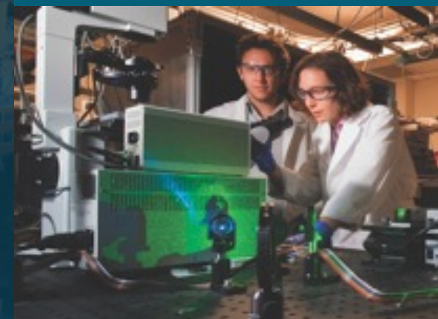
Sandia National Laboratories

Stanford ME469: The Role of Mass Flow Rate



PRESENTED BY

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Computational Thermal and Fluid Mechanics

Sandia National Laboratories SAND2018-4536 PE

U.S. DEPARTMENT OF ENERGY NNSA

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Advection Operators: Outline

- The Advection Operator and the Role of $\dot{m}$
- Pressure Stabilization Effects
- Revisit the Projected Nodal Gradient

First, The Role of $\dot{m}$

- For an equal-order, low-Mach approximate projection scheme, explicit pressure stabilization was added. How does this manifest itself in the advection operator?

$$\int w \frac{\partial \rho u_j Z}{\partial x_j} dV = \int \rho u_j Z \, n_j \, dS \approx \sum_{ip} \dot{m} Z_{ip}$$

Recall discrete continuity solve

$$D\hat{u} = \tau(Lp^{n+1} - DGp^{n})$$

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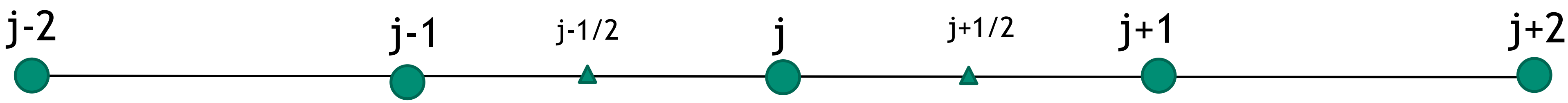

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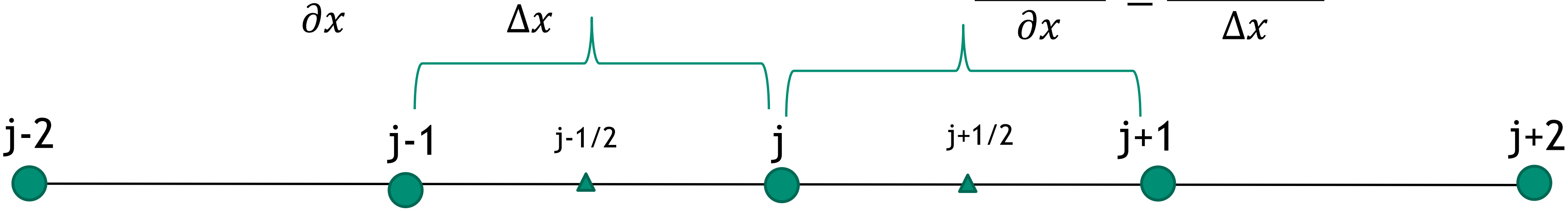

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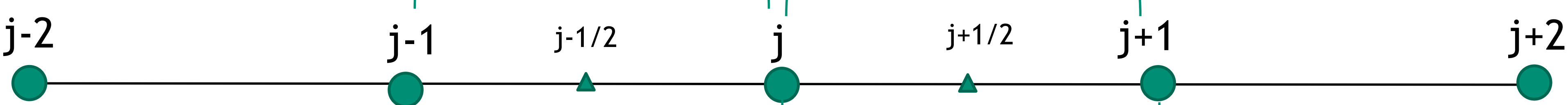

$$G_x P_{j-1} = \frac{P_j - P_{j-2}}{2\Delta x}$$

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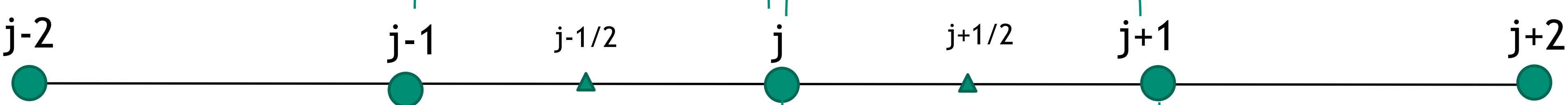


$$G_x P_{j-1} = \frac{P_j - P_{j-2}}{2\Delta x} \qquad G_x P_j = \frac{P_{j+1} - P_{j-1}}{2\Delta x} \qquad G_x P_{j+1} = \frac{P_{j+2} - P_j}{2\Delta x}$$

- Using the above equations, we can derive the actual continuity equation that we are solving:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} \neq 0 \propto \tau \frac{\partial^4 P}{\partial x_j^4} \Delta x_j{}^3$$

Now, the hard choice….

- We have already seen that the continuity equation includes a fourth-order pressure derivative, at least when using the standard fine-scale momentum residual stabilization approach
- Moreover, even order derivatives are stabilizing (suppresses growth via dissipation)
- Two notions of velocity:

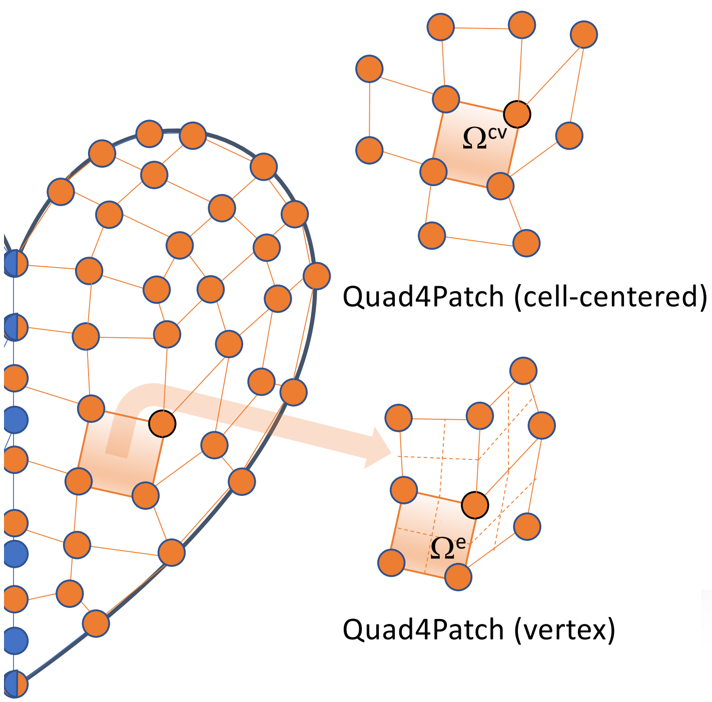
1. The nodal velocity whose divergence (in a uniform density flow) is proportional to the fourth-order pressure derivative
2. The integration point velocity, or convecting velocity, that includes the L-DG term

$$(\rho u_j)_{ip} = \sum_{k=1}^{npe} N_{ip,k}(\rho u_j) - \tau_{ip}\left(\sum_{k=1}^{npe} \frac{\partial N_{ip,k}}{\partial x_j} P_k - \sum_{k=1}^{npe} N_{ip,k} G_j P_k\right)$$

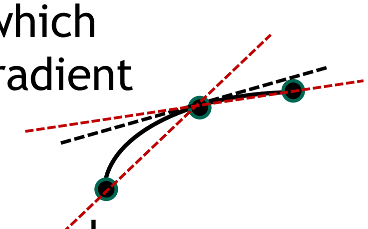
- Inclusion of the pressure stabilization terms in the mass flow rate ensures that the DOF is conserved, however, in some parts of the literature, this is not clearly represented and sometimes these terms are dropped.
- Inclusion of these pressure stabilization terms in momentum also provides a dissipative quality and, as such the system is not perfectly kinetic energy conserving, You et al., PoF, 2008, https://doi.org/10.1063/1.3006077

Re-visiting Nodal Gradient Operator: An Alternative View

- Recall, that the edge- and element-based diffusion operator, and for some choices for the advection operator, and pressure stabilization, a nodal gradient is required
- What is a nodal gradient? As formerly described in the *Computing Gradients* lecture, in a cell-centered context we obtained this via a number of ways, e.g., Green-Gauss, Least Squares, etc.
- Let's take another view…



First, the gradient of a function for ordinary finite element basis functions are discontinuous, i.e., the value at a shared element face depends on which element is used to compute the gradient



Therefore, we can view the nodal gradient a continuous at the nodes and discontinuous within the elements

Let's minimize this difference:

$$\frac{1}{2}\left(\frac{\partial \phi}{\partial x_j} - G_j\phi\right)^2.$$

by solving:

$$\int wG_j\phi dV = \int w\frac{\partial \phi}{\partial x_j}dV. \xrightarrow[\text{Lumped-mass}]{} G_j\phi = \frac{\sum_{ip}\phi_{ip}n_j dS}{V}.$$

Re-visiting Nodal Gradient Operator: Convergence

- On a Quadratic Unstructured Quad element, we see that second-order convergence of the projected nodal gradient is only noted when a consistent mass matrix is used (requires a matrix inversion, or an outer iteration for an explicit scheme)
- Lack of design-order projected nodal gradients affect the design-order expectation for the DOFs velocity and pressure! No real extension of reconstruction schemes to higher-order