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**Algorithm design and analysis**

# **Recursion – Two pointer**

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# CONTENT

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**(1) – Brute Force - Exhaustive**

**(2) – Recursion**

**(3) – Two Pointer**

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# Brute Force

- A straightforward approach usually based on problem statement and definitions
- The simplest to apply
- General approach: applicable to a very wide variety of problems
- No limitation on instance size
- Acceptable speed
- Useful for solving small-size instances of a problem



# Brute Force

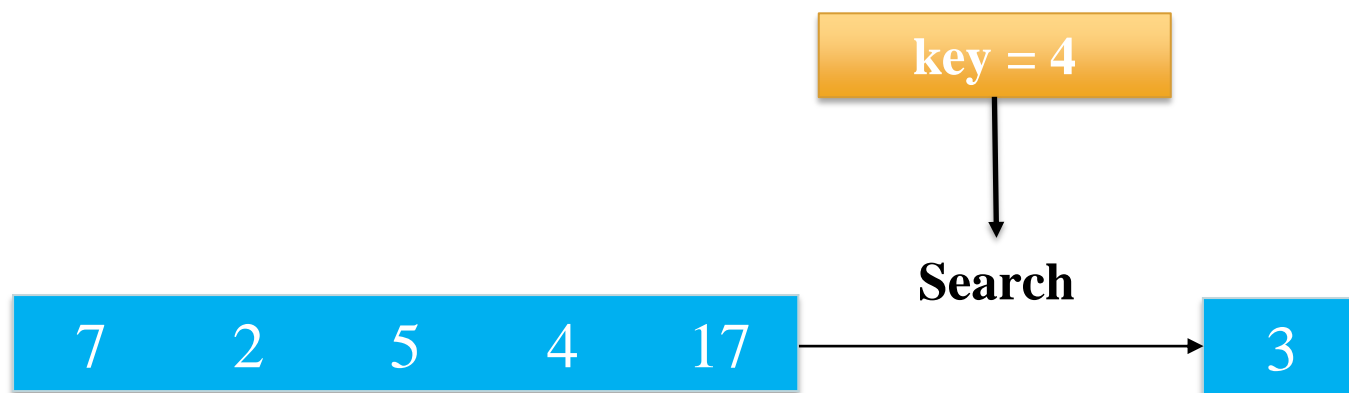
- A straightforward approach usually based on problem statement and definitions
- Example:
  - Computing  $a^n$  ( $a, b > 0$ ,  $n$  integer)
  - Computing  $n!$
  - Multiple two  $b$  by  $b$  matrices
  - Sequential search
  - Selection Sort

# Brute Force

## Sequential Search (Linear Search)

Input: a sequence of n number  $\langle a_1, a_2, \dots, a_n \rangle$  and key

Output: index of key in the sequence if exist, -1 if not exist

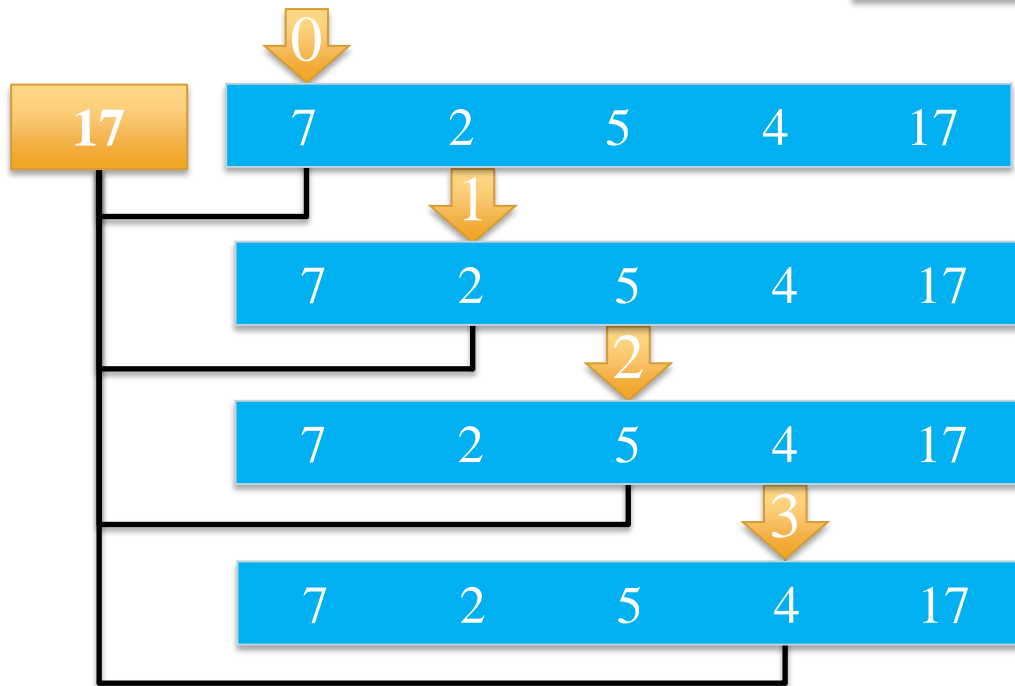
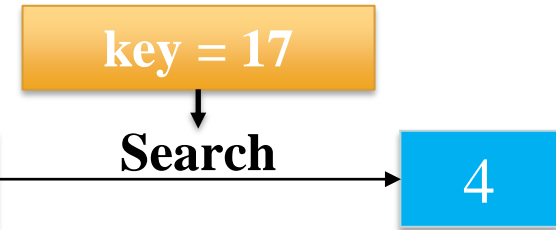




# Brute Force

## Sequential Search (Linear Search)

7 2 5 4 17



Stop => Output = 3

```
[15] def linear_search(arr, key):  
  
    n = len(arr)  
    for idx in range(n):  
  
        element = arr[idx]  
        if element == key:  
            return idx  
  
    return -1
```

$T(n)$  is  $O(n)$



# Brute Force

## Selection Sort

Input: a sequence of n number  $\langle a_1, a_2, \dots, a_n \rangle$  and *key*

Output: a permutation (reordering)  $\langle a'_1, a'_2, \dots, a'_n \rangle$

such that  $a'_1 \leq a'_2 \leq \dots \leq a'_n$





# Brute Force

## Selection Sort

Input: a sequence of n number  $\langle a_1, a_2, \dots, a_n \rangle$  and *key*

Output: a permutation (reordering)  $\langle a'_1, a'_2, \dots, a'_n \rangle$

such that  $a'_1 \leq a'_2 \leq \dots \leq a'_n$

- Select the smallest element from an unsorted list in each iteration
- Place that element at the beginning of the unsorted list







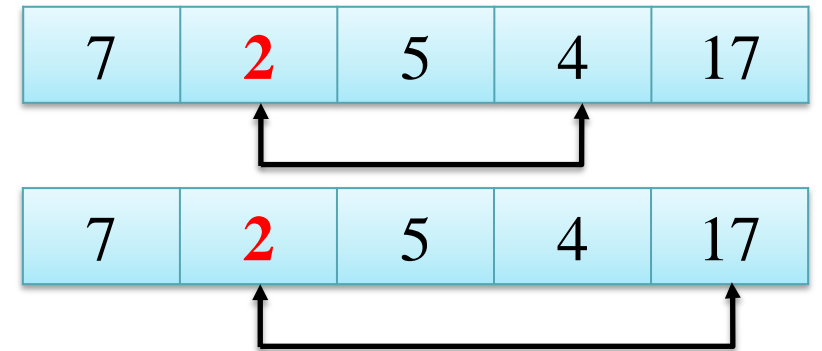
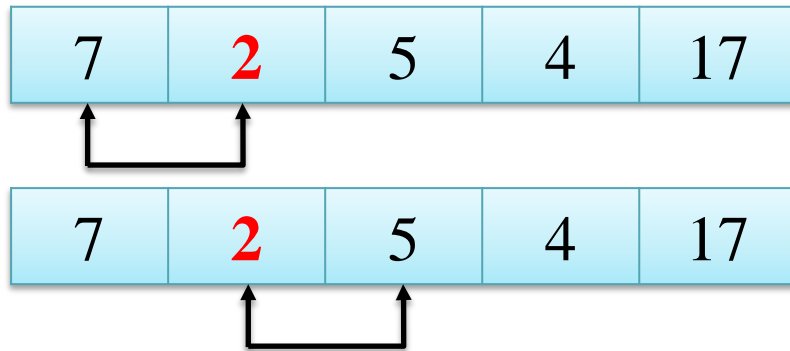
# Brute Force

## Selection Sort

- Set the first element as minimum



- Compare minimum with the other element. If the other element is smaller than minimum, assign the second element as minimum.



- After each iteration, minimum is placed in the front of the unsorted list





# Brute Force

## Selection Sort

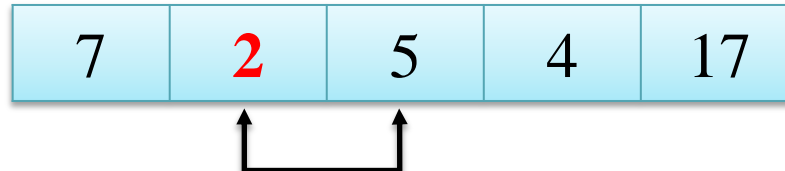
Step 0

i = 1



min value at index = 1

i = 2



min value at index = 1

i = 3



min value at index = 1

i = 4



min value at index = 1



swapping

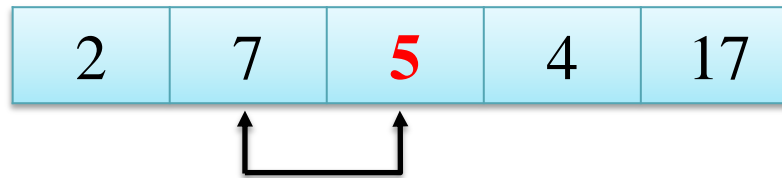


# Brute Force

## Selection Sort

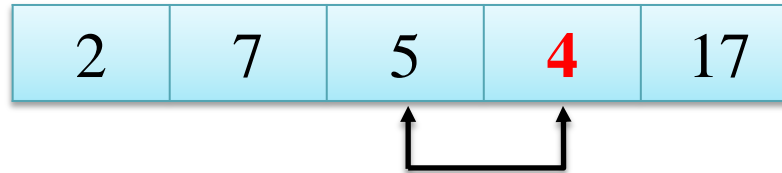
### Step 1

$i = 2$



min value at index = 2

$i = 3$



min value at index = 3

$i = 4$



min value at index = 3



swapping

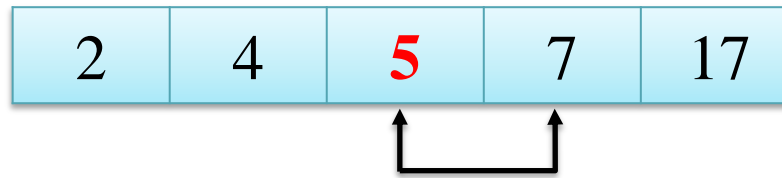


# Brute Force

## Selection Sort

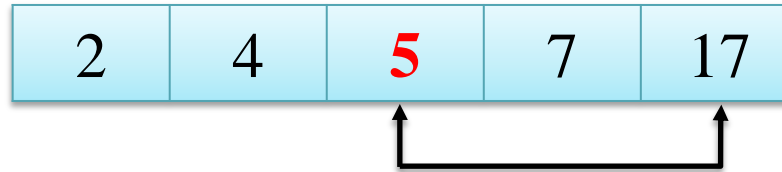
Step 2

$i = 3$



min value at index = 2

$i = 4$



min value at index = 2



already in place



# Brute Force

## Selection Sort

### Step 3

$i = 4$



min value at index = 3



already in place

### Step 4

$i = 4$



min value at index = 4



already in place



# Brute Force

## Selection Sort

```
[3] def selection_sort(array):  
    n = len(array)  
  
    for step in range(n):  
        min_idx = step  
  
        for i in range(step + 1, n):  
  
            if array[i] < array[min_idx]:  
                min_idx = i  
  
        array[step], array[min_idx] = array[min_idx], array[step]  
  
    return array  
  
arr = [7, 2, 5, 4, 17]  
selection_sort(arr)
```

[2, 4, 5, 7, 17]



# Brute Force

## Selection Sort

```
[3] def selection_sort(array):  
    n = len(array)  
  
    for step in range(n):  
        min_idx = step  
  
        for i in range(step + 1, n):  
  
            if array[i] < array[min_idx]:  
                min_idx = i  
  
        array[step], array[min_idx] = array[min_idx], array[step]  
  
    return array  
  
arr = [7, 2, 5, 4, 17]  
selection_sort(arr)
```

[2, 4, 5, 7, 17]

$T(n)$  is  $O(n^2)$

Worst case  
Best case  
Average case



# Brute Force

## Summary

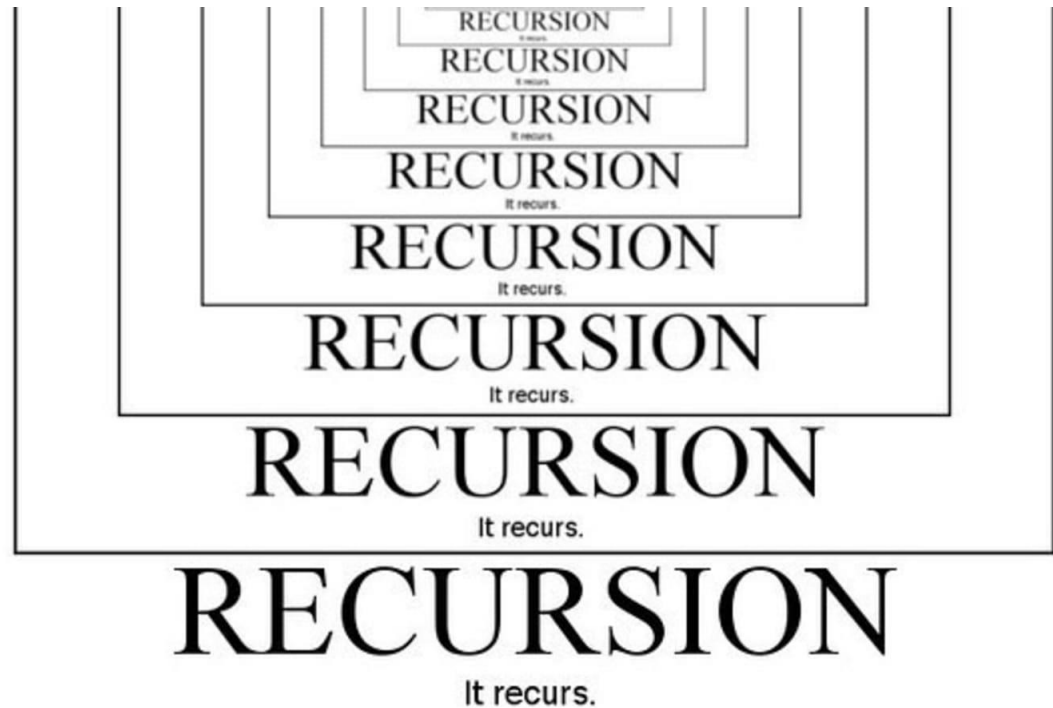
- Scanning all element (objective)
- Each element: check with the condition
  - If yes, stop
  - If no, continue
- Simple
- Not optimized
- The first problem solving model to think for every problem





# Recursion

- Recursion: a function makes one or more calls to itself during execution
- Powerful for performing repetitive tasks
- Example:
  - Factorial Function
  - Binary Search
  - Fibonacci Number





# Recursion

## Factorial Function

- For any integer  $n \geq 0$

$$n! = \begin{cases} 1 \\ n * (n - 1) * (n - 2) * \dots * 2 * 1 \end{cases}$$

- Recursive definition:

$$n! = \begin{cases} 1 \\ n * (n - 1)! \end{cases}$$

- Input: integer  $n \geq 0$
- Output: value of  $n!$

if  $n = 0$   
if  $n \geq 1$

if  $n = 0$   
if  $n \geq 1$

Base case



# Recursion

## Factorial Function

➤ Recursive definition:

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n * (n - 1)! & \text{if } n \geq 1 \end{cases}$$

```
[4] def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n*factorial(n-1)  
  
factorial(5)
```

120



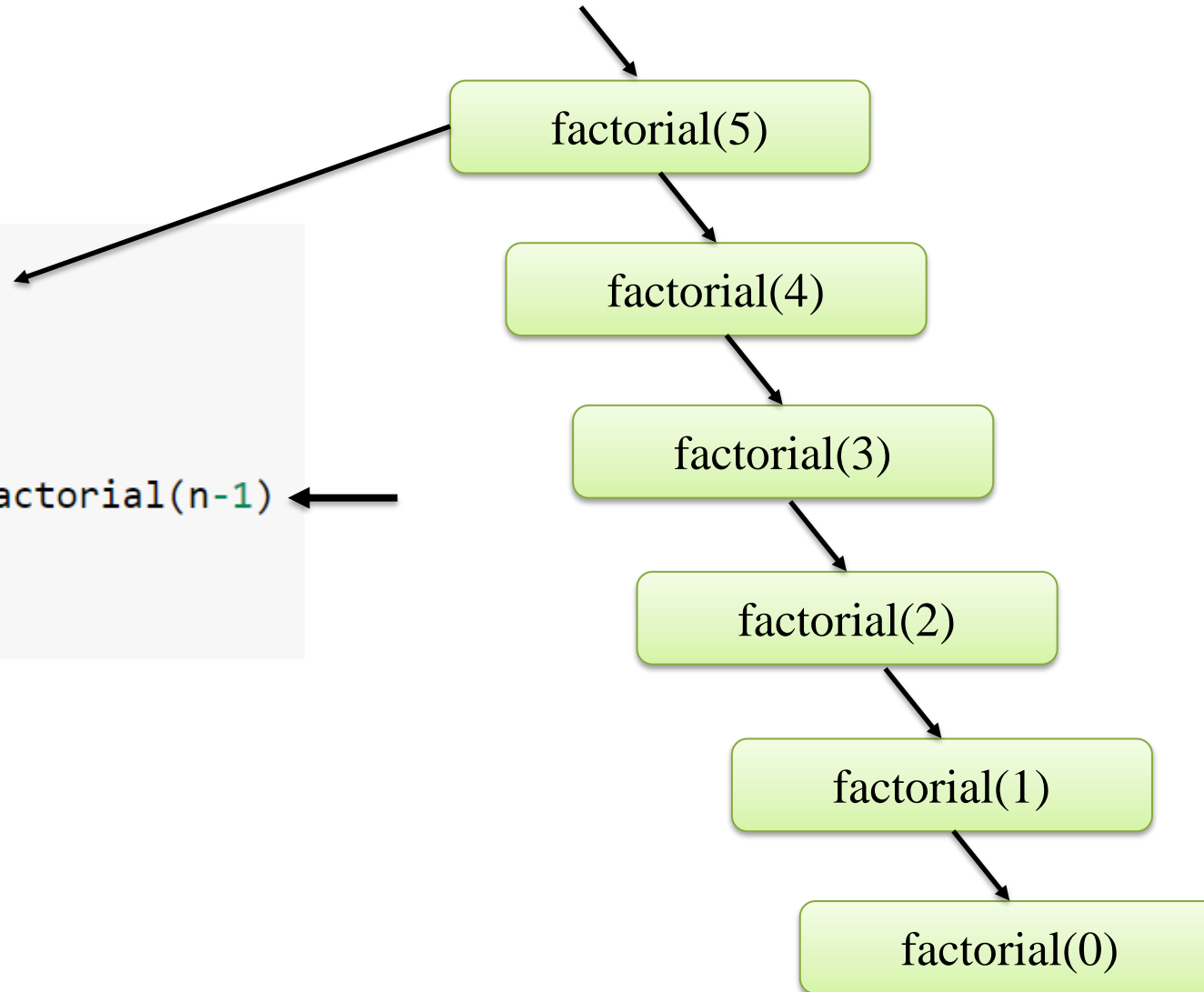
# Recursion

## Factorial Function

```
[4] def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n*factorial(n-1)
```

factorial(5)

120



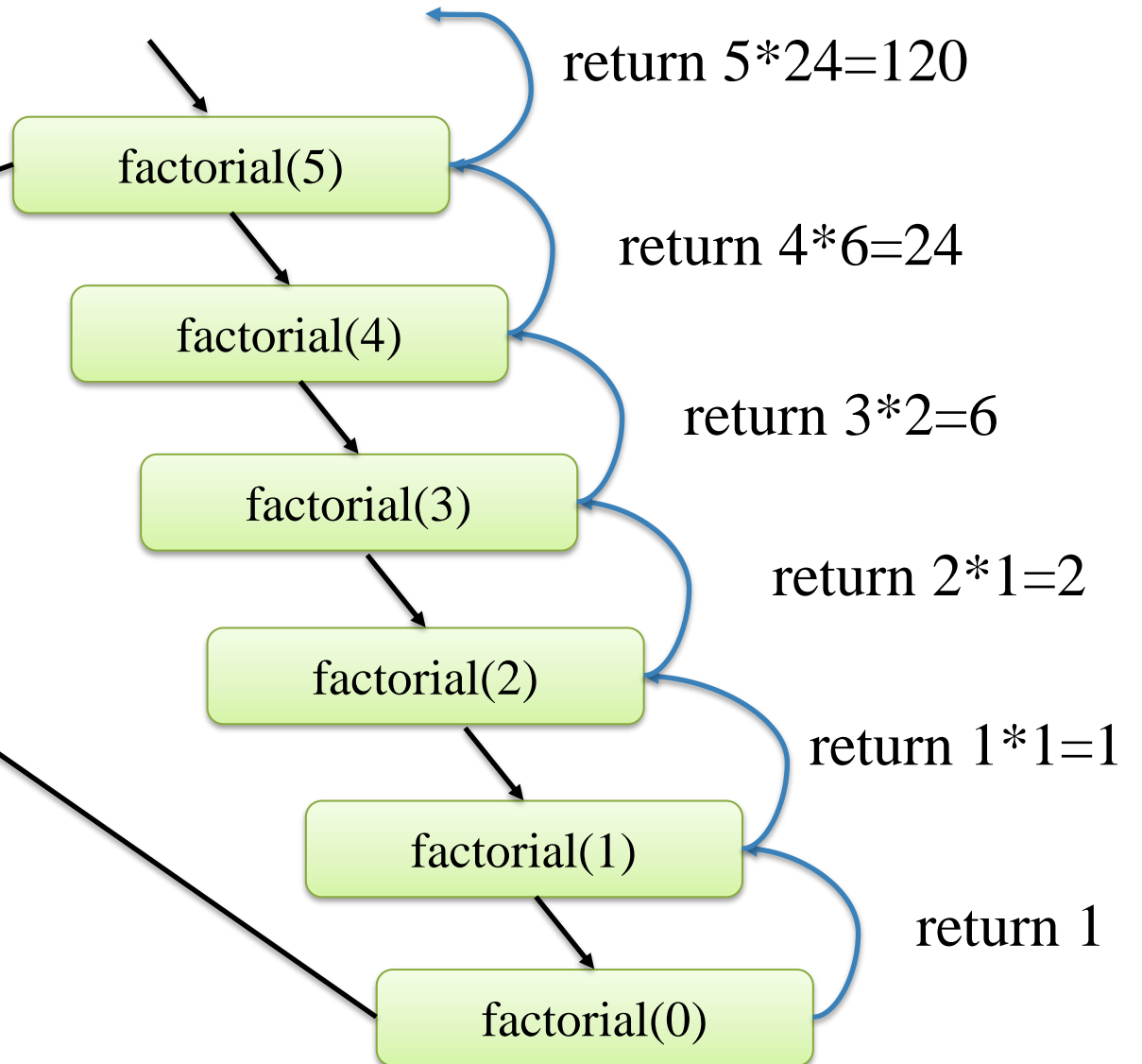
# Recursion

## Factorial Function

```
[4] def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n*factorial(n-1)
```

factorial(5)

120



# Recursion

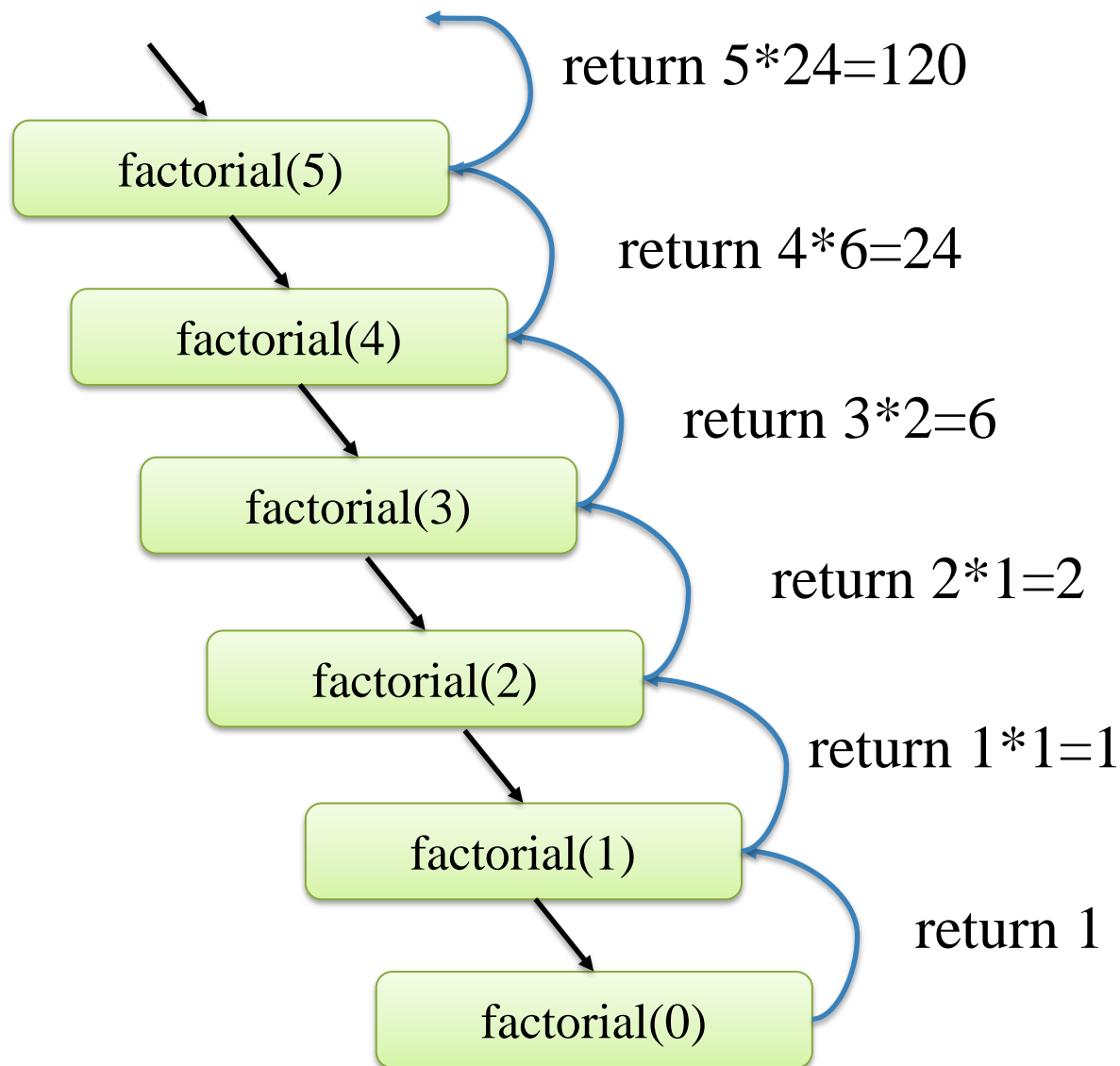
## Factorial Function

```
[4] def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n*factorial(n-1)
```

factorial(5)

120

The number of recursive call



# Recursion

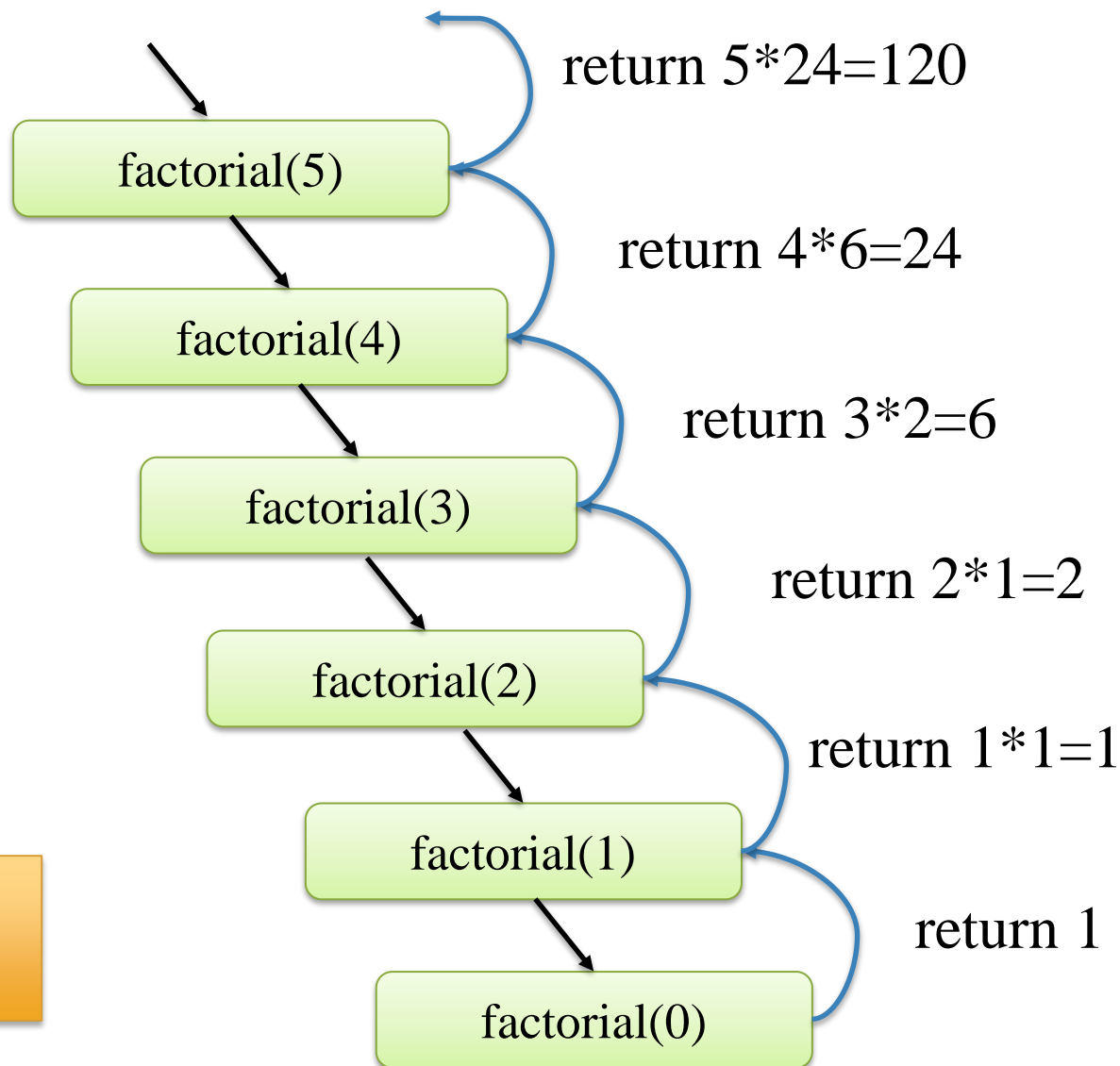
## Factorial Function

```
[4] def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n*factorial(n-1)
```

factorial(5)

120

The number of recursive call is  $n + 1$   
 $T(n)$  is  $O(n)$





# Recursion

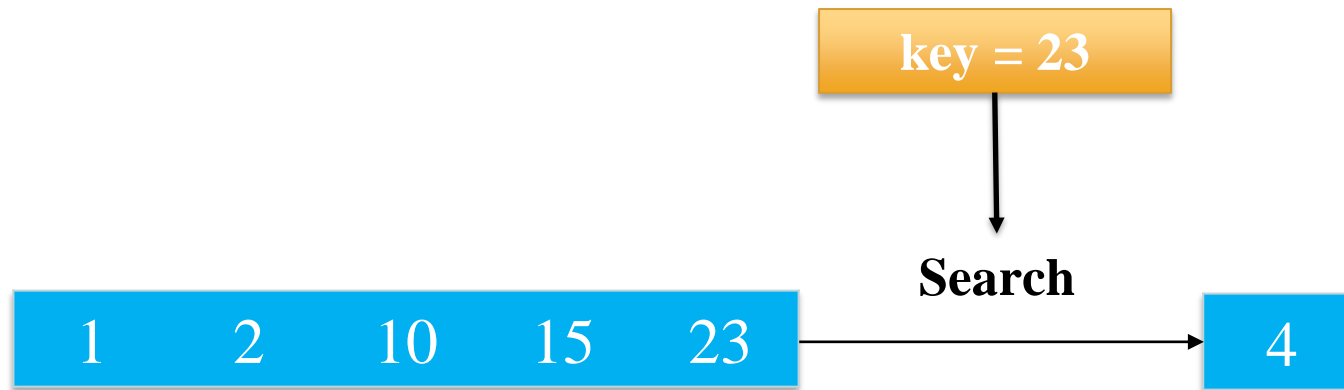
## Binary Search

Searching problem

Input: a sorted sequence of n number  $\langle a_1, a_2, \dots, a_n \rangle$ , key

Output: index of key in the sequence if exist, -1 if not exist

=> Always searched in the middle of a portion of a sequence







# Recursion

## Binary Search

Initially,  $low = 0$ ,  $high = len(arr) - 1$

Always searched in the middle of a portion of a sequence

$$mid = \lfloor (low + high) / 2 \rfloor$$

Consider three case:

- $key = arr[mid] \Rightarrow$  search successfully  $\Rightarrow$  return  $mid$
- $key < arr[mid] \Rightarrow$  find key in  $arr[low, mid-1]$
- $key > arr[mid] \Rightarrow$  find key in  $arr[mid+1, high]$



# Recursion

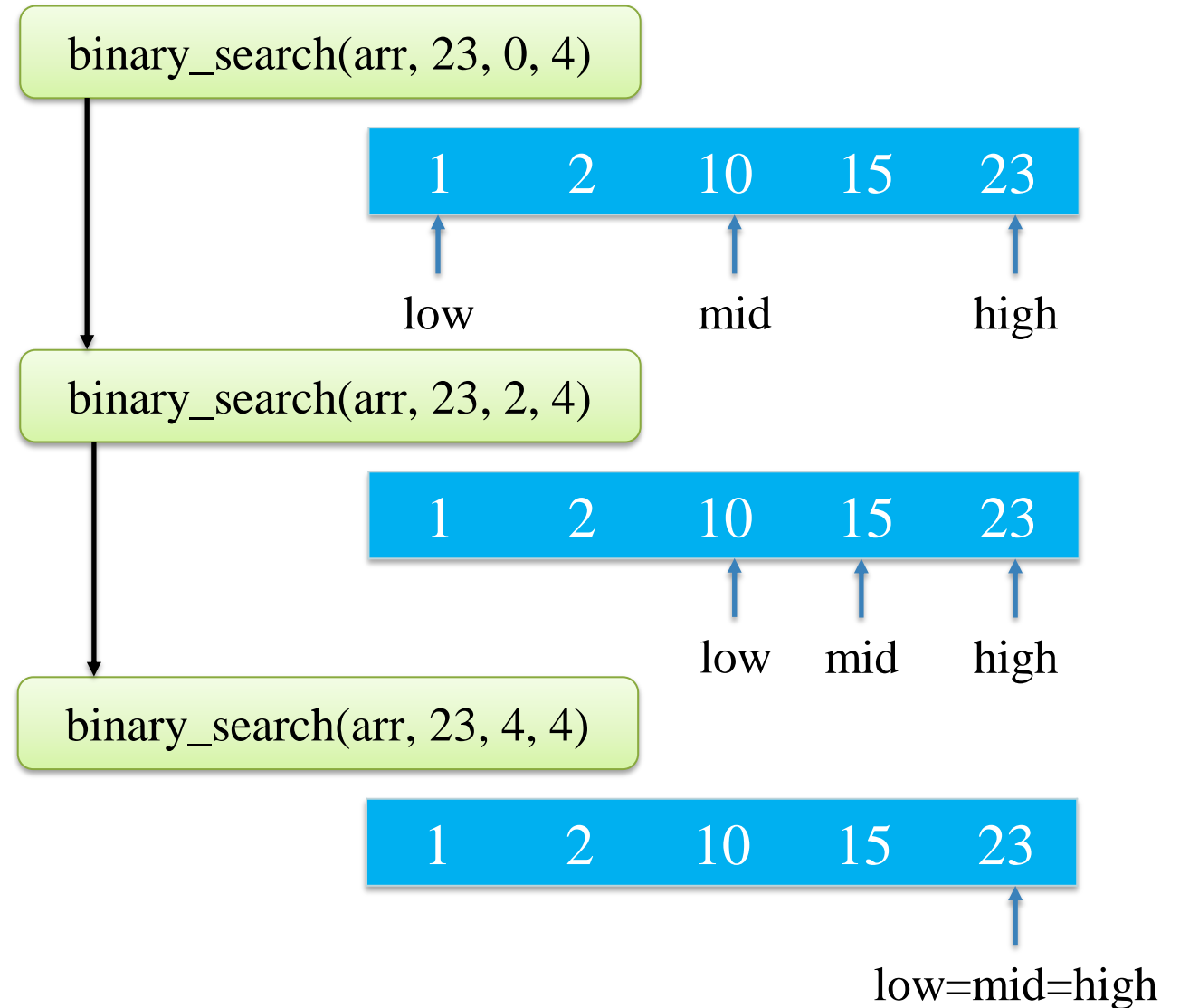
## Binary Search

```
▶ def binary_search(array, key, low, high):  
    if high >= low:  
        mid = low + (high - low)//2  
        if array[mid] == key:  
            return mid  
        elif array[mid] > key:  
            return binary_search(array, key, low, mid-1)  
        else:  
            return binary_search(array, key, mid + 1, high)  
    else:  
        return -1  
  
arr = [1, 2, 10, 15, 23]  
key = 23  
low = 0  
high = len(arr) - 1  
binary_search(arr, key, low, high)
```

# Recursion

## Binary Search

```
def binary_search(array, key, low, high):  
    if high >= low:  
        mid = low + (high - low)//2  
        if array[mid] == key:  
            return mid  
        elif array[mid] > key:  
            return binary_search(array, key, low, mid-1)  
        else:  
            return binary_search(array, key, mid + 1, high)  
    else:  
        return -1  
  
arr = [1, 2, 10, 15, 23]  
key = 23  
low = 0  
high = len(arr) - 1  
binary_search(arr, key, low, high)
```

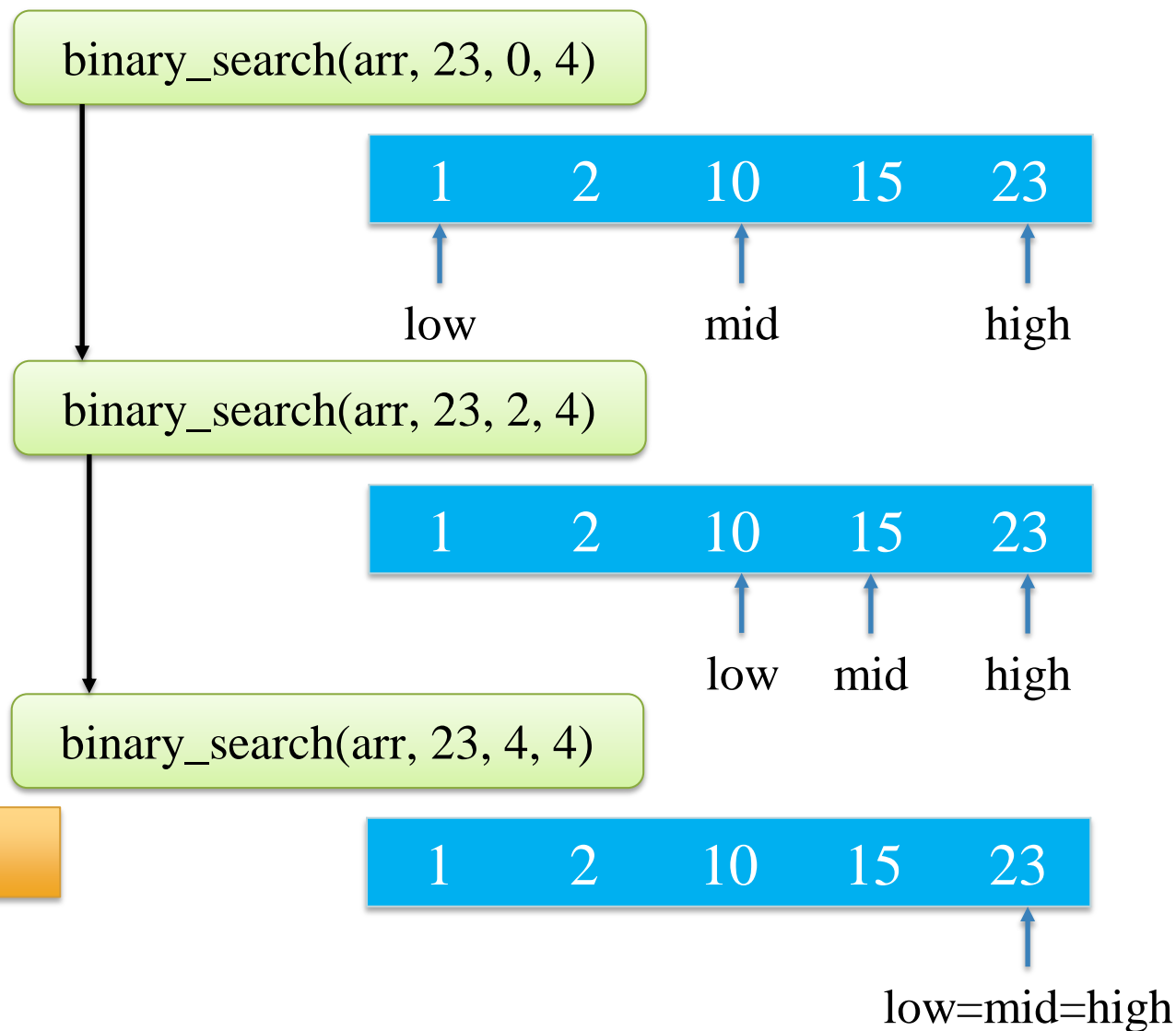


# Recursion

## Binary Search

```
def binary_search(array, key, low, high):  
    if high >= low:  
        mid = low + (high - low)//2  
        if array[mid] == key:  
            return mid  
        elif array[mid] > key:  
            return binary_search(array, key, low, mid-1)  
        else:  
            return binary_search(array, key, mid + 1, high)  
    else:  
        return -1  
  
arr = [1, 2, 10, 15, 23]  
key = 23  
low = 0  
high = len(arr) - 1  
binary_search(arr, key, low, high)
```

The number of recursive call ?



# Recursion

## Binary Search

```
def binary_search(array, key, low, high):  
    if high >= low:  
        mid = low + (high - low)//2  
        if array[mid] == key:  
            return mid  
        elif array[mid] > key:  
            return binary_search(array, key, low, mid-1)  
        else:  
            return binary_search(array, key, mid + 1, high)  
    else:  
        return -1
```

```
arr = [1, 2, 10, 15, 23]
```

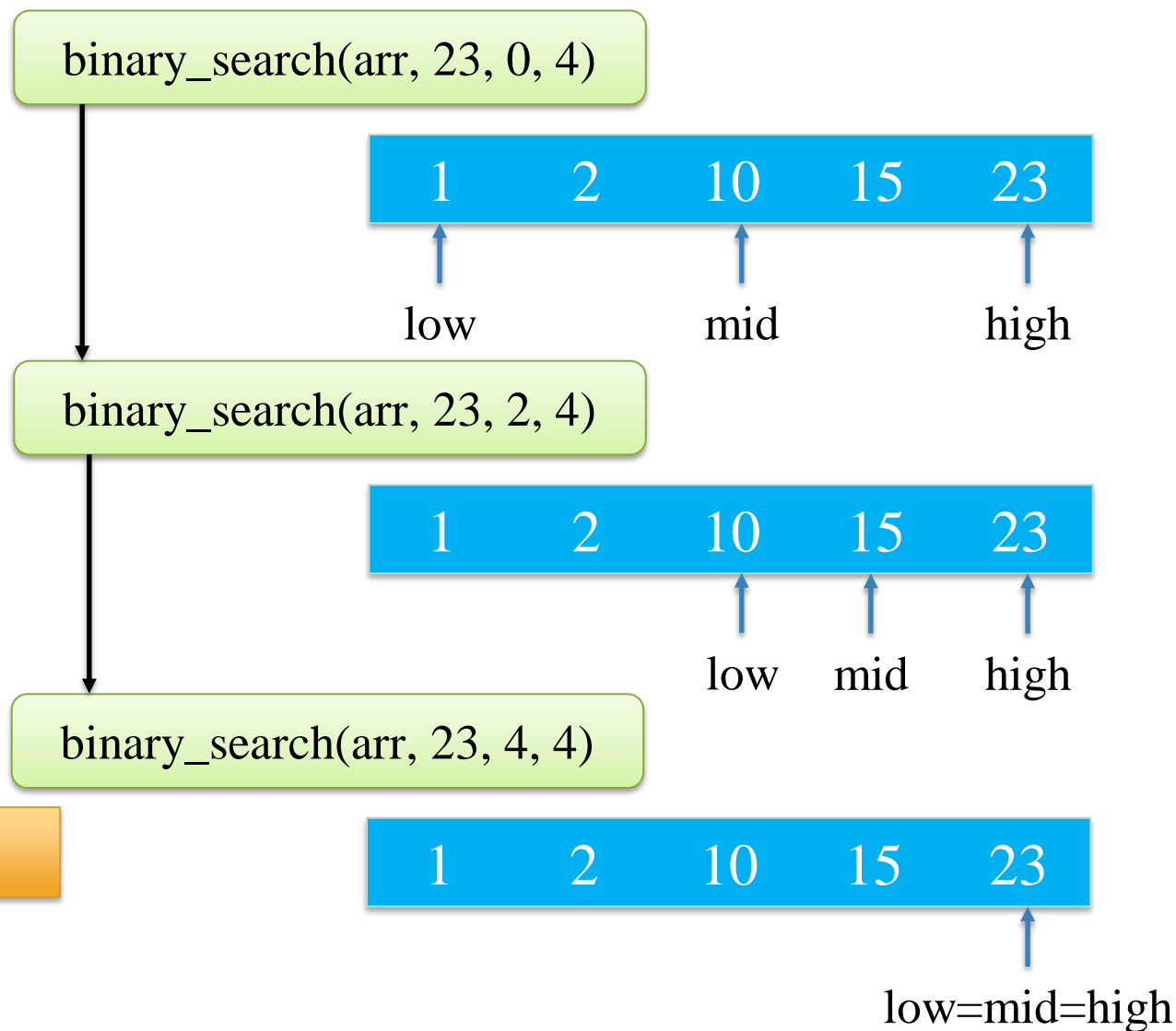
```
key = 23
```

```
low = 0
```

```
high = len(arr) - 1
```

```
binary_search(arr, key, low, high)
```

**$T(n)$  is  $O(\log n)$**





# Recursion

## Fibonacci Number [[509](#)] Leetcode

$$F_0 = 0$$

$$F_1 = 1$$

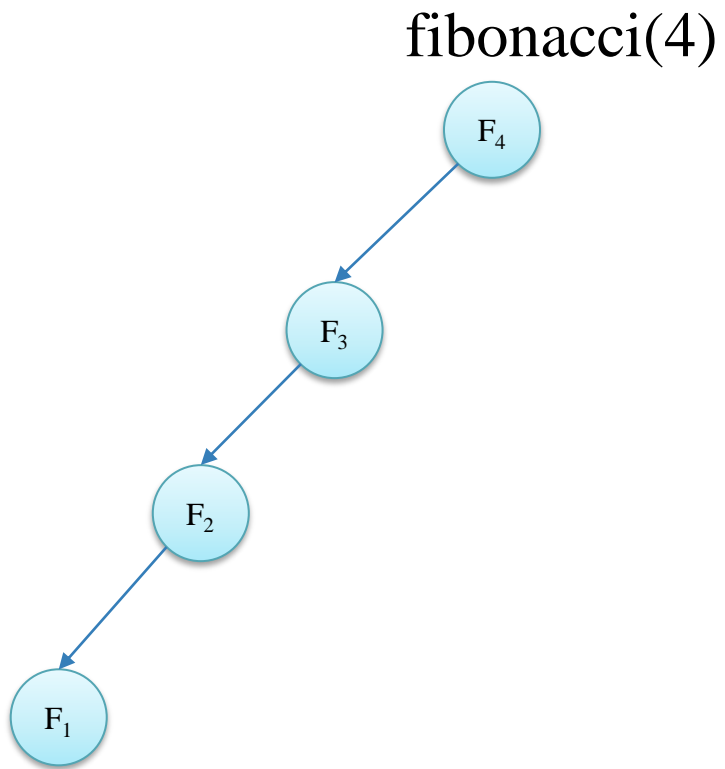
$$F_n = F_{n-2} + F_{n-1} \quad \text{for } n > 1$$

```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```



# Recursion

## Fibonacci Number



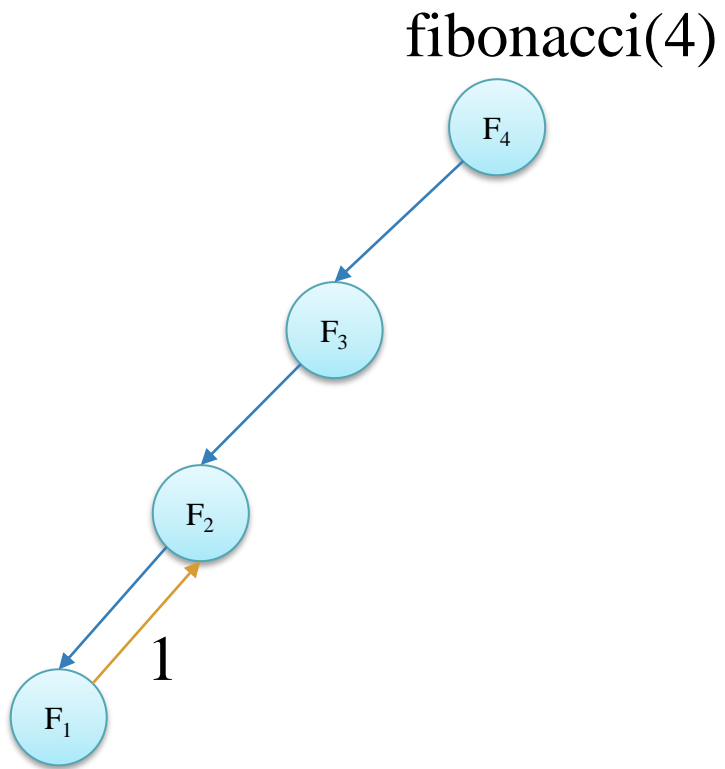
```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

3



# Recursion

## Fibonacci Number



```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

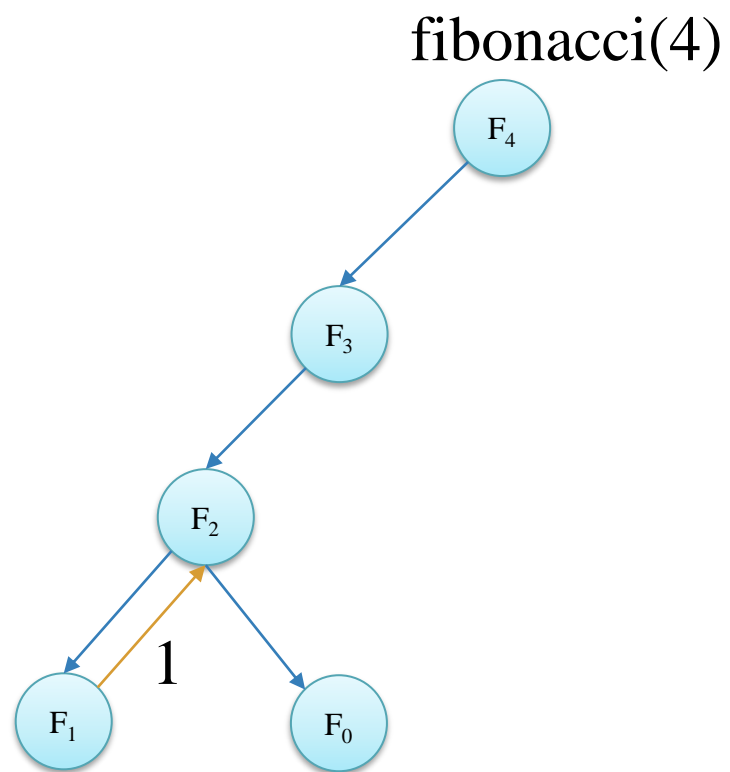
3





# Recursion

## Fibonacci Number



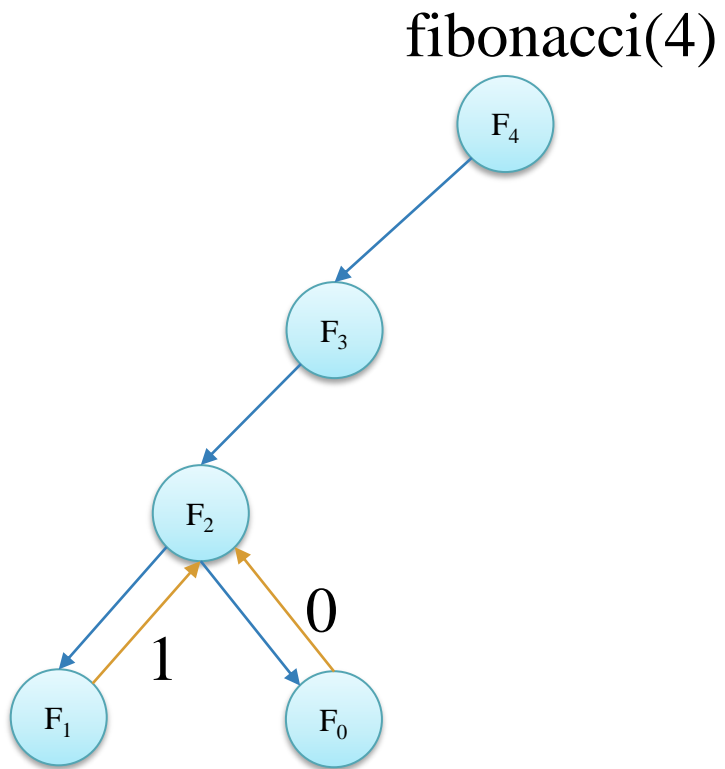
```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

3



# Recursion

## Fibonacci Number



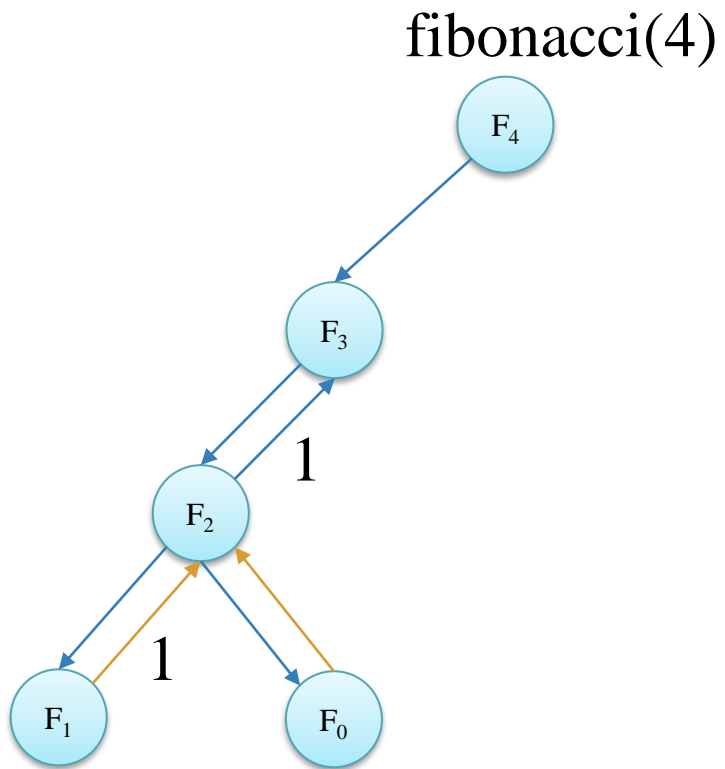
```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

3



# Recursion

## Fibonacci Number



```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

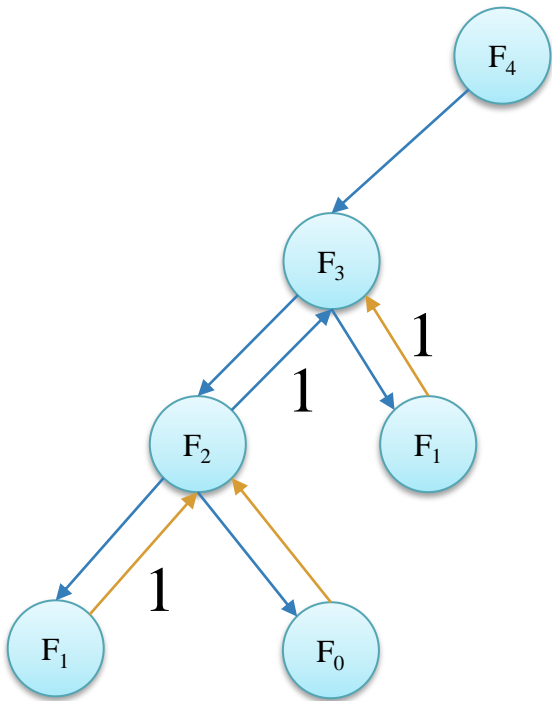
3



# Recursion

## Fibonacci Number

fibonacci(4)



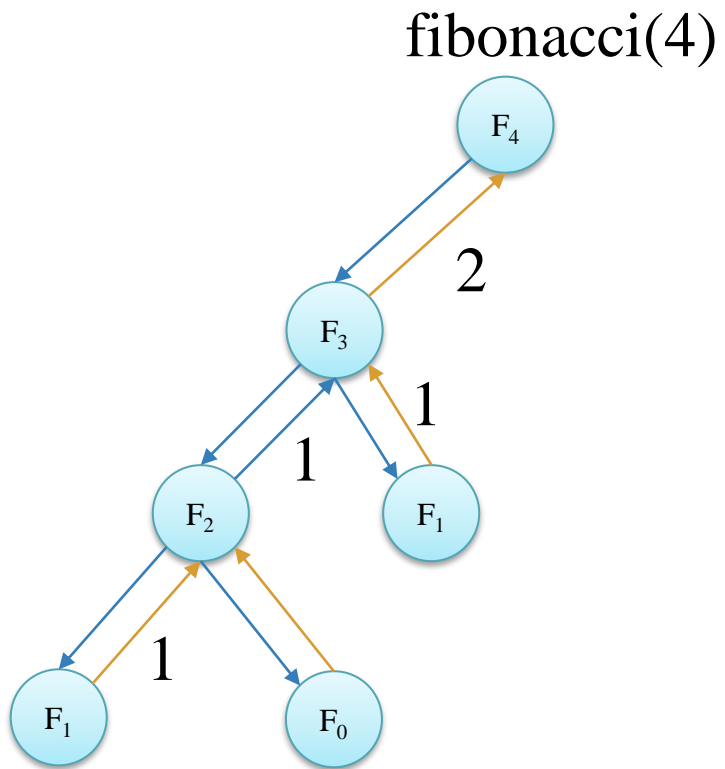
```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

3



# Recursion

## Fibonacci Number



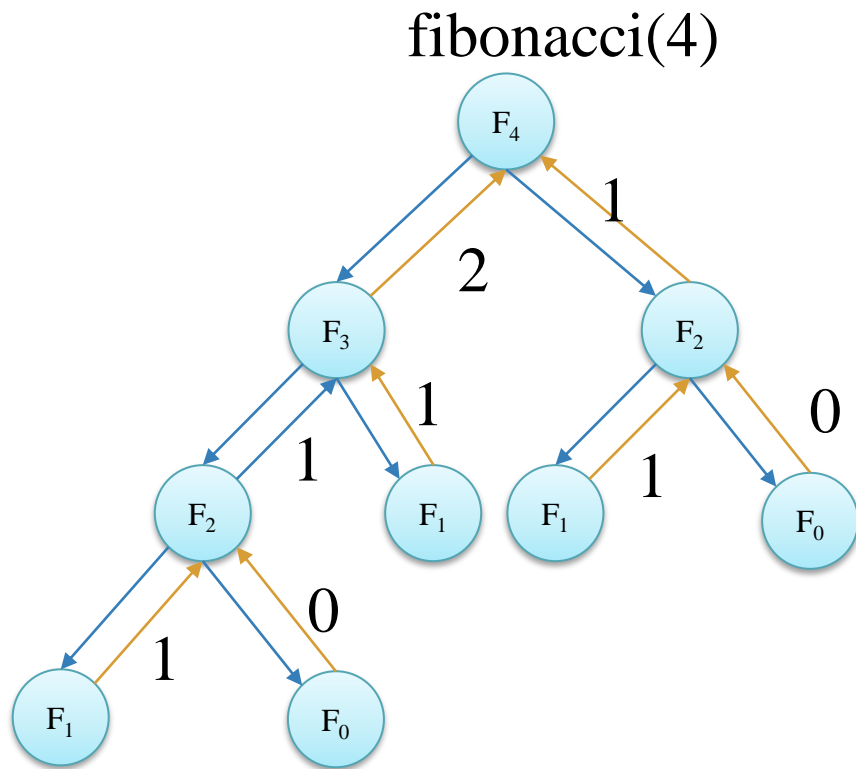
```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

3



# Recursion

## Fibonacci Number



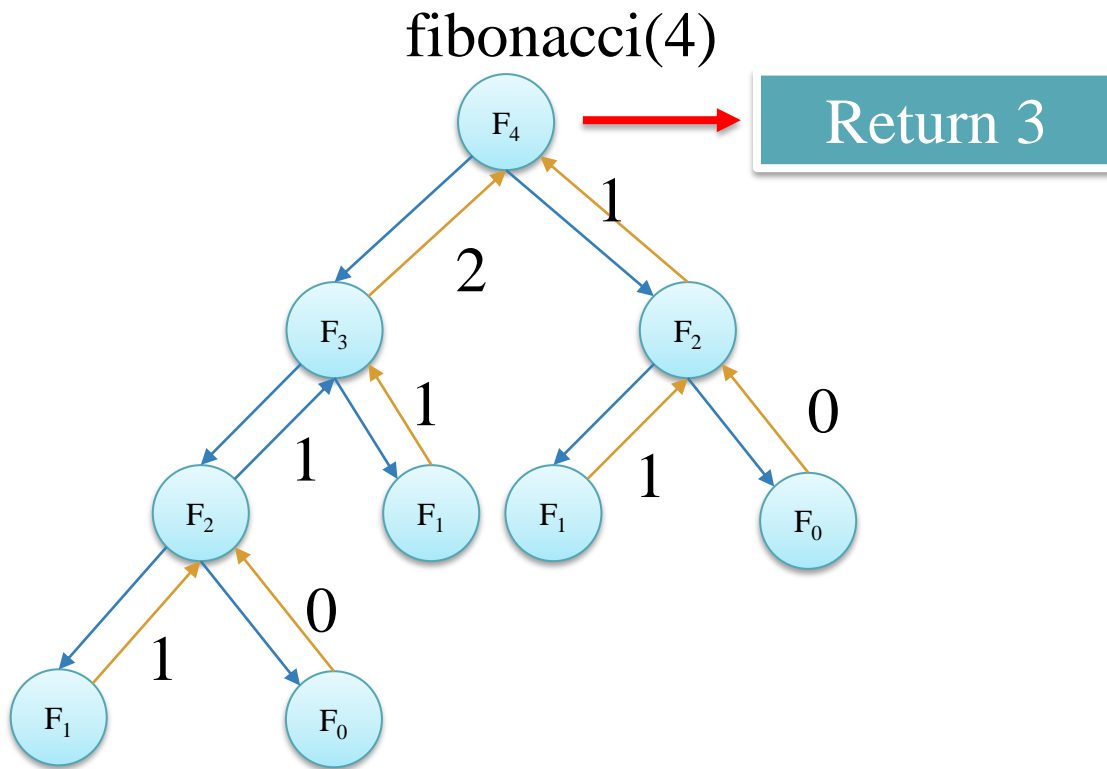
```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

3



# Recursion

## Fibonacci Number



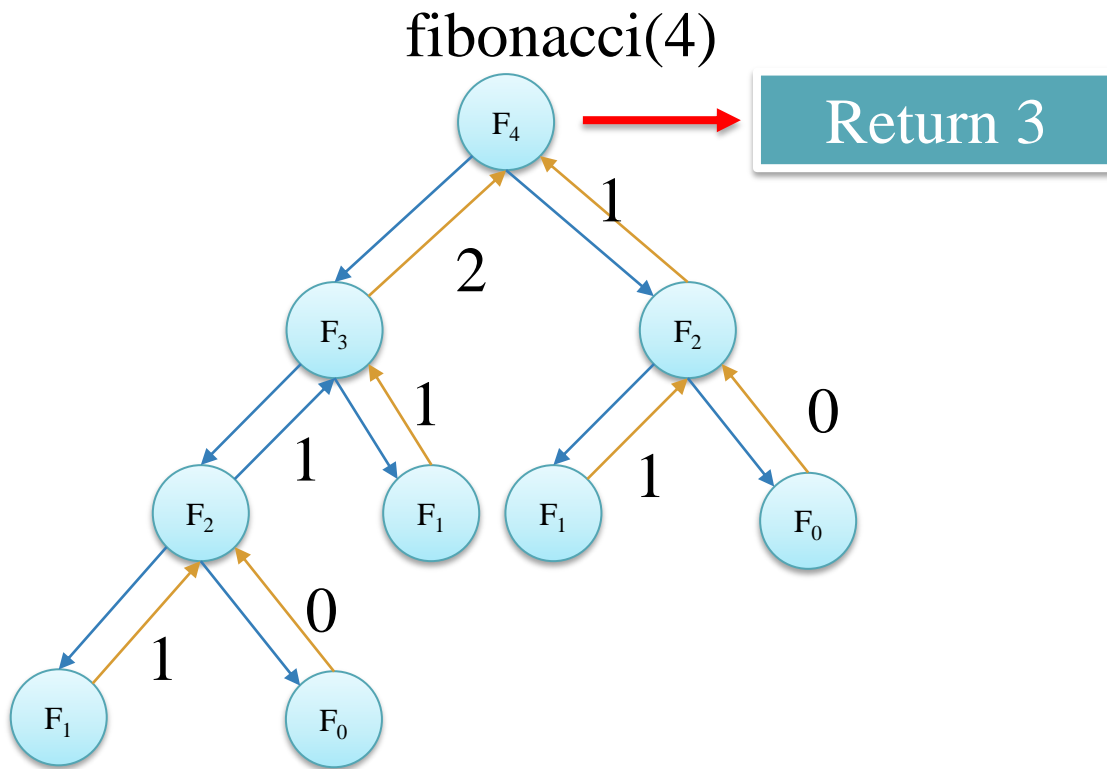
```
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    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

3



# Recursion

## Fibonacci Number



```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

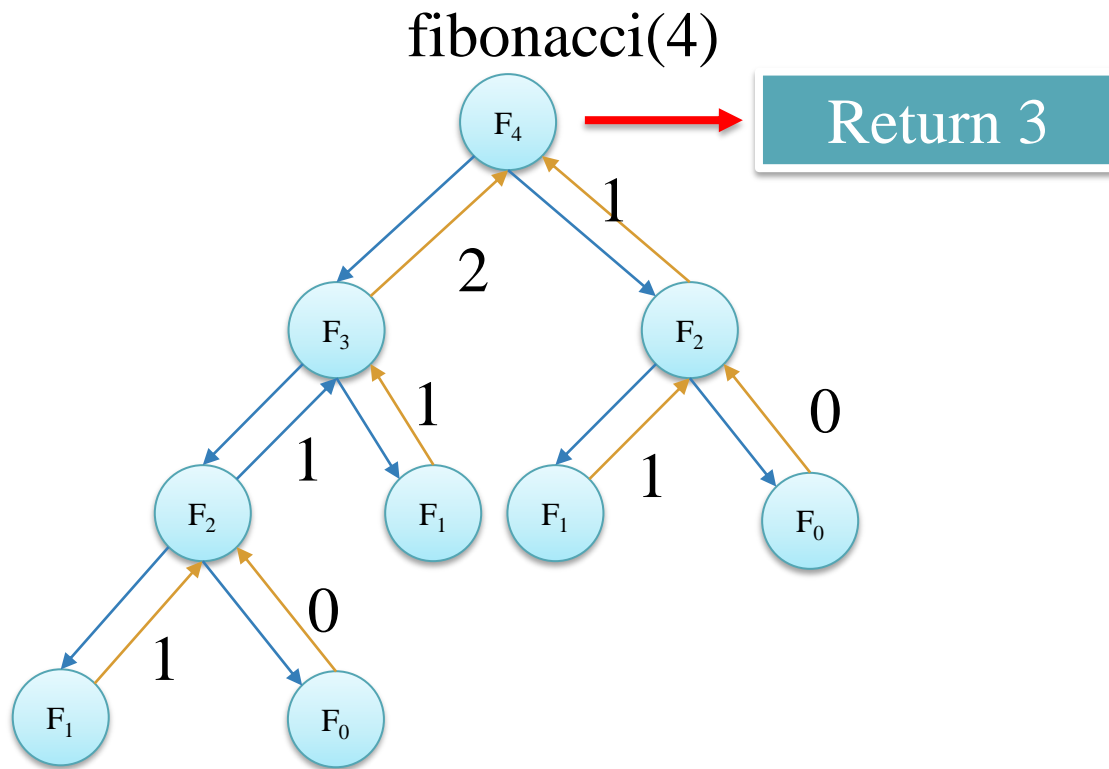
3

The number of recursive call is ?



# Recursion

## Fibonacci Number



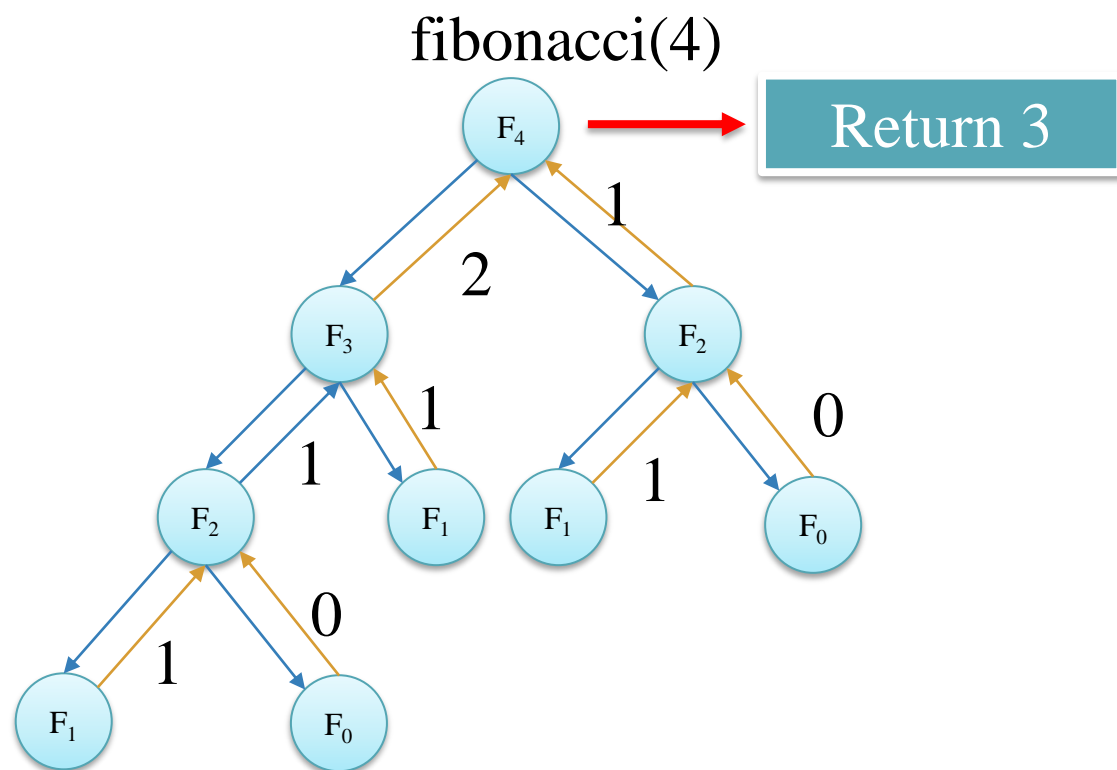
```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

3

$T(n)$  is  $O(2^n)$

# Recursion

## Fibonacci Number



```
def fibonacci(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)  
  
fibonacci(4)
```

3

$T(n)$  is  $O(2^n)$

$T(n)$  is  $O(n)$  ???



# Recursion

## Fibonacci Number

Efficient recursion with  $O(n)$

```
[34] def good_fibonacci(n):  
    if n == 0:  
        return (0, 0)  
    elif n == 1:  
        return (1, 0)  
    else:  
        (a, b) = good_fibonacci(n-1)  
        return (a+b, a)  
  
good_fibonacci(4), good_fibonacci(4)[0]  
  
((3, 2), 3)
```



# Recursion

## SUMMAR

Base case  
Avoid infinite recursion

```
[34] def good_fibonacci(n):  
      if n == 0:  
          return (0, 0)  
      elif n == 1:  
          return (1, 0)  
      else:  
          (a, b) = good_fibonacci(n-1)  
          return (a+b, a)  
  
      good_fibonacci(4), good_fibonacci(4)[0]  
  
      ((3, 2), 3)
```

Call to itself

Smaller instance size



# Two pointer

## Reverse String [[344](#)] Leetcode

### Example 1:

Input: `s = ["h","e","l","l","o"]`

Output: `["o","l","l","e","h"]`

### Example 2:

Input: `s = ["H","a","n","n","a","h"]`

Output: `["h","a","n","n","a","H"]`

### Constraints:

- `1 <= s.length <= 105`
- `s[i]` is a printable ascii character.



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# Two Pointer

## Reverse String

Using for loop

```
[68] def reverse_str_1(s):  
    tg_s = ''  
    for i in range(len(s)-1, -1, -1):  
        tg_s += s[i]  
    return tg_s
```

```
s = 'abcde'  
reverse_str_1(s)
```

```
'edcba'
```



# Two Pointer

## Reverse String

Using recursion

```
[67] def reverse_str_2(s, low, high):  
        if low < high - 1:  
            s[low], s[high-1] = s[high-1], s[low]  
            reverse_str_2(s, low + 1, high - 1)  
  
s = 'abcde'  
s = list(s)  
low = 0  
high = len(s)  
reverse_str_2(s, low, high)  
"".join(s)  
  
'edcba'
```



# Two Pointer

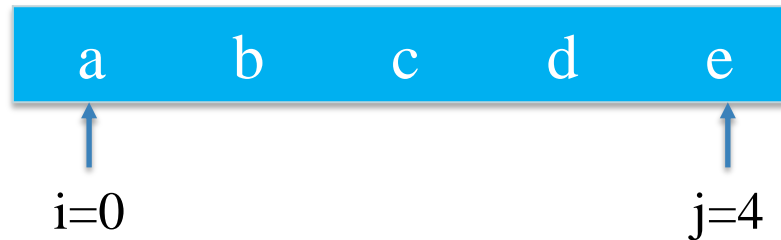
## Reverse String

Using two pointer

- Convert string to list



- Init two pointer:  $i$ : index of first element,  $j$ : index of last element



- While loop with  $i \leq j$ . swapping value of  $i$  and  $j$ , add  $i$  and 1. sub  $j$  and 1



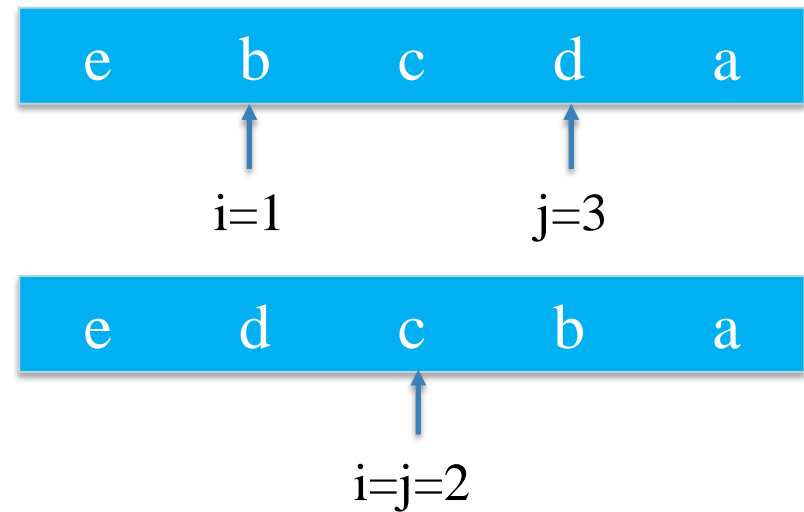
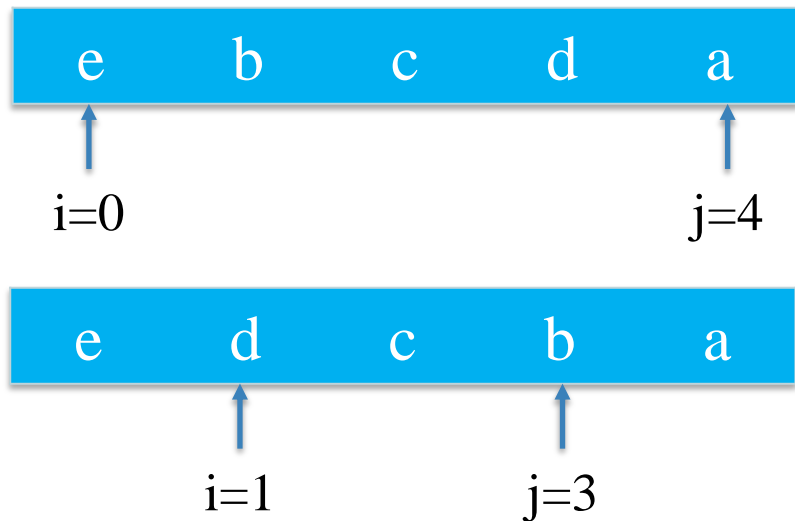


# Two Pointer

## Reverse String

Using two pointer

- While loop with  $i \leq j$ . swapping value of  $i$  and  $j$ , add  $i$  and 1. sub  $j$  and 1



- Convert list to string





# Two Pointer

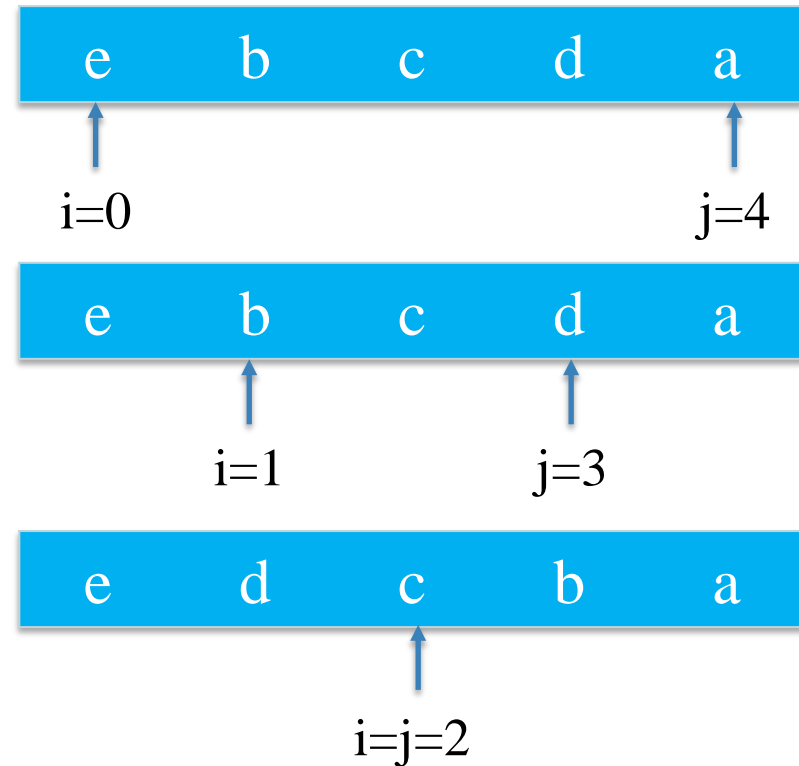
## Reverse String

Using two pointer

```
def reverse_str_3(s):  
    i = 0  
    j = len(s) - 1  
    while i <= j:  
        s[i], s[j] = s[j], s[i]  
        i = i + 1  
        j = j - 1  
    return "".join(s)
```

```
s = 'abcde'  
s = list(s)  
reverse_str_3(s)
```

➞ 'edcba'





# Two Pointer

## Find Subarray with Given Sum [[560](#)] Leetcode

Given an array of integers *nums* and an integer *k*, return the total number of subarrays whose sum equals to *k*.

A subarray is a contiguous non-empty sequence of elements within an array.

### Example 1:

**Input:** `nums = [1,1,1], k = 2`

**Output:** 2

### Example 2:

**Input:** `nums = [1,2,3], k = 3`

**Output:** 2

### Constraints:

- `1 <= nums.length <= 2 * 104`
- `-1000 <= nums[i] <= 1000`
- `-107 <= k <= 107`

# Two Pointer

## Find Subarray with Given Sum

Simple solution

1	2	3	5	6
---	---	---	---	---

$T(n)$  is ?

```
def subarray_sum(nums, key):  
    n = len(nums)  
    count = 0  
    for i in range(n):  
        total = 0  
        for j in range(i, n):  
            total += nums[j]  
            if total == key:  
                count += 1  
    return count
```

```
nums = [1, 2, 3, 5, 6]  
key = 5  
subarray_sum(nums, key)
```

# Two Pointer

## Find Subarray with Given Sum

Simple solution

1	2	3	5	6
---	---	---	---	---

$T(n)$  is  $O(n^2)$

```
def subarray_sum(nums, key):  
    n = len(nums)  
    count = 0  
    for i in range(n):  
        total = 0  
        for j in range(i, n):  
            total += nums[j]  
            if total == key:  
                count += 1  
    return count
```

```
nums = [1, 2, 3, 5, 6]  
key = 5  
subarray_sum(nums, key)
```



# Two Pointer

## Find Subarray with Given Sum

Using two pointer

1	2	3	5	6
---	---	---	---	---

i = ?  
j = ?  
stop condition ?

```
def subarray_sum(nums, key):  
    n = len(nums)  
    count = 0  
    for i in range(n):  
        total = 0  
        for j in range(i, n):  
            total += nums[j]  
            if total == key:  
                count += 1  
    return count
```

```
nums = [1, 2, 3, 5, 6]  
key = 5  
subarray_sum(nums, key)
```



# Two Pointer

## Find Subarray with Given Sum

Using two pointer

1	2	3	5	6
---	---	---	---	---

i = ?  
j = ?  
stop condition ?

```
def subarray_sum(nums, key):  
    n = len(nums)  
    count = 0  
    for i in range(n):  
        total = 0  
        for j in range(i, n):  
            total += nums[j]  
            if total == key:  
                count += 1  
    return count
```

```
nums = [1, 2, 3, 5, 6]  
key = 5  
subarray_sum(nums, key)
```



# Two Pointer

## Find Subarray with Given Sum

Using two pointer

1	2	3	5	6
---	---	---	---	---

$i = ?$

$j = ?$

stop condition ?

total : current subarray sum

count: number of subarray

```
def subarray_sum(nums, key):  
    n = len(nums)  
    count = 0  
    for i in range(n):  
        total = 0  
        for j in range(i, n):  
            total += nums[j]  
            if total == key:  
                count += 1  
    return count
```

```
nums = [1, 2, 3, 5, 6]  
key = 5  
subarray_sum(nums, key)
```

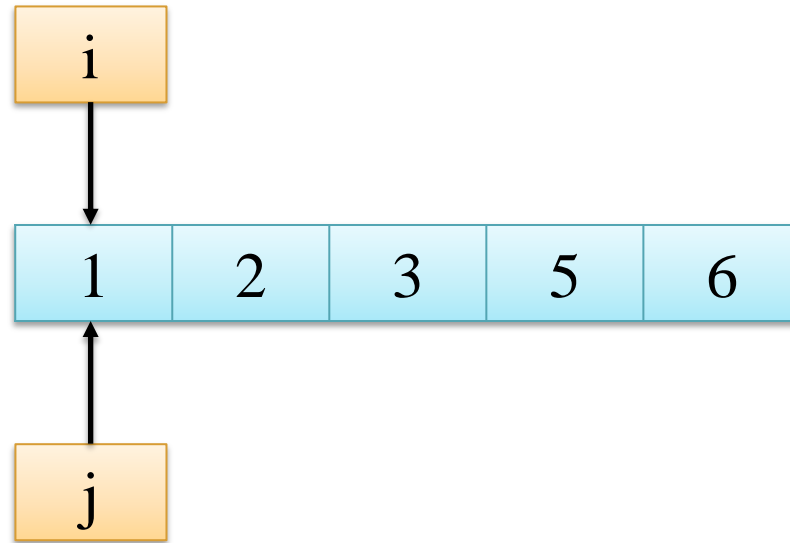




# Two Pointer

## Find Subarray with Given Sum

Using two pointer



total: 0

count: 0

$$\text{total} + \text{array}[i] = 0 + 1 = 1 < 5$$



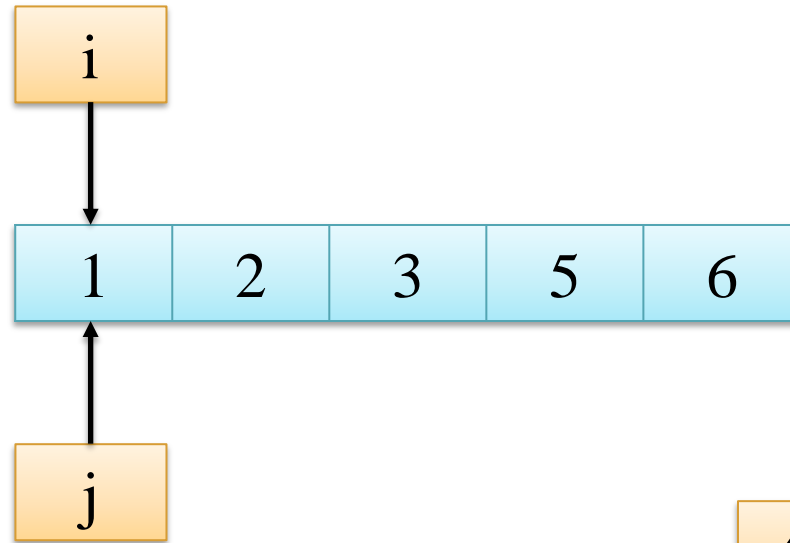
# Two Pointer

## Find Subarray with Given Sum

Using two pointer

total: 0

count: 0



$\text{total} = \text{total} + \text{array}[i] = 0 + 1 = 1$

$\text{total} + \text{array}[i] = 0 + 1 = 1 < 5$

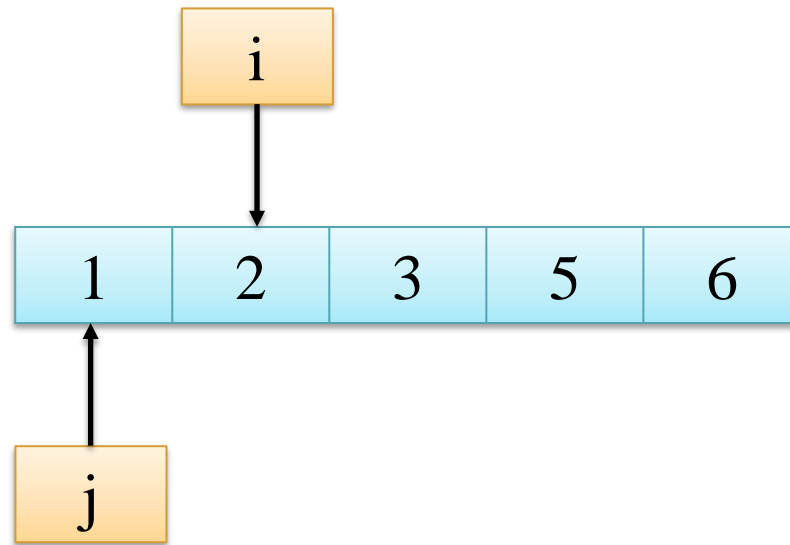
$i = i + 1 = 0 + 1 = 1$



# Two Pointer

## Find Subarray with Given Sum

Using two pointer



total: 1

count: 0



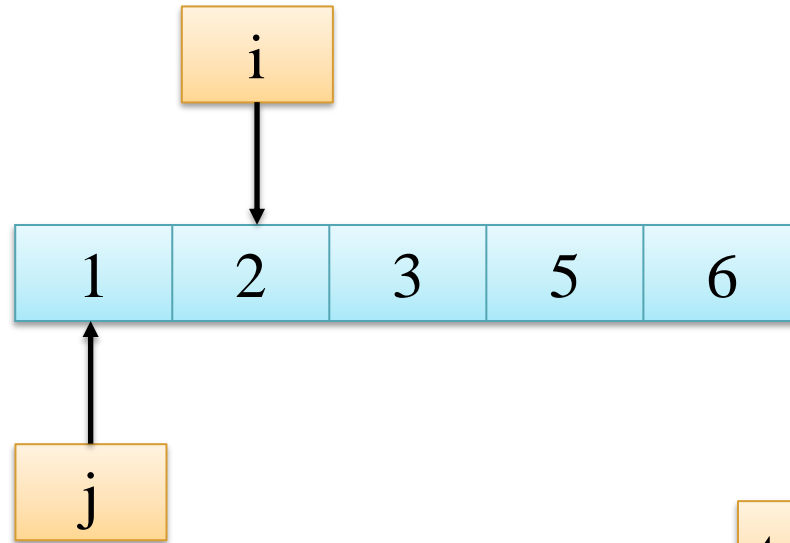
# Two Pointer

## Find Subarray with Given Sum

Using two pointer

total: 1

count: 0



$\text{total} = \text{total} + \text{array}[i] = 1 + 2 = 3$

$\text{total} + \text{array}[i] = 1 + 2 = 3 < 5$

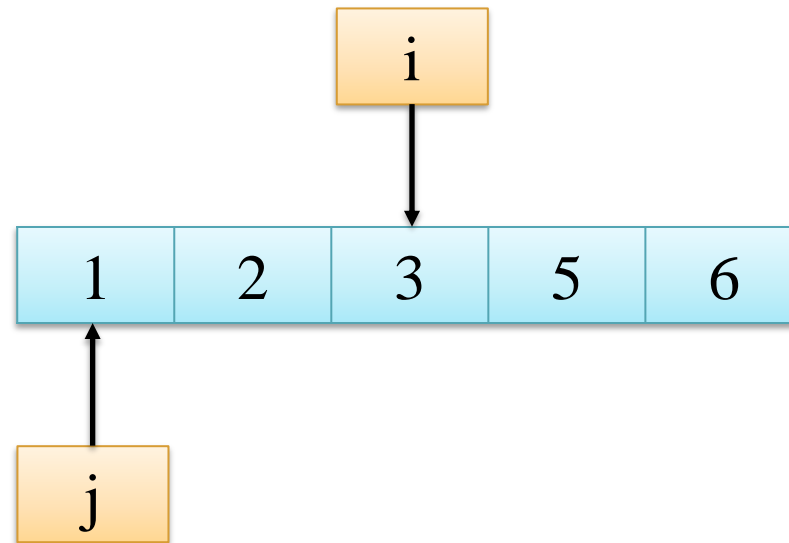
$i = i + 1 = 1 + 1 = 2$



# Two Pointer

## Find Subarray with Given Sum

Using two pointer



total: 3

count: 0



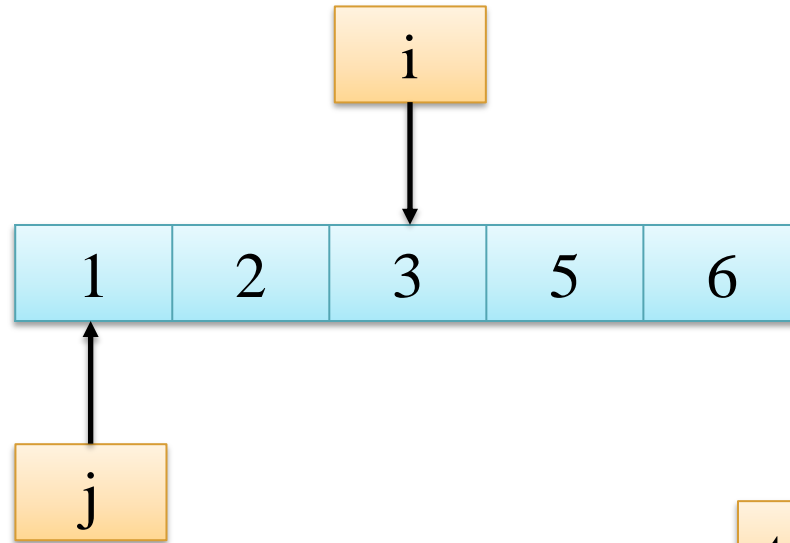
# Two Pointer

## Find Subarray with Given Sum

Using two pointer

total: 3

count: 0



$$\text{total} = \text{total} - \text{array}[j] = 3 - 1 = 2$$

$$\text{total} + \text{array}[i] = 3 + 3 = 6 > 5$$

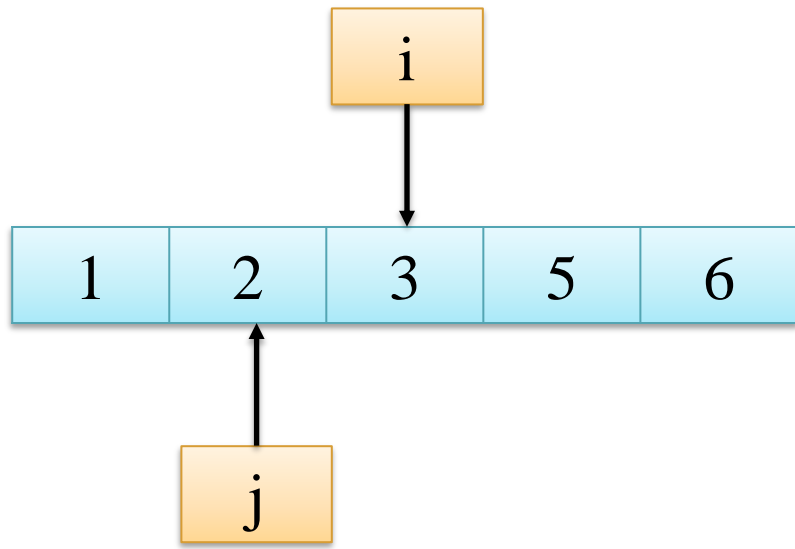
$$j = j + 1 = 0 + 1 = 1$$



# Two Pointer

## Find Subarray with Given Sum

Using two pointer



total: 2

count: 0



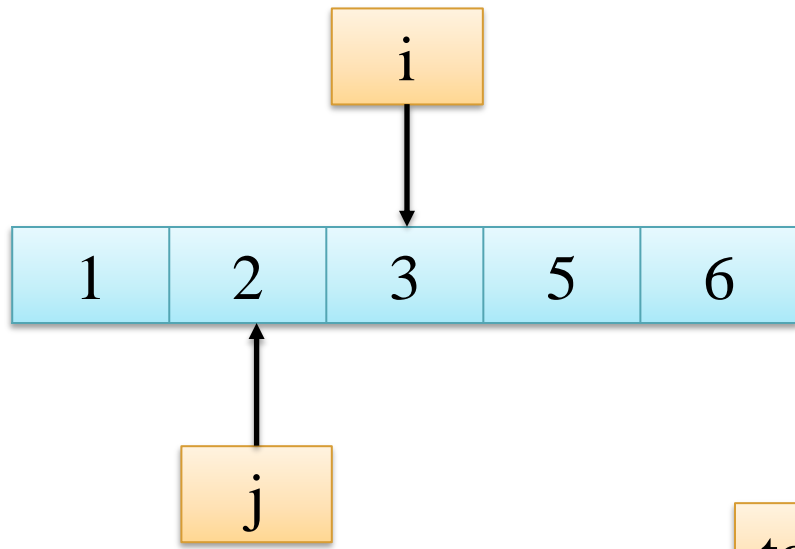
# Two Pointer

## Find Subarray with Given Sum

Using two pointer

total: 2

count: 0



$$\text{total} + \text{array}[i] = 2 + 3 = 5 = 5$$

$$\text{total} = \text{total} + \text{array}[i] = 2 + 3 = 5$$

$$\text{count} = \text{count} + 1 = 0 + 1 = 1$$

$$i = i + 1 = 2 + 1 = 3$$

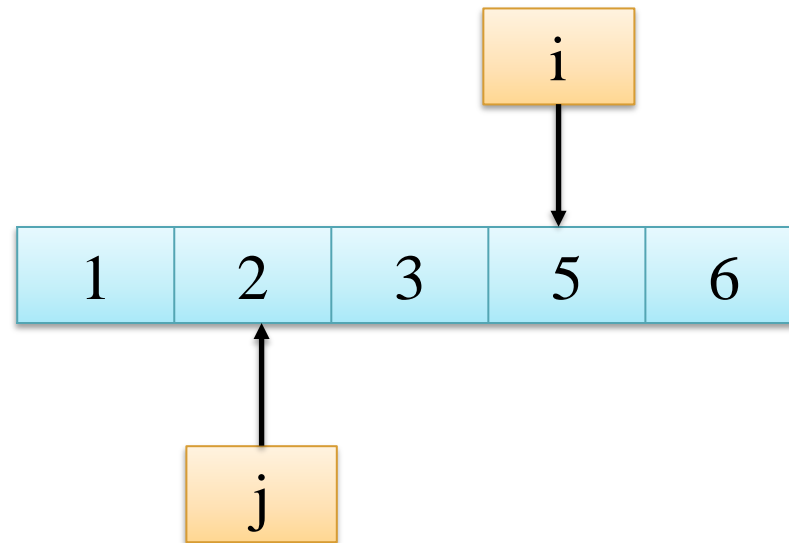




# Two Pointer

## Find Subarray with Given Sum

Using two pointer



total: 5

count: 1



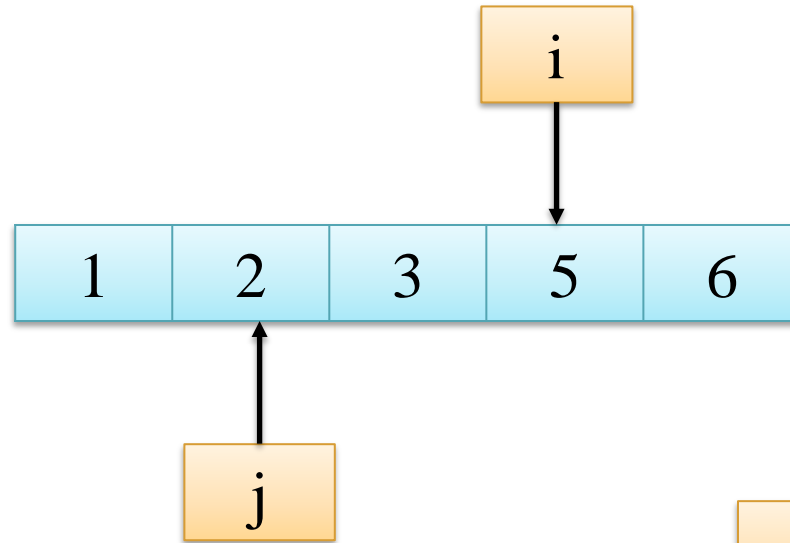
# Two Pointer

## Find Subarray with Given Sum

Using two pointer

total: 5

count: 1



$$\text{total} = \text{total} - \text{array}[j] = 5 - 2 = 3$$

$$\text{total} + \text{array}[i] = 5 + 5 = 10 > 5$$

$$j = j + 1 = 1 + 1 = 2$$



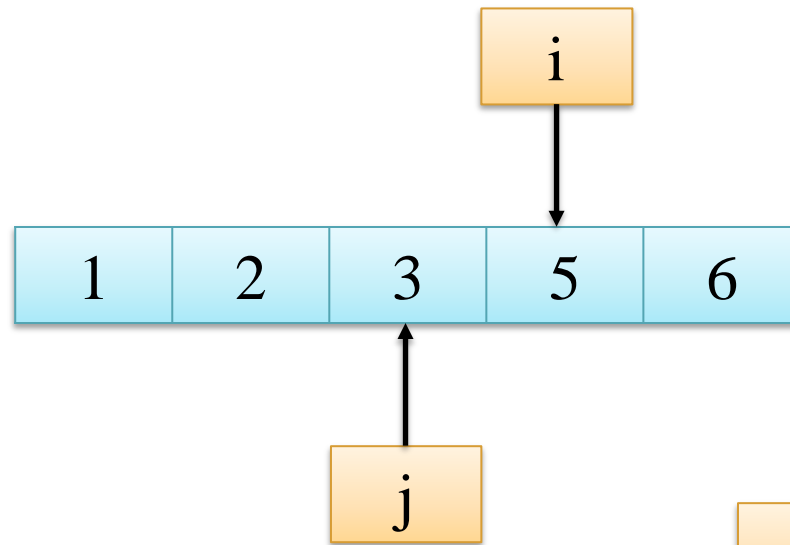
# Two Pointer

## Find Subarray with Given Sum

Using two pointer

total: 3

count: 1



$$\text{total} = \text{total} - \text{array}[i] = 3 - 3 = 0$$

$$\text{total} + \text{array}[i] = 3 + 5 = 8 > 5$$

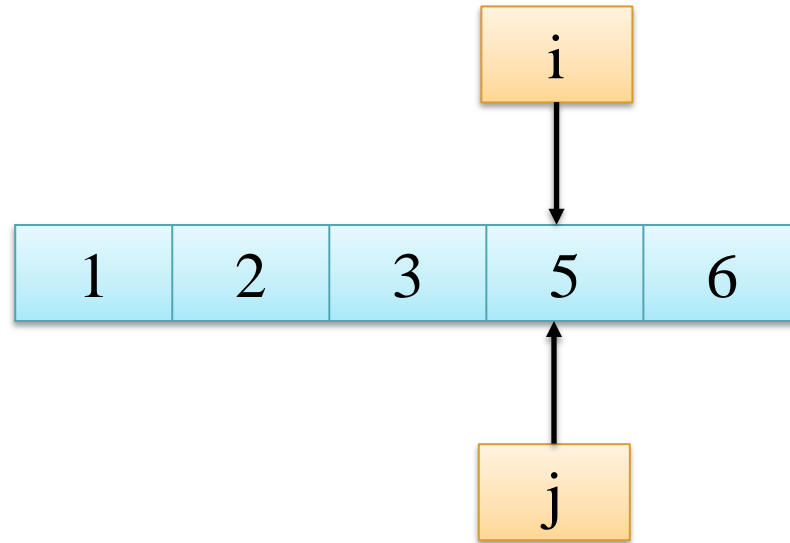
$$j = j + 1 = 2 + 1 = 3$$



# Two Pointer

## Find Subarray with Given Sum

Using two pointer



total: 0

count: 1



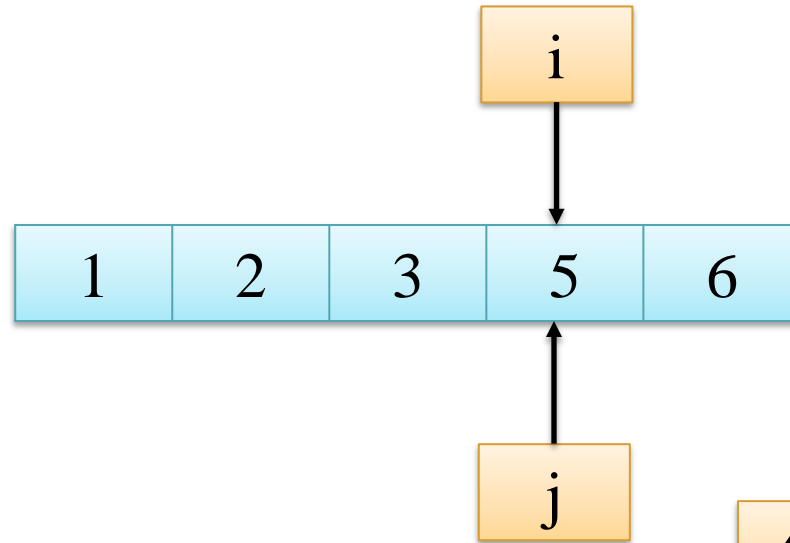
# Two Pointer

## Find Subarray with Given Sum

Using two pointer

total: 0

count: 1



$$\text{total} = \text{total} + \text{array}[i] = 0 + 5 = 5$$

$$\text{total} + \text{array}[i] = 0 + 5 = 5 = 5$$

$$\text{count} = \text{count} + 1 = 1 + 1 = 2$$

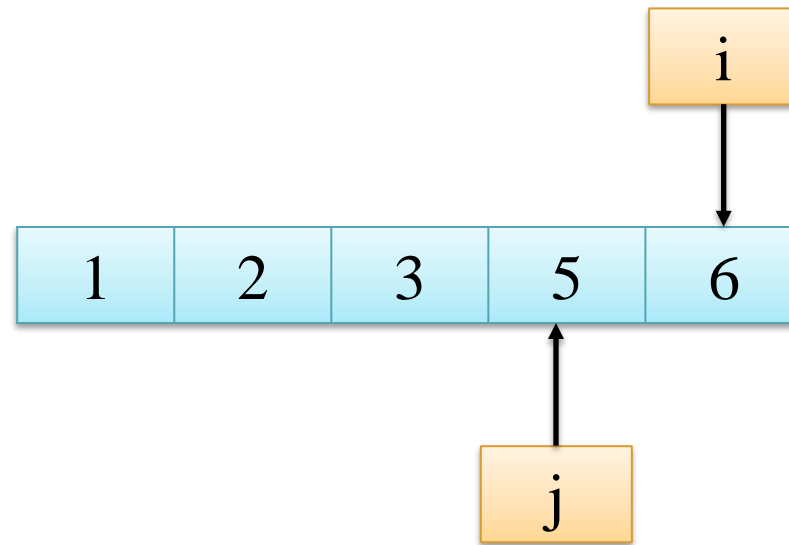
$$i = i + 1 = 3 + 1 = 4$$



# Two Pointer

## Find Subarray with Given Sum

Using two pointer



total: 5

count: 2



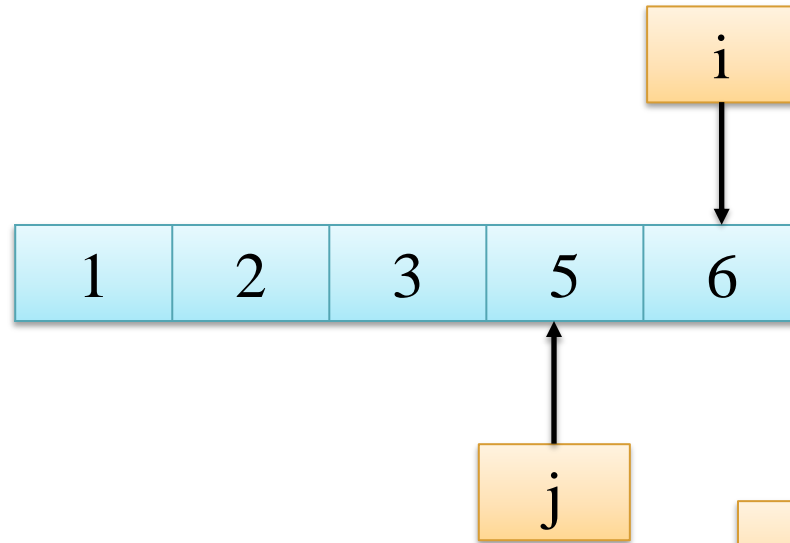
# Two Pointer

## Find Subarray with Given Sum

Using two pointer

total: 5

count: 2



$$\text{total} = \text{total} - \text{array}[j] = 5 - 5 = 0$$

$$\text{total} + \text{array}[i] = 5 + 6 = 11 > 5$$

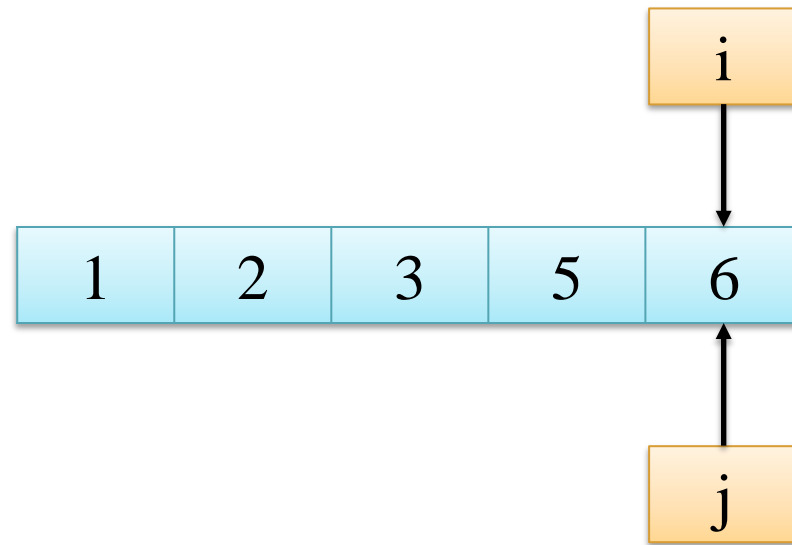
$$j = j + 1 = 3 + 1 = 4$$



# Two Pointer

## Find Subarray with Given Sum

Using two pointer



total: 0

count: 2

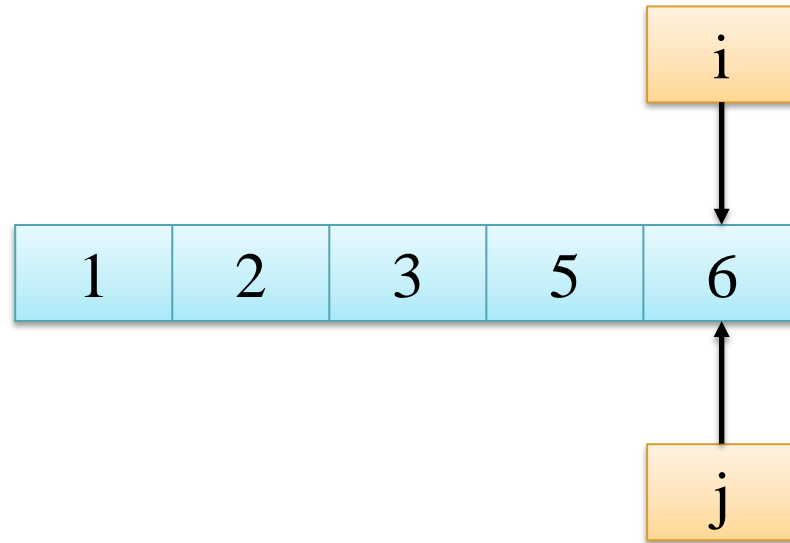




# Two Pointer

## Find Subarray with Given Sum

Using two pointer



total: 0

count: 2

$$\text{total} + \text{array}[i] = 0 + 6 = 6 > 5$$

Stop loop



# Two Pointer

## Find Subarray with Given Sum

Using two pointer

```
def subarray_sum_two_pointer(nums, key):  
    i = 0  
    j = 0  
    count = 0  
    total = 0  
    while i < len(nums):  
        if total + nums[i] < key:  
            total += nums[i]  
            i += 1  
        elif total + nums[i] > key:  
            total -= nums[j]  
            j += 1  
        else:  
            count += 1  
            total += nums[i]  
            i += 1  
    return count  
  
nums = [1, 2, 3, 5, 6]  
key = 5  
subarray_sum_two_pointer(nums, key)
```

# Two Pointer

## Find Subarray with Given Sum

Using two pointer

$T(n)$  is ?

```
def subarray_sum_two_pointer(nums, key):  
    i = 0  
    j = 0  
    count = 0  
    total = 0  
    while i < len(nums):  
        if total + nums[i] < key:  
            total += nums[i]  
            i += 1  
        elif total + nums[i] > key:  
            total -= nums[j]  
            j += 1  
        else:  
            count += 1  
            total += nums[i]  
            i += 1  
    return count  
  
nums = [1, 2, 3, 5, 6]  
key = 5  
subarray_sum_two_pointer(nums, key)
```

# Two Pointer

## Find Subarray with Given Sum

Using two pointer

$T(n)$  is  $O(n)$

```
def subarray_sum_two_pointer(nums, key):  
    i = 0  
    j = 0  
    count = 0  
    total = 0  
    while i < len(nums):  
        if total + nums[i] < key:  
            total += nums[i]  
            i += 1  
        elif total + nums[i] > key:  
            total -= nums[j]  
            j += 1  
        else:  
            count += 1  
            total += nums[i]  
            i += 1  
    return count  
  
nums = [1, 2, 3, 5, 6]  
key = 5  
subarray_sum_two_pointer(nums, key)
```



# SUMMARY

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**(1) – Brute Force – Exhaustive**  
**Sequential Search**  
**Selection Sort**  
**(Optional) Bubble Sort**

**(2) – Recursion**  
**Factorial Function**  
**Fibonacci Number**

**(3) – Two Pointer**  
**Reverse String**  
**Find Subarray With Given Sum**

---



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# (Optional) Exercise

- Leetcode [[1](#)] Two Sum
- Leetcode [[15](#)] 3Sum
- Leetcode [[16](#)] 3Sum Closet
- Leetcode [[867](#)] Transpose Matrix



# (Optional) Exercise

## ➤ Fibonacci Word

$$F(0) = A$$

$$F(1) = B$$

$$F(n) = F(n-1) + F(n-2)$$

$$\text{EX: } F(2) = F(1) + F(0) = BA \Rightarrow \text{len}(F(n)) = 2$$

$$F(3) = F(2) + F(1) = BAB \Rightarrow \text{len}(F(n)) = 3$$

Given a integer *target*. Returns the value indexed at (*target* - 1) in the string  $F(n)$  found, such that the longest length of string  $F(n)$  is closest to *target*.



# (Optional) Exercise

## ➤ Fibonacci Word

$$F(0) = A$$

$$F(1) = B$$

$$F(n) = F(n-1) + F(n-2)$$

$$\text{EX: } F(2) = F(1) + F(0) = BA \Rightarrow \text{len}(F(n)) = 2$$

$$F(3) = F(2) + F(1) = BAB \Rightarrow \text{len}(F(n)) = 3$$

Given a integer *target*. Returns the value indexed at (*target* - 1) in the string  $F(n)$  found, such that the longest length of string  $F(n)$  is closest to *target*.





# (Optional) Exercise

## ➤ Fibonacci Word

Input: target = 2

Output: A

Explanation: target = 2  $\Rightarrow$  n = 2 because  $F(2) = BA$ ,  $\text{len}(F(2)) = 2 \geq \text{target} \Rightarrow F(2)[\text{target}-1] = F(2)[1] = A$

Input: target = 4

Output: B

Explanation: target = 4  $\Rightarrow$  n = 4 because  $F(4) = BABBA$ ,  $\text{len}(F(4)) = 5 \geq \text{target} \Rightarrow F(4)[\text{target}-1] = F(4)[3] = B$



# (Optional) Exercise

## ➤ Fibonacci Word

Input: target = 5

Output: A

Explanation: target = 5  $\Rightarrow$  n = 4 because  $F(4) = \text{BABBA}$ ,  $\text{len}(F(4)) = 5 \geq \text{target} \Rightarrow F(4)[\text{target}-1] = F(4)[4] = \text{A}$

Input: target = 7

Output: A

Input: target = 1000000000

Output: A



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# Thanks!

## Any questions?