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ME469: Numerical Abstractions

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Useful Abstractions

Real World

- The intended application

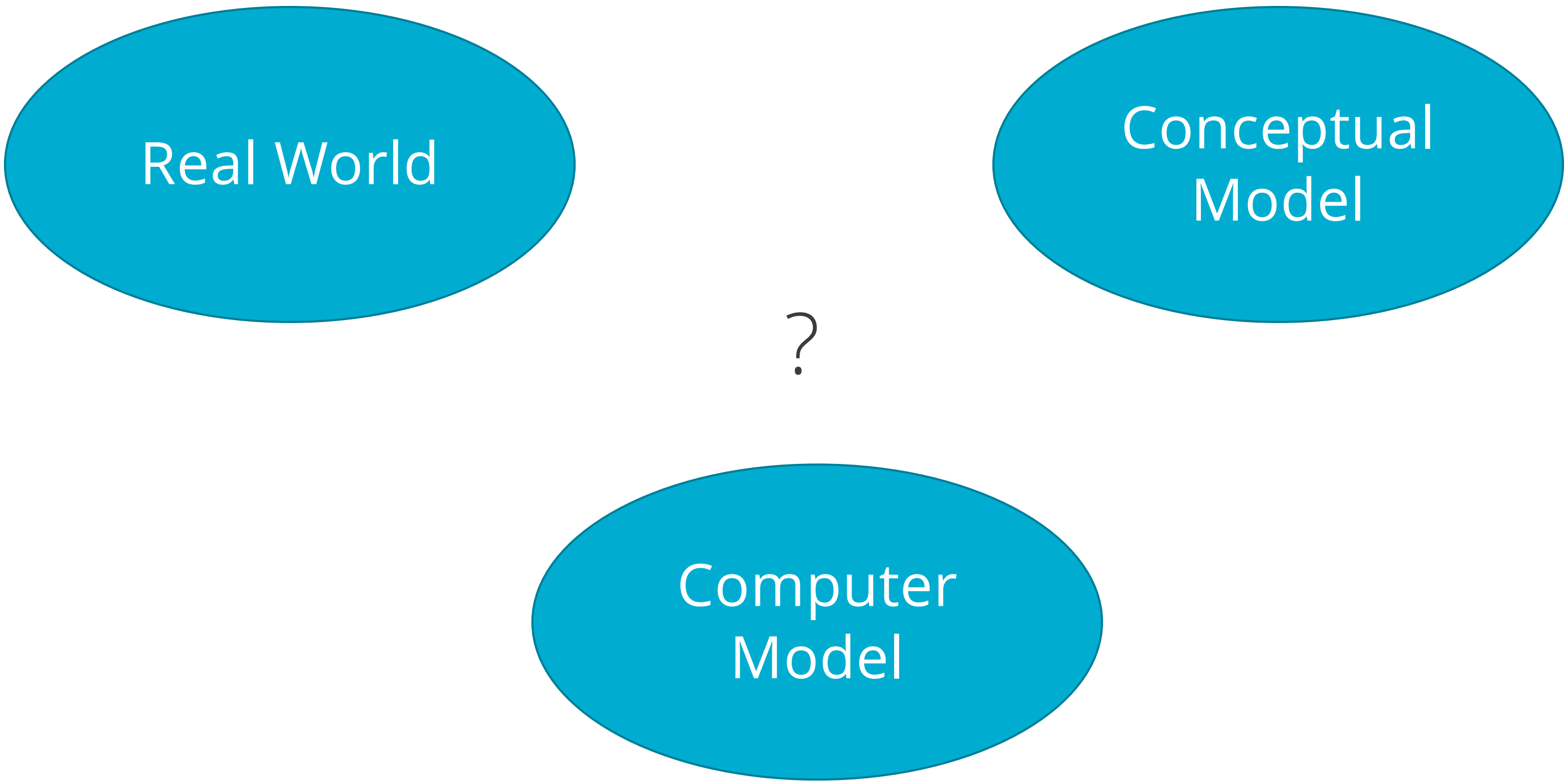
Conceptual Model

- Physical laws, hypothesis and models (e.g. a set of PDEs) and initial and boundary conditions

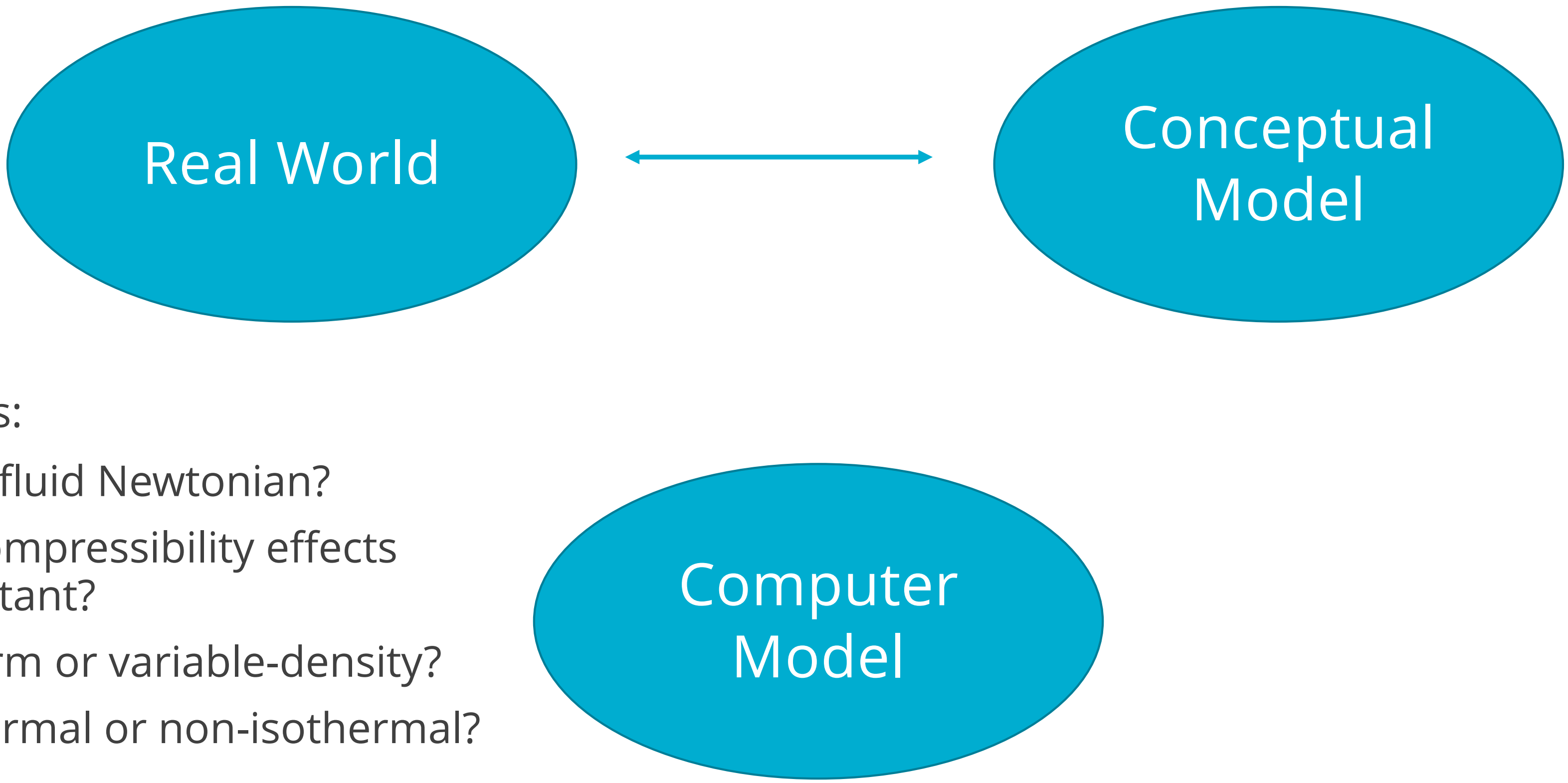
Computer Model

- A set of computational algorithms that allow to build a numerical, or approximated solution to the conceptual model

Mappings:



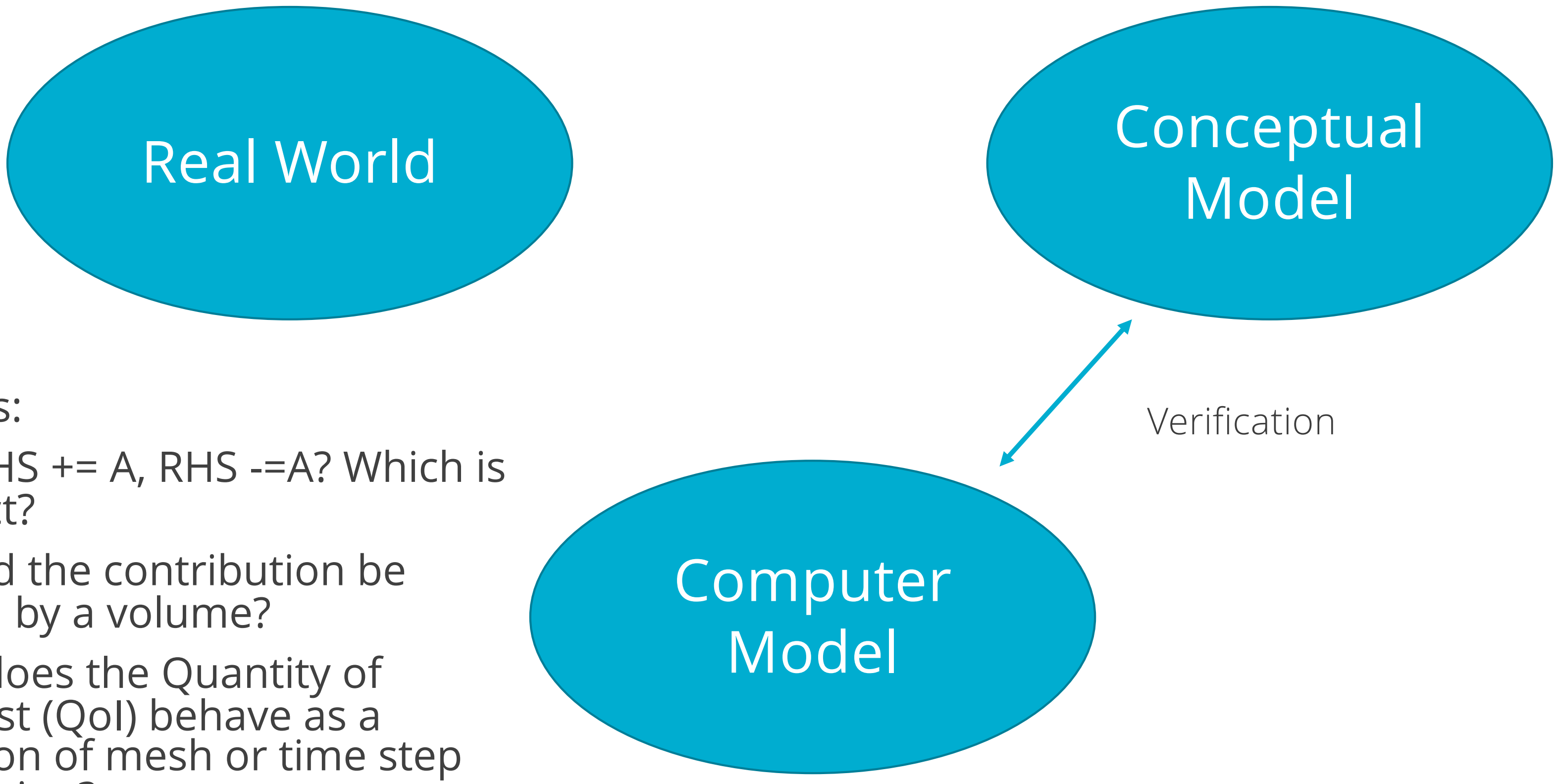
<u>Qualification</u>: ***Adequacy of the conceptual model to provide an acceptable level of agreement for the intended application***



Examples:

- Is the fluid Newtonian?
- Are compressibility effects important?
- Uniform or variable-density?
- Isothermal or non-isothermal?

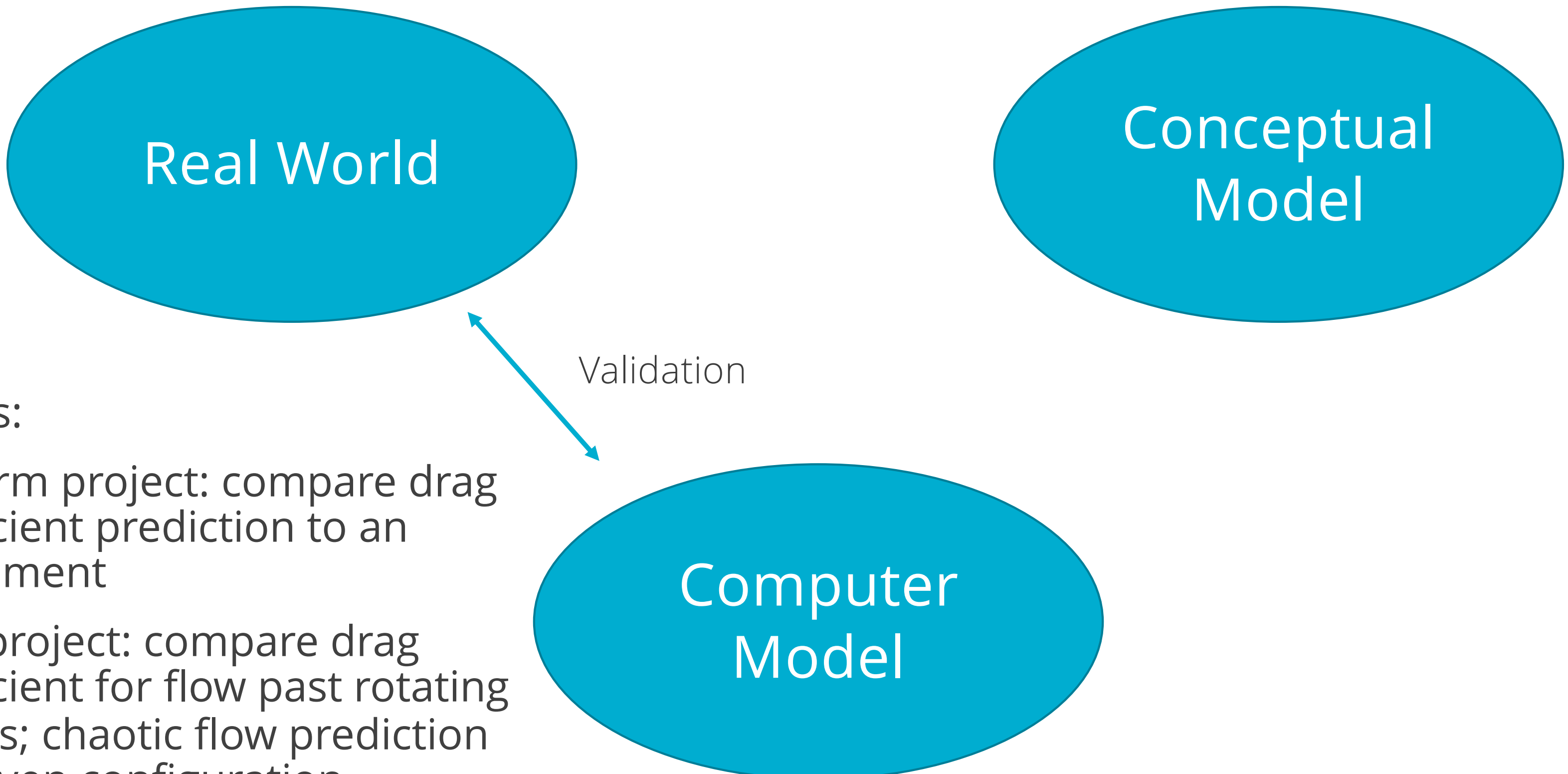
<u>Verification</u>: ***Process of determining that (1) the model implementation faithfully represents the conceptual model (without mistakes) and (2) the solution to the model is accurate***



Examples:

- Is it RHS += A, RHS -=A? Which is correct?
- Should the contribution be scaled by a volume?
- How does the Quantity of Interest (QoI) behave as a function of mesh or time step resolution?

Validation: ***Process of determining the degree to which the numerical solutions (hence the model) is an accurate representation of reality from the perspective of the intended application***



Examples:

- Midterm project: compare drag coefficient prediction to an experiment
- Final project: compare drag coefficient for flow past rotating shapes; chaotic flow prediction at a given configuration

Verification vs Validation (V&V)? The Formal Lexicon

<u>Verification</u>: Are we solving the equations correctly?

- Represents an exercise in computational mathematics
- Given an equation, is the solution converging at known rates?

<u>Validation</u>: Are we solving the correct equations?

- Represents an exercise in understanding the physics associated with the real world use case

In this course, we will strongly focus on *verification*

- Establishing the correctness of the numerical implementation is key
- Comparisons of the numerical results to reality is not the primary objective

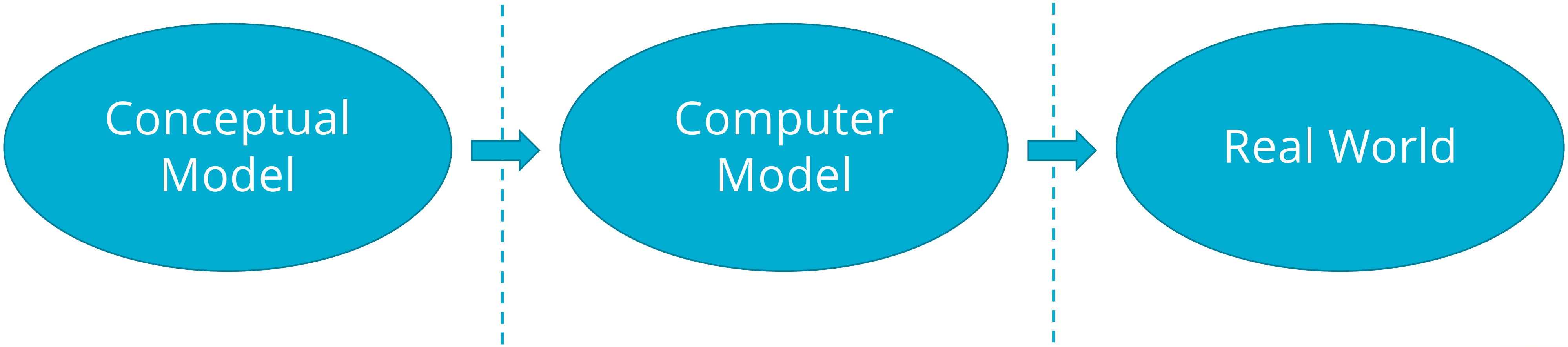
Verification challenges?

- Knowledge of the true solution, i.e., exact analytical solutions
- How many exact solutions exist for our class of physics? Not many!
- Hint: Method of Manufactured Solutions (MMS)

The Construction of a Numerical Method: Discretization and Solution

Mathematical Formulation

- Partial differential equations (PDE)
- Initial Conditions (IC)
- Boundary Conditions (BC)



The Construction of a Numerical Method: Discretization and Solution

Mathematical Formulation

- Partial differential equations (PDE)
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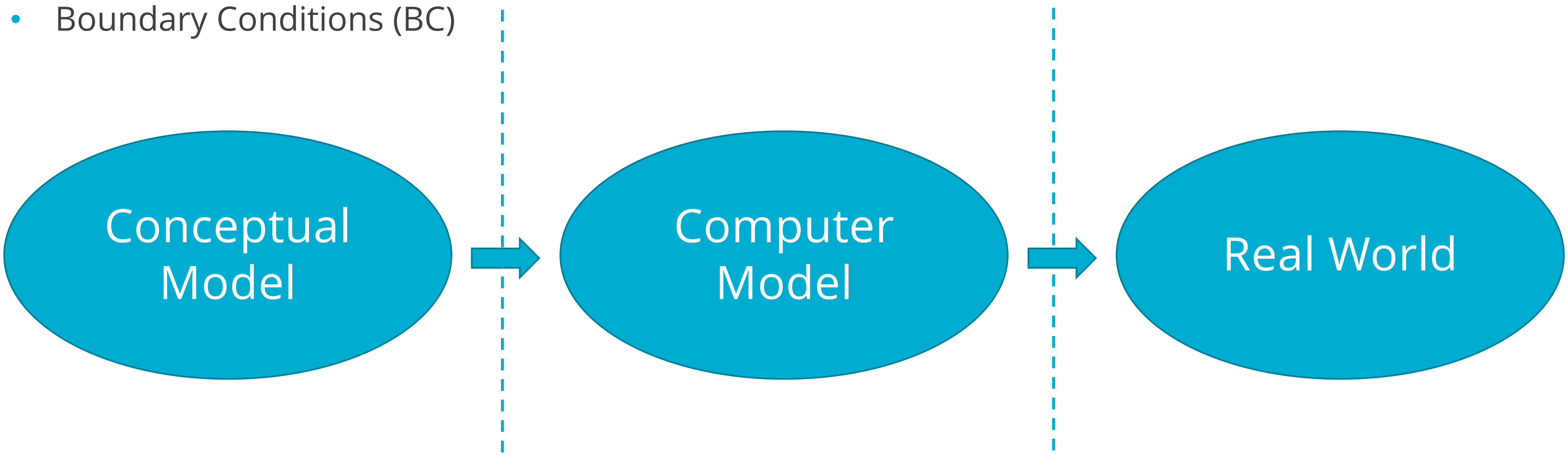
- Boundary Conditions (BC)

Numerical Formulation

- Discretize in space and time
- Algebraic Differential Equations (ADE)



Discretization step

The Construction of a Numerical Method: Discretization and Solution

Mathematical Formulation

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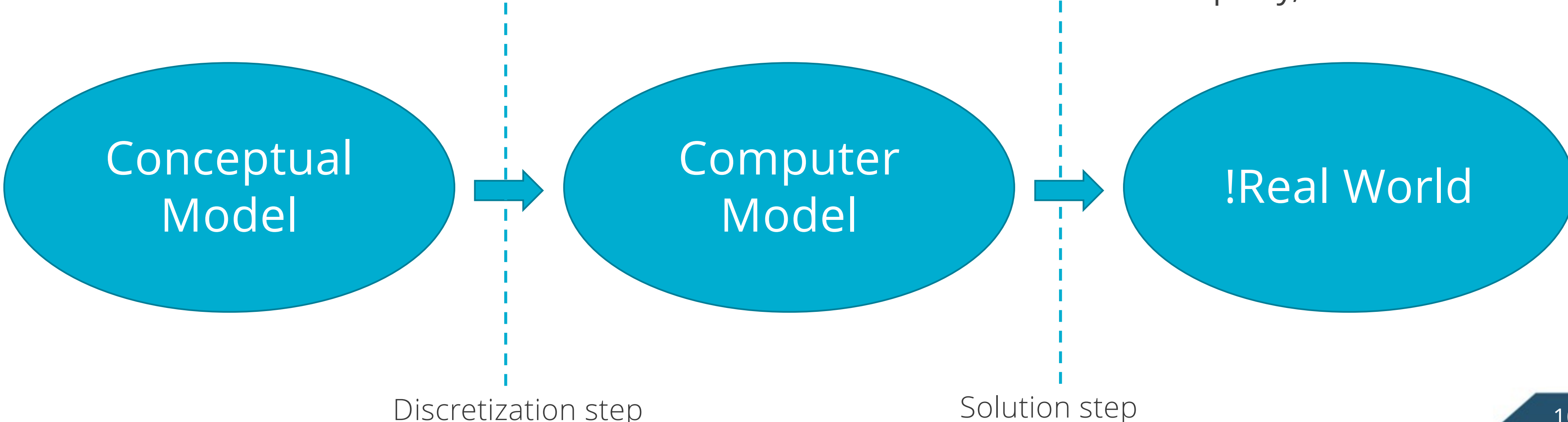
- Boundary Conditions (BC)

Numerical Formulation

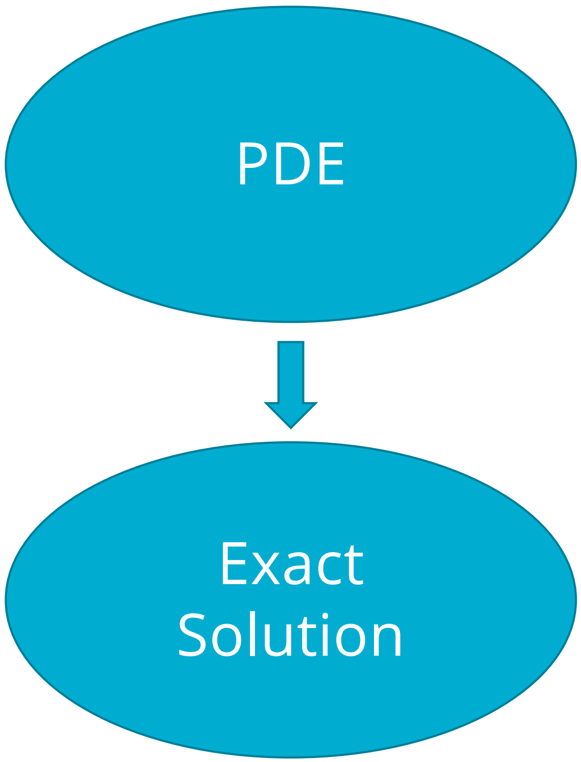
- Discretize in space and time
- Algebraic Differential Equations (ADE)

Numerical Solution

- Algebraic Differential Equations (ADE)
- Linear solvers
- Convergence, mesh adequacy, ...?

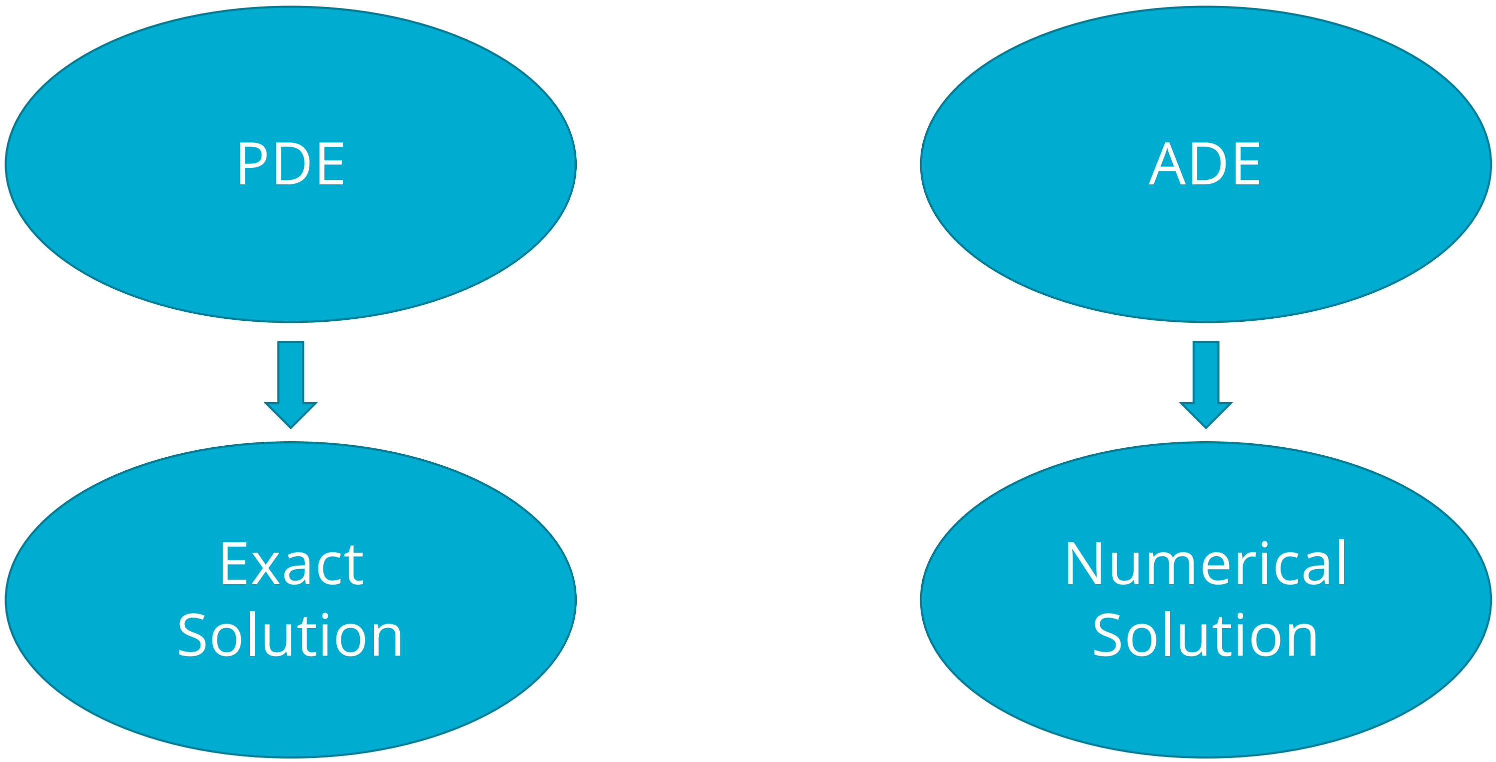


Numerical Simulation Conundrum



What we would like to have…

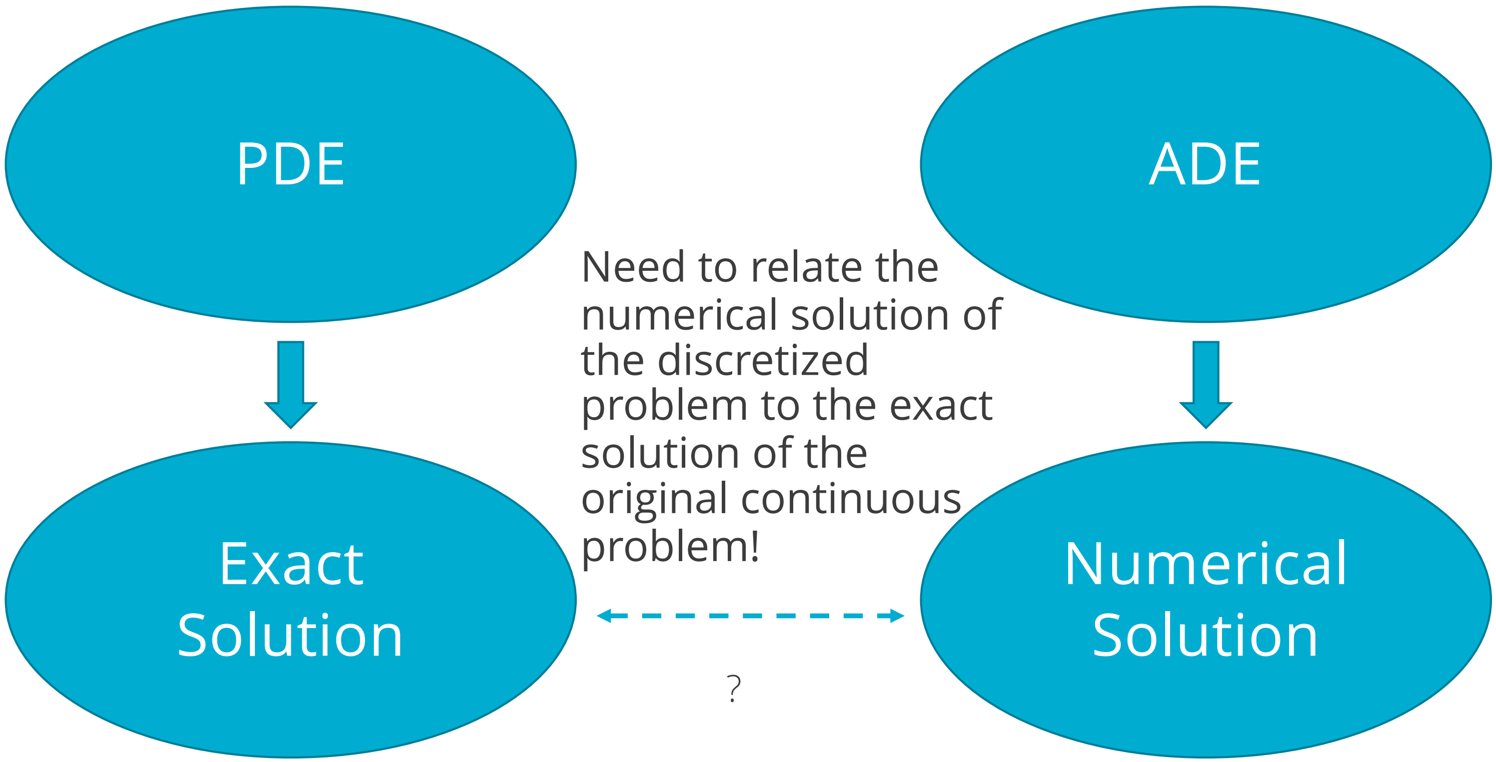
Numerical Simulation Conundrum



What we would like to have...

What we have....

Numerical Simulation Conundrum

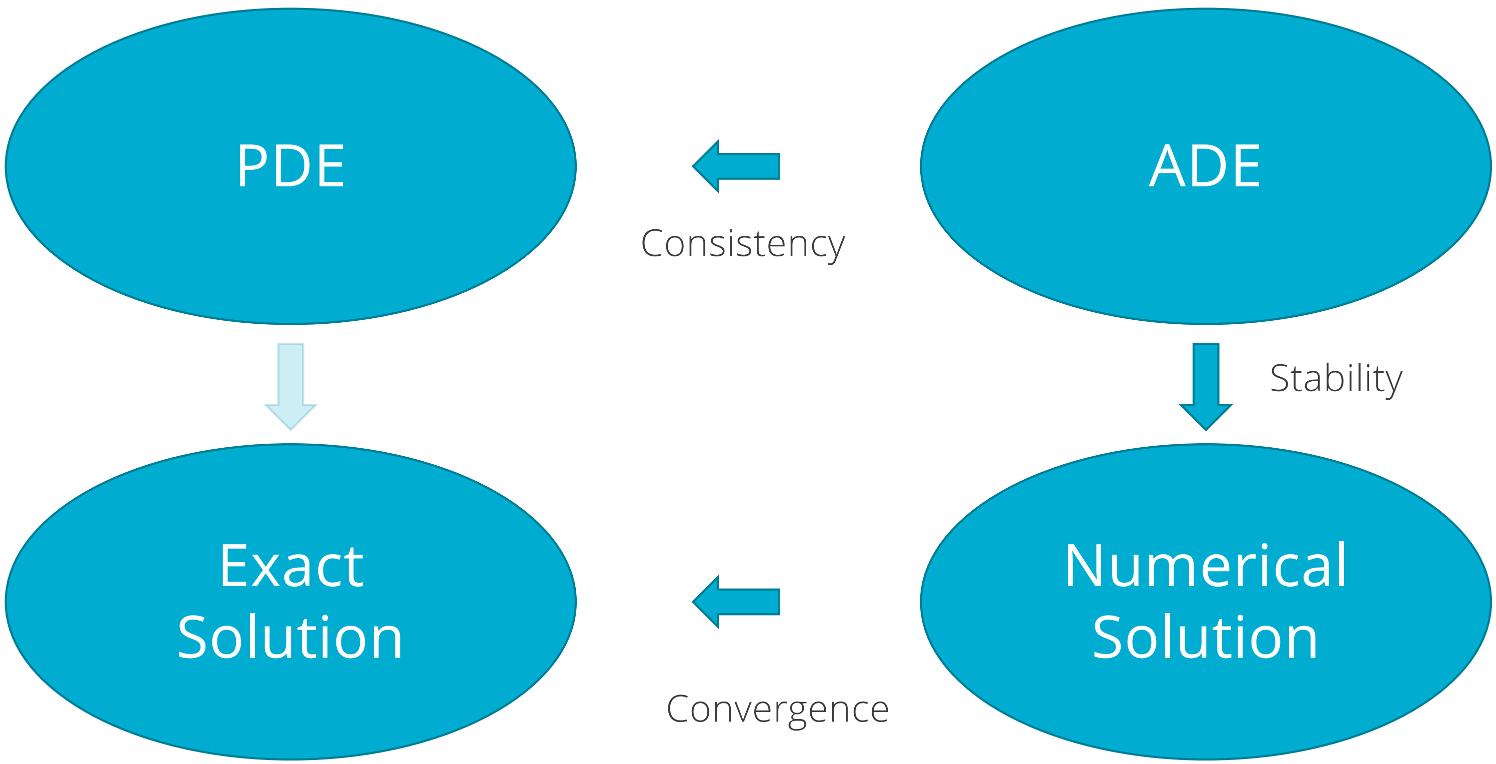


Need to relate the numerical solution of the discretized problem to the exact solution of the original continuous problem!

What we would like to have…

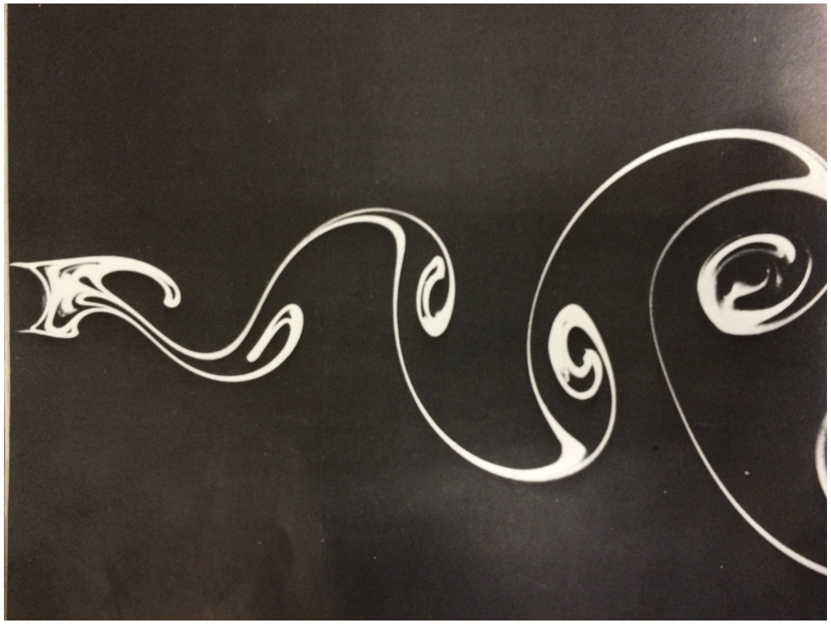
What we have….

Numerical Simulation Conundrum



- For a well-posed, linear PDE, Lax-Richtmyer Equivalence Theorem, AKA the *Fundamental Theorem of Numerical Analysis*: Consistency + Stability = Convergence

Example 1: Vortex Street



Conceptual Model

Computer Model

Real World?

Example 2: Modeling a Luminaria – In the Snow!

Conceptual Model

Computer Model

Real World?