8 The ac-side quantities in the converter of Fig. 6-5.

Effect of line inductance

- In practice the supplying ac system contains inductance
  - transformers, wiring, sometimes additional commutation inductance to limit di/dt of thyristors
- Current cannot change instantaneously fast
- Figure represents commutation from thyristor TC (T2) to TA (T1) in Fig. 4.15

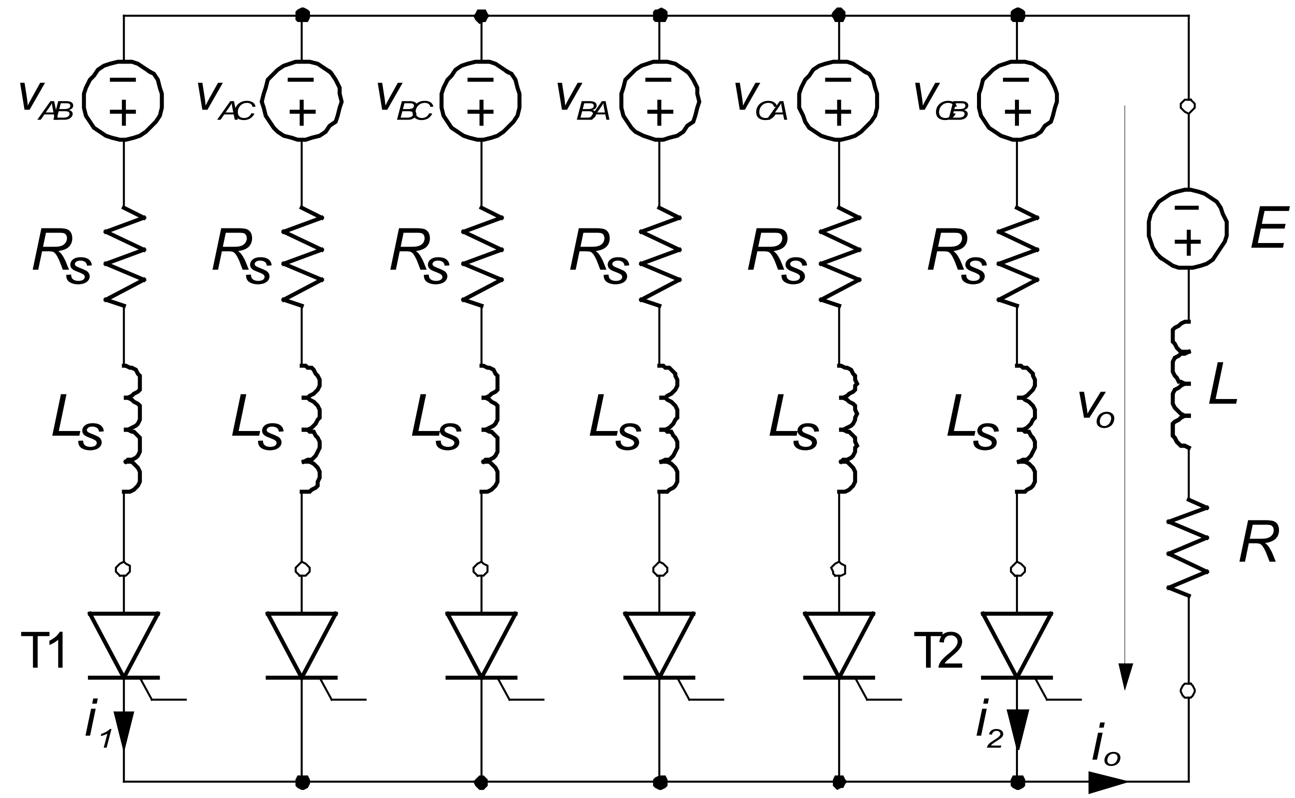
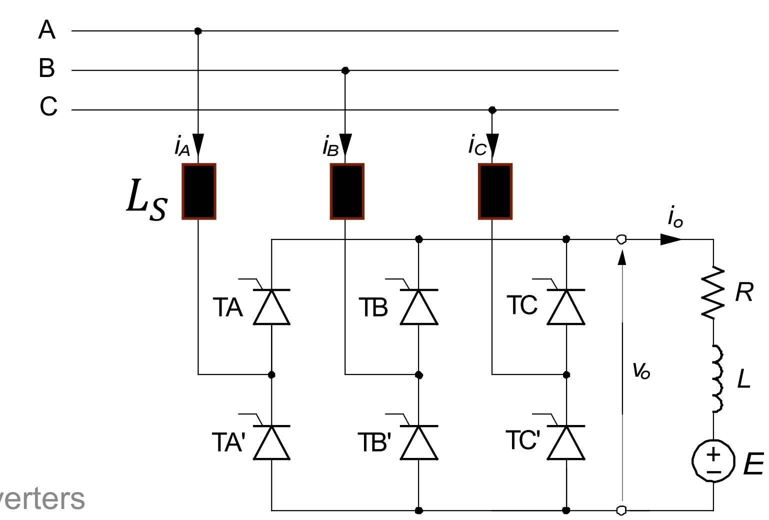


Fig. 4.25 Equivalent circuit of a phase-controlled six-pulse rectifier supplied from a practical dc voltage source



Waveforms of voltage and current in a phase-controlled six-pulse rectifier during commutation

- During commutation two thyristors in the positive side conduct simultaneously
- Current is changing in the line inductances and therefore output dc-voltage is not any of the line-to-line voltages but an average of two

$$v_o = v_{AB} - L_s \frac{di_1}{dt} \qquad v_o = v_{CB} - L_s \frac{di_2}{dt}$$

$$2v_o = v_{AB} + v_{CB} - L_s \left(\frac{di_1}{dt} + \frac{di_2}{dt} \right) = v_{AB} + v_{CB},$$

- Derivatives of currents i_1 and i_2 are equal as the sum of these currents is constat I_d, however, signs are opposite and thus the sum of derivatives is zero

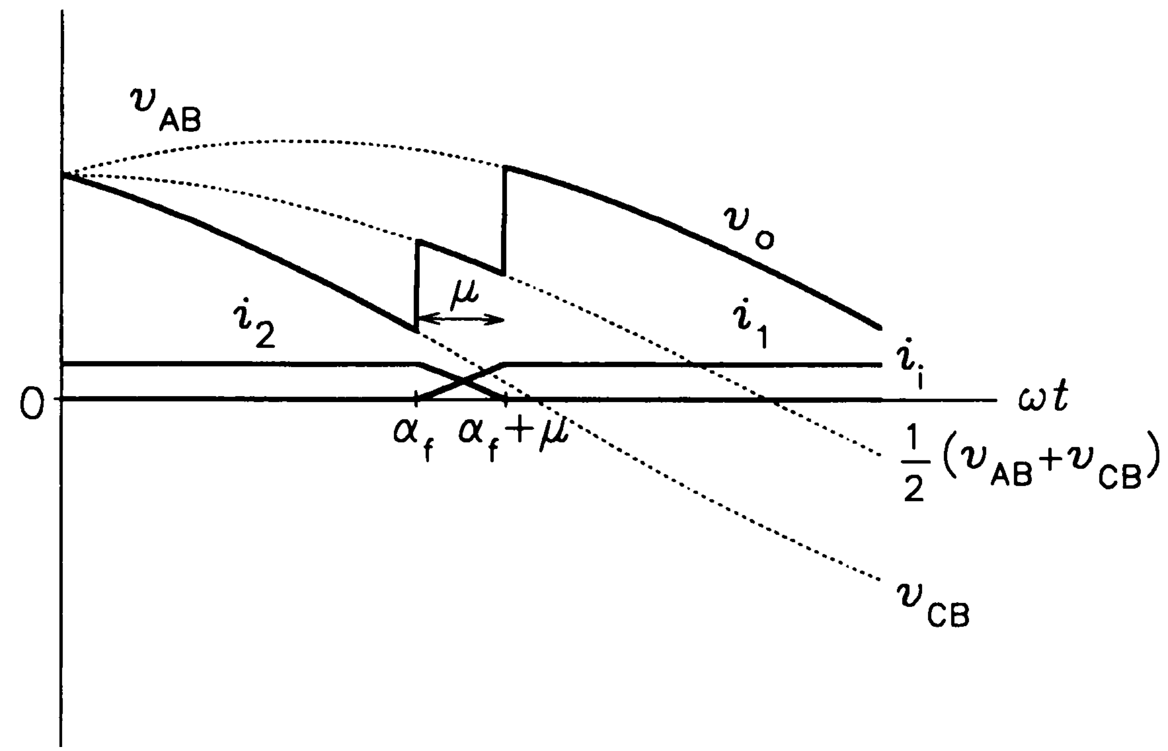


Fig. 4.26 During the commutation interval μ current i_2 decreases to zero and i_1 inceases to I_d

Commutation angle μ

- Current during commutation can be integrated from

$$\frac{di_1}{dt} = -\frac{di_2}{dt} = \frac{1}{2L_s}(v_{AB} - v_{CB})$$
$$= \frac{1}{2L_s}\left[V_{LL,p}\sin\left(\omega t + \frac{1}{3}\pi\right) - V_{LL,p}\sin\left(\omega t + \frac{2}{3}\pi\right)\right]$$
$$= \frac{V_{LL,p}}{2L_s}\sin(\omega t).$$

- Current i_1 increases from zero to $I_{o,dc}$ during commutation interval μ, ie after the delay of α_f and ending at $\alpha_f + \mu$. Therefore, after integration of previous equation

- $$\frac{V_{LL,p}}{2X_s}[\cos(\alpha_f) - \cos(\alpha_f + \mu)] = I_{o,dc}$$ or $$\mu = \left|\cos^{-1}\left[\cos(\alpha_f) - 2\frac{X_s I_{o,dc}}{V_{LL,p}}\right] - \alpha_f\right|$$

Voltage drop due to commutation

- Commutation causes a reduction in the obtained dc-voltage, instead of being one line-to-line voltage it is an average of two voltages as shown before
- Reduction can be calculated from

$$\Delta V_{\mathrm{o,dc}} = \frac{3}{\pi}\int_{\alpha_{\mathrm{f}}}^{\alpha_{\mathrm{f}}+\mu} v_{\mathrm{AB}} - \frac{v_{\mathrm{AB}}+v_{\mathrm{CB}}}{2}\,d\omega t = \frac{3}{\pi}\int_{\alpha_{\mathrm{f}}}^{\alpha_{\mathrm{f}}+\mu} \frac{v_{\mathrm{AB}}-v_{\mathrm{CB}}}{2}\,d\omega t$$

$$= \frac{3}{2\pi}\int_{\alpha_{\mathrm{f}}}^{\alpha_{\mathrm{f}}+\mu} V_{\mathrm{LL,p}} \sin(\omega t)\,d\omega t$$

$$= \frac{3}{2\pi} V_{\mathrm{LL,p}}[\cos(\alpha_{\mathrm{f}}) - \cos(\alpha_{\mathrm{f}}+\mu)] = \frac{3}{\pi} X_{\mathrm{s}} I_{\mathrm{o,dc.}}$$

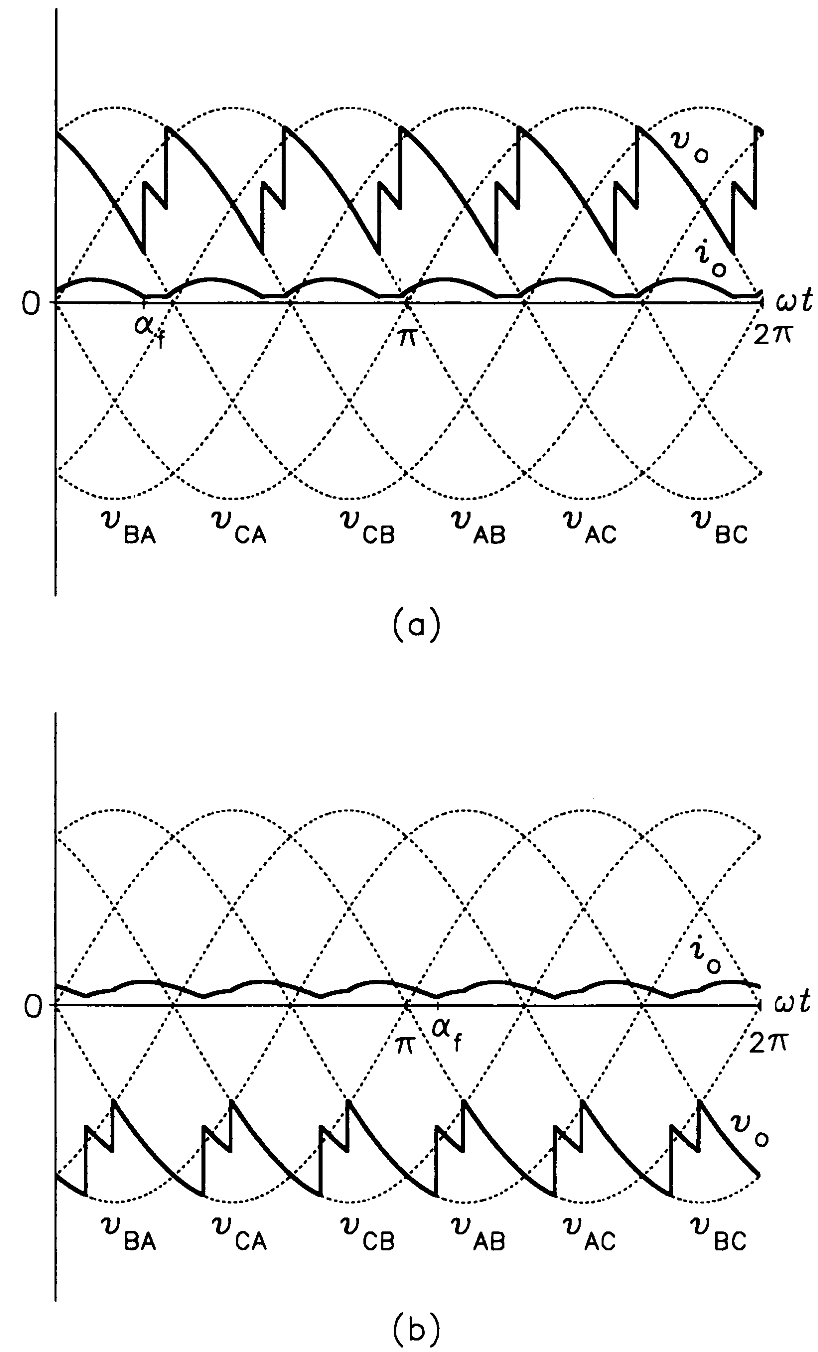
- This reduction can be modeled as a resistive voltage drop. It depends directly from the current and reactace of the supply system. All similar components can be added

$$R_{\mathrm{r}} = \frac{3}{\pi} X_{\mathrm{s}} + R_{\mathrm{s}} + 2R_{\mathrm{ON}} + R_{\mathrm{w}}$$

- The dc output voltage is then $$V_{\mathrm{o,dc}} = \frac{3}{\pi} V_{\mathrm{LL,p}} \cos(\alpha_{\mathrm{f}}) - R_{\mathrm{r}} I_{\mathrm{o,dc}}$$

Waveforms of output voltage and current in a phase-controlled six-pulse rectifier supplied from a source with inductance: (a) rectifier mode, (b) inverter mode

Fig. 4.27



Notched waveform of input voltage in a phase-controlled six-pulse rectifier supplied from a source with inductance

- During commutation in Fig. 4.26 the converter voltage is not the ideal system voltages v_{AB} or v_{CB}
- The two phases after the line inductances are connected together and voltages are equal

$$v_{ab} = v_{bc} = v_o = \frac{1}{2}(v_{AB} + v_{CB})$$

- Any other three-phase load connected next to the rectifier will experience this notched supply voltage and its operation can be disturbed

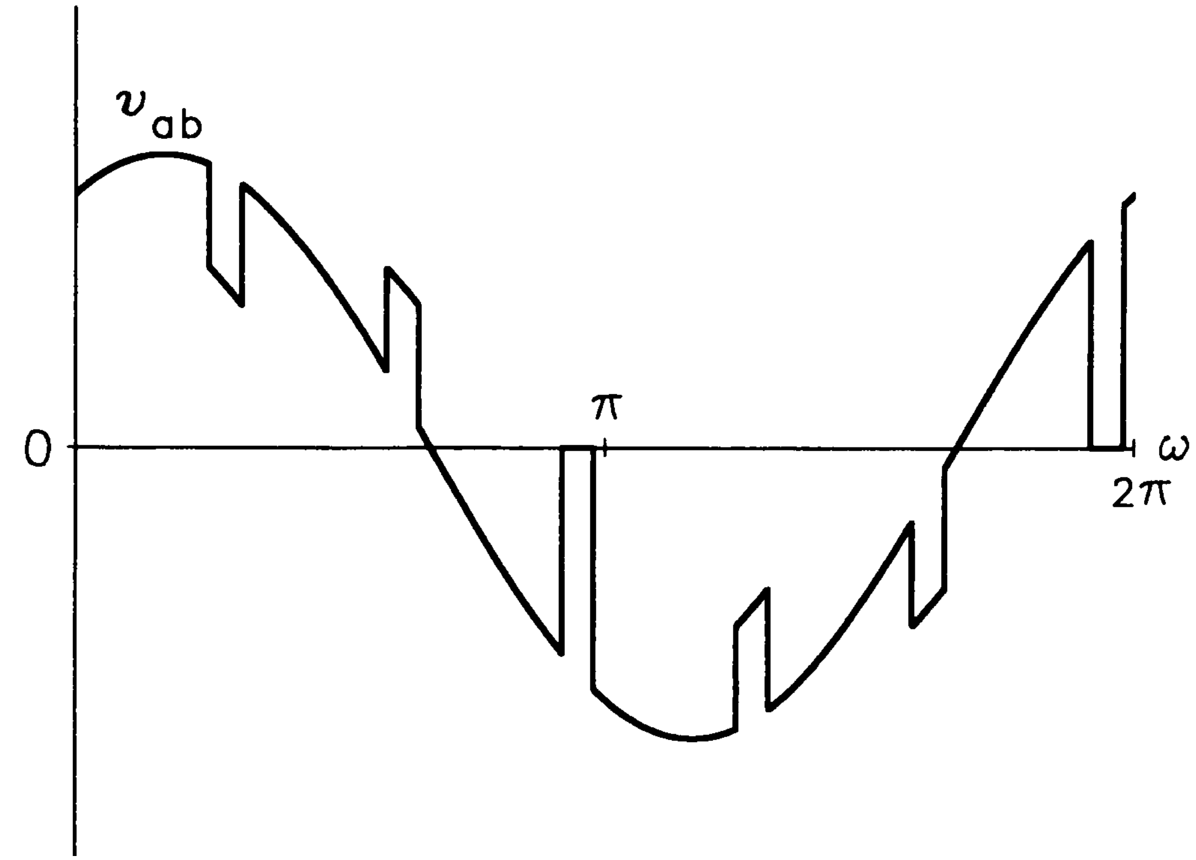
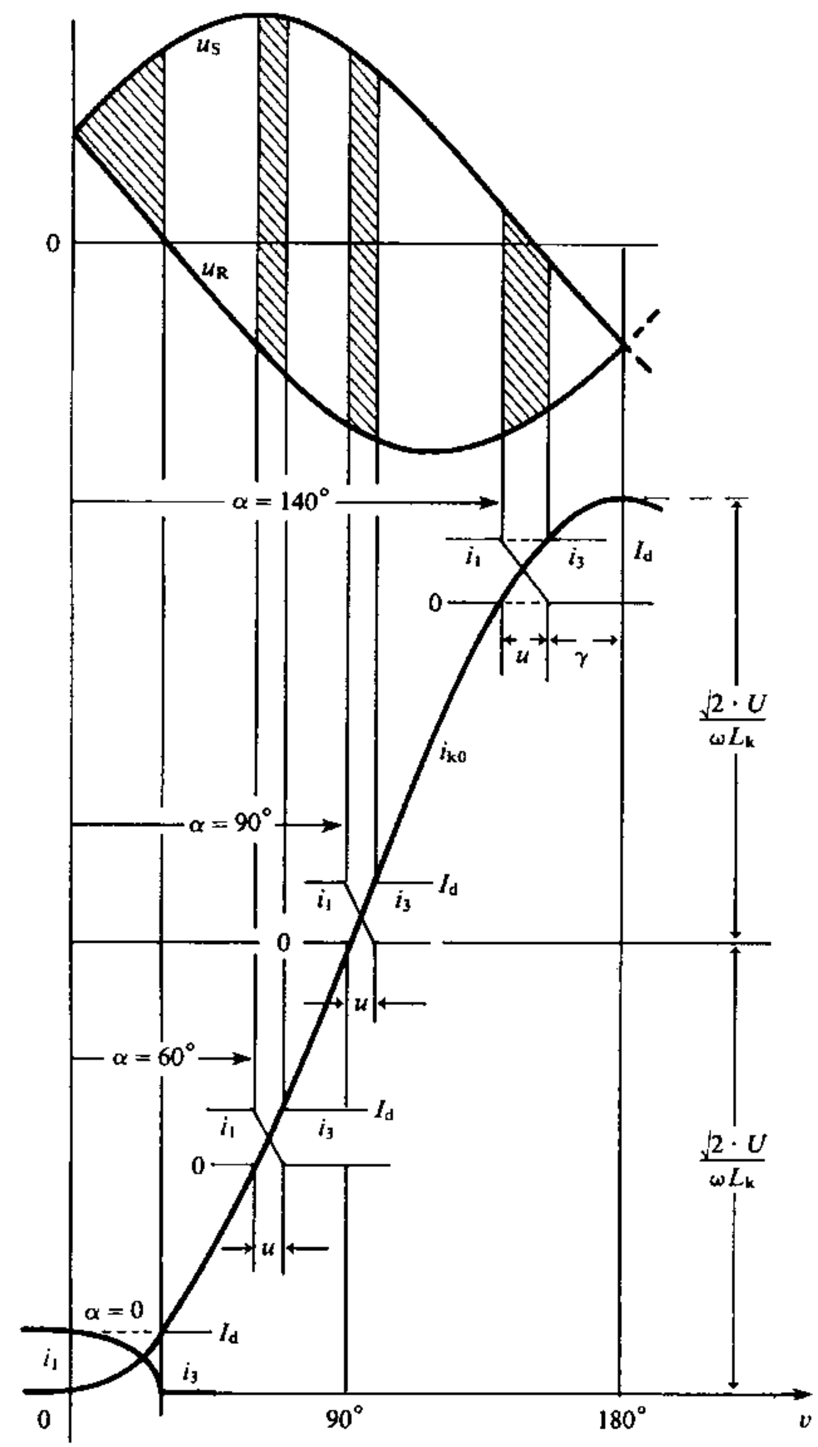


Fig. 4.28

Effect of firing angle on commutation

- The voltage that turns of the thyristor, is one of the line-to-line voltages as demonstrated in the attached figure
  - If firing angle is zero, this voltage builds up slower and it takes longer to change the line currents
  - If firing angle is 90 degree, the difference in two phase voltages is highest and shortest time is needed to change the same current in the same inductances
- If the firing angle approaches 180 degrees, the positive value of the driving voltage changes its polarity. Therefore, commutation needs to end so that $\alpha + \mu < 180^{o}$
- Otherwise, there is commutation failure, and the currents start to increase, and the dc-voltage will be reversing from negative to a positive value as the thyristor that was expected to turn off continues to conduct
- iPES animations
  - [iPES: Phase-Controlled Thyristor Converters: Commutation / Converter Equivalent Circuit (ethz.ch)]
  - [iPES: Phase-Controlled Thyristor Converters: Loss of Commutation (ethz.ch)]



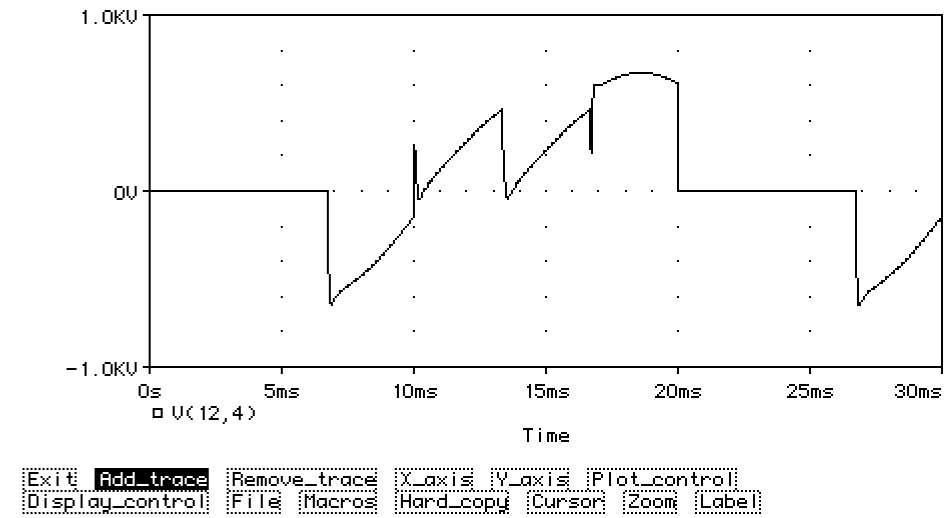
Voltage over a conducting thyristor

- When a thyristor conducts, voltage over it is naturally zero
  - This is highlighted in the two attached figures
- After the conduction, voltage over the thyristor is negative
  - If the firing angle approaches 180 degrees, this negative period becomes shorter and shorter
- Thyristor needs a recovery time, during which the voltage over it needs to be negative

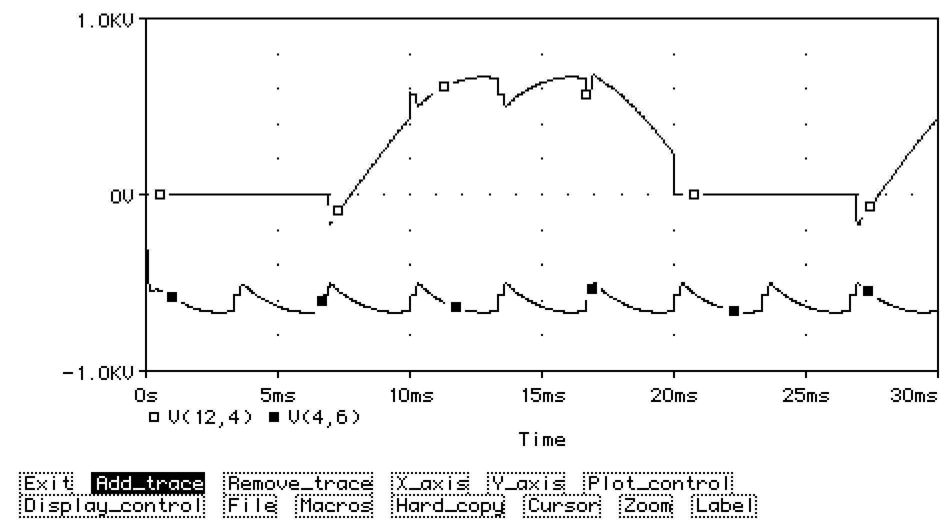
$$\gamma = 180^{o} - \alpha - \mu$$

- Which is often even 18 degrees, corresponding to around 1 ms
- This means that the firing angle needs to be limited to be less than around 160 degrees depending on the thyristors and commutation angle

$\alpha = 110^{o}$



$\alpha = 160^{o}$



Plane of operation, operating area, and operating quadrants of a rectifier

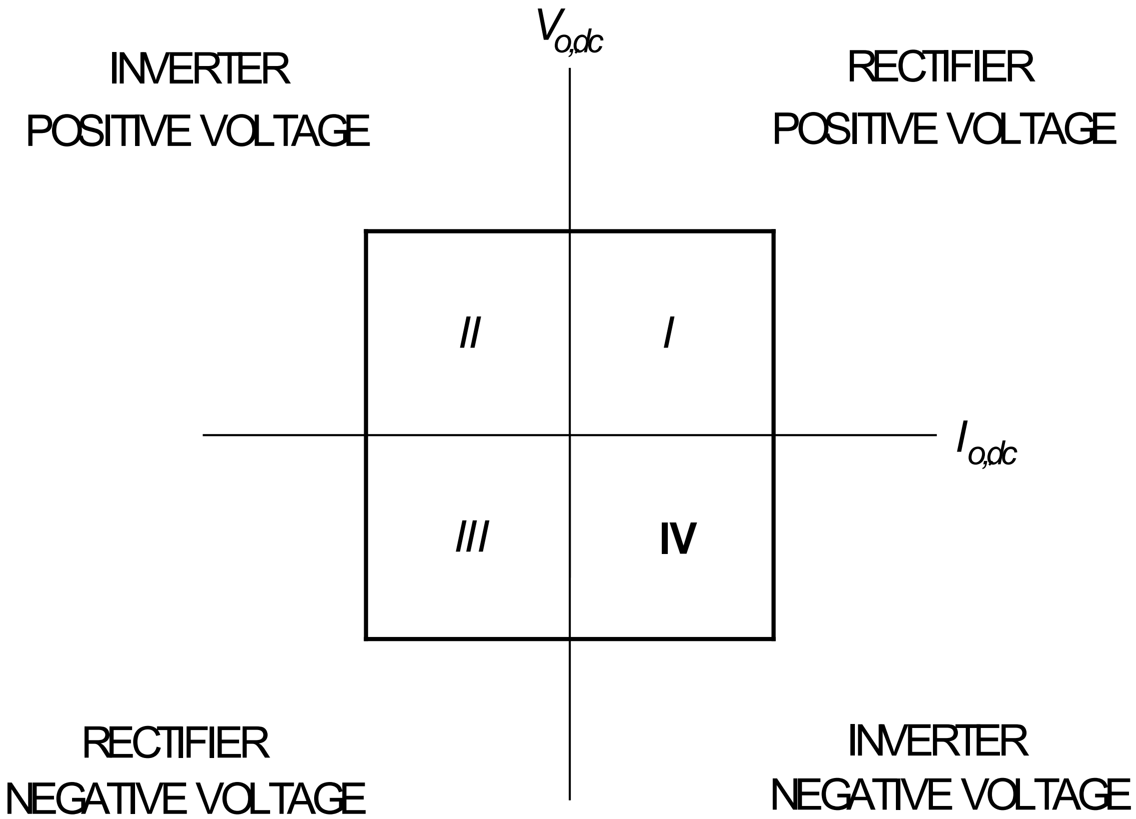


Fig. 4.29

Controlled rectifier with a cross-switch

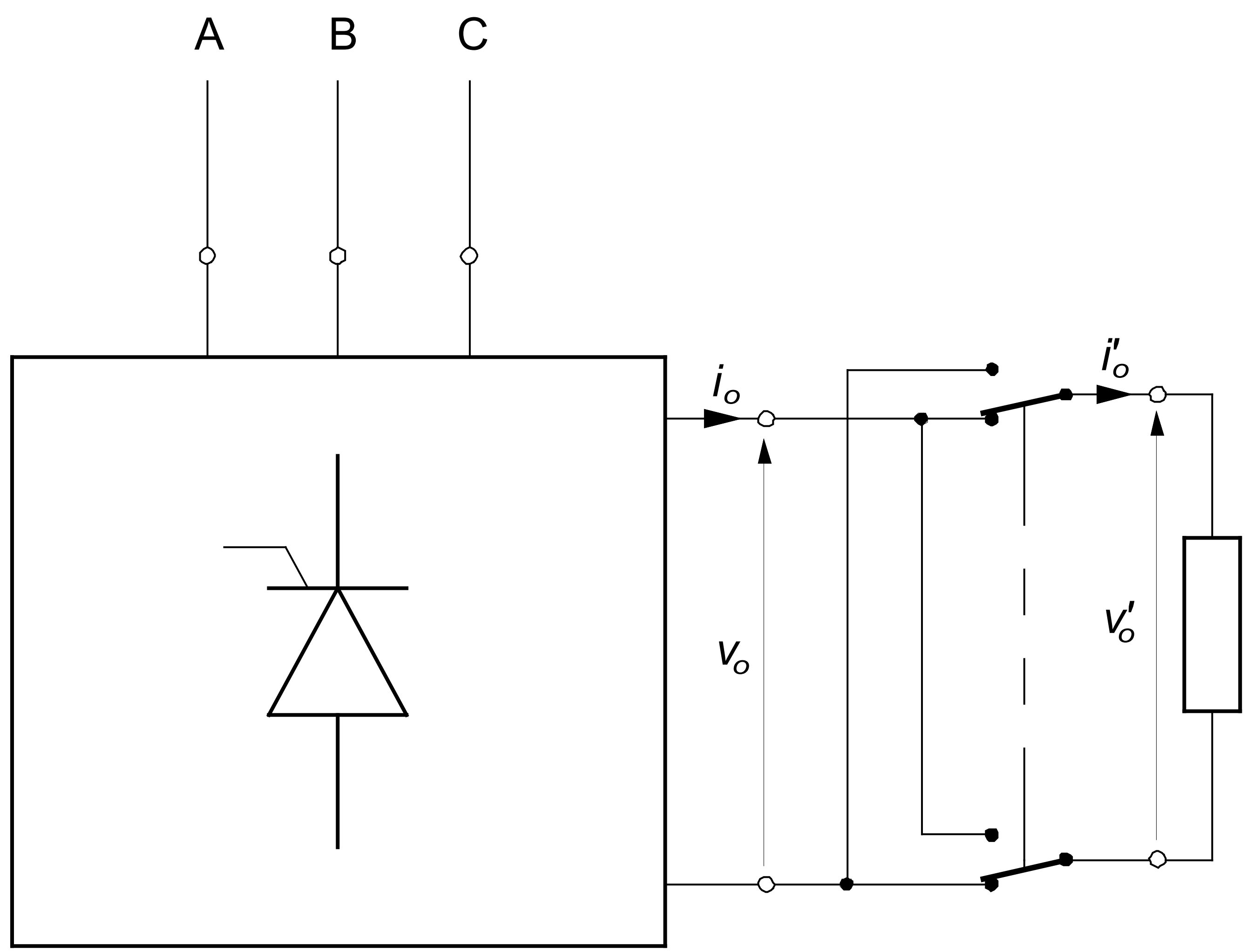


Fig. 4.30

Antiparallel connection of two controlled rectifiers

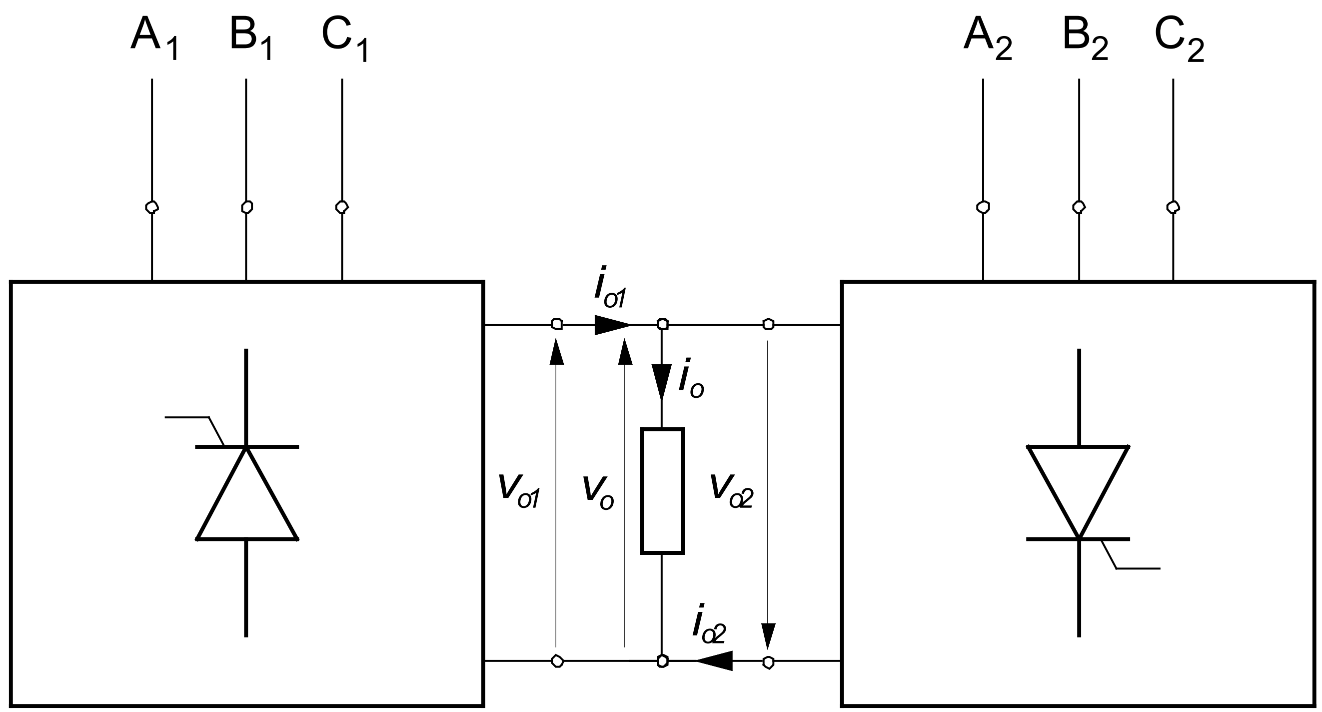


Fig. 4.31

Summary of the module

- In diode rectifiers average of the dc output voltage cannot be adjusted.
- In thyristor converters dc-voltage output can be controlled by phase-modulation, ie by using a delay angle. Voltage average depends on the cos of the delay.
- As the delay increases also the reactive power needed from the ac system increases. Reactive power depends on the sin of the delay angle.
- Line-current of the rectifiers is not sinusoidal. The harmonics are on the frequencies that are multiples of line frequency. These are of six plus/minus one, i.e., 5, 7, 11, 13,… etc. if the rectifier is three-phase six-pulse rectifier. Amplitudes of the harmonics are decreasing as the harmonic order increases. They are the fundamental divided by the harmonic order number.
- In many cases, the filtering on the dc-side is not infinite, ie the inductance on the dc-side is small or it is missing totally. In these cases, the dc-current can be continuous or discontinuous. Also, possible voltage source or large dc-capacitor has an effect on the operating mode.
- Inductance on the ac-side of the rectifiers slows down the commutation of the current, line-current cannot drop to zero instantaneously. This causes commutation delay but also reduces di/dt of thyristors and helps in this respect. Commutation delay increases if line-inductance increases. Commutation causes a small drop on the dc-voltage too.
- In diode rectifiers polarity of dc-side voltage and current are always positive and power flows from the ac-grid to the dc-side.
- In thyristor converters polarity of dc-current is always positive but dc-voltage can be reversed by increasing control delay to be more than 90 degrees. In this case power flows from dc to ac-grid and the dc side needs to have a source that provides this power.
- For example, in dc-machine drives we need four quadrant operation, and this is achieved by antiparallel connection of two thyristor converters. One is for positive current and the other one for negative current. Voltage is controlled by the delay angle.