Exceptional service in the national interest

ME469: Mesh Motion Approaches

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SAND2018-4536 PE

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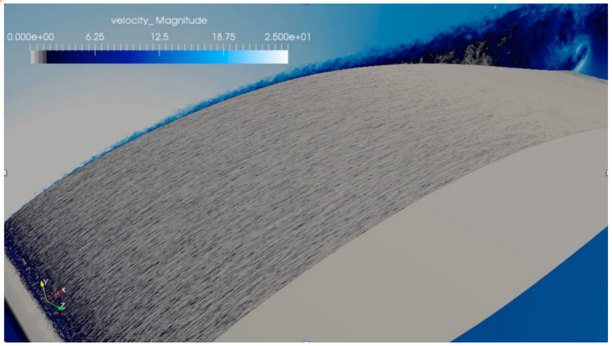
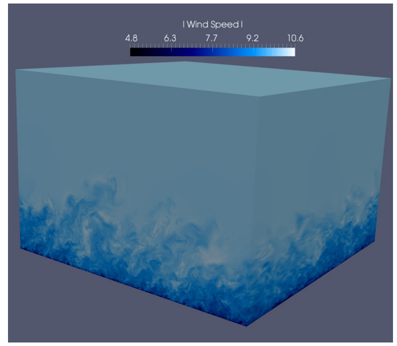
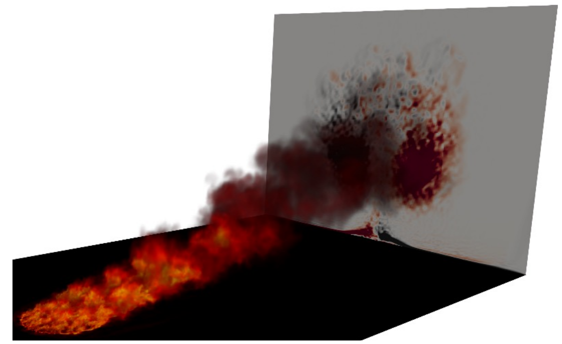
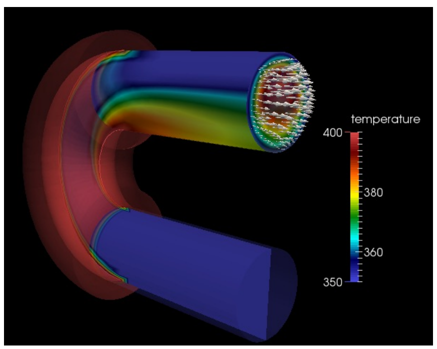
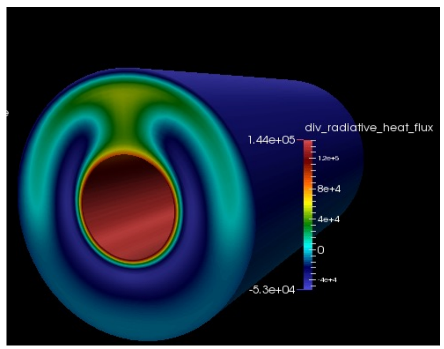
Thus Far, Many Complex Applications: Multi-physics, ***Static-Mesh***

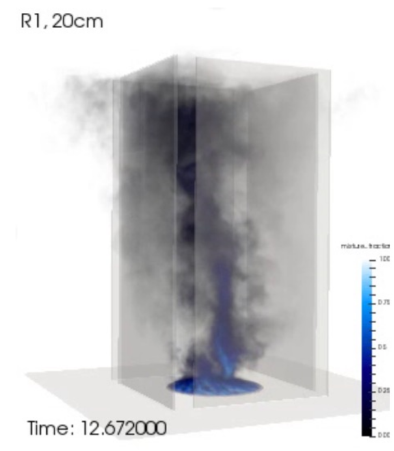


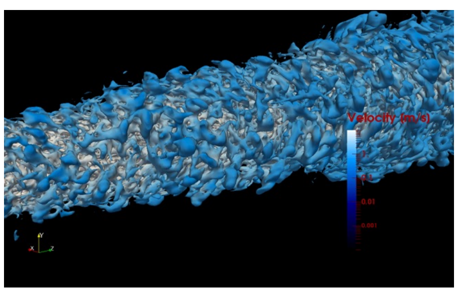


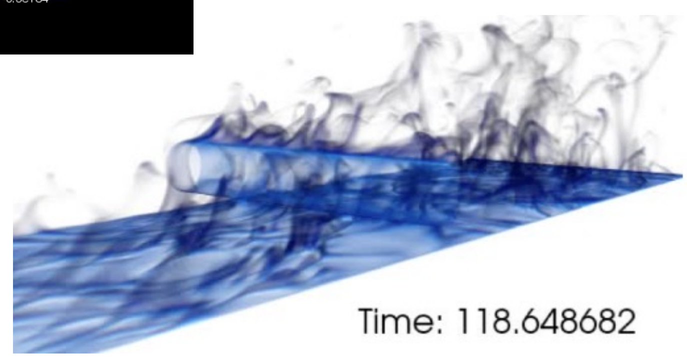




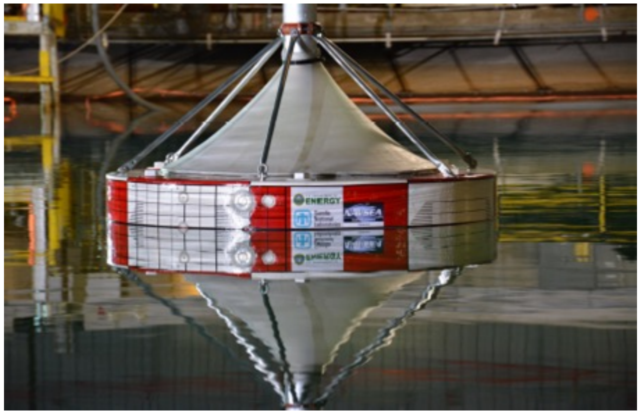








However, Many Applications in Which A Body Moves.....



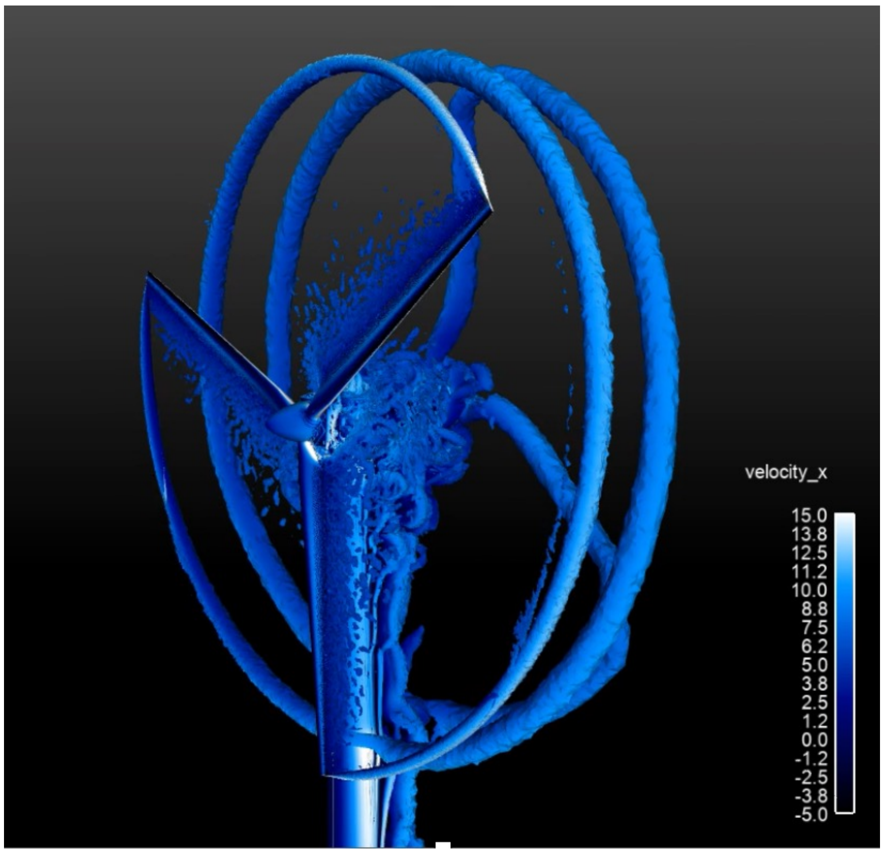
Wave, Coe et al., SAND2016-10094

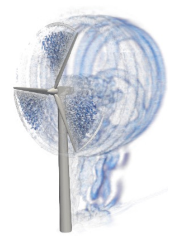
Wind, Sandia SWIFT facility, https://energy.sandia.gov

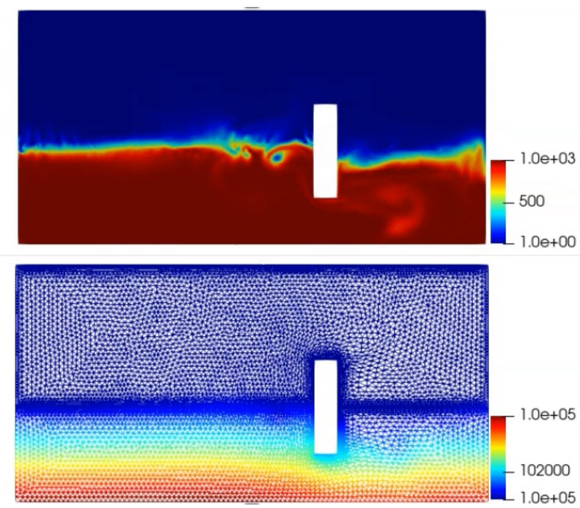
https://web.stanford.edu/group/frg/

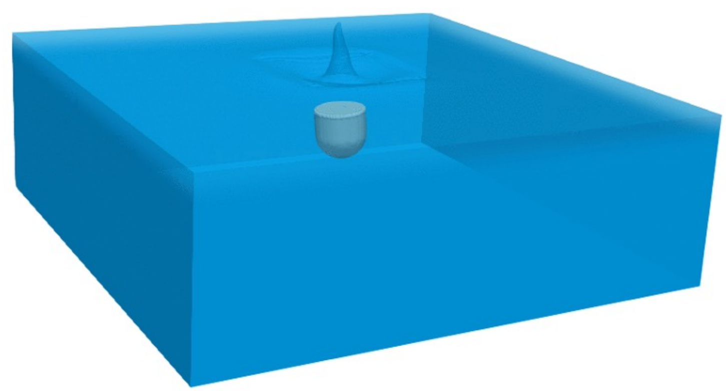
Moving Mesh Approaches

- Options: mesh deformation, sliding mesh and overset









Mesh Deformation and Mesh Smoothing

The mesh remains conformal and distorts (compresses and is stretched) as the solid body moves.

Elements encounter a volumetric time-rate of change

Flow past an erratic airfoil

A Note on Fluid/Structure Interaction (FSI)

- FSI drives coupling between the solid body structural response, the fluid mechanics (force and loadings), and the mesh smoothing scheme

❑ *Linear Elastic Structural Mechanics*

$$\rho \frac{\partial^2 \xi_i}{\partial t^2} - \frac{\partial \sigma_{ij}}{\partial x_j} = F_i$$

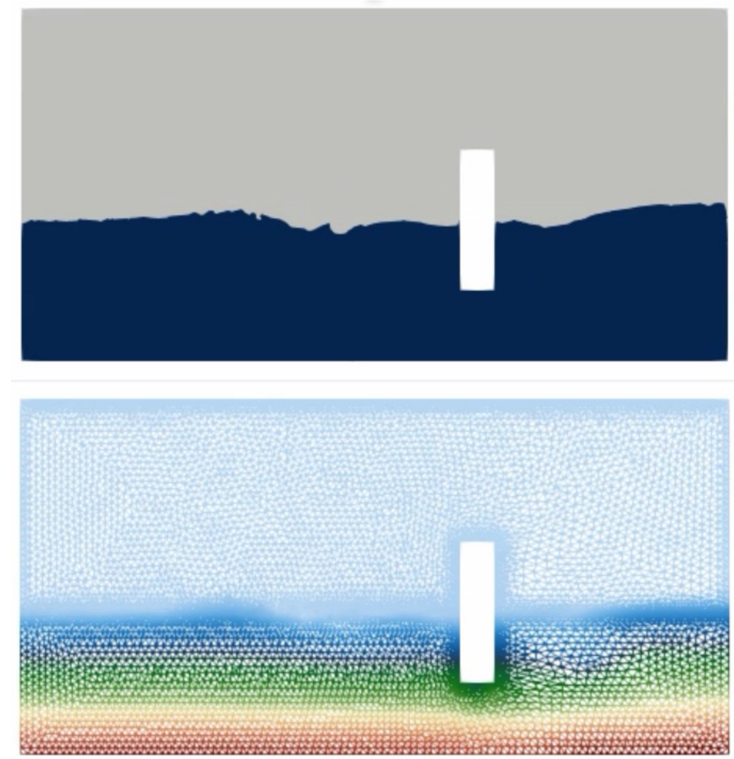
$$\sigma_{ij} = \mu \left(\frac{\partial \xi_i}{\partial x_j} + \frac{\partial \xi_j}{\partial x_i} \right) + \lambda \frac{\partial \xi_k}{\partial x_k} \delta_{ij}$$

$BC:$

$$\int p n_i \, dS$$

Fluids solve provides static $n^{1/2}$ pressure (surface)

Info:
density (top; water is blue)
static pressure (bottom)
– roughly ten seconds



Wave Generator

A Note on Fluid/Structure Interaction (FSI)

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❑ *Linear Elastic Structural Mechanics*

❑ *Mesh Motion Mechanics*

$$\cancel{\rho \frac{\partial^2 \xi_i}{\partial t^2}} - \frac{\partial \sigma_{ij}}{\partial x_j} = \cancel{F_i}$$

Lame' constants $\propto \frac{1}{V}$

$$\sigma_{ij} = \mu\left(\frac{\partial \xi_i}{\partial x_j} + \frac{\partial \xi_j}{\partial x_i}\right) + \lambda \frac{\partial \xi_k}{\partial x_k}\delta_{ij}$$

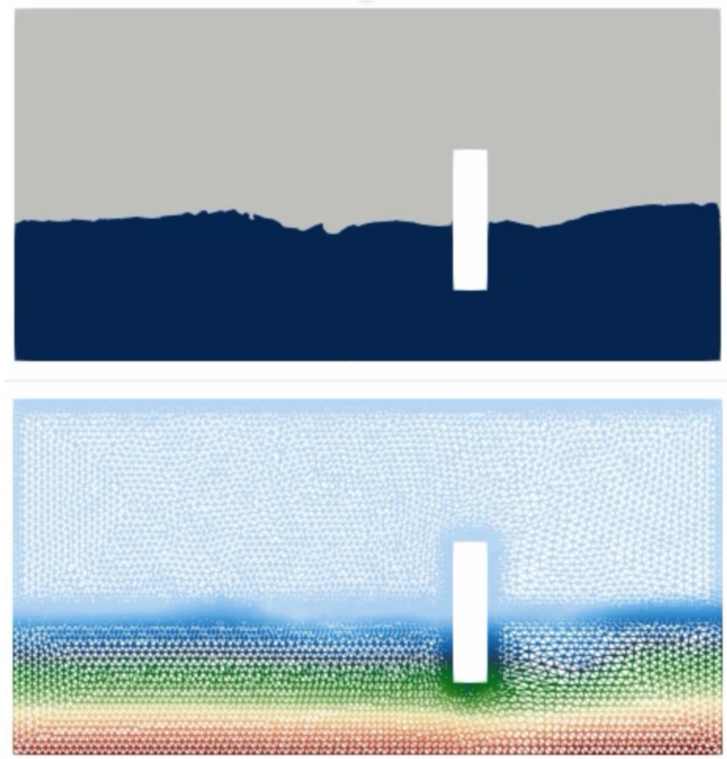
$BC:$

$\xi_i = \xi_i^{spec}$

SM solve provides displacements (surface)

PDE-based where stress is modeled (above elastic) with special properties

Info:
density (top; water is blue)
static pressure (bottom)
– roughly ten seconds



Wave Generator

A Note on Fluid/Structure Interaction (FSI)

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Info:
density (top; water is blue)
static pressure (bottom)
– roughly ten seconds

Smith et al. “Stable Neo-Hookean Flesh Simulation”, Pixar Animation Studios, and “spring” approaches that connect opposing vertices of the element

Wave Generator

Physical Analogy: Model the mesh by connected springs

- Rather than solve a PDE with a modified stress model, the elements can be tied together by “springs”, see Yang et al, “Improvement in the spring analogy mesh deformation method through the cell-center concept”, Aero. Science Tech. (2021)
  - Physics Analogy (as apposed to PDE-based)

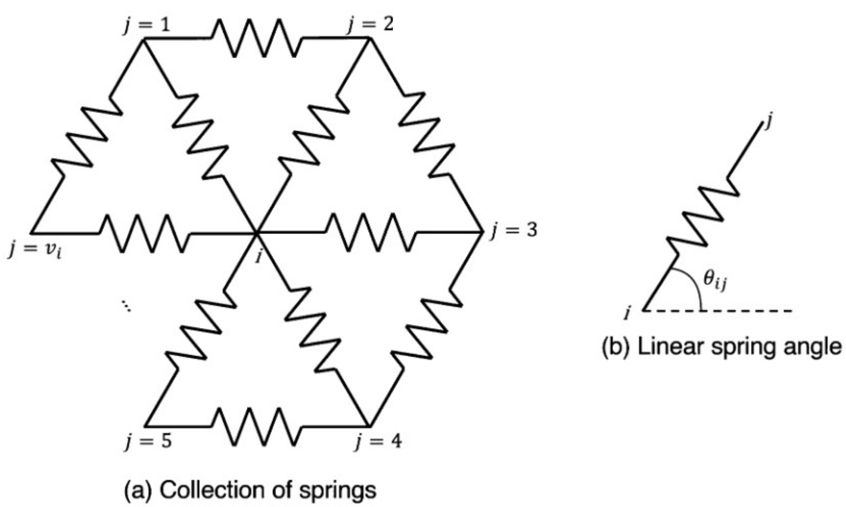
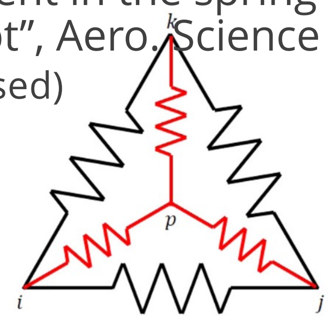


Fig. 1. Spring analogy illustration around a nodal point.



ustration of the cell-center spring analogy in a triangular cell.

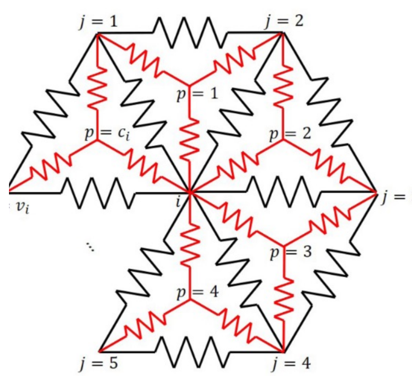


Fig. 4. Assemble of fictitious spring surrounding nodal point i in cell-center concept. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

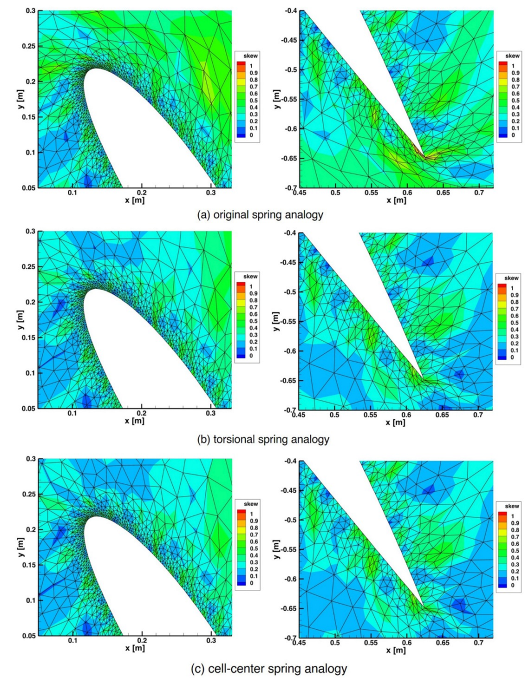


Fig. 7. Deformed meshes obtain from various spring analogy methods for rotating airfoil with inviscid mesh.

A Note on Fluid/Structure Interaction (FSI)

- FSI drives coupling between the solid body structural response, the fluid mechanics (force and loadings), and the mesh smoothing scheme

❑ *Linear Elastic Structural Mechanics*

❑ *Mesh Motion Mechanics*

$BC:$

$\xi_i = \xi_i^{spec}$

❑ *Fluid Mechanics*

$$\frac{\partial \rho u_i}{\partial t} + \rho u_i \frac{\partial v_k}{\partial x_k} + \frac{\partial \rho u_j u_i}{\partial x_j} - \frac{\partial \sigma_{ij}}{\partial x_j} = 0$$

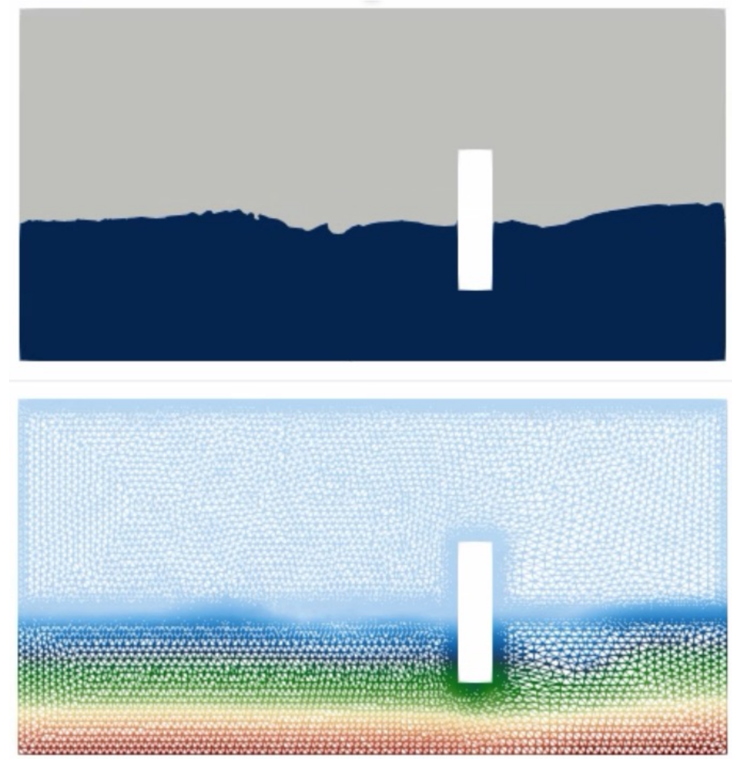
$$\sigma_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - p\delta_{ij}$$

$BC:$

$u_i = u_i^{spec}$

MM solve provides velocity (surface and volume) and displacements (volume)

Info:
density (top; water is blue)
static pressure (bottom)
– roughly ten seconds



Wave Generator

Geometric Conservative Law (GCL)

- For moving mesh applications, care must be taken such that a moving mesh does not create spurious flow
- The Geometric Conservative Law states, simply, (below V is the volume, and v_j is the mesh velocity):

$$\frac{\partial V}{\partial t} = \int v_j n_j dS$$

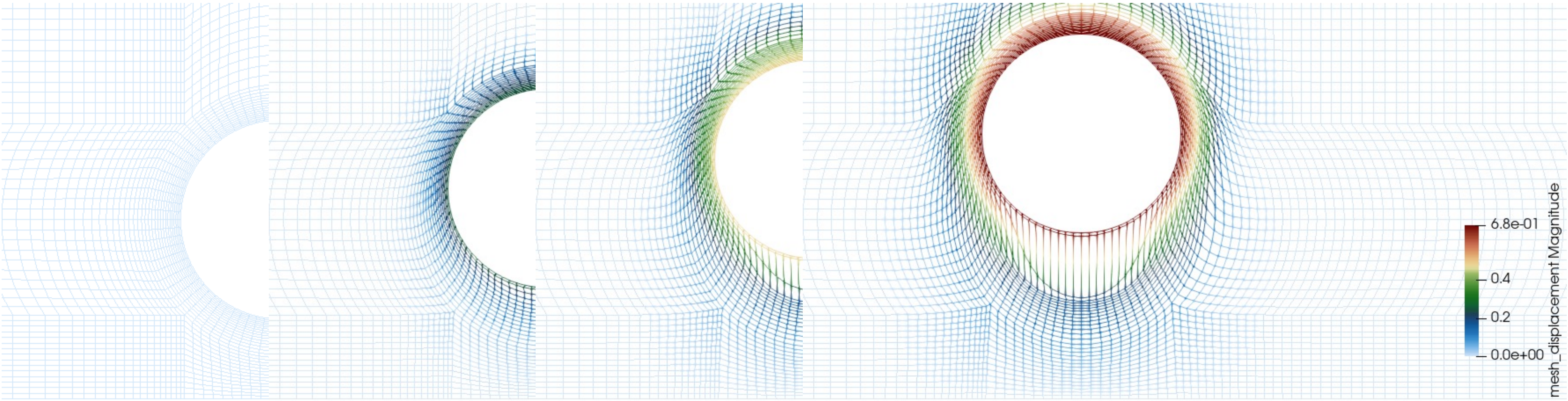
- The conservation equation for scalar ϕ is

$$\frac{d}{dt}\int \rho\phi dV + \int \rho\phi(u_j - v_j)n_j dS = 0$$

- In general, CGL does not come for free and there are varying levels of sophistication that one can employ:
  - Discrete–GCL, Guillard and Farhat, Comput. Method. App;. Mech. Engr. 190 (2000)
  - "Geometry Correction" Ma et al., Proc. Engr. 126 (2015)
  - Lineage from Arbitrary Lagrangian Eulerian (ALE)

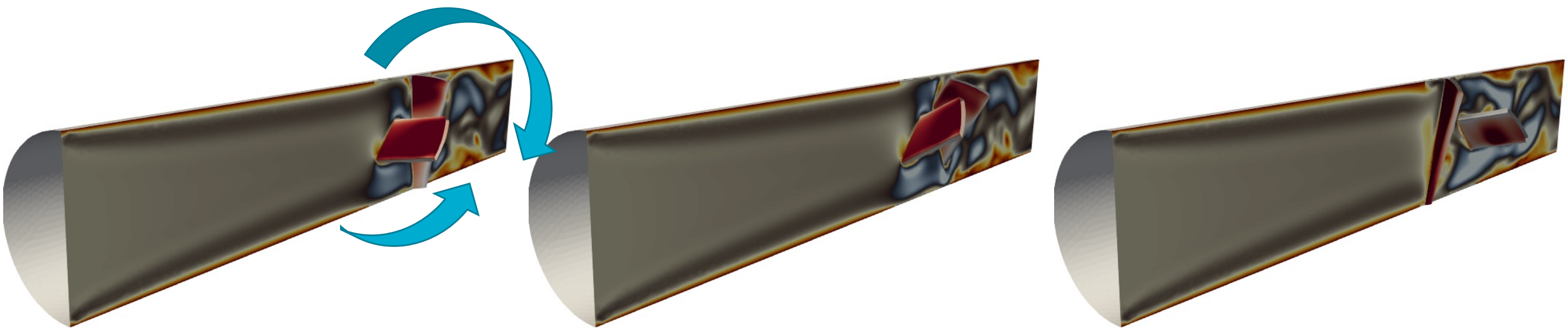
Complexity: Mesh Inversion

In general, there may be a maximum mesh displacement that can not be supported



Sliding Mesh: Conceptual Description: Review

- A generalized non-conformal scheme supports will fixed rotation
- Conceptually, multiple sliding blocks can be included
- Below, we note:
  - Front blade, COUNTER-CLOCKWISE
  - Back blade, CLOCKWISE



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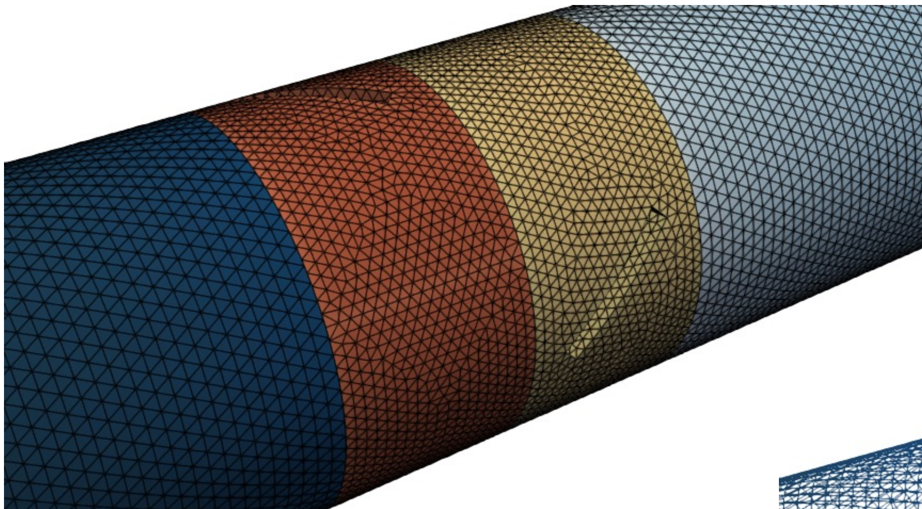
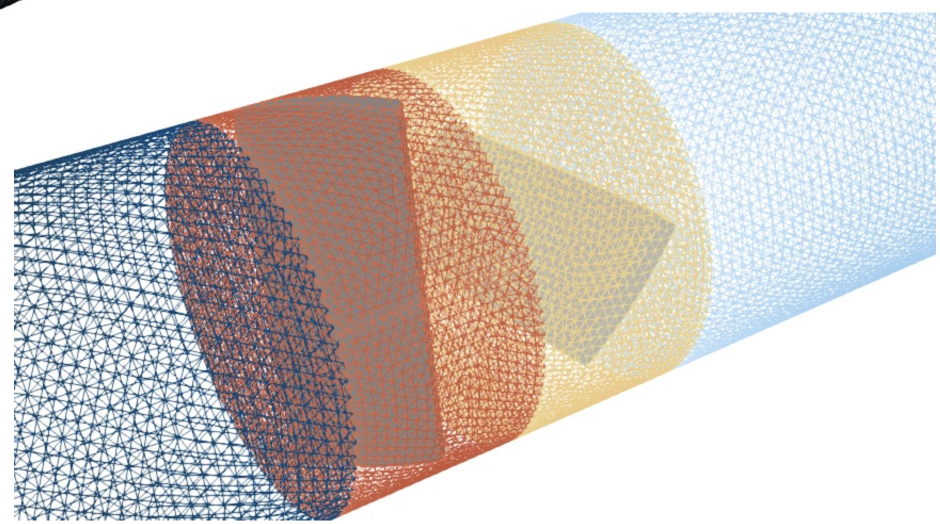
Time: 4.190354 secor

Time: 4.589262 seconds

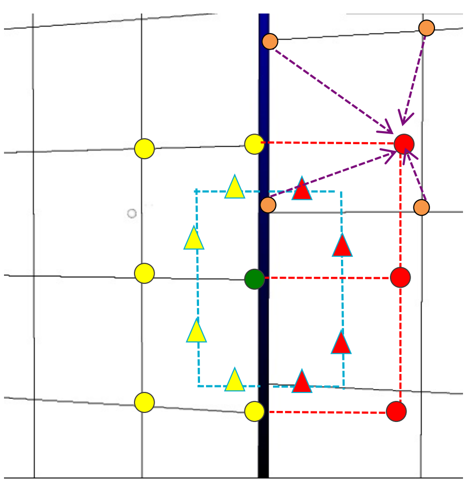
Multiple Sliding Mesh: Geometry

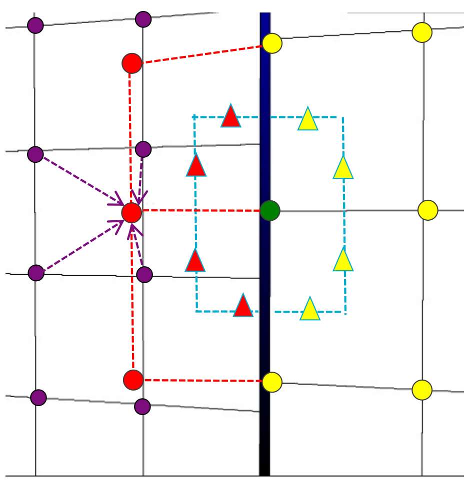




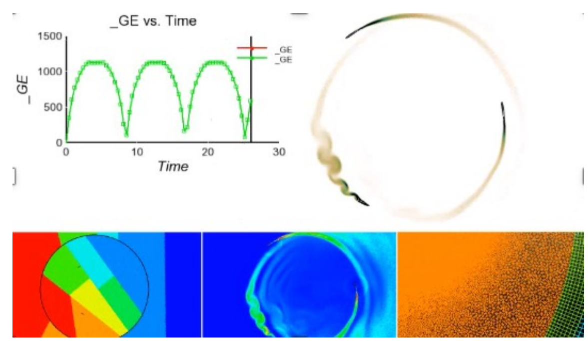
Halo-based Approach for Non-Conformal Interfaces: Vertex-Based

- Has the “flavor” of an overset method; i.e., extrusion of surface mesh to opposing domain with reconstruction of states
- Two-pass algorithm: surface_1:block_2 and surface_2:block_1
- See Stejil, 2008 and extrusion-based of Blades, 2006 and Dodds, 2016
- First implemented in Nalu for wind-energy applications, Domino, JCP 2018





- **Requires prescribed movement, e.g., VAWT**



Halo-based Approach for Non-Conformal Interfaces; CC-based

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- See Stejil, 2008 and extrusion-based of Blades, 2006 and Dodds, 2016
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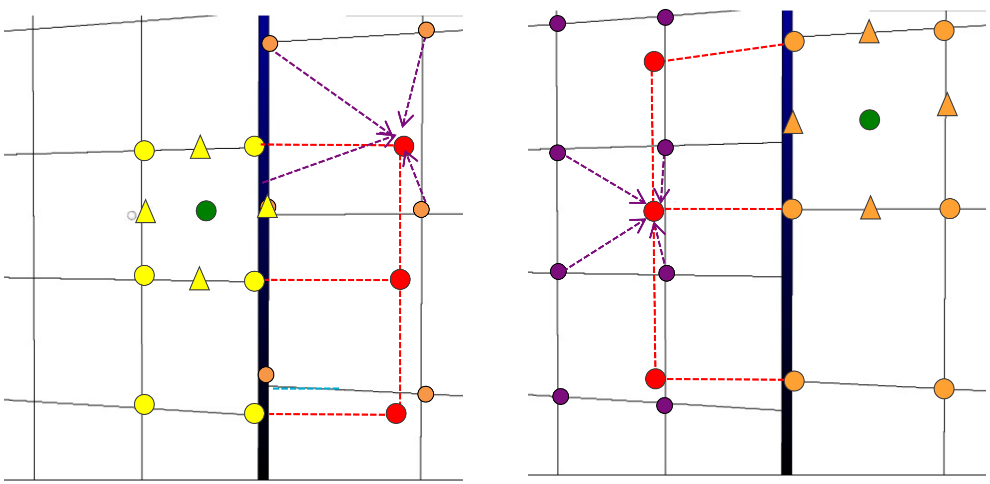
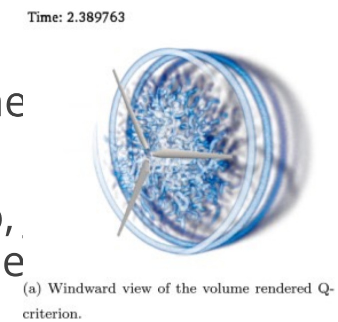


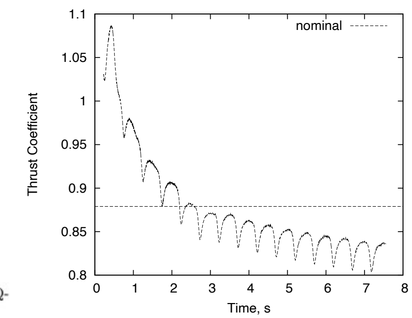
- **Requires prescribed movement**

Design-order Mesh Constructs for Non-Conformal Interfaces: Requires Prescribed Mesh Movement

- Non-conformal and overset methods provides ease of me and, when required, mesh motion
- Design-order hybrid DG/CVFEM established, see Domino, Comput. Phys. (2018) based on classic discontinuous Gale work of Arnold et al. SIAM (2002)



(a) Windward view of the volume rendered Q-criterion.



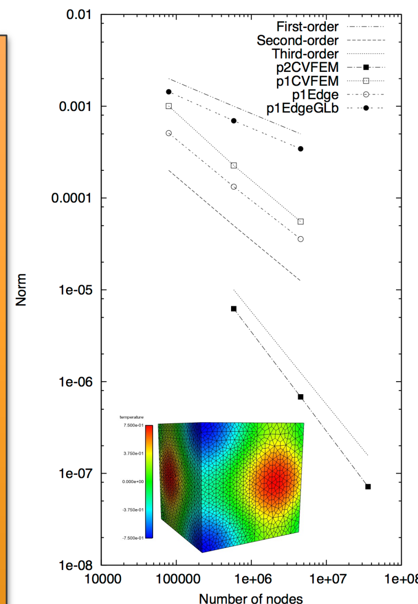
$$
\begin{aligned}
\int_{\Omega_A} w^A \left(\frac{\partial \rho Z}{\partial t} - S_i\right) d\Omega - \int_{\Omega_A} \frac{\partial w^A}{\partial x_j} q_j d\Omega + \int_{\Gamma \setminus \Gamma_{AB}} w^A q_j n_j d\Gamma \\
+ \int_{\Gamma_{AB}} w^A \hat{Q}_n(A,B) d\Gamma \qquad (2.8) \\
+ \int_{\Gamma_{AB}} \frac{\partial w^A}{\partial x_j} n_j \lambda^{IP} (Z^A - Z^B) d\Gamma,
\end{aligned}
$$

and

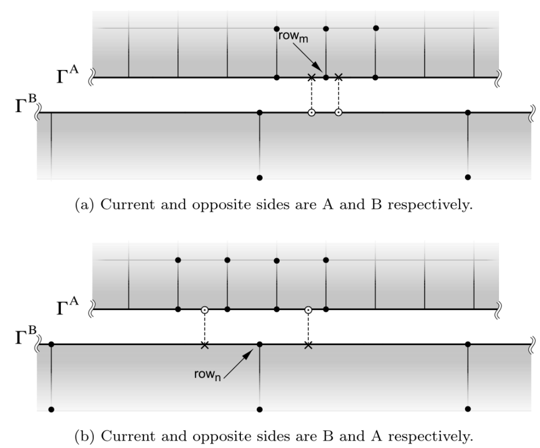
$$
\begin{aligned}
\int_{\Omega_B} w^B \left(\frac{\partial \rho Z}{\partial t} - S_i\right) d\Omega - \int_{\Omega_B} \frac{\partial w^B}{\partial x_j} q_j d\Omega + \int_{\Gamma \setminus \Gamma_{AB}} w^B q_j n_j d\Gamma \\
+ \int_{\Gamma_{AB}} w^B \hat{Q}_n(B,A) d\Gamma \qquad (2.9) \\
+ \int_{\Gamma_{AB}} \frac{\partial w^B}{\partial x_j} n_j \lambda^{IP} \left(Z^B - Z^A\right) d\Gamma.
\end{aligned}
$$

The numerical fluxes, $\hat{Q}_n(\alpha,\beta)$, are

$$
\hat{Q}_n(\alpha,\beta) = \frac{1}{2}\left[(q_j^\alpha n_j^\alpha - q_j^\beta n_j^\beta) + \lambda(Z^\alpha - Z^\beta)\right]. \qquad (2.10)
$$



Non-conformal MMS



(a) Current and opposite sides are A and B respectively.

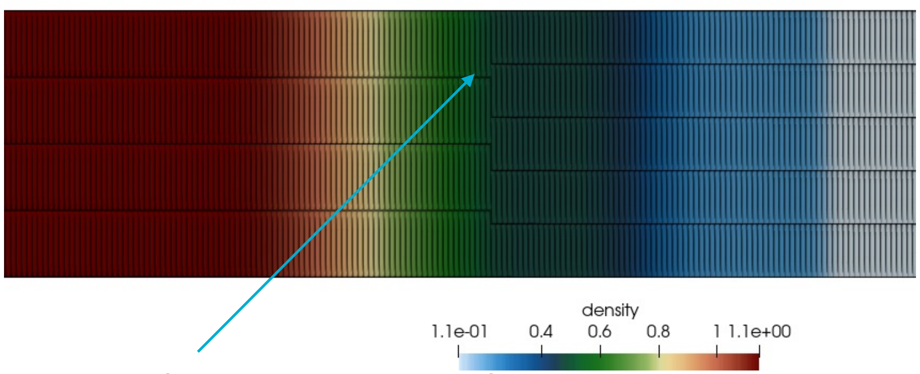
(b) Current and opposite sides are B and A respectively.

Current/Opposite

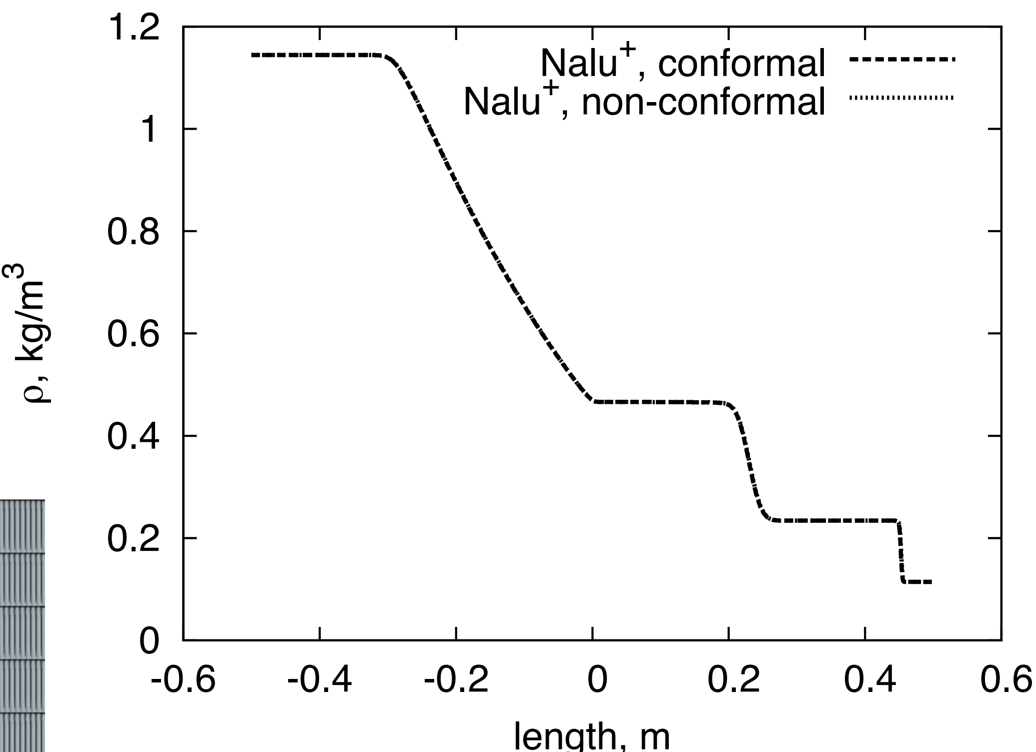
Shock Tube: Non-conformal Mesh

Hybrid DG/CVFEM-based non-conformal algorithm originally developed for the low-Mach regime

- Domino, S. P., "*Design-order, non-conformal low-Mach fluid algorithms using a hybrid CVFEM/DG approach*", J. Comput. Physics, 2018,
  - https://doi.org/10.1016/j.jcp.2018.01.00
- Compressible AUSM regime



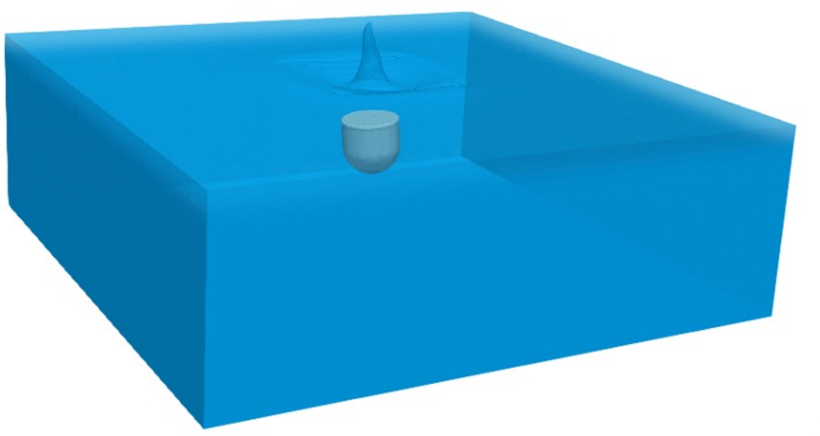
Non-conformal (static) interface



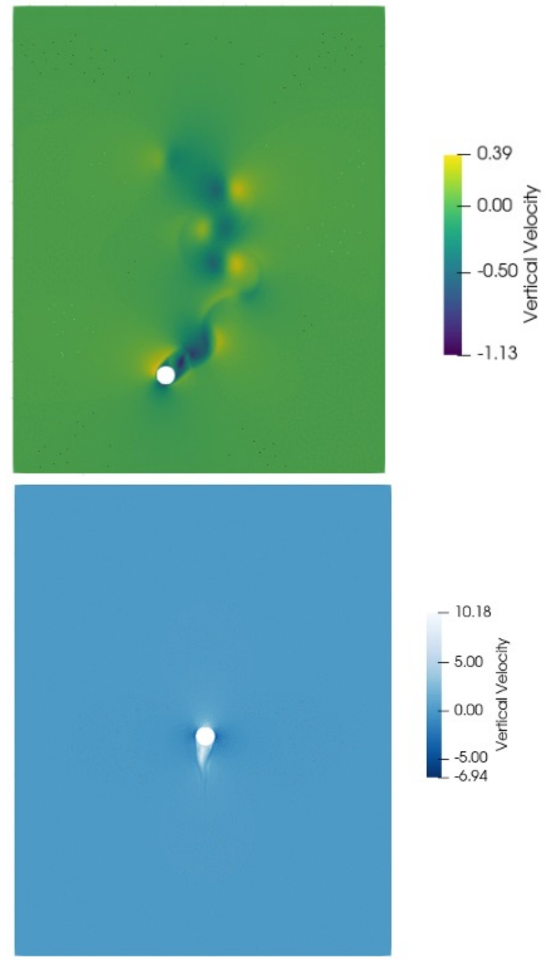
What is there is non-prescribed mesh motion?

- In many applications, the movement of the body is coupled to the fluid flow

What then?



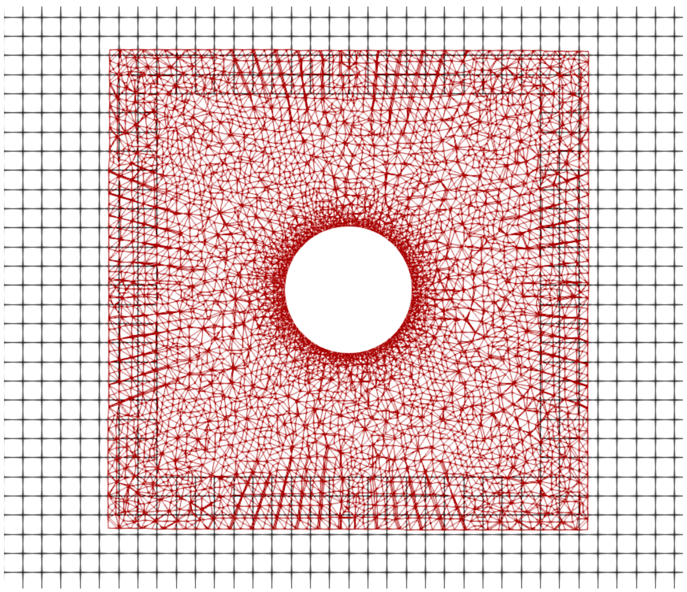
Buoy drop of Ransley et al., Renew. Ener. (2017)

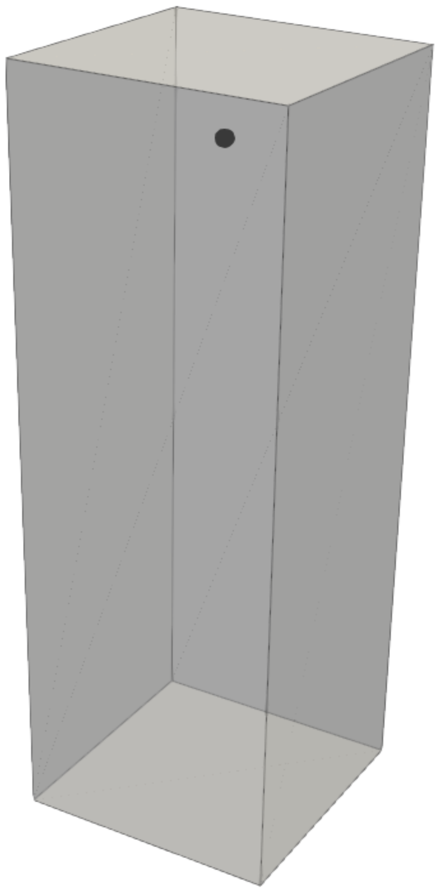


Falling/rising sphere of Horne and Mahesh, J. Comput. Fluids, (2019)

Overset!

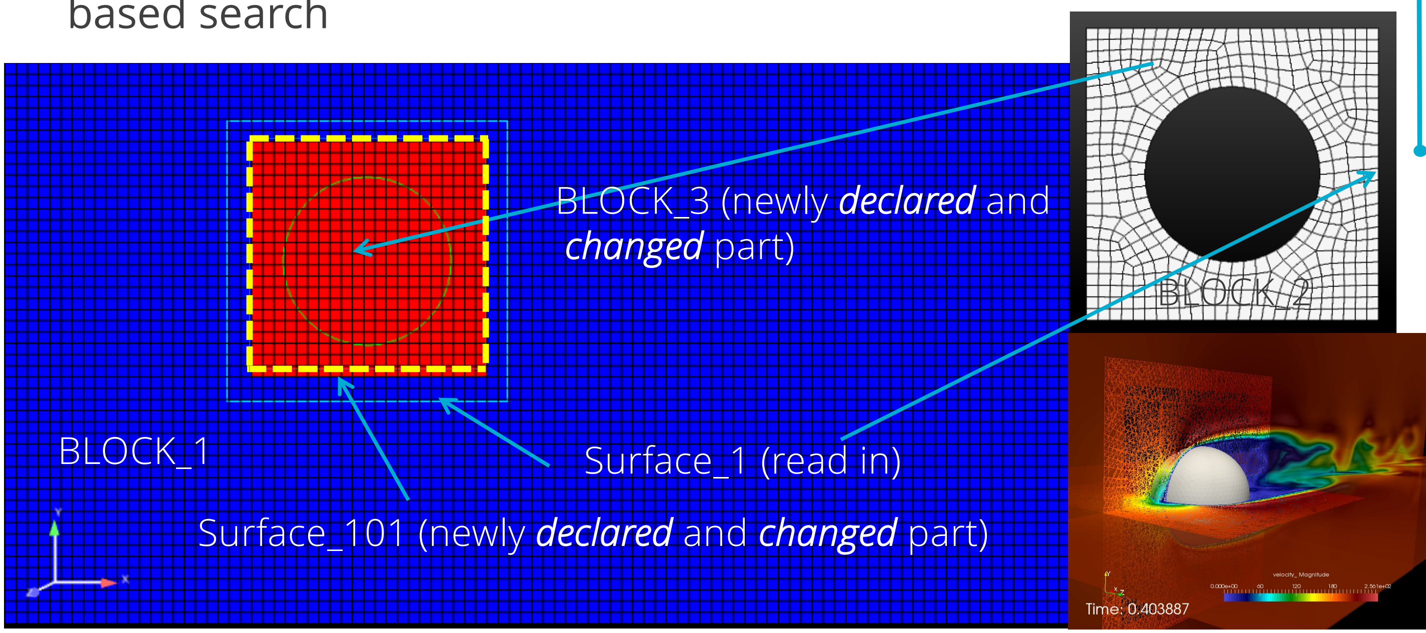
Background and overset meshes are defined



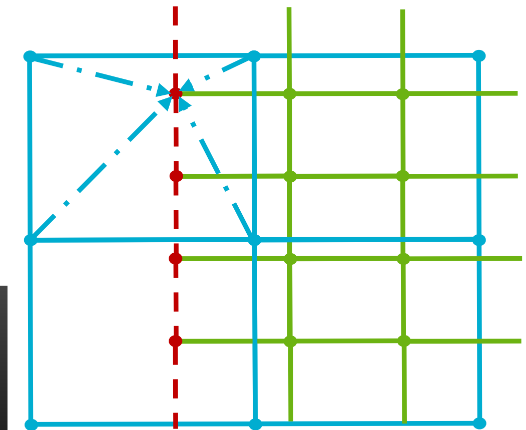


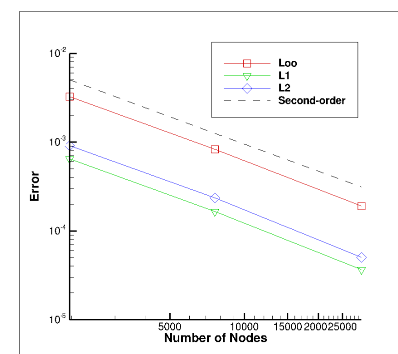
Overset: Freedom for Generalized Motion

- Implicit, constraint-based overset-based approach, AKA, Chimera”, Chesshire and Henshaw, J. Comput. Phys. 90 (1990)
- **Does not require presumed movement**
- Similar to Sharma et al, J. Comput. Phys. (2021), however, STK-based search



Fluids Hex/Tet


Constraint-based



MMS

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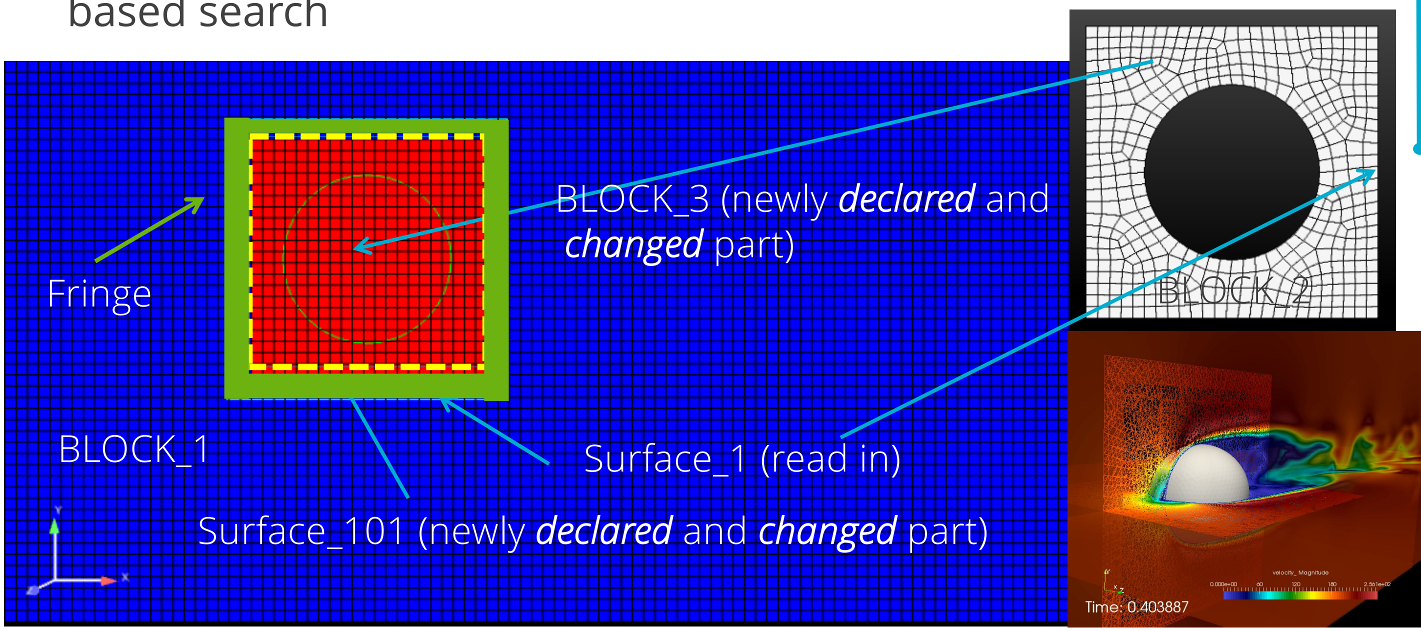
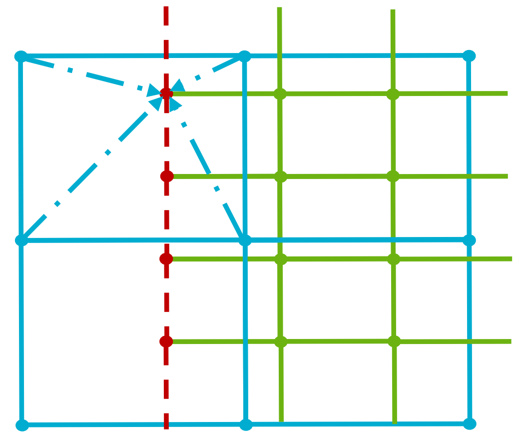


Fluids Hex/Tet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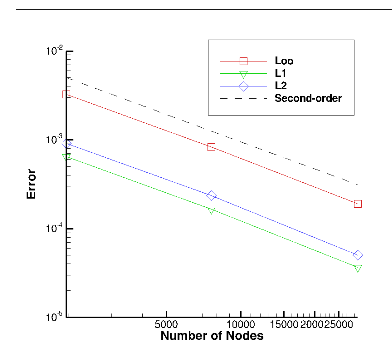


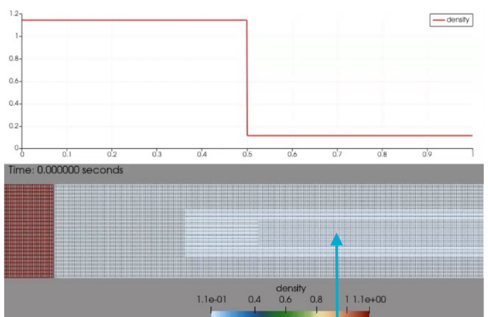
MMS

New!

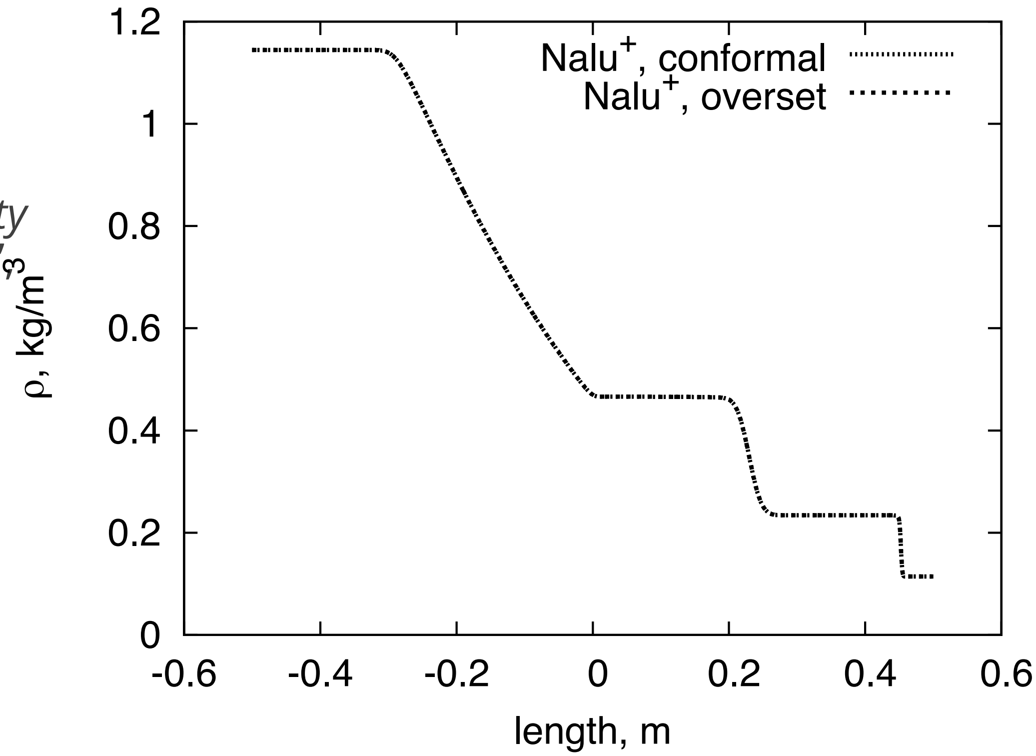
Shock Tube: Overset Mesh

Based on implicit constraint-based approach originally developed for the low-Mach regime

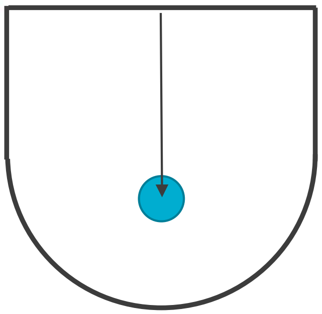
- Domino, S. P. and Horne, W., "*Development and deployment of a credible unstructured, six-DOF, implicit low-Mach overset high-fidelity simulation tool for wave energy applications*", submitted to Renewable Energy (11/2021)
- *<u>Extended</u>* to compressible regime



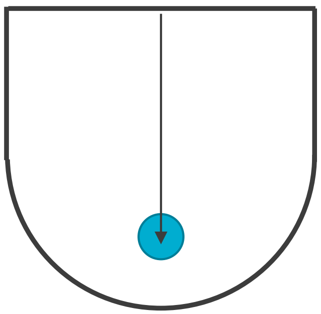
Overset (static) interface



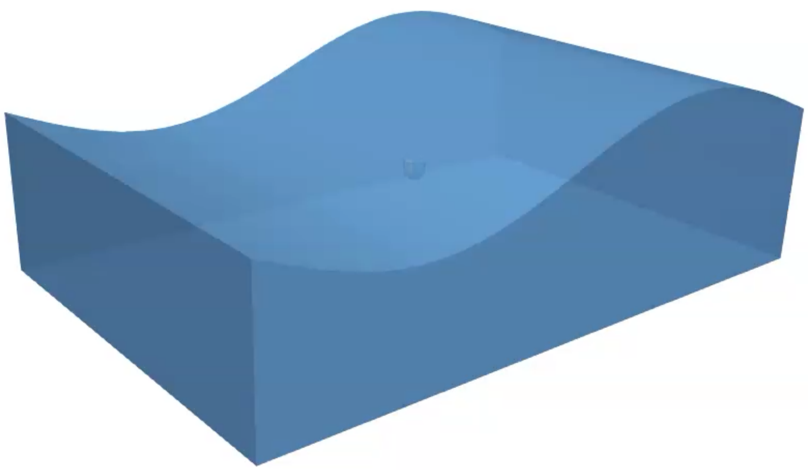
Buoy Subjected to large Wave-Form Validation Benchmark: Explore Center of Mass

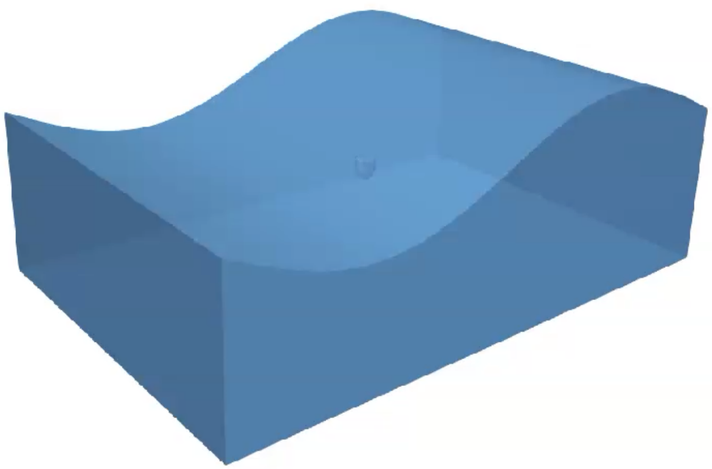


High COM



Low COM





Mesh Motion Review

Low-to-Moderate Displacement:

- Solid mechanics analogies, either PDE or physical (springs)
- Added GCL requirements, matrix is constant

Prescribed Mesh Motion via sliding:

- Halo, or DG-based
- No added GCL, however, matrix changes (along well-defined surfaces)

Full six-DOF motion:

- Overset
- Everything changes!