ME6404 Fall 2017

Lab 3 - Model Reference Control

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1. Introduction

The first objective of this lab was to implement a model reference controller in a Matlab simulation and investigate the controller's performance and robustness by varying the model damping, natural frequency, and the system natural frequency. The second objective of the lab was to implement the model reference controller on the portable bridge crane and investigate the controller performance in increasingly unrealistic models. For this, the performance is observed while varying the model damping, model natural frequency, system natural frequency, and response to external disturbances.

2. Matlab simulation

The model is implemented in Matlab with a state space representation of the model and plant. The system is then implemented in Simulink. The system performance is investigated for varying damping, $\zeta_m = 0.1,\ 0.5,\ 0.8$. Then, the modeling error is investigated with varying natural frequency (suspension length).

2.1. Model Reference Controller Design

The design of a model reference controller is based on Lyapunov's Direct Method, i.e. the goal is to make $\dot{V}(e) = -e^T Q e + 2M$ with $M = e^T P(Ax - f(x,u,t) + Bv$ negative definite. A positive semi-definite, symmetric, real matrix $Q = [1,\ 0;\ 0,\ 1\ ]$ is chosen. As a conservative constraint M = 0 is demanded. Then the Lyapunov equation

$$-Q = A^T P + PA$$

is solved for P, which leads to $P = [\frac{\omega_m^4 + \omega_m^2 + 4\omega_n^2\zeta_m^2}{4\,\omega_m^2\omega_n\zeta_m},\ \frac{1}{2\omega_m^2};\ \frac{1}{2\omega_m^2},\ \frac{\omega_m^2+1}{4\omega_m^2\omega_n\zeta_m}\ ]$

Herewith, $M = -(P_2 e_1 + P_3 e_2)(u - v + \omega_m^2 x_1 - \omega_n^2 x_1 + 2\omega_n x_2 \zeta_m - 2\omega_n x_2 \zeta_n$

In order to achieve M = 0, a controller u is computed as

$$u = v - \omega_m^2 x_1 + \omega_n^2 x_1 - 2\omega_m x_2 \zeta_m + 2\omega_n x_2 \zeta_n$$

and implemented in a Matlab function.

2.2. Model Damping and System Performance

For all simulations the system input is a velocity pulse of $t_{on} = 1s$, $t_{off} = 5\ s$. First, three different damping ratios for the desired model are examined, while the natural frequency of the model remains unchanged. Figure 1 shows that an increased damping ratio leads to a decrease in the amplitude of the oscillation. This results in a reduced settling time. In the case of $\zeta_m =$ 0.1 there is significant residual oscillation. Accordingly, also the error increases with smaller damping ratios, as shown in Figure 2.

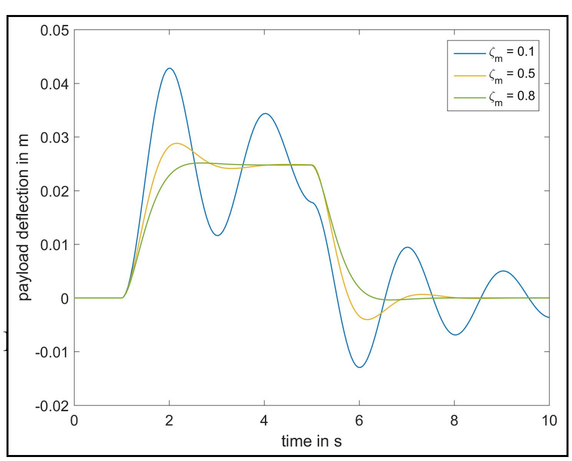

Figure 1: Matlab simulation payload deflection for various damping coefficients

Figure 2: Matlab simulation payload deflection error for various damping coefficients

2.3. Model Error and System Frequency

Second, the model natural frequency is changed, which occurs in practice when the suspension length varies. Figure 3 shows the influence of changed cable lengths on the deflection error. While the error is negligible for the nominal length L = 1 m, it is significant for other cable lengths. At the beginning of the time series, it is recognizable that the plant deviates from the model one side for smaller suspension lengths and to the other side for larger cable lengths.

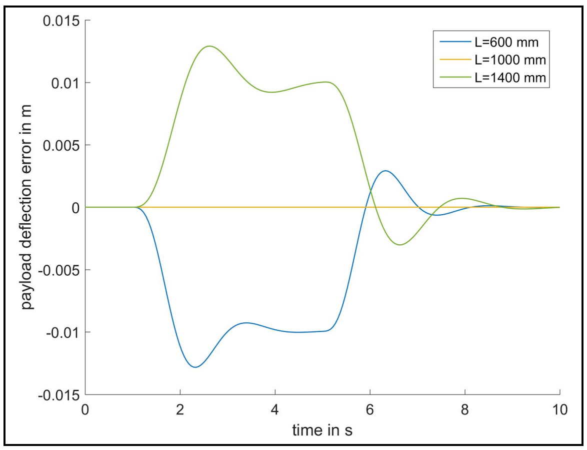

Figure 3: Matlab simulation payload deflection error for various suspension lengths

3. Bridge Crane Implementation

The controller is implemented on the PBC. The controller performance is investigated for increasing damping values, starting with $\zeta_m = 0.5$. Next, the controller performance is investigated for increasing model natural frequency, starting at $\omega_m = \omega_n$. Then, the robustness of the controller is investigated by varying the natural frequency of the plant (suspension cable length), without recalculating the state space matrices. Finally, the controller performance response to external disturbances is investigated. Results are discussed in following sections.

3.1. Controller Performance - Model Damping

The model reference controller started out well with the lowest damping ratio of 0.5, by driving the oscillation down to zero after the initial command velocity pulse. However, when the damping was increased to about 1.75, we began to see a decrease in controller performance and at a damping of 2.5, the oscillations started to become uncontrollable as seen in Figure 4 below.

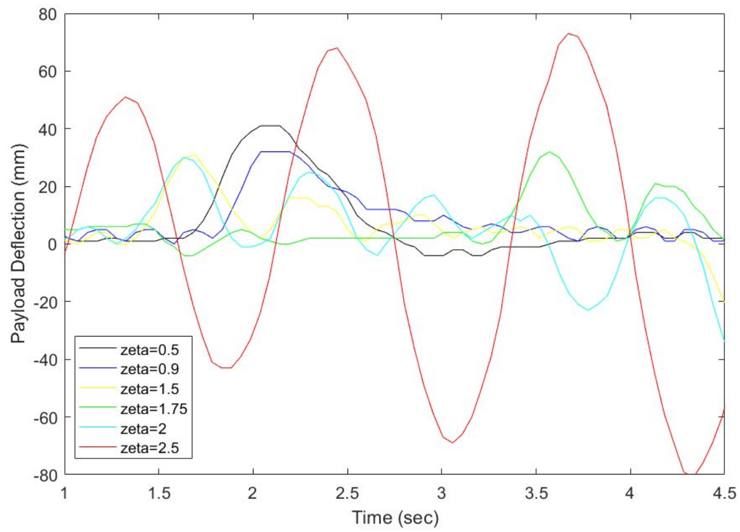

Figure 4: Oscillation time history when model damping ratio is varied

From these results as well as our initial simulations, we can see that inaccurately setting the model's damping ratio too far away from the plant's can have a disastrous effect on the performance of our model reference controller.

3.2. Controller Performance - Model Frequency

It is observed that when the model's damping ratio is held at 0.5 but natural frequency is increased to even slightly above the calculated value of 3.1321, we begin to see large and frequent oscillations in the system which takes the controller some time to stabilize. This sensitivity can be attributed to the fact that the model's natural frequency contributes to the control law as a second degree polynomial (compared to first degree for the damping ratio) and thus small changes can lead to decreased controller performance.

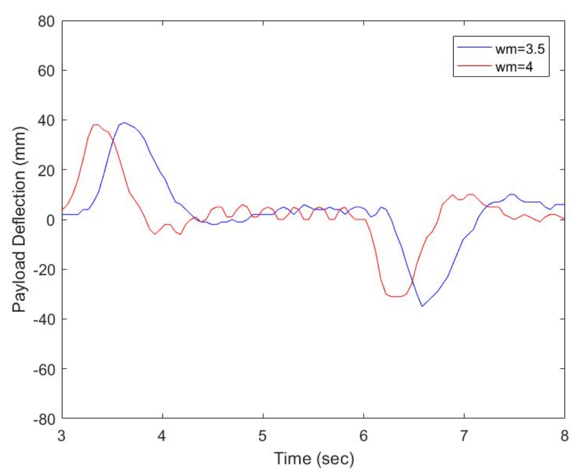

Figure 5: Oscillation time history when model natural frequency is varied

3.3. Controller Robustness - System Frequency

In order to assess the controller robustness, the natural frequency of the system was varied by changing the suspension length, without recalculating the state matrices. In Figure 6 it can be seen that the performance of the controller degrades with higher and lower cable lengths. As the actual system differs from the model implementation, the controller performance gets worse. Therefore the controller robustness is limited by the accuracy of the controller implementation, i.e. it requires an accurate model of the plant.

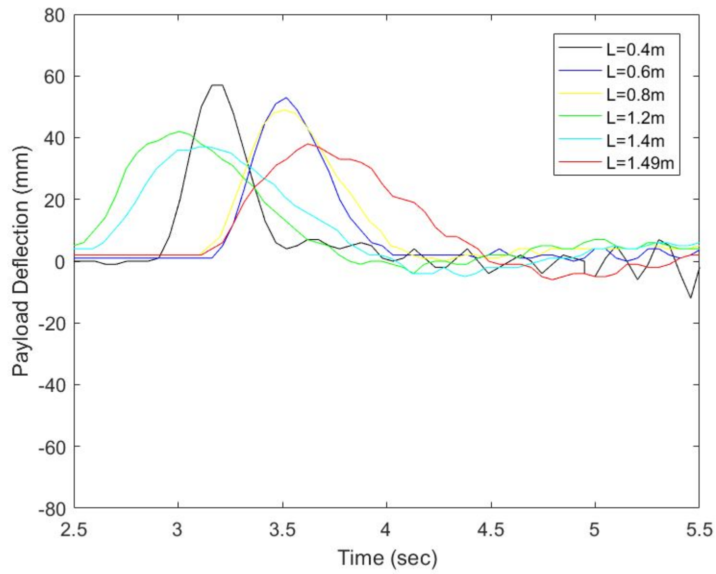

Figure 6: Oscillation time history when suspension cable length is varied

3.4. Controller Performance - External Disturbances

The controller performance is observed after an external disturbance with zero command signal. The payload was rapidly pushed, and as shown in Figure 7, the controller returns the payload to its nominal position in about 2 seconds. Small residual oscillations can be seen around 0 mm deflection, as the controller cannot perfectly track and control the response. This could be due to modeling error or actuator limitations in response to an impulse disturbance.

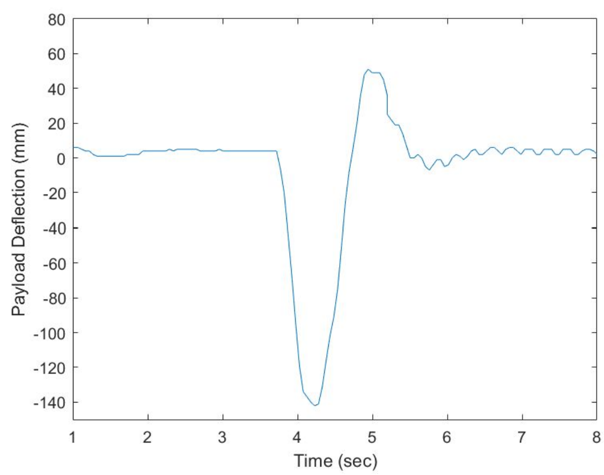

Figure 7: Payload oscillation in response to an external disturbance with no command signal

4. Controller Implementation Challenges

Assumptions in the controller implementation can limit the accuracy or performance of the controller. For example, the state space formulation of the model and plant uses a small angle approximation for the payload deflection angle. This approximation is a source of error, which affects the accuracy of the controller.

The system simulations also assumed a continuous system where control computations like integrals and derivatives for calculating the system states were done in continuous time. However, in the actual crane, digital signals are sampled at a particular sampling rate and all computations are performed in distinct intervals by a sample-and-hold method. These discrete time calculations are performed in the Z-domain as opposed to our state space representation in the S-domain. It is also necessary that when we sample data from the crane, we ensure that the sampling frequency satisfies the Nyquist sampling criterion to prevent signal aliasing, otherwise the signal must be reconstructed using an interpolation method like Shannon's Sampling Theorem.

Additionally, the feedback control relies on sensors for state information. This is a potential source for error with imperfect measurements on the state. Inaccurate state tracking will obviously limit the controller's performance and ability to track a response. There could also be delay time for the sensors to transmit information to the controller, which is not accounted for in the controller implementation.

Actuator limitations can also limit the controller's performance. In response to an external disturbance, the crane system throws an error. The alarm likely results from limitations on the actuators, as they are unable to respond to the impulse input quickly enough. As expected, there are physical limitations on the controller performance that are not captured in the model implementation.