实验三 - 马尔科夫决策

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2018 年 10 月 26 日

马尔科夫决策过程

马尔科夫随机过程（MDP, Markov Decision Process）是一种离散时间随机控制过程。

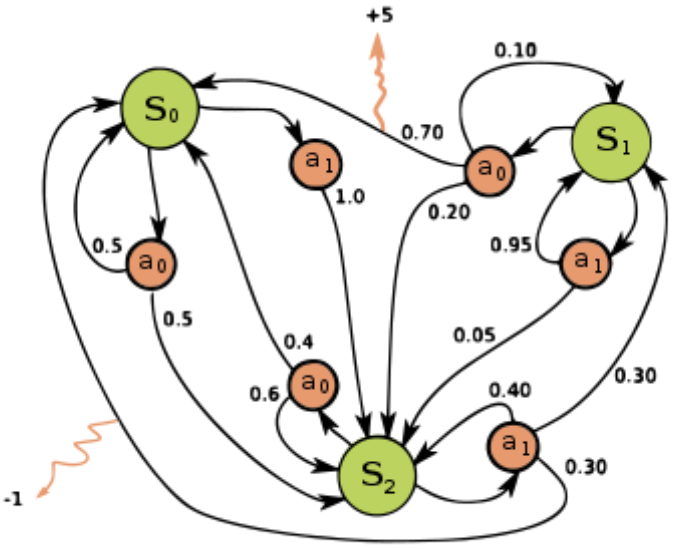
在每一步中，随机过程处于状态 s，决策者此时采取一个可行的决策 a。在下一步随机过程将转移到新的状态 s'，并给决策者相应的回报 $R_a(s, s')$。

随机过程进入新状态 s' 的概率取决于所采取的决策。具体来说，其概率大小有转移矩阵中的 $P_a(s, s')$ 定义。因此，下一个状态 s' 取决于当前状态 s 和决策者采取的决策 a。在已知 s 和 a 的情况之下，状态 s' 与所有之前的状态都是（条件）独立的。也就是说 MDP 具有马尔科夫性质，即随机过程后续的状态的条件概率分布只与当前状态有关，与之前的状态无关。

定义

马尔科夫过程是一个 5 元组 $(S, A, P_a, R_a; \gamma)$，其中：

- S 是有穷状态空间集合
- A 是有穷决策集合，A_s 表示状态 s 下可以采取的决策集合
- $P_a(s, s') = Pr(s_{t+1} = s' | s_t = s, a_t = a)$ 表示在 t 时刻处于状态 s 并采取决策 a 将在 $t+1$ 进入状态 s' 的概率，此概率与时间 t 无关
- $R_a(s, s')$ 是从状态 s 经过决策 a 到达状态 s' 时的期望瞬时收益
- $\gamma \in [0, 1)$ 是决策因子，表示将来状态与当前状态的重要度差别



三个状态、两个决策的 MDP

问题

假定决策的方案：一个函数 π，其中 $\pi(s)$ 表示在状态 s 下采取的决策。一旦有了决策方案 π，再联合在给定方案下到达下一状态的概率 $Pr(s_{t+1} = s' | s_t = s, a_t = a)$，我们可以得到到达下一状态的概率 $Pr(s_{t+1} = s' | s_t = s)$，这就是一个马尔科夫转移矩阵。

MDP 的核心问题在于找到最优的决策方案。我们的目标是选择最优的 π，使得期望收益最大化，通常我们会将将来的收益打一个折扣，因为无穷远的时间收益即使很大也没有意义：

$$W = \sum_{t=0}^{\infty} \gamma^t R_{at}(s_t, s_t + 1) \quad \text{这里 } a_t = \pi(s_t)\text{，也就是我们采取的决策方案} \tag{1}$$

其中 γ 用来表示折扣因子并且满足 $\gamma \in [0, 1)$。例如，当折扣率为 r 时，即后一时刻 $1 + r$ 块钱的收益只相当于当前时刻 1 块钱的收益，那么就有 $\gamma = \frac{1}{1+r}$。

由于 MDP 具有马尔科夫性质，W 的值与时间是无关的，因此其只是 s 的函数。

算法

已知：转移函数 P，收益函数 R

求解：最优决策方案以最大化期望收益

动态规划算法：

定义两个数组 V 和 π，其中 V 作为暂存数组，π 包含采取的决策。在算法结束时，π 将包含最优决策方案，$V(s)$ 将包含折后收益和。递归计算如下：

$$\pi(s) := \arg\max_a \left\{ \sum_{s'} P_a(s,s') \left(R_a(s,s') + \gamma V(s') \right) \right\} \tag{2}$$

这一步主要是寻求一个决策 a 使得收益最大化。

$$V(s) := \sum_{s'} P_{\pi(s)}(s,s') \left(R_{\pi(s)}(s,s') + \gamma V(s') \right) \tag{3}$$

这一步根据决策累积计算收益。

随机需求的单商品存贮决策

问题描述

每个月，仓库经理都会清点某种商品的当前库存量，从而决定是否要从供应商那里进货，进货的话要进多少。在此过程中，他需要权衡该商品库存带来的成本，和不能满足消费者对该商品的需求所带来的损失。他的目标就是最大化各月所得收益和期望值。我们设商品的需求量是一个已知概率分布的随机变量，且积压订单是不允许的，故库存量不会为负数。

- s_t 是第 t 个月的初库存量，它是状态变量
- a_t 是第 t 个月的订货量，它是决策变量
- D_t 是第 t 个月的随机需求量，假定该需求满足一个时间齐次的分布 $p_j = p(D_t = j), j = 0, 1, 2...$，也就是说需求量的分布与时间 t 无关

由于库存量非负，得到状态转移方程：

$$s_{t+1} = max\{s_t + a_t - D_t, 0\} \equiv [s_t + a_t - D_t]^+ \tag{4}$$

假设：

- 每个月月初做出是否订货和订货数量的决策，并假定订货可以及时送到
- 对商品的需求贯穿整个月，但是在该月的最后一天所有订单必须得到满足
- 如果顾客对某商品的需求超过该商品的库存量，即顾客的需求得不到满足，顾客可以到别处去购买他所需的商品。因此不会有因供货不足而造成订单积压的问题
- 收益、成本和需求分布不会按月改变
- 产品售出量都是整数
- 仓库容量为 M 个单位

建立模型

决策阶段：

$$t = 1, 2, ..., T \tag{5}$$

状态空间：

$$S = \{0, 1, 2, ..., s\} \tag{6}$$

决策集合：

$$A(i) = \{0, 1, 2, ..., s - i\}, i \in S \tag{7}$$

表示在状态 i 下可供选择的有限个决策的集合，由于最多只能存有 $s = M$ 件商品，在状态 i 时最多购进 $s - i$ 件商品。$A = \cup_{i \in S} A(i)$ 表示决策集合。

转移概率：

$$P_a(i, j) = \begin{cases} 0, & j \in (i + a, M], i + a < M \\ p_{i+a-j}, & j \in (0, i + a], i + a \leq M \\ q_{i+a}, & j = 0, i + a \leq M \end{cases} \tag{8}$$

解释如下：

1. 因为购进了 a 件商品，那么在下一个状态最多只能有 $i + a$ 件商品，这就是需求量 $d = 0$ 的情况，发生的概率记为 p_{i+a}。
2. 如果在下一个状态只剩下了 $j \in (0, i + a]$ 件商品，说明卖出了 $d = i + a - j$ 件商品，此事件概率记为 p_{i+a-j}。
3. 如果下一个状态只剩下 0 件商品，这可能是刚好需求量是 $d = i + a$ 全部卖出了，也可能是需求量 $d > i + a$ 但是由于供不应求，顾客转到其他商家去购买了，此事件概率记为 $q_{i+a} = p_{i+a} + p_{i+a+1} + ... + p_{\infty} = \sum_{d=i+a}^{\infty} p_d$。

期望报酬：

$$\sum_j R_a(i, j) = \begin{cases} F(i + a) - O(a) - h(i + a), & t \in [1, T - 1] \\ g(i), t = T \end{cases} \tag{9}$$

解释如下：

1. 从状态 i 选择策略 a 进入下一个状态的总收益等于总营业额 $F(i + a)$ 减去订购 a 件商品的总成本 $O(a)$，再减去 $h(i + a)$，对应于 $i + a$ 件商品的每个月的库存费用。

   其中：

   $$F(u) = \sum_{j=0}^{u-1} p_j f(j) + q_u f(u) \tag{10}$$

   其中又有 $f(u)$ 表示卖出 u 件商品时的收入。

   $O(a)$ 表示当前订购 a 件商品的成本。

   $h(i + a)$ 表示库存量为 $i + a$ 的库存费用。

2. 当处于最后一个时刻时，我们即使采取任何策略也得不到收益了，因此收益就等于 $g(i)$ 表示库存量为 i 时的剩余库存价值。

策略：

选取每个阶段决策的规则为一个策略。一个有限阶段的马尔科夫策略可以写成：

$$\pi = (d_1(i), d_2(i), ..., d_T(i)) \tag{11}$$

其中 $d_t(i)$ 是阶段 t 下状态为 i 时采用的决策。

动态规划递归方程：

- $u_t^*(i)$ 表示第 t 阶段状态是 i 时，采取最优策略，从第 t 阶段到第 T 阶段的最大总期望收益。

$$u_t^*(i) = \begin{cases} max_{a \in A_s} \{\sum_j R_a(i,j) + \sum_{j=0}^{s} P_a(i,j) u_{t+1}^*(j)\}, & t = T-1, T-2, ..., 1 \\ g(i), & t = T \end{cases} \tag{12}$$

  可以看出我们想要计算 $u_1^*(i)$，必须先计算 $u_2^*(i)$，如此递推到需要最先计算 $u_T^*(i) = g(i)$。

- $a_t^*(t)$ 表示使式 (12) 最大化的决策。
- $v^*(i)$ 表示当第 1 阶段状态为 i 时，采用最优策略获得的第 T 阶段最大总期望收益。

实例计算

对参数赋值，令

$$\begin{gathered} o(u) = 2u, \quad g(u) = 0, \quad h(u) = u, \quad s = 3, \quad T = 3, \quad f(u) = 8u \\ p_j = \begin{cases} \frac{1}{4}, & d = 0 \\ \frac{1}{2}, & d = 1 \\ \frac{1}{4}, & d = 2 \end{cases} \end{gathered} \tag{13}$$

用自然语言解释为：

库存量不能多于 3 件，所有成本和收益都是线性的，这意味着每订购一件商品花费为 2，每件商品每月的库存费用为 1，每单位商品售出的收益为 8。根据 (10) 式，可向顾客供应的商品数量为 u 时的期望收益 $F(u)$ 如下所示：

$$F(u) = \begin{cases} 0, & u = 0 \\ \frac{1}{4} \times 0 + (\frac{1}{2} + \frac{1}{4}) \times 8 = 6, & u = 1 \\ \frac{1}{4} \times 0 + \frac{1}{2} \times 8 + \frac{1}{4} \times 16 = 8, & u = 2 \\ \frac{1}{4} \times 0 + \frac{1}{2} \times 8 + \frac{1}{4} \times 16 = 8, & u = 3 \end{cases} \tag{14}$$

如果在第 t 月初库存量为 s_t，购进 a 件新商品，结合订购商品的花费以及库存持有成本，我们可以得到期望收益。

先计算转移概率表：

$$P_a(s_t, j)$$

s_t+a\j	0	1	2	3
0	1	0	0	0
1	$\frac{3}{4}$	$\frac{1}{4}$	0	0

s +a\j	0	1	2	3
2	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

表 1

$\sum_j R_a(i,j)$

s_t\a	0	1	2	3
0	0-0-0=0	6-2-1=3	8-4-2=2	8-6-3=-1
1	6-0-1=5	8-2-2=4	8-4-3=1	8-6-4=-2 ✗
2	8-0-2=6	8-2-3=3	8-4-4=0 ✗	8-6-5=-3 ✗
3	8-0-3=5	8-2-4=2 ✗	8-4-5=-1 ✗	8-6-6=-4 ✗

表二

因为仓库容量为 3 所以打 ✗ 的决策是不可能出现的。

表中每一项都写成了营业额-订购成本-库存成本的形式。营业额通过查询式 (13) 得来，订购成本通过查询 $o(u)$ 得来，库存成本通过查询 $h(u)$ 得来。

Step 1

令 $t=4$

$$u_4^*(i)=g(i)=0, i\in[0,3] \tag{15}$$

Step 2

令 $t=3$

$$\begin{aligned} u_3^*(i)&=\max_{a\in A_s}\{\sum_j R_a(i,j)+\sum_{j=0}^{s}P_a(i,j)u_4^*(j)\} \\ u_3^*(i)&\xlongequal{u_4^*=0}\max_{a\in A_s}\{\sum_j R_a(i,j)\} \\ &i\in[0,3] \end{aligned} \tag{16}$$

根据期望收益表 $\sum_j R_a(i,j)$ 得到如下的决策表：

i	$a_3^*(t)$	$u_3^*(t)$
0	1	3
1	0	5
2	0	6
3	0	5

表三

Step 3

令 $t=2$

$$u_2^*(i)=\max_{a\in A_s}\{\sum_j R_a(i,j)+\sum_{j=0}^{s}P_a(i,j)u_3^*(j)\} \tag{17}$$

例如，对 $u_2(0)$ 在 $a=0$ 时的计算过程如下：

$$u_2(0)=0+1\times u_3^*(0)=3 \tag{18}$$

对 $u_2(1)$ 在 $a=1$ 时的计算过程如下：

$$\begin{aligned} u_2(1)&=3+\frac{1}{4}\times u_3^*(0)+\frac{1}{2}\times u_3^*(1)+\frac{1}{4}\times u_3^*(2)\\ &=3+\frac{1}{4}\times 3+\frac{1}{2}\times 5+\frac{1}{4}\times 6\\ &=7.75 \end{aligned} \tag{19}$$

对每个 i 取不同的 a 计算得到最大的 $u_2^*(i)$，将此时的 a 记为 $a_2^*(i)$。

Step 4

令 $t=1$，重复上述过程，得到 $u_1^*(i)$ 和 $a_1^*(i)$。当初始状态为 i 时，最优策略 $\pi(i)=(a_1^*(i),a_2^*(i),a_3^*(i))$，从初始时刻到结束时刻的总报酬 $v^*(i)$。

实验结果

执行 `make` 得到如下的计算结果：

T = 3 测试

虽然老师给的 PPT 中 P 和 R 的得来是错误的，但是之后根据 P 和 R 进行计算的过程是正确的。使用 PPT 中老师所给的 P 和 R，结果如下：

```
T = 3

Transition probablity P[before_sold][after_sold] =
[[1.   0.   0.   0.  ]
 [0.75 0.25 0.   0.  ]
 [0.25 0.5  0.25 0.  ]
 [0.   0.25 0.5  0.25]]

Reward matrix R[current_stock][action] =
[[ 0 -1 -2 -5]
 [ 5  0 -3  0]
 [ 6 -1  0  0]
 [ 5  0  0  0]]

Calculating time: 2
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 0.0
                if take action: 1 the cumulative reward will be: -1.0
                if take action: 2 the cumulative reward will be: -2.0
                if take action: 3 the cumulative reward will be: -5.0
```

```
        so we choose action: 0 and will get max reward: 0
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 5.0
                if take action: 1 the cumulative reward will be: 0.0
                if take action: 2 the cumulative reward will be: -3.0
        so we choose action: 0 and will get max reward: 5.0
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 6.0
                if take action: 1 the cumulative reward will be: -1.0
        so we choose action: 0 and will get max reward: 6.0
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 5.0
        so we choose action: 0 and will get max reward: 5.0


Calculating time: 1
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 0.0
                if take action: 1 the cumulative reward will be: 0.25
                if take action: 2 the cumulative reward will be: 2.0
                if take action: 3 the cumulative reward will be: 0.5
        so we choose action: 2 and will get max reward: 2.0
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 6.25
                if take action: 1 the cumulative reward will be: 4.0
                if take action: 2 the cumulative reward will be: 2.5
        so we choose action: 0 and will get max reward: 6.25
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 10.0
                if take action: 1 the cumulative reward will be: 4.5
        so we choose action: 0 and will get max reward: 10.0
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 10.5
        so we choose action: 0 and will get max reward: 10.5


Calculating time: 0
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 2.0
                if take action: 1 the cumulative reward will be: 2.0625
                if take action: 2 the cumulative reward will be: 4.125
                if take action: 3 the cumulative reward will be: 4.1875
        so we choose action: 3 and will get max reward: 4.1875
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 8.0625
                if take action: 1 the cumulative reward will be: 6.125
                if take action: 2 the cumulative reward will be: 6.1875
        so we choose action: 0 and will get max reward: 8.0625
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 12.125
                if take action: 1 the cumulative reward will be: 8.1875
        so we choose action: 0 and will get max reward: 12.125
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 14.1875
        so we choose action: 0 and will get max reward: 14.1875


Cumulative maximum reward u[time][current_stock] =
[[ 4.1875  8.0625 12.125  14.1875]
 [ 2.      6.25   10.     10.5   ]
 [ 0.      5.      6.      5.    ]
 [ 0.      0.      0.      0.    ]]

Best action a[time][current_stock] =
[[3. 0. 0. 0.]
 [2. 0. 0. 0.]
 [0. 0. 0. 0.]
 [0. 0. 0. 0.]]
```

这与 PPT 中的计算过程完全相符。

T = 3

```
T = 3

Transition probablity P[before_sold][after_sold] =
[[1.   0.   0.   0.  ]
 [0.75 0.25 0.   0.  ]
 [0.25 0.5  0.25 0.  ]
 [0.   0.25 0.5  0.25]]

Reward matrix R[current_stock][action] =
[[ 0.  3.  2. -1.]
 [ 5.  4.  1.  0.]
 [ 6.  3.  0.  0.]
 [ 5.  0.  0.  0.]]

Calculating time: 2
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 0.0
                if take action: 1 the cumulative reward will be: 3.0
                if take action: 2 the cumulative reward will be: 2.0
                if take action: 3 the cumulative reward will be: -1.0
        so we choose action: 1 and will get max reward: 3.0
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 5.0
                if take action: 1 the cumulative reward will be: 4.0
                if take action: 2 the cumulative reward will be: 1.0
        so we choose action: 0 and will get max reward: 5.0
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 6.0
                if take action: 1 the cumulative reward will be: 3.0
        so we choose action: 0 and will get max reward: 6.0
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 5.0
        so we choose action: 0 and will get max reward: 5.0


Calculating time: 1
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 3.0
                if take action: 1 the cumulative reward will be: 6.5
                if take action: 2 the cumulative reward will be: 6.75
                if take action: 3 the cumulative reward will be: 4.5
        so we choose action: 2 and will get max reward: 6.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 8.5
                if take action: 1 the cumulative reward will be: 8.75
                if take action: 2 the cumulative reward will be: 6.5
        so we choose action: 1 and will get max reward: 8.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 10.75
                if take action: 1 the cumulative reward will be: 8.5
        so we choose action: 0 and will get max reward: 10.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 10.5
```

```
        if take action: 0 the cumulative reward will be: 10.5
    so we choose action: 0 and will get max reward: 10.5


Calculating time: 0
    if current stock is: 0
        if take action: 0 the cumulative reward will be: 6.75
        if take action: 1 the cumulative reward will be: 10.25
        if take action: 2 the cumulative reward will be: 10.75
        if take action: 3 the cumulative reward will be: 9.1875
    so we choose action: 2 and will get max reward: 10.75
    if current stock is: 1
        if take action: 0 the cumulative reward will be: 12.25
        if take action: 1 the cumulative reward will be: 12.75
        if take action: 2 the cumulative reward will be: 11.1875
    so we choose action: 1 and will get max reward: 12.75
    if current stock is: 2
        if take action: 0 the cumulative reward will be: 14.75
        if take action: 1 the cumulative reward will be: 13.1875
    so we choose action: 0 and will get max reward: 14.75
    if current stock is: 3
        if take action: 0 the cumulative reward will be: 15.1875
    so we choose action: 0 and will get max reward: 15.1875


Cumulative maximum reward u[time][current_stock] =
[[10.75   12.75   14.75   15.1875]
 [ 6.75    8.75   10.75   10.5   ]
 [ 3.      5.      6.      5.    ]
 [ 0.      0.      0.      0.    ]]

Best action a[time][current_stock] =
[[2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [1. 0. 0. 0.]
 [0. 0. 0. 0.]]
```

其中 $u[0][0]$ 表示在第 1 天库存量为 0 时最大的收益，即前文所说的 $u_1^*(0)$，这时需要采取的决策是 $a[0][0]$，也就是购进 2 件商品。

从整体上看 $u[v][m]$ 表示第 $v+1$ 天库存量为 m 时采取最佳策略 $a[v][m]$ 得到的最大累计收益。

可以看到计算结果中 P、R 与手动计算的表一、二完全相符，同时 u 和 a 中第 3 行与表三完全相符。不难看出计算结果完全正确。

T = 20

```
T = 20

Transition probablity P[before_sold][after_sold] =
[[1.   0.   0.   0.  ]
 [0.75 0.25 0.   0.  ]
 [0.25 0.5  0.25 0.  ]
 [0.   0.25 0.5  0.25]]
```

```
Reward matrix R[current_stock][action] =
[[ 0.  3.  2. -1.]
 [ 5.  4.  1.  0.]
 [ 6.  3.  0.  0.]
 [ 5.  0.  0.  0.]]

Calculating time: 19
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 0.0
                if take action: 1 the cumulative reward will be: 3.0
                if take action: 2 the cumulative reward will be: 2.0
                if take action: 3 the cumulative reward will be: -1.0
        so we choose action: 1 and will get max reward: 3.0
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 5.0
                if take action: 1 the cumulative reward will be: 4.0
                if take action: 2 the cumulative reward will be: 1.0
        so we choose action: 0 and will get max reward: 5.0
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 6.0
                if take action: 1 the cumulative reward will be: 3.0
        so we choose action: 0 and will get max reward: 6.0
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 5.0
        so we choose action: 0 and will get max reward: 5.0


Calculating time: 18
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 3.0
                if take action: 1 the cumulative reward will be: 6.5
                if take action: 2 the cumulative reward will be: 6.75
                if take action: 3 the cumulative reward will be: 4.5
        so we choose action: 2 and will get max reward: 6.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 8.5
                if take action: 1 the cumulative reward will be: 8.75
                if take action: 2 the cumulative reward will be: 6.5
        so we choose action: 1 and will get max reward: 8.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 10.75
                if take action: 1 the cumulative reward will be: 8.5
        so we choose action: 0 and will get max reward: 10.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 10.5
        so we choose action: 0 and will get max reward: 10.5


Calculating time: 17
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 6.75
                if take action: 1 the cumulative reward will be: 10.25
                if take action: 2 the cumulative reward will be: 10.75
                if take action: 3 the cumulative reward will be: 9.1875
        so we choose action: 2 and will get max reward: 10.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 12.25
                if take action: 1 the cumulative reward will be: 12.75
                if take action: 2 the cumulative reward will be: 11.1875
        so we choose action: 1 and will get max reward: 12.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 14.75
                if take action: 1 the cumulative reward will be: 13.1875
        so we choose action: 0 and will get max reward: 14.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 15.1875
        so we choose action: 0 and will get max reward: 15.1875


Calculating time: 16
```

Calculating time: 16
	if current stock is: 0
		if take action: 0 the cumulative reward will be: 10.75
		if take action: 1 the cumulative reward will be: 14.25
		if take action: 2 the cumulative reward will be: 14.75
		if take action: 3 the cumulative reward will be: 13.359375
	so we choose action: 2 and will get max reward: 14.75
	if current stock is: 1
		if take action: 0 the cumulative reward will be: 16.25
		if take action: 1 the cumulative reward will be: 16.75
		if take action: 2 the cumulative reward will be: 15.359375
	so we choose action: 1 and will get max reward: 16.75
	if current stock is: 2
		if take action: 0 the cumulative reward will be: 18.75
		if take action: 1 the cumulative reward will be: 17.359375
	so we choose action: 0 and will get max reward: 18.75
	if current stock is: 3
		if take action: 0 the cumulative reward will be: 19.359375
	so we choose action: 0 and will get max reward: 19.359375

Calculating time: 15
	if current stock is: 0
		if take action: 0 the cumulative reward will be: 14.75
		if take action: 1 the cumulative reward will be: 18.25
		if take action: 2 the cumulative reward will be: 18.75
		if take action: 3 the cumulative reward will be: 17.40234375
	so we choose action: 2 and will get max reward: 18.75
	if current stock is: 1
		if take action: 0 the cumulative reward will be: 20.25
		if take action: 1 the cumulative reward will be: 20.75
		if take action: 2 the cumulative reward will be: 19.40234375
	so we choose action: 1 and will get max reward: 20.75
	if current stock is: 2
		if take action: 0 the cumulative reward will be: 22.75
		if take action: 1 the cumulative reward will be: 21.40234375
	so we choose action: 0 and will get max reward: 22.75
	if current stock is: 3
		if take action: 0 the cumulative reward will be: 23.40234375
	so we choose action: 0 and will get max reward: 23.40234375

Calculating time: 14
	if current stock is: 0
		if take action: 0 the cumulative reward will be: 18.75
		if take action: 1 the cumulative reward will be: 22.25
		if take action: 2 the cumulative reward will be: 22.75
		if take action: 3 the cumulative reward will be: 21.4130859375
	so we choose action: 2 and will get max reward: 22.75
	if current stock is: 1
		if take action: 0 the cumulative reward will be: 24.25
		if take action: 1 the cumulative reward will be: 24.75
		if take action: 2 the cumulative reward will be: 23.4130859375
	so we choose action: 1 and will get max reward: 24.75
	if current stock is: 2
		if take action: 0 the cumulative reward will be: 26.75
		if take action: 1 the cumulative reward will be: 25.4130859375
	so we choose action: 0 and will get max reward: 26.75
	if current stock is: 3
		if take action: 0 the cumulative reward will be: 27.4130859375
	so we choose action: 0 and will get max reward: 27.4130859375

Calculating time: 13
	if current stock is: 0
		if take action: 0 the cumulative reward will be: 22.75
		if take action: 1 the cumulative reward will be: 26.25
		if take action: 2 the cumulative reward will be: 26.75
		if take action: 3 the cumulative reward will be: 25.415771484375
	so we choose action: 2 and will get max reward: 26.75

```
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 28.25
                if take action: 1 the cumulative reward will be: 28.75
                if take action: 2 the cumulative reward will be: 27.415771484375
        so we choose action: 1 and will get max reward: 28.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 30.75
                if take action: 1 the cumulative reward will be: 29.415771484375
        so we choose action: 0 and will get max reward: 30.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 31.415771484375
        so we choose action: 0 and will get max reward: 31.415771484375


Calculating time: 12
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 26.75
                if take action: 1 the cumulative reward will be: 30.25
                if take action: 2 the cumulative reward will be: 30.75
                if take action: 3 the cumulative reward will be: 29.41644287109375
        so we choose action: 2 and will get max reward: 30.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 32.25
                if take action: 1 the cumulative reward will be: 32.75
                if take action: 2 the cumulative reward will be: 31.41644287109375
        so we choose action: 1 and will get max reward: 32.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 34.75
                if take action: 1 the cumulative reward will be: 33.41644287109375
        so we choose action: 0 and will get max reward: 34.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 35.41644287109375
        so we choose action: 0 and will get max reward: 35.41644287109375


Calculating time: 11
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 30.75
                if take action: 1 the cumulative reward will be: 34.25
                if take action: 2 the cumulative reward will be: 34.75
                if take action: 3 the cumulative reward will be: 33.41661071777344
        so we choose action: 2 and will get max reward: 34.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 36.25
                if take action: 1 the cumulative reward will be: 36.75
                if take action: 2 the cumulative reward will be: 35.41661071777344
        so we choose action: 1 and will get max reward: 36.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 38.75
                if take action: 1 the cumulative reward will be: 37.41661071777344
        so we choose action: 0 and will get max reward: 38.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 39.41661071777344
        so we choose action: 0 and will get max reward: 39.41661071777344


Calculating time: 10
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 34.75
                if take action: 1 the cumulative reward will be: 38.25
                if take action: 2 the cumulative reward will be: 38.75
                if take action: 3 the cumulative reward will be: 37.41665267944336
        so we choose action: 2 and will get max reward: 38.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 40.25
                if take action: 1 the cumulative reward will be: 40.75
                if take action: 2 the cumulative reward will be: 39.41665267944336
        so we choose action: 1 and will get max reward: 40.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 42.75
```

```
                if take action: 0 the cumulative reward will be: 42.75
                if take action: 1 the cumulative reward will be: 41.41665267944336
        so we choose action: 0 and will get max reward: 42.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 43.41665267944336
        so we choose action: 0 and will get max reward: 43.41665267944336


Calculating time: 9
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 38.75
                if take action: 1 the cumulative reward will be: 42.25
                if take action: 2 the cumulative reward will be: 42.75
                if take action: 3 the cumulative reward will be: 41.41666316986084
        so we choose action: 2 and will get max reward: 42.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 44.25
                if take action: 1 the cumulative reward will be: 44.75
                if take action: 2 the cumulative reward will be: 43.41666316986084
        so we choose action: 1 and will get max reward: 44.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 46.75
                if take action: 1 the cumulative reward will be: 45.41666316986084
        so we choose action: 0 and will get max reward: 46.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 47.41666316986084
        so we choose action: 0 and will get max reward: 47.41666316986084


Calculating time: 8
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 42.75
                if take action: 1 the cumulative reward will be: 46.25
                if take action: 2 the cumulative reward will be: 46.75
                if take action: 3 the cumulative reward will be: 45.41666579246521
        so we choose action: 2 and will get max reward: 46.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 48.25
                if take action: 1 the cumulative reward will be: 48.75
                if take action: 2 the cumulative reward will be: 47.41666579246521
        so we choose action: 1 and will get max reward: 48.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 50.75
                if take action: 1 the cumulative reward will be: 49.41666579246521
        so we choose action: 0 and will get max reward: 50.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 51.41666579246521
        so we choose action: 0 and will get max reward: 51.41666579246521


Calculating time: 7
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 46.75
                if take action: 1 the cumulative reward will be: 50.25
                if take action: 2 the cumulative reward will be: 50.75
                if take action: 3 the cumulative reward will be: 49.4166664481163
        so we choose action: 2 and will get max reward: 50.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 52.25
                if take action: 1 the cumulative reward will be: 52.75
                if take action: 2 the cumulative reward will be: 51.4166664481163
        so we choose action: 1 and will get max reward: 52.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 54.75
                if take action: 1 the cumulative reward will be: 53.4166664481163
        so we choose action: 0 and will get max reward: 54.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 55.4166664481163
        so we choose action: 0 and will get max reward: 55.4166664481163
```

```
Calculating time: 6
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 50.75
                if take action: 1 the cumulative reward will be: 54.25
                if take action: 2 the cumulative reward will be: 54.75
                if take action: 3 the cumulative reward will be: 53.416666612029076
        so we choose action: 2 and will get max reward: 54.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 56.25
                if take action: 1 the cumulative reward will be: 56.75
                if take action: 2 the cumulative reward will be: 55.416666612029076
        so we choose action: 1 and will get max reward: 56.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 58.75
                if take action: 1 the cumulative reward will be: 57.416666612029076
        so we choose action: 0 and will get max reward: 58.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 59.416666612029076
        so we choose action: 0 and will get max reward: 59.416666612029076


Calculating time: 5
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 54.75
                if take action: 1 the cumulative reward will be: 58.25
                if take action: 2 the cumulative reward will be: 58.75
                if take action: 3 the cumulative reward will be: 57.41666665300727
        so we choose action: 2 and will get max reward: 58.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 60.25
                if take action: 1 the cumulative reward will be: 60.75
                if take action: 2 the cumulative reward will be: 59.41666665300727
        so we choose action: 1 and will get max reward: 60.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 62.75
                if take action: 1 the cumulative reward will be: 61.41666665300727
        so we choose action: 0 and will get max reward: 62.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 63.41666665300727
        so we choose action: 0 and will get max reward: 63.41666665300727


Calculating time: 4
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 58.75
                if take action: 1 the cumulative reward will be: 62.25
                if take action: 2 the cumulative reward will be: 62.75
                if take action: 3 the cumulative reward will be: 61.41666666325182
        so we choose action: 2 and will get max reward: 62.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 64.25
                if take action: 1 the cumulative reward will be: 64.75
                if take action: 2 the cumulative reward will be: 63.41666666325182
        so we choose action: 1 and will get max reward: 64.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 66.75
                if take action: 1 the cumulative reward will be: 65.41666666325182
        so we choose action: 0 and will get max reward: 66.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 67.41666666325182
        so we choose action: 0 and will get max reward: 67.41666666325182


Calculating time: 3
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 62.75
                if take action: 1 the cumulative reward will be: 66.25
                if take action: 2 the cumulative reward will be: 66.75
                if take action: 3 the cumulative reward will be: 65.41666666581295
```

```
        so we choose action: 2 and will get max reward: 66.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 68.25
                if take action: 1 the cumulative reward will be: 68.75
                if take action: 2 the cumulative reward will be: 67.41666666581295
        so we choose action: 1 and will get max reward: 68.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 70.75
                if take action: 1 the cumulative reward will be: 69.41666666581295
        so we choose action: 0 and will get max reward: 70.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 71.41666666581295
        so we choose action: 0 and will get max reward: 71.41666666581295


Calculating time: 2
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 66.75
                if take action: 1 the cumulative reward will be: 70.25
                if take action: 2 the cumulative reward will be: 70.75
                if take action: 3 the cumulative reward will be: 69.41666666645324
        so we choose action: 2 and will get max reward: 70.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 72.25
                if take action: 1 the cumulative reward will be: 72.75
                if take action: 2 the cumulative reward will be: 71.41666666645324
        so we choose action: 1 and will get max reward: 72.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 74.75
                if take action: 1 the cumulative reward will be: 73.41666666645324
        so we choose action: 0 and will get max reward: 74.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 75.41666666645324
        so we choose action: 0 and will get max reward: 75.41666666645324


Calculating time: 1
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 70.75
                if take action: 1 the cumulative reward will be: 74.25
                if take action: 2 the cumulative reward will be: 74.75
                if take action: 3 the cumulative reward will be: 73.41666666661331
        so we choose action: 2 and will get max reward: 74.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 76.25
                if take action: 1 the cumulative reward will be: 76.75
                if take action: 2 the cumulative reward will be: 75.41666666661331
        so we choose action: 1 and will get max reward: 76.75
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 78.75
                if take action: 1 the cumulative reward will be: 77.41666666661331
        so we choose action: 0 and will get max reward: 78.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 79.41666666661331
        so we choose action: 0 and will get max reward: 79.41666666661331


Calculating time: 0
        if current stock is: 0
                if take action: 0 the cumulative reward will be: 74.75
                if take action: 1 the cumulative reward will be: 78.25
                if take action: 2 the cumulative reward will be: 78.75
                if take action: 3 the cumulative reward will be: 77.41666666665333
        so we choose action: 2 and will get max reward: 78.75
        if current stock is: 1
                if take action: 0 the cumulative reward will be: 80.25
                if take action: 1 the cumulative reward will be: 80.75
                if take action: 2 the cumulative reward will be: 79.41666666665333
        so we choose action: 1 and will get max reward: 80.75
        if current stock is: 2
```

```
        if current stock is: 2
                if take action: 0 the cumulative reward will be: 82.75
                if take action: 1 the cumulative reward will be: 81.41666666665333
        so we choose action: 0 and will get max reward: 82.75
        if current stock is: 3
                if take action: 0 the cumulative reward will be: 83.41666666665333
        so we choose action: 0 and will get max reward: 83.41666666665333


 Cumulative maximum reward u[time][current_stock] =
[[78.75       80.75       82.75       83.41666667]
 [74.75       76.75       78.75       79.41666667]
 [70.75       72.75       74.75       75.41666667]
 [66.75       68.75       70.75       71.41666667]
 [62.75       64.75       66.75       67.41666666]
 [58.75       60.75       62.75       63.41666665]
 [54.75       56.75       58.75       59.41666661]
 [50.75       52.75       54.75       55.41666645]
 [46.75       48.75       50.75       51.41666579]
 [42.75       44.75       46.75       47.41666317]
 [38.75       40.75       42.75       43.41665268]
 [34.75       36.75       38.75       39.41661072]
 [30.75       32.75       34.75       35.41644287]
 [26.75       28.75       30.75       31.41577148]
 [22.75       24.75       26.75       27.41308594]
 [18.75       20.75       22.75       23.40234375]
 [14.75       16.75       18.75       19.359375  ]
 [10.75       12.75       14.75       15.1875    ]
 [ 6.75        8.75       10.75       10.5       ]
 [ 3.          5.          6.          5.        ]
 [ 0.          0.          0.          0.        ]]

 Best action a[time][current_stock] =
[[2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [2. 1. 0. 0.]
 [1. 0. 0. 0.]
 [0. 0. 0. 0.]]
```

自然语言解释：当初始状态库存是 3 时，不购进，之后同样依据最优决策矩阵采取最优决策，整个过程的平均收益是 83.41666667。

源代码

upupming/Lab3-markov-decision-process

参考文献

1. Markov decision process - Wikipedia