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ME469: A Verification and Validation (V&V) Methodology: Laboratory

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Focus on Three Nalu/reg_tests/test_files/laboratory

- 2d_quad9_couette
- 2d_quad4_channel
- 3d_tet4_pipe



Nalu/reg_tests/test_files/laboratory/2d_quad9_couette: Setup

- Top, imposed velocity
- Bottom, no-slip wall
- Left and right sides, periodic

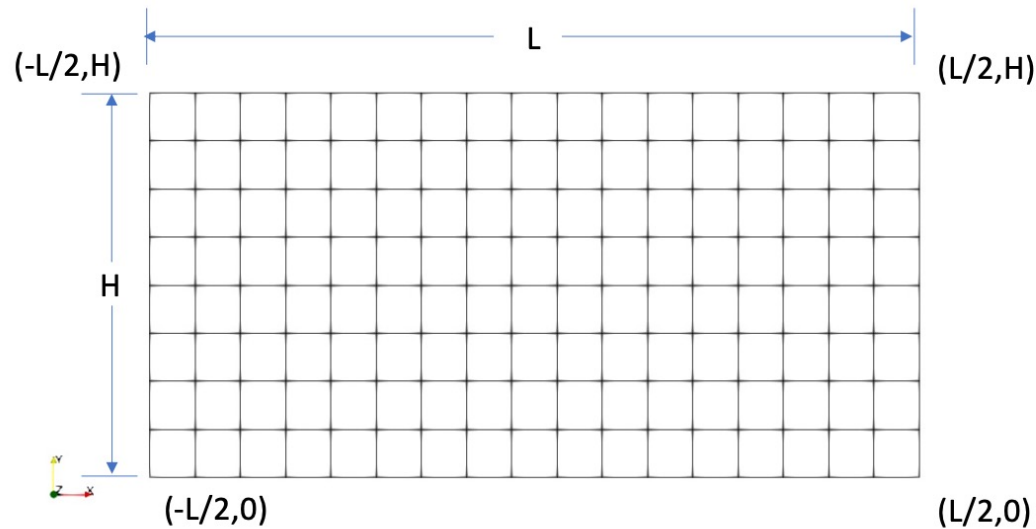


Figure 1: Two-dimensional couette flow in which the height is 2 m and length, 1 m



Nalu/reg_tests/test_files/laboratory/2d_quad9_couette: Equations

- Subject to the imposed conditions, the momentum equation reduces to:

$$\mu \frac{d^2 u_x}{dy^2} = 0$$

- With analytical solutions (at a given Reynolds number): $Re = \frac{\rho u_b H}{\mu}$

$$u_x(y) = k_1 y + k_2 \quad u_x(y) = \frac{u_b}{H} y$$

- Simple, and a linear profile



Nalu/reg_tests/test_files/laboratory/2d_quad9_couette: Findings

- Start-up adds additional complexity to the solution, however, at steady state, our profile is predicted to be linear

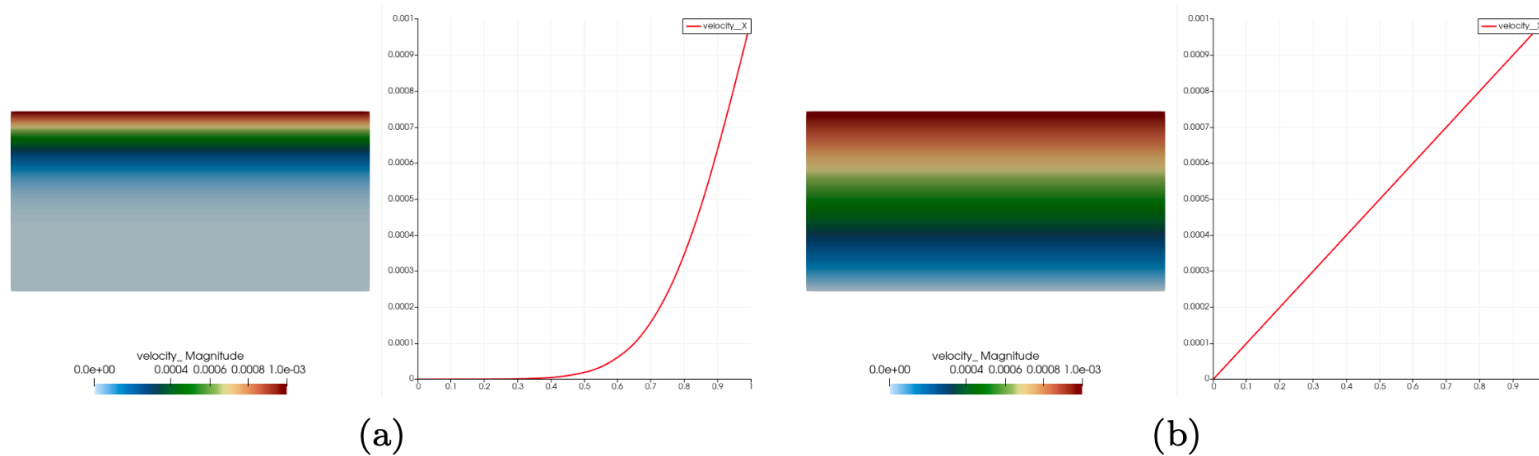


Figure 2: Velocity shadings (left) and velocity profile (right) for the $Re = 1236$ case.



Nalu/reg_tests/test_files/laboratory/2d_quad4_channel: Setup

- Top and bottom, no-slip wall
- Left and right sides, either open (specified pressure drop) or with a body force, periodic

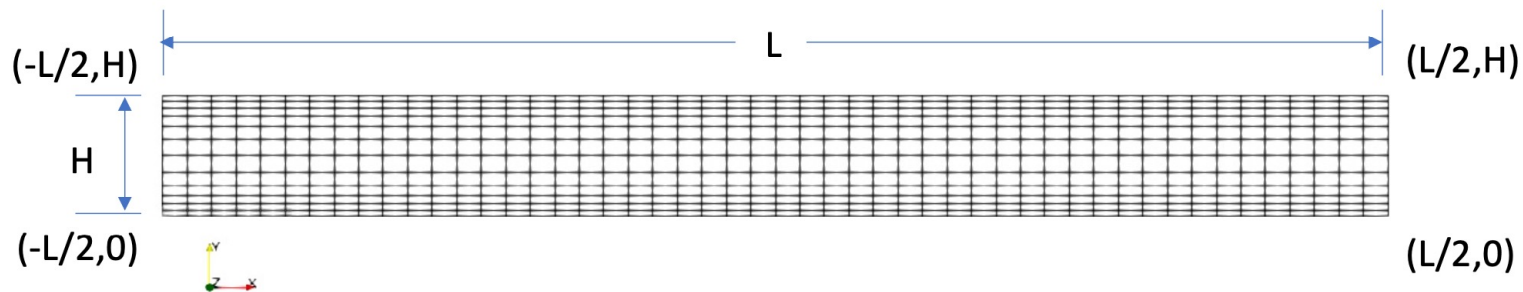


Figure 1: Two-dimensional channel flow in which the height is unity and length, 10.



Nalu/reg_tests/test_files/laboratory/2d_quad4_channel: Equations

- Subject to the imposed conditions, the momentum equation reduces to:

$$\mu \frac{d^2 u_x}{dy^2} = 0$$

- With analytical solutions (at a given Reynolds number): $Re_\tau = \frac{\rho u_\tau H/2}{\mu}$

$$u_x(y) = \frac{1}{\mu} \frac{dP}{dx} \frac{y^2}{2} + k_1 y + k_2 \quad u_x(y) = \frac{1}{2\mu} \frac{dP}{dx} [y^2 - Hy] \quad u_x^{max} = \frac{1}{8\mu} \frac{dP}{dx} H^2$$

- Quadratic profile
- Please review write-up for specification of Reynolds number, wall shear stress, and pressure gradient balance

$$\int \frac{dP}{dx} dV = \int \tau_w dA$$



Nalu/reg_tests/test_files/laboratory/2d_quad9_channel: Findings

- Quadratic solution, using a second-order scheme, is captured at the degree-of-freedom locations, without differences wrt the analytical solution
- However, the wall shear stress is not – why?

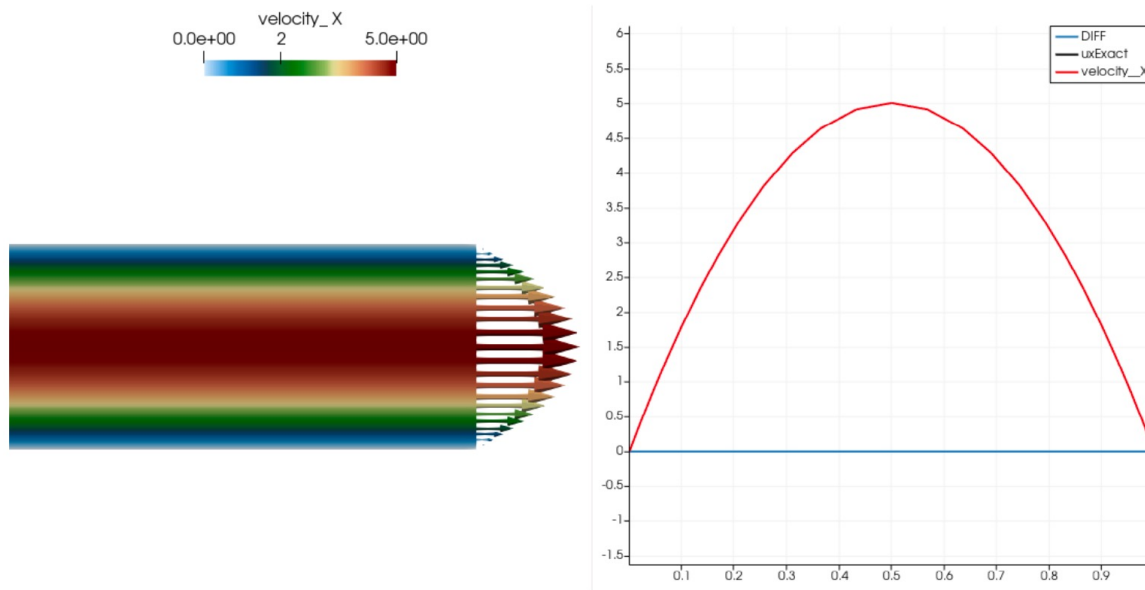


Figure 2: Velocity shadings and profile for the $Re^\tau = 10$ case.



Nalu/reg_tests/test_files/laboratory/3d_tet4_pipe: Setup

- Outer cylinder, no-slip wall
- Front and back sides, either open (specified pressure drop) or with a body force, periodic
- Tet4 and Hex8 mesh shown below

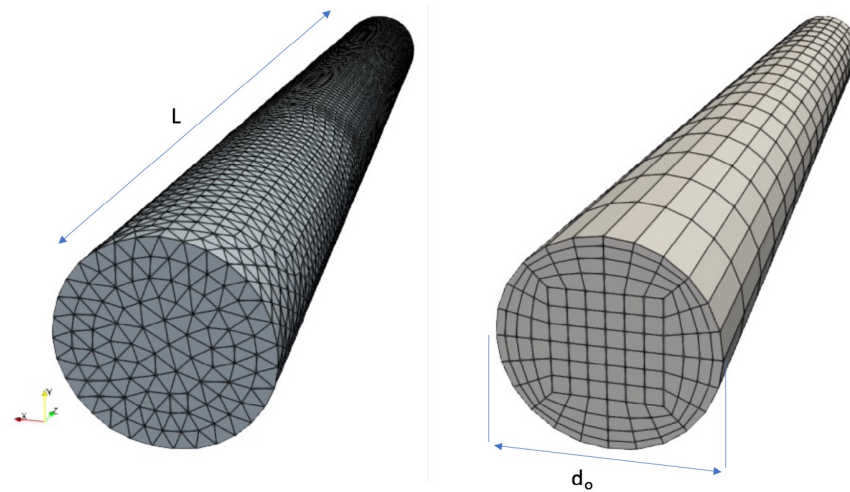


Figure 1: Three-dimensional pipe flow configuration of length L and diameter d_o outlining the Tet4 (left) and Hex8 (right) topology.



Nalu/reg_tests/test_files/laboratory/ 3d_tet4_pipe : Equations

- Subject to the imposed conditions, the axial momentum equation (cylindrical coordinates) reduces to:

$$\frac{dP}{dz} = \frac{1}{r} \frac{\partial r \tau_{rz}}{\partial r} \quad \frac{\partial u_z}{\partial r} \Big|_{r=0} = 0 \quad u_z \Big|_{r=R} = 0$$

- With analytical solutions (at a given Reynolds number): $Re_\tau = \frac{\rho u_\tau D}{\mu}$

$$\frac{r^2}{2} \frac{dP}{dz} = r \mu \frac{\partial u_z}{\partial r} + k_1 \quad u_z(r) = \frac{1}{4\mu} \frac{dP}{dz} [r^2 - R^2] \quad u_x^{max} = -\frac{R^2}{4\mu} \frac{dP}{dZ}$$

- Quadratic profile
- Please review write-up for specification of Reynolds number, wall shear stress, and pressure gradient balance

$$\int \frac{dP}{dz} dV = \int \tau_w dA$$



Nalu/reg_tests/test_files/laboratory/2d_quad9_channel: Findings

- Quadratic solution, using a second-order scheme now demonstrates error as compared to the analytical solution, why?
- The lab exercise overviews the difference in error when using Tet4 and Hex8 mesh.

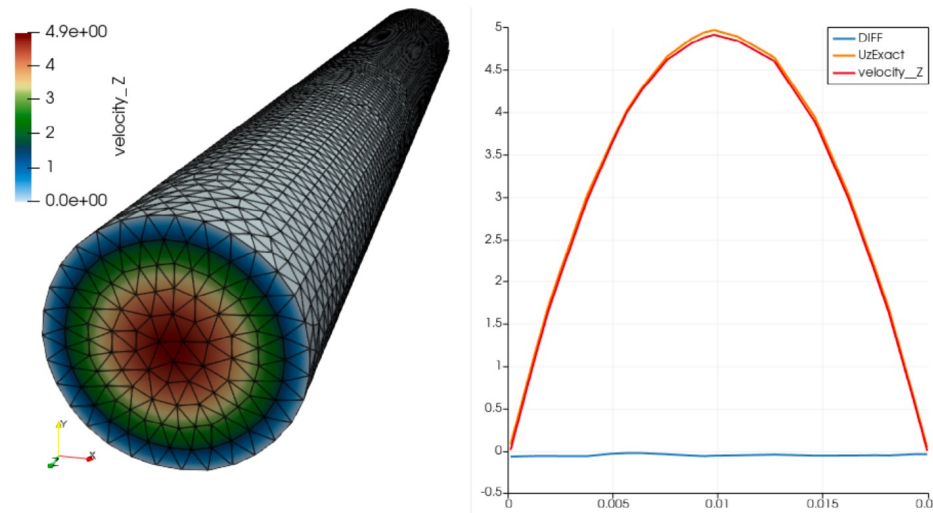


Figure 2: Axial velocity shadings (left) and radial profile (right) for the $Re^\tau = 20$ case.



Conclusions

- In low-Mach fluid mechanics, we have a limited set of analytical solutions that can be used to ensure that neither coding nor conceptual discretization-like mistakes have been made
- In this module, we have reviewed three of the simplest cases
- As complexity increases, the method of manufactured solution (MMS) becomes. Powerful tool to ensure that the numerical scheme is converging as expected

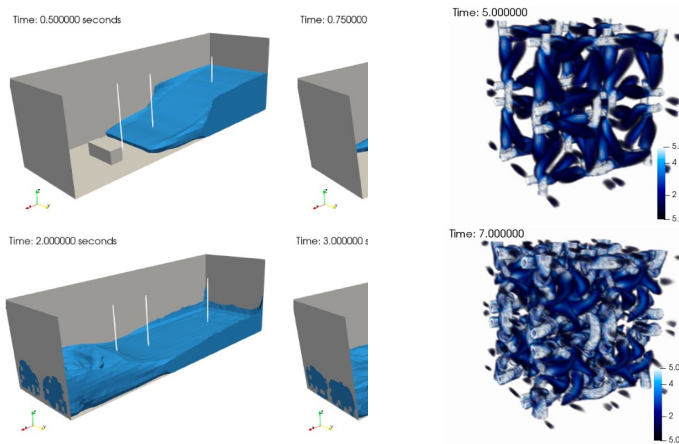


Figure 2: Volume of fluid evolution

Figure 1: Three-dimensional Q-criterion time series at 5, 6, 7, and 8 seconds.

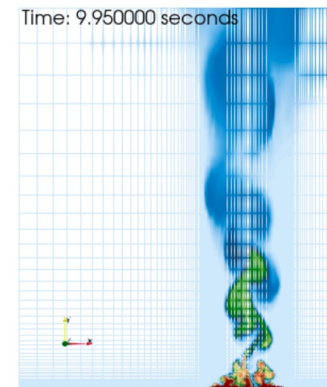


Figure 1: Two-dimensional helium plume configuration. The plot shows the quadratic Quad9 mesh and density shading.

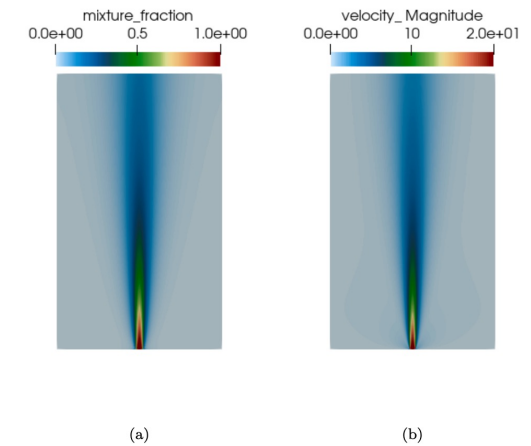


Figure 2: Planar mixture fraction (a) and velocity magnitude (b) shadings for the $Re = 100$ jet case.

Other labs: dam break, Taylor Green decay, helium plume, open jet (see /laboratory for more)