

Tutorial 1: DFA, NFA

pre tutorial

① Define NFA and DFA normally:

DFA:

for each input symbol one can determine the state to which the machine will move as it has a finite number of states the machine is deterministic finite Automata

DFA can be represented by 5-tuple

(Q, E, S, q_0, F) where

Q - finite set of states

E - input alphabet

S - transition function

q_0 is initial state

F is non empty set of final state

NFA:

The finite automata are called NFA when there exist many paths for specific input from the current state to next state each NFA can be translated into DFA but every NFA is not DFA

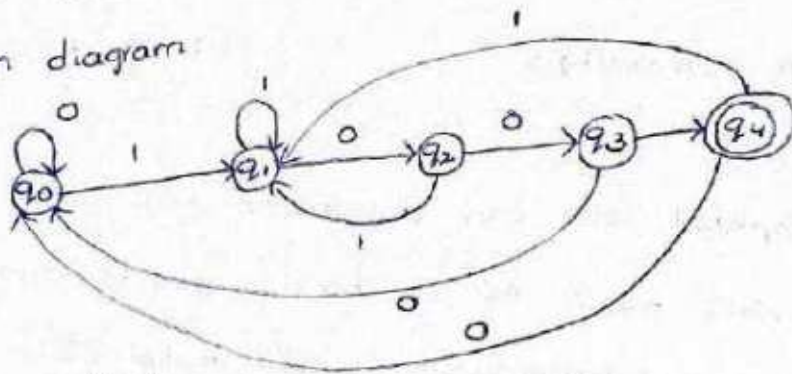
The two exceptions are:

- * It contains multiple next states

- * It contains ϵ transitions

- ② construct a DFA that accepts the language $L = \{\omega \in \{0,1\}^* / \omega \text{ contains } 1001 \text{ or } 0110\}$

Transition diagram



Transition table:

| $q \backslash \epsilon$ | 0 | 1 |
|-------------------------|-------|-------|
| $\rightarrow q_0$ | q_0 | q_1 |
| q_1 | q_2 | q_1 |
| q_2 | q_3 | q_0 |
| q_3 | q_0 | q_1 |
| q_4 | q_0 | q_2 |

- ③ write the steps for converting E-NFA to DFA and vice versa with an example for each:

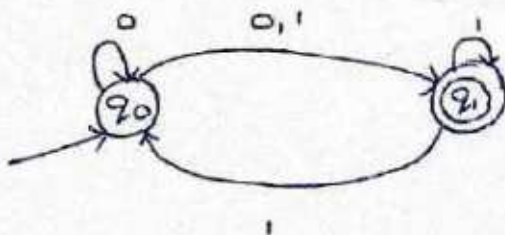
A. Step ①: - Initially $Q' = \emptyset$

Step ②: - Add q_0 of NFA to Q' then find the transitions from this start state

Step ③: In Q' find possible set of states for each input symbol. if this set of states is not in Q' , then add it to Q'

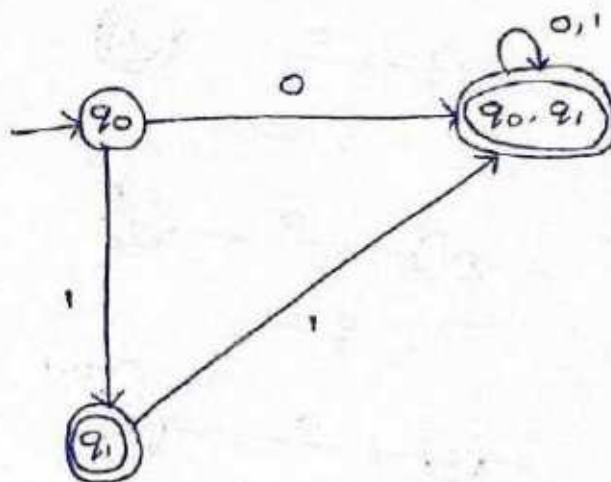
Step ④: In DFA, the final state will be all states which contain f (final states of NFA)

Eg:-



| | 0 | 1 |
|---------|----------------|----------------|
| q_0 | $\{q_0, q_1\}$ | $\{q_1\}$ |
| *q_1 | ϕ | $\{q_0, q_1\}$ |

| | 0 | 1 |
|----------------|----------------|----------------|
| q_0 | $\{q_0, q_1\}$ | $\{q_1\}$ |
| *q_1 | ϕ | $\{q_0, q_1\}$ |
| $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ |



steps for converting DFA to NFA

step ①: Let's assume DFA D has state set

$$Q = \{q_0, q_1, \dots, q_n\}$$

step ②: Now we build NFA N as follows

(i) start with DFA D

(ii) Add an additional accepting state for NFA N , such

that N will have $n+1$ total no. of states

lets call new accepting state q_{n+1}

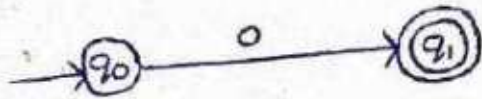
(iii) Now, add an epsilon ϵ transition from all accepting state q_{n+1} and make all the original accepting states just normal states

Eg:- $\epsilon = \{0, 1\}$ starts with '0'

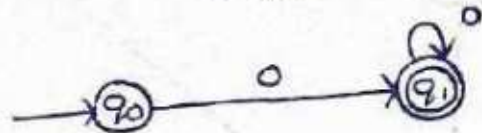
$L = \{ 0, 00, 01, 000, 001, 010, 0010, 0101, \dots \}$



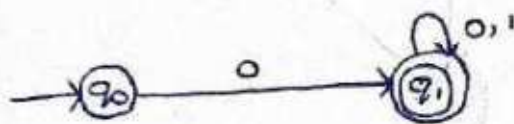
$\epsilon = 0$



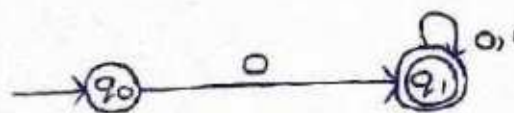
$\epsilon = 00$



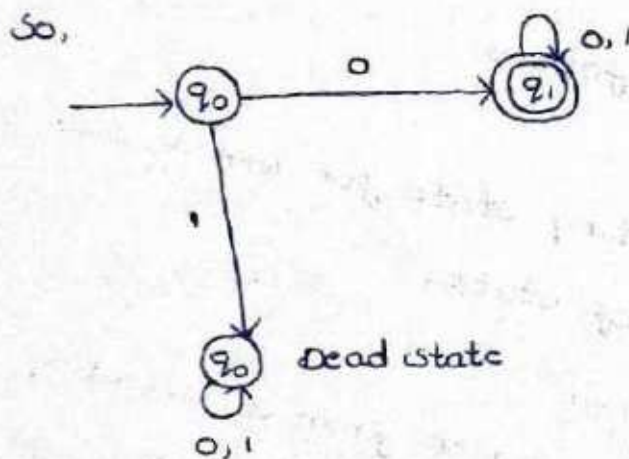
$\epsilon = 01$



$\epsilon = 000$

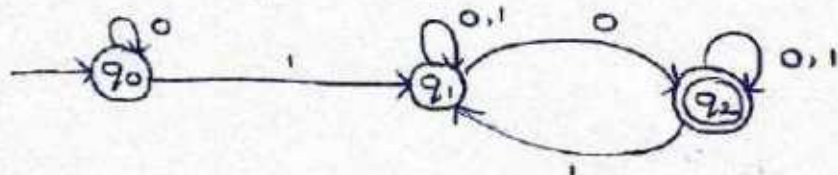


$\epsilon = 001$ (we cannot draw)



In tutorial

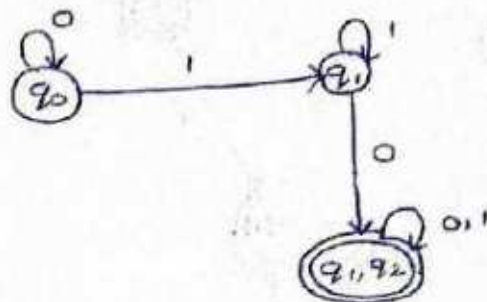
① convert the following NFA to DFA



| | 0 | 1 |
|-------|----------------|----------------|
| q_0 | q_0 | q_1 |
| q_1 | $\{q_1, q_2\}$ | q_1 |
| q_2 | q_2 | $\{q_1, q_2\}$ |

Now,

| | 0 | 1 |
|----------------|----------------|----------------|
| q_0 | q_0 | q_1 |
| q_1 | $\{q_1, q_2\}$ | q_1 |
| $\{q_1, q_2\}$ | $\{q_1, q_2\}$ | $\{q_1, q_2\}$ |



② Write the algorithm that converts NFA to DFA. explain your algorithm works using the below NFA?

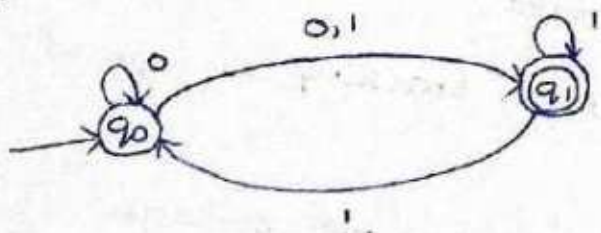
A- Algorithm :-

step ①: initially $Q' = \phi$

step ②: Add q_0 of NFA to Q' of DFA

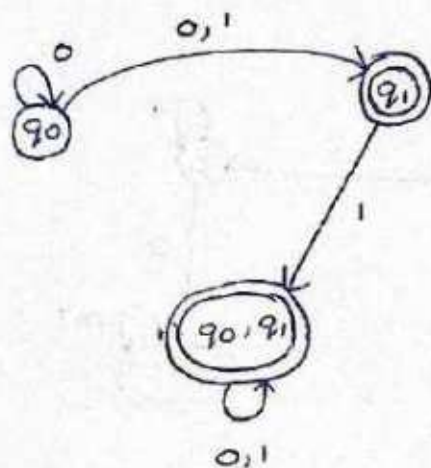
step ③: In Q' , find possible states for each input symbol if this set of states is not in Q' , then add it to Q'

step ④: In DFA, the final state will be all statements which contain F (final state of NFA)



| | 0 | 1 |
|---------|----------------|----------------|
| q_0 | $\{q_0, q_1\}$ | $\{q_1\}$ |
| q_1^* | ϕ | $\{q_0, q_1\}$ |

| | 0 | 1 |
|----------------|----------------|----------------|
| q_0 | $\{q_0, q_1\}$ | $\{q_1\}$ |
| q_1 | ϕ | $\{q_0, q_1\}$ |
| $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_1\}$ |



post tutorial :

① Differentiate NFA and DFA .

NFA

① $(Q, \Sigma, \delta, q_0, F)$

$\delta = Q \times \Sigma \rightarrow 2^Q$

② $Q = \{q_0, q_1\}, \Sigma = \{0, 1\}$

$q_0 \rightarrow 0 \quad q_1 \rightarrow 1$

③ Transition may leads to multiple states

④ Back Tracking is not required

⑤ practical implementation of DFA is feasible

DFA

① $(Q, \Sigma, \delta, q_0, F)$

$\delta = Q \times \Sigma \rightarrow Q$

② $Q = \{q_0, q_1\}, \Sigma = \{0, 1\}$



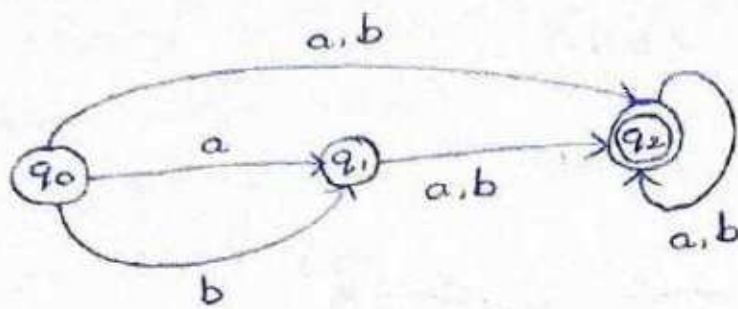
③ Transition leads to unique state

④ Back tracking is required

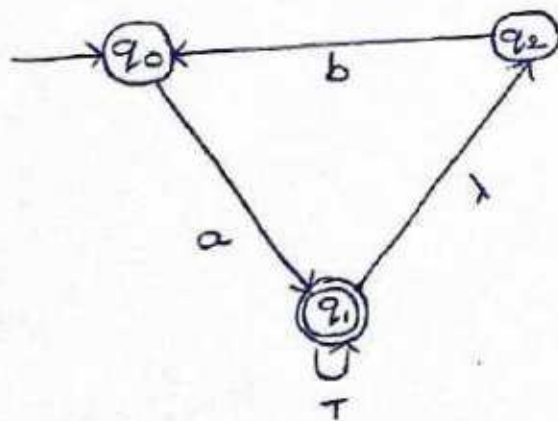
⑤ Not feasible, convert NFA to DFA

② construct NFA for language $L = \{\omega \in \{0,1\}^* / a^* + b^*\}$

$$L = \{a, aa, aaa, b, bb, bbb, aaaa, \dots\}$$

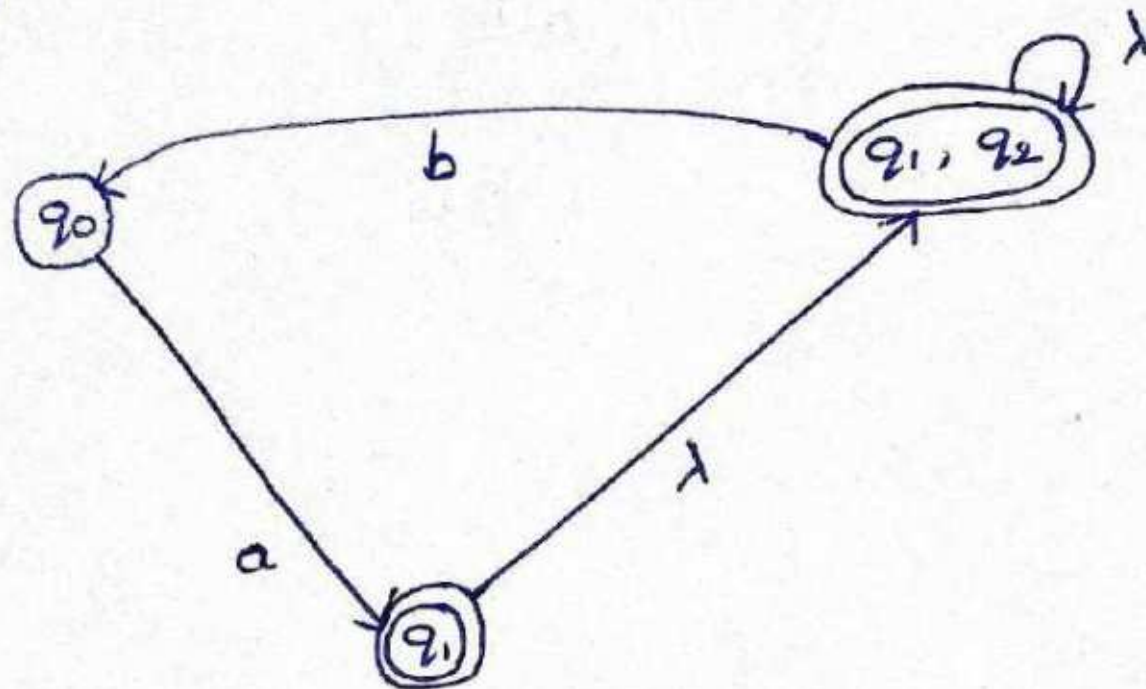


③ convert the following E-NFA to DFA



| | a | b | λ |
|-----------------------------|----------------|----------------|------------------------------------|
| q ₀ | q ₁ | φ | φ |
| q ₁ | φ | φ | {q ₁ , q ₂ } |
| q ₂ [*] | φ | q ₀ | φ |

| | a | b | λ |
|------------------|--------|--------|----------------|
| q_0 | q_1 | ϕ | ϕ |
| q_1 | ϕ | ϕ | $\{q_1, q_2\}$ |
| $\{q_1, q_2\}^*$ | ϕ | q_0 | $\{q_1, q_2\}$ |



Tutorial 2: Regular expression

pre tutorial:

- ① Explain regular and name some of identity rules for the regular expression? Assume a, b and c are regular expressions in the identity rules

Regular expression:

it is used for representing certain sets of strings in algebraic fashion

identity rules:

$$\phi + r = r$$

$$\phi \cdot r = r \cdot \phi = \phi$$

$$\epsilon \cdot r = r \cdot \epsilon = r$$

$$r + r = r$$

$$r^* \cdot r^* = r^*$$

$$r^* r = r r^* = r^*$$

$$(r^*)^* = r^*$$

$$R R^* = R^* R = R$$

$$\epsilon + r^* r = \epsilon + r r^* = r^*$$

$$(ab)^* a = a (ba)^*$$

$$(a+b)^* = (a^* b^*)^* \\ = (a^* + b^*)^*$$

$$(a+b)c = ac + bc$$

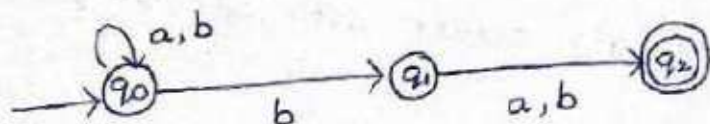
$$\epsilon^* = \epsilon$$

$$\phi^* = \epsilon$$

② consider language L given by regular expression $(a+b)^* b(a+b)$ over the alphabet $\{a, b\}$ design a DFA that accepts L

convert RE into NFA and find DFA from NFA

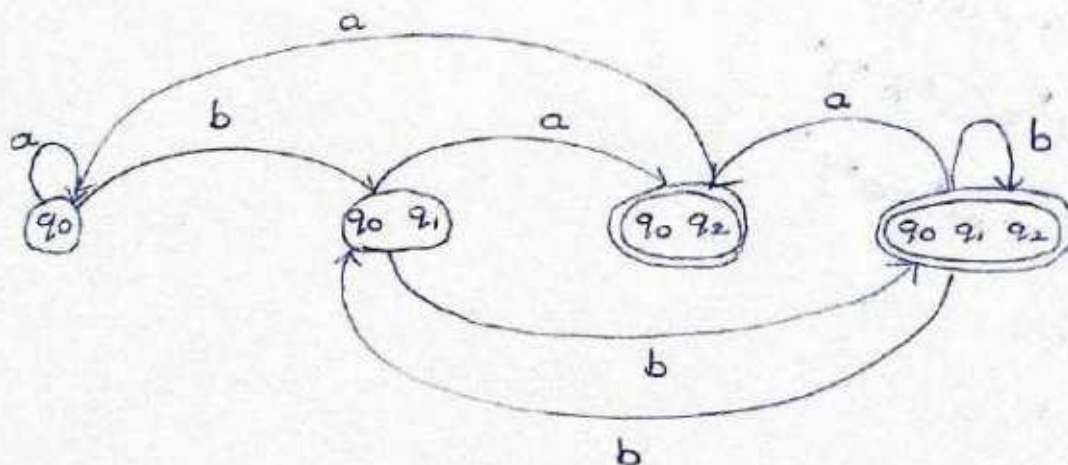
$$RE = (a+b)^* b(a+b)$$



| | a | b |
|----------------|----------------|------------------------------------|
| q ₀ | q ₀ | {q ₀ , q ₁ } |
| q ₁ | q ₂ | q ₂ |
| q ₂ | φ | φ |

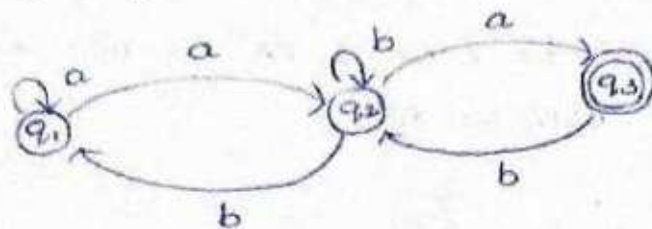
DFA table from NFA

| | a | b |
|--|------------------------------------|---|
| q ₀ | q ₀ | {q ₀ , q ₁ } |
| {q ₀ , q ₁ } | {q ₀ , q ₂ } | {q ₀ , q ₁ , q ₂ } |
| *{q ₀ , q ₂ } | q ₀ | {q ₀ , q ₁ } |
| *{q ₀ , q ₁ , q ₂ } | {q ₀ , q ₂ } | {q ₀ , q ₁ , q ₂ } |



In-tutorial

- ① provide algorithm to convert NFA into RE
create a RE for following NFA



$$q_1 = q_1 a + q_2 b$$

$$q_2 = q_2 b + q_1 a$$

$$q_3 = q_2 a$$

Now,

$$q_1 = q_1 a + q_2 b + \epsilon \quad \text{--- ①}$$

$$q_2 = q_1 a + q_2 b \quad \text{--- ②}$$

$$q_3 = q_2 a \quad \text{--- ③}$$

Sub ② in ①

$$q_1 = q_2 + \epsilon$$

$$q_3 = (q_1 a + q_2 b + q_3 b) a$$

$$q_3 = q_1 a a + q_2 a b + q_3 a b \quad \text{--- ④}$$

from eq ②

$$q_2 = q_1 a + q_2 b + q_3 b$$

$$q_2 = q_1 a + q_2 b + (q_2 a) b$$

$$q_2 = q_1 a + q_2 (b + ab)$$

$$\begin{matrix} R & Q & R & P \\ q_2 & = & q_1 a & + q_2 (b + ab) \end{matrix}$$

$$q_2 = (q_1 a) (b + ab)^* \quad \text{--- ⑤}$$

from eq ③

$$q_1 = \epsilon + q_1 a + q_2 b$$

$$q_1 = \epsilon + q_1 a + ((q_1 a) (b + ab)^*) b$$

$$q_1 = \epsilon + q_1 (a (b + ab)^*) b$$

$$\begin{matrix} R & Q & R & P \\ q_1 & = & \epsilon & + q_1 (a (b + ab)^*) b \end{matrix}$$

$$q_1 = \epsilon ((a + a (b + ab)^*) b)^*$$

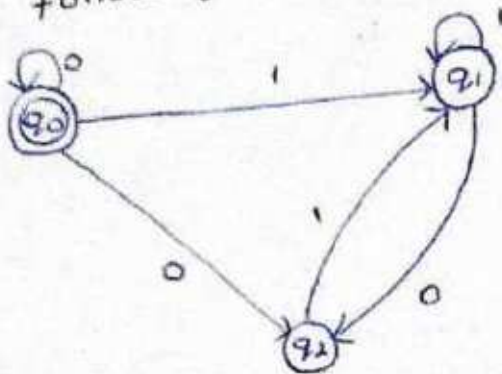
$$q_1 = (a + a (b + ab)^* b)^* \quad \text{--- ⑥}$$

$$\boxed{ER = R}$$

final state $q_3 \Rightarrow q_3 = q_2 a$

$$q_3 = (a + a(b+ab)^*b)^* a(b+ab)^* a$$

② explain arden's theorem to convert FA to RE use algorithm to convert following DFA to RE



Arden's theorem:-

If P and Q are 2 RE over Σ and if P doesn't contain ϵ then the following equation in R by $R = Q + RP$ has a unique solution

$$R = QP^*$$

Steps

- 1) for each state q_1, q_2, q_3, \dots all expts that comes into state written in equation format
- 2) Add epsilon to initial state
- 3) calculate all equations
- 4) Result is value of final state

Solution:

$$q_0 = q_0 0 + \epsilon \quad \text{--- (1)}$$

$$q_1 = q_0 1 + q_1 1 \quad \text{--- (2)}$$

$$q_2 = q_0 0 + q_1 0 \quad \text{--- (3)}$$

from (2) and (3)

$$q_1 = q_01 + q_11 + q_101$$

$$q_1 = q_01 + q_1(1+01)$$

$$R \quad Q \quad R \quad P$$

$$q_1 = q_01(1+01)^* \quad \text{--- (4)}$$

from (1) and (3)

$$q_0 = q_00 + q_10 + \epsilon \quad \text{--- (5)}$$

from (4) and (5)

$$q_0 = q_00 + q_01(1+01)^*00 + \epsilon$$

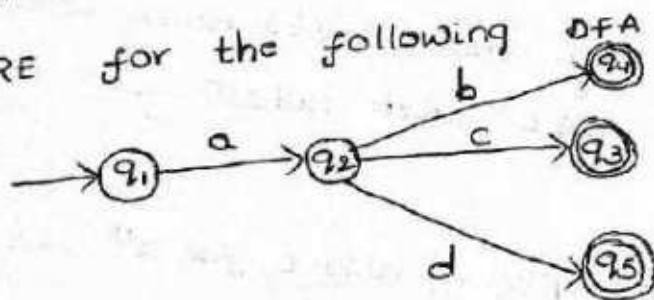
$$q_0 = \epsilon + q_0(0+1(1+01)^*00)$$

$$q_0 = \epsilon(0+1(1+01)^*00)$$

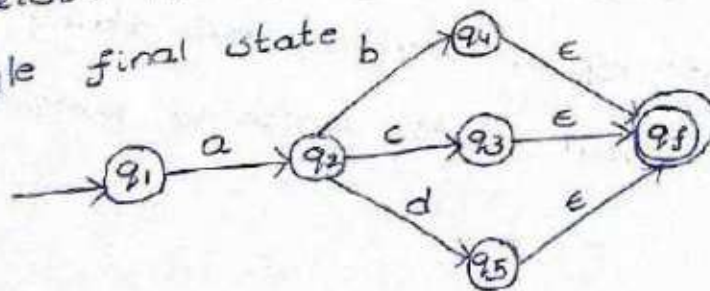
$$q_0 = 0+1(1+01)^*00$$

past tutorial

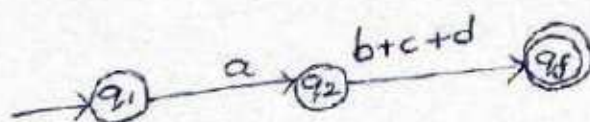
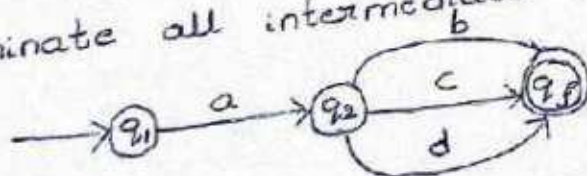
① find RE for the following DFA



There exists multiple final states so we convert them into a single final state



eliminate all intermediate stages q_4, q_3, q_5



lets eliminate q_2

~~$\rightarrow q_1 a(b+c+d) q_f$~~

Regular expression = $a(b+c+d)$

② for $\Sigma = \{a, b\}$ let us consider the regular language
 $L = \{x/x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$ what could be the minimum
pumping length (the constant guaranteed by the pumping
lemma) for L ?

A) $L = \{x/x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$

$L = \{a^2, a^5, \dots\} \cup \{b^{10}, b^{22}, \dots\}$

pumping Lemma:

let L be an infinite RL then there exists some positive
integer n such that any $w \in L$ with $|w| \geq n$ can be
decomposed as $w = xyz$

with $|xy| \leq n$ such that $w_i = xy^i z$ is also L for all $i = 0, 1, 2, \dots$

\therefore minimum pumping length should be 11, because string with
length 10 ($w = b^{10}$) does not repeat anything, but string with
length 11 (i.e., $w = b^{11}$) will repeat states length of pumping lemma

Tutorial - 3:

pre tutorial:

- ① what is context free grammar. explain with an example?
context free grammar is a formal grammar which is used to generate all possible strings in a given formal language
it can be defined by four steps

$$G = (V, T, P, S)$$

where G = Grammar

V = Set of non terminal symbols

T = finite set of terminal symbols

P = production rules

S = start symbol

Example:

$$L = \{wcw^R \mid w \in (a,b)^*\}$$

production rules: $s \rightarrow asa$

$s \rightarrow bsb$

$s \rightarrow c$

Now check the string $abcbba$ string can be derived from the given CFG

$s \rightarrow asa$

$s \rightarrow absba$

$s \rightarrow abbsbba$

$s \rightarrow abcbba$

by applying production $s \rightarrow asa$, $s \rightarrow bsb$ recursively and finally

the production $s \rightarrow c$ we get string $abcbba$

In tutorial:

- ① construct a CFG for a language

$$L = \{wcw^R / w \in (a,b)^*\}$$

$$L = \{aa, bb, abba, aabbba, ababba, \dots\}$$

production rules

$$S \rightarrow asa$$

$$S \rightarrow bsb$$

$$S \rightarrow \epsilon$$

Now check the string abbcbbba string can be derived from the given CFG

$$S \rightarrow asa$$

$$S \rightarrow absbba$$

$$S \rightarrow abbsbba$$

$$S \rightarrow abbcbbba$$

$$\text{string} = abbcbbba$$

- ② Derive the string "aabbabba" for leftmost derivation and right most derivation using a CFG

$$S \rightarrow aB/bA$$

$$A \rightarrow a/as/bAA$$

$$B \rightarrow b/bs/aBB$$

"aabbabba"

Left most derivation

S

aB

$$S \rightarrow aB$$

aaBB

$$B \rightarrow aBB$$

aabbB

$$B \rightarrow b$$

aabbbs

$$B \rightarrow bs$$

aabbbaB

$$S \rightarrow aB$$

aabbabs

$B \rightarrow bs$

aabbabba

$S \rightarrow bA$

aabbabba

$A \rightarrow a$

Right most derivation:

S

$S \rightarrow aB$

aB

$B \rightarrow aBB$

aaBB

$B \rightarrow bs$

aaBbs

$S \rightarrow bA$

aaBbba

$A \rightarrow a$

aaBbba

$B \rightarrow bs$

aabsbba

$S \rightarrow bA$

aabbabba

$A \rightarrow a$

aabbabba

post tutorial

① Generate CFG for the language

$$L = \{0^i 1^j 0^k \mid i \geq j + k\}$$

$$L = \{0^i 1^j 0^k \mid i \geq j + k, i, k \geq 1\}$$

0 1 1 1 0

$S \rightarrow XYZ$

$X \rightarrow 0X1 \mid 01$

$Y \rightarrow 1Y \mid 1$

$Z \rightarrow 1Z0 \mid 10$

$S \rightarrow XYZ$

$\rightarrow 0X1YZ$

$X \rightarrow 0X1$

$\rightarrow 0011YZ$

$X \rightarrow 01$

$\rightarrow 00111YZ$

$Y \rightarrow 1Y$

$\rightarrow 001111Z$

$Y \rightarrow 1$

$\rightarrow 00111110$

$Z \rightarrow 10$

$i=2, j=5, k=1$

$S \rightarrow 3$

$i \geq j + k$

Tutorial 4 - parse tree, ambiguity CFG

pre tutorial

① Differentiate ambiguous and unambiguous grammar

ambiguous

- ① The leftmost and rightmost derivations are not same
- ② Amount of non-terminals in ambiguous grammar is less than unambiguous grammar
- ③ length of parse tree is short
- ④ it generates more than one parse tree
- ⑤ it contains ambiguity

unambiguous

- ① The leftmost and rightmost derivations are same
- ② Amount of non-terminals in unambiguous grammar is greater than ambiguous grammar
- ③ length of parse tree is large
- ④ it generates only one parse tree
- ⑤ it does not contain ambiguity

In tutorial

① consider the following grammar

$$S \rightarrow ASB|c$$

$$A \rightarrow \epsilon | aA$$

$$B \rightarrow \epsilon | bB$$

Derive the string acb using leftmost and rightmost derivation
show the parse tree for your derivation

LMD:

S

ASB

$S \rightarrow ASB$

aASB

$A \rightarrow aA$

aESB

$A \rightarrow E$

aECB

$S \rightarrow C$

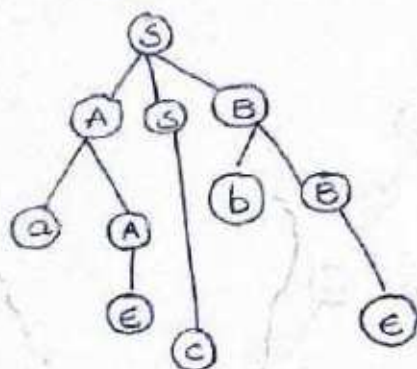
aECbB

$B \rightarrow bB$

aECbE

$B \rightarrow E$

acb



RMD

S

ASB

$S \rightarrow ASB$

ASbB

$B \rightarrow bB$

ASbE

$B \rightarrow E$

AcbE

$S \rightarrow C$

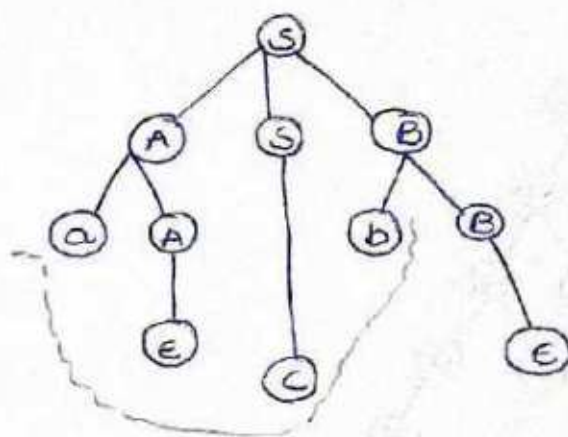
aAcbE

$A \rightarrow aA$

aECbE

$A \rightarrow E$

acb



② consider the following grammar

$S \rightarrow as/e$

The language generated by this grammar

$L = \{a^n, n \geq 0\} \text{ or } a^*$

i) find the LMD and RMD

ii) Also, prove All the strings generated from this grammar have their LMD and RMD exactly same draw the parse of the same

Let us consider string $w = aaa$

i) LMD

$S \rightarrow as/E$

S

as

$S \rightarrow as$

aaas

$S \rightarrow as$

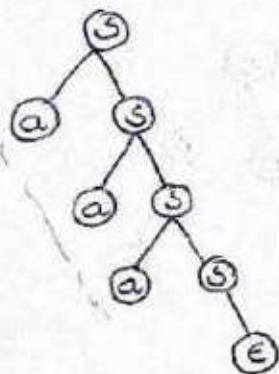
aaaa

$S \rightarrow as$

aaaE

$S \rightarrow E$

aaa



aaa

RMD

$S \rightarrow as/E$

S

as

$(S \rightarrow as)$

aaas

$S \rightarrow as$

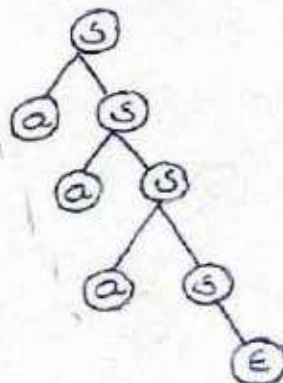
aaaa

$S \rightarrow as$

aaaE

$S \rightarrow E$

aaa



aaa

clearly

LMD parse tree = RMD parse tree

past tutorial:

① consider the following grammar

$S \rightarrow sas/b$

it is an ambiguous grammar? generate the string babab from this grammar to prove your point?

The given grammar

$$S \rightarrow sas/b$$

$$w = babab$$

LMD

S

sas

$$S \rightarrow sas$$

sasas

$$S \rightarrow sas$$

basas

$$S \rightarrow b$$

babas

$$S \rightarrow b$$

babab

$$S \rightarrow b$$

RMD

S

sas

$$(S \rightarrow sas)$$

sasas

$$S \rightarrow sas$$

sasab

$$S \rightarrow b$$

sabab

$$S \rightarrow b$$

babab

$$S \rightarrow b$$

LMD

S

sas

$$S \rightarrow sas$$

bas

$$S \rightarrow b$$

basas

$$S \rightarrow sas$$

babas

$$S \rightarrow b$$

babab

$$S \rightarrow b$$

RMD

S

sas

$$S \rightarrow sas$$

sab

$$S \rightarrow b$$

sasab

$$S \rightarrow sas$$

sabab

$$S \rightarrow b$$

babab

$$S \rightarrow b$$

more than one left most parse tree
more than one right most parse tree
more than one parse tree

\therefore The given grammar is ambiguous

Tutorial-5: Simplification of CFG, normal forms

pre tutorial

- ① Explain simplification of grammar: mention its use: elaborate the steps that are followed in simplification process:

Simplification of grammar

It means reduction of grammar by removing useless symbols each variable and each terminal of G appears in the derivation of some word in L

* There should not be any production as $x \rightarrow y$ where x and y are non-terminal

* If ϵ is not in language L there not be production $x \rightarrow \epsilon$

Reduced grammar

→ Removal of useless grammar

→ elimination of ϵ production

→ Removal of unit production

Removal of useless grammar:

A variable can be useless if it does not take part in derivation of any string take part in derivation

eg: $T \rightarrow aaB / aBA / aAT$

$A \rightarrow aA$

$B \rightarrow ab / d$

$C \rightarrow ad$

here $C \rightarrow ad$ is useless, $A \rightarrow aA$ is useless, $A \rightarrow aA$ is useless

To remove $A \rightarrow aA$ we will find first all variables which will never lead to a terminal string such as variable 'A'

Then we will remove all productions in which the variable 'B' occurs

elimination of ϵ production.

$S \rightarrow \epsilon$ are called ϵ productions

step ① find out all nullable non-terminal variable which derives ϵ

step ②: for each production $A \rightarrow \alpha$ construct all production $A \rightarrow \alpha'$ where α' is obtained from α by removing one or more non-terminal from step ①

step ③. Now combine the result of step 2 with original production and remove ϵ productions

Eg. $S \rightarrow xyx$

$x \rightarrow \epsilon x | \epsilon$

$y \rightarrow y | \epsilon$

Let us take

$S \rightarrow xyx$

↓

ϵyx

yx

$S \rightarrow xyx$

↓

$x \epsilon x$

xx

if both x are ϵ

$S \rightarrow y$

Now,

$S \rightarrow xy | yx | xx | x | y$

consider $x \rightarrow \epsilon x$

$S \rightarrow xyx$

↓

$\epsilon y \epsilon$

xy

if y and x are ϵ $S \rightarrow x$

replace ϵ at RHS for x then

$$x \rightarrow 0$$

$$x \rightarrow 0x10$$

Similarly $y \rightarrow 1y1$

Rewrite the CFG as

$$S \rightarrow xy | yx | xy | xy$$

$$x \rightarrow 0x$$

$$y \rightarrow 1y1$$

Removing unit productions

There are productions in which one non-terminal gives another non-terminal

Step ①: To remove $x \rightarrow y$ add production $x \rightarrow a$ to grammar rule whenever $y \rightarrow a$ occurs in this grammar

Step ②: Now, delete $x \rightarrow y$ from grammar

Step ③: Repeat step 1 and step 2 until all unit productions are removed

Eg. $S \rightarrow 0A | 1B | C$

$$A \rightarrow 0s | 00$$

$$B \rightarrow 1/A$$

$$C \rightarrow 01$$

$S \rightarrow C$ is unit production, by removing $S \rightarrow C$ add a rule to S

$$S \rightarrow 0A | 1B | 01$$

$B \rightarrow A$ is also a unit production

$$B \rightarrow 1/0s | 00$$

Rewrite CFG

$$S \rightarrow 0A/1B/01$$

$$A \rightarrow 00/00$$

$$B \rightarrow 1/00/00$$

$$C \rightarrow 01$$

② find a reduced grammar equivalent to the grammar G having production rules

$$S \rightarrow AC/B$$

$$A \rightarrow a$$

$$C \rightarrow C/BC$$

$$E \rightarrow aA/E$$

phase 1-

$$T = \{a, c, e\}$$

$$W_1 = \{A, C, E\} \text{ from rules } A \rightarrow a, C \rightarrow C, E \rightarrow aA$$

$$W_2 = \{A, C, E\} \cup \{S\} \text{ from rule } S \rightarrow AC$$

$$W_3 = \{A, C, E, S\} \cup \emptyset$$

Since $W_2 = W_3$ we can derive G_1 as

$$G_1 = \{\{A, C, E, S\}; \{a, c, e\}, P\{S\}\}$$

where $P: S \rightarrow AC, A \rightarrow a, C \rightarrow C, E \rightarrow aA/C$

phase 2-

$$Y_1 = \{S\}$$

$$Y_2 = \{S, A, C\} \quad S \rightarrow AC$$

$$Y_3 = \{S, A, C, a, c\} \quad A \rightarrow a \text{ \& } C \rightarrow C$$

$$Y_4 = \{S, A, C, a, c\}$$

Since $Y_3 = Y_4$ we can derive G_2

$$G_2 = \{\{A, C, S\}, \{a, c\}, P\{S\}\}$$

where $P: S \rightarrow AC, A \rightarrow a, C \rightarrow C$

In tutorial:-

① Remove unit production from following grammar

$$S \rightarrow AC$$

$$A \rightarrow a$$

$$C \rightarrow x/b$$

$$x \rightarrow y$$

$$y \rightarrow z$$

$$z \rightarrow a$$

There are 3 unit productions in the grammar

$$C \rightarrow x, x \rightarrow y \text{ and } y \rightarrow z$$

At first remove $y \rightarrow z$

As $z \rightarrow a$, we add $y \rightarrow a$ and $y \rightarrow z$ is removed

So

$$S \rightarrow AC, A \rightarrow a, C \rightarrow x/b, x \rightarrow y, y \rightarrow a, x \rightarrow A$$

Now remove $x \rightarrow y$

As $y \rightarrow a$ we add $x \rightarrow a$ and $x \rightarrow y$ is removed

So

$$S \rightarrow AC, A \rightarrow a, C \rightarrow x/b, x \rightarrow a, y \rightarrow a, z \rightarrow a$$

Now remove $C \rightarrow x$

As $x \rightarrow a$ we add $C \rightarrow a$ and $C \rightarrow y$ is removed

$$S \rightarrow AC, A \rightarrow a, C \rightarrow a/b, x \rightarrow a, y \rightarrow a, z \rightarrow a$$

Now x, y, z are unreachable hence we can remove those

The final CFG is

$$S \rightarrow AC, A \rightarrow a, C \rightarrow a/b$$

② A grammar G_1 is defined with rules $S \rightarrow xA/BB$, $B \rightarrow b/AB$, $x \rightarrow b$, $A \rightarrow a$ write productions obtained after normalized G_{1NF} of G_1

$$S \rightarrow xA/BB$$

$$B \rightarrow b/AB$$

$$x \rightarrow b$$

$$A \rightarrow a$$

step ① convert grammar into CNF

step ②: if grammar exists left recursion, eliminate it

step ③: convert production rule into G_{1NF} form in the grammar

step ① and step ② already exist in question so skipping it

$$S \rightarrow xA/BB$$

$$B \rightarrow b/xAB/BBB$$

$$A \rightarrow a$$

$$x \rightarrow b$$

$S \rightarrow xA$ and $A \rightarrow xAB$ is not in G_{1NF} so substitute $x \rightarrow b$ in production rule $S \rightarrow xA$ and $A \rightarrow xAB$ as

$$S \rightarrow bA/BB$$

$$B \rightarrow b/bAB/BBB$$

$$A \rightarrow a$$

$$x \rightarrow a$$

Now remove left recursion ($B \rightarrow BBB$)

$$S \rightarrow bA/BB$$

$$B \rightarrow bc/bABC$$

$$C \rightarrow BBC/\epsilon$$

$$A \rightarrow a$$

$$x \rightarrow b$$

Now remove null production $\epsilon \rightarrow \epsilon$

$$S \rightarrow bA \mid BB$$

$$B \rightarrow bc \mid bABC \mid b \mid bAB$$

$$C \rightarrow BBC \mid BB$$

$$A \rightarrow a$$

$$x \rightarrow b$$

$S \rightarrow BB$ is not in GNF, substitute $B \rightarrow bc \mid bABC \mid b \mid bAB$ in production rule $S \rightarrow BB$

$$S \rightarrow bA \mid bcB \mid bABCb \mid b \mid bABB$$

$$B \rightarrow bc \mid bABC \mid b \mid bAB$$

$$C \rightarrow BBC$$

$$C \rightarrow bcB \mid bABCb \mid bb \mid bABB$$

$$A \rightarrow a$$

$$x \rightarrow b$$

$C \rightarrow BBC$ is not in GNF, substitute $B \rightarrow bc \mid bABC \mid b \mid bAB$ in production rule

$$C \rightarrow BBC \text{ as}$$

$$S \rightarrow bA \mid bcB \mid bABCb \mid bb \mid bABB$$

$$B \rightarrow bc \mid bABC \mid b \mid bAB$$

$$C \rightarrow bcB \mid bABCb \mid bb \mid bABB$$

$$A \rightarrow a$$

$$x \rightarrow b$$

Hence, this is GNF form for grammar G

post tutorial:

① convert the following

a) CFG into CNF

$S' \rightarrow S$

$S \rightarrow ASA \mid aB$

$A \rightarrow B \mid \epsilon$

$B \rightarrow b \mid \epsilon$

Now ϵ is removed

$S' \rightarrow S$

$S \rightarrow ASA \mid aB$

$A \rightarrow B \mid \epsilon$

$B \rightarrow b$

Remove unit productions $S \rightarrow S$, $S' \rightarrow S$ and $A \rightarrow B$, $A \rightarrow \epsilon$:

after removing $S \rightarrow S$:

$P: S' \rightarrow S, S \rightarrow ASA \mid aB$

$A \rightarrow B \mid \epsilon$

$B \rightarrow b$

After removing $S' \rightarrow S$:

$P: S' \rightarrow ASA \mid aB \mid a \mid AS \mid SA,$

$S \rightarrow ASA \mid aB \mid a \mid AS \mid SA,$

$A \rightarrow B \mid \epsilon, B \rightarrow b$

After removing $A \rightarrow B$:

$P: S' \rightarrow ASA \mid aB \mid a \mid AS \mid SA,$

$S \rightarrow ASA \mid aB \mid a \mid AS \mid SA,$

$A \rightarrow b \mid \epsilon, B \rightarrow b$

After removing $A \rightarrow S$:

$$P: S' \rightarrow ASA / aB / a / AS / SA,$$

$$S \rightarrow ASA / aB / a / AS / SA,$$

$$A \rightarrow b / ASA / aB / a / AS / SA,$$

$$B \rightarrow b$$

Now find out the productions that has more than two variables in RHS

$$S' \rightarrow ASA, S \rightarrow ASA \text{ and } A \rightarrow ASA$$

$$P: S' \rightarrow Ax / aB / a / AS / SA,$$

$$S \rightarrow Ax / aB / a / AS / SA,$$

$$A \rightarrow b / Ax / aB / a / AS / SA,$$

$$B \rightarrow b,$$

$$x \rightarrow SA$$

change the productions

$$S' \rightarrow aB, S \rightarrow aB \text{ and } A \rightarrow aB$$

finally we get

$$P: S' \rightarrow Ax / yB / a / AS / SA,$$

$$S \rightarrow Ax / yB / a / AS / SA,$$

$$A \rightarrow b / Ax / yB / a / AS / SA,$$

$$B \rightarrow b,$$

$$x \rightarrow SA,$$

$$y \rightarrow a$$

which is the required chomsky normal form for the given CFG

② write the steps for removing null productions and unreachable symbols: example with your own:

1) Remove null productions

→ A production is considered null if its right hand side is empty

→ for example consider a CFG with the following production

$A \rightarrow \epsilon$ the production can be removed

Eg. the null production ($S \rightarrow \epsilon$) can be removed

$$G = (N, T, P, S)$$

$$\text{where } N = \{S, A, B\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow A, A \rightarrow B, B \rightarrow a\}$$

$$S = S$$

2) Remove unreachable symbols:

→ A symbol is considered unreachable if it can never appear in any thing generated by the CFG

→ To identify unreachable symbols, start from start symbol and mark all symbols reachable from it

Eg. - The unreachable symbols can be removed new grammar

$$G = (N, T, P, S)$$

$$\text{where } N = \{S, B\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow A, B, a\}$$

$$S = S$$