Module 4

Analysis of power electronics based on space-vector theory

Learning outcomes of the module

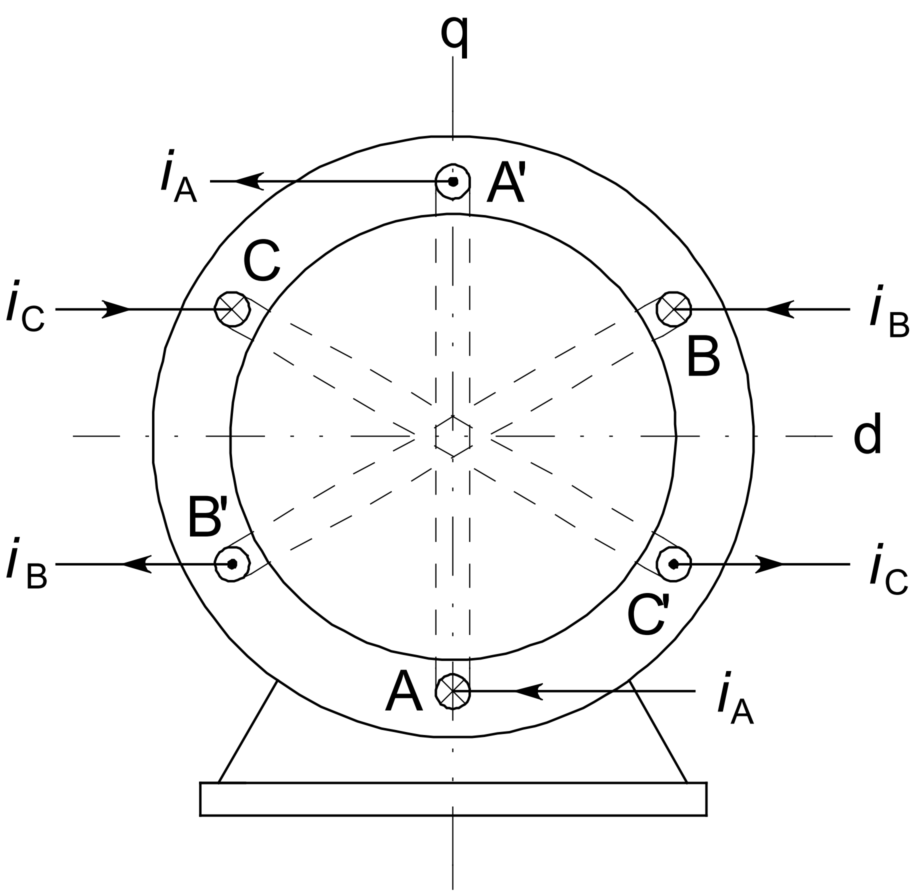
After the module you will be able to:

- understand the concept of space-vector
- can analyze three-phase circuits with space-vector
- can follow space-vector pulse-width modulation discussed in later modules

Introduction

- Space-vector is an instantaneous complex valued vector describing the three-phase system
- Space-vector is used
  - analysis of ac-machines, transients
  - necessary in the control of ac-machines, vector control methods
  - used in the analysis of VSI, Pulse Width Modulation
  - in some methods modulation and motor control are combined, DTC (direct torque control)

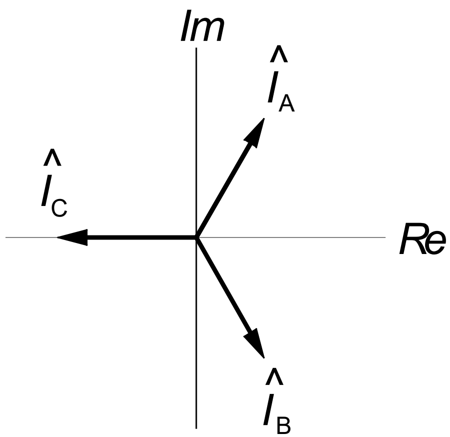
Stator of a three-phase electric ac machine



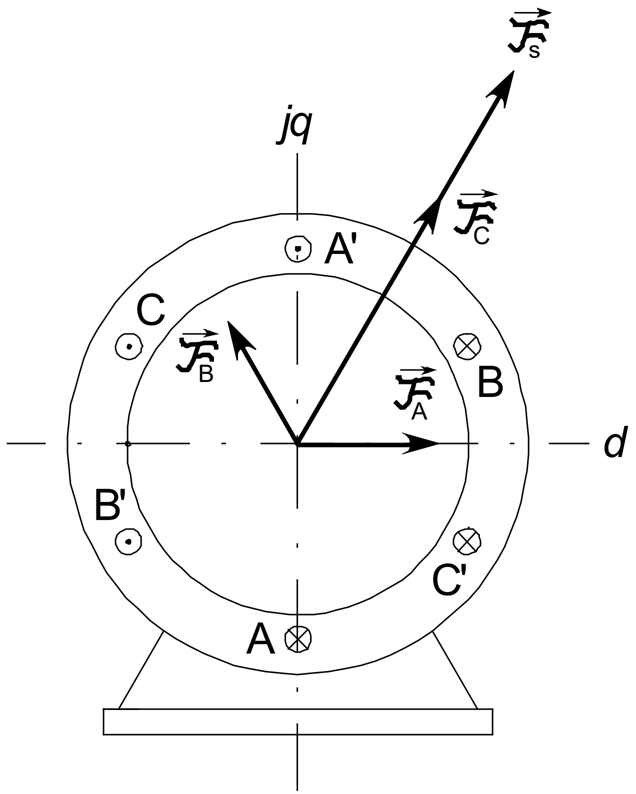
Generation of a space vector in a three-phase electric ac machine

a) phasor diagram of stator currents

b) vectors of magnetomotive force, Fs



(a)



(b)

Space-vector

The space vector $\vec{\mathcal{F}}_s$ is given by

$$\vec{\mathcal{F}}_s = \mathcal{F}_{as} + \mathcal{F}_{bs} e^{j120^{\mathrm{o}}} + \mathcal{F}_{cs} e^{j240^{\mathrm{o}}}$$

An MMF is a product of number of turns in a coil and current in the coil. Therefore, dividing an MMF space vector by the turn number gives a current space vector, $\vec{i}$:

$$\vec{i} = \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}.$$

The concept of current space vectors can be extended on voltages, e.g.,

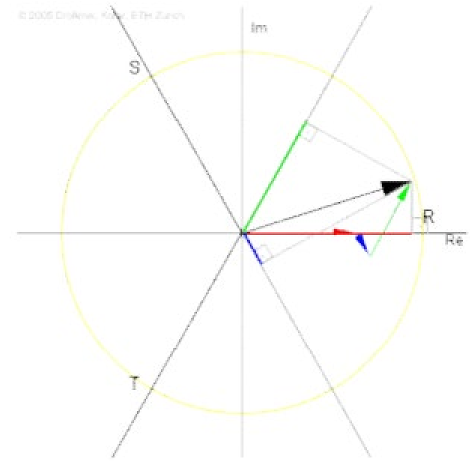
$$\vec{v} = \begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_{AN} \\ v_{BN} \\ v_{CN} \end{bmatrix}$$

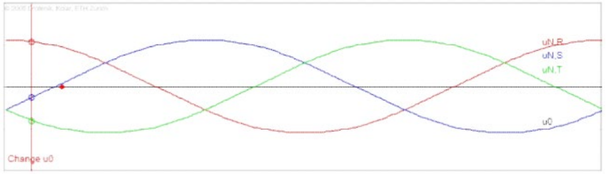
Note 1: More often in litterature space vector is scaled with 2/3 so that its length is equal to the peak value

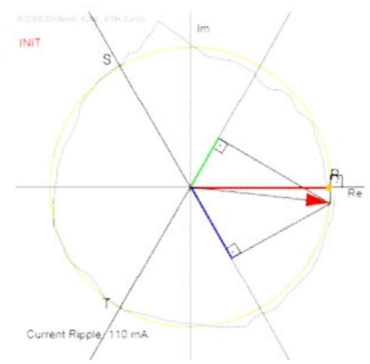
Note 2: Very often the real and imaginary parts are represented with a and b and not with d and q as here

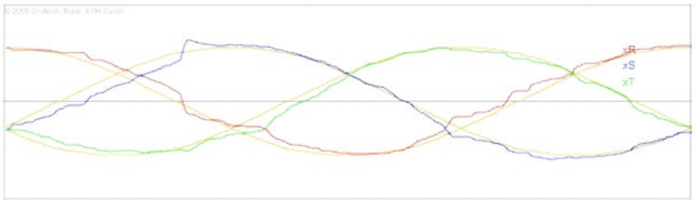
Animation on space-vector

- An animation showing the creation of space-vector from three sinusoidal voltages is shown here (enroll as a guest)
  - In the animation it is possible to add a common mode voltage to all of the three phases, i.e. voltage which is the same in all phases
  - Negative frequency means that the system is rotating in counter clock direction
- Back-transformation from space-vector to phase quantities is shown in this animation
  - Space vector is instantaneous, i.e. therefore its amplitude doesn't need to be constant neither speed of rotation => results are not sinusoidal

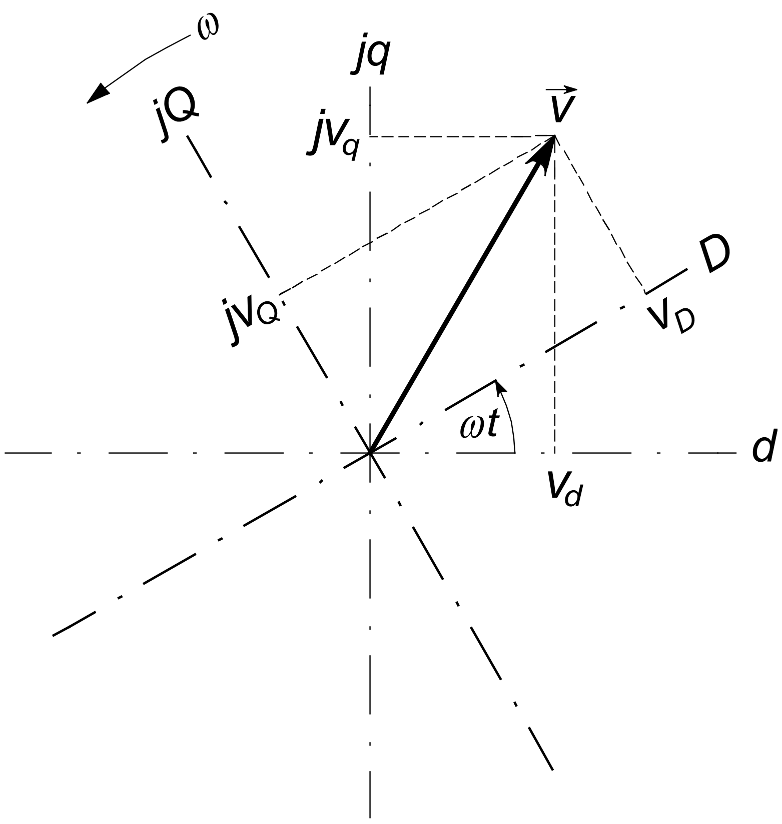








Voltage space-vector in the stationary and rotating reference frames



- Reference frame marked with d and q is fixed, i.e. it is not rotating
- Reference frame marked with D and Q rotates with angular speed ω
- Coordinate transformation is only an exponential multiplication by the angle
- Sinusoidal ac voltages and currents are creating a rotating vector in dq-coordinate system but in DQ-coordinates result is constant, i.e. dc
  - **Different control methods are easier to implement with dc-quantities**

Coordinate transformation

- Consider a voltage space vector $\vec{v}$ expressed in the stationary *dq* reference frame as $\vec{v} = v_d + jv_q$.
- The same vector in a reference frame *DQ*, rotating with the speed of ω is given by $\vec{v}^e = \vec{v}e^{-j\omega t} = v_D + jv_Q$
- and the relation between *dq* and *DQ* components is

$$\begin{bmatrix} v_D \\ v_Q \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} \begin{bmatrix} v_d \\ v_q \end{bmatrix}.$$

- As $\vec{v} = \vec{v}^e e^{j\omega t}$ and then

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{bmatrix} \begin{bmatrix} v_D \\ v_Q \end{bmatrix}$$

- The described concept of rotating reference frame is illustrated in the previous page.

Summary of the module

- Space-vector is a tool used in the analysis of three-phase ac machines and power electronics. Most advanced control methods in these applications are based on space-vectors.
- Space-vector is an instantaneous vector in the complex plane and cannot present common-mode voltages of the system.
- Especially in control applications in is advantageous to do the analysis in synchronous coordinate system as in steady state the controlled variables are dc and thus there is no delay in the system. For this, we need the coordinate transformation.