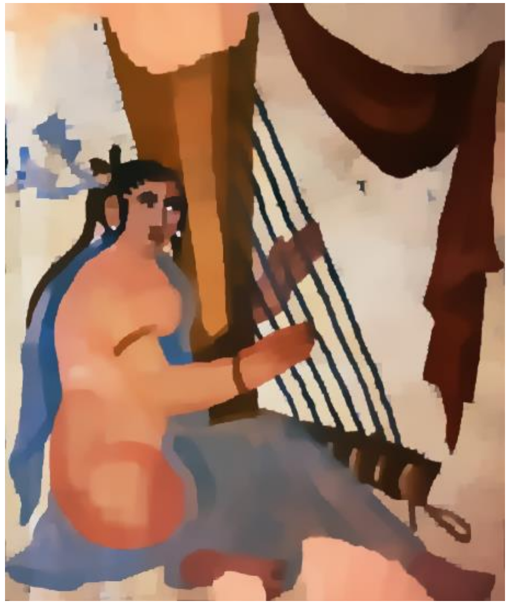
KOOC W5

Variational optimization in Image Understanding (II)

Prof. Changick Kim

EE, KAIST

Outline

- Implementation of TV regularization models
  - ROF model (1992)
  - Linearization for EL equation (1996)
  - Duality-based algorithm (2004) → primal-dual algorithm
  - TV/L1 model
  - Splitting techniques → ADMM
- Weighted least squares (WLS, 2008)
- L0 smoothing (2011)
- Experimental result
- Discussion

Primal-Dual algorithm

- Since ADMM introduces some auxiliary variables and requires the solution of some linear systems, the iterative procedure can be complicated.
- Chambolle and Pock considered solving the minimax problem [ref A]:

$$\min_{x \in X} \varphi(Kx) + \phi(x) \longrightarrow \min_{\boldsymbol{x} \in X} \max_{\boldsymbol{z} \in Z} \phi(\boldsymbol{x}) + \langle K\boldsymbol{x}, \boldsymbol{z} \rangle - \psi(\boldsymbol{z}).$$

Primal | Primal-dual

- They solve the problem by a first-order primal-dual algorithm as follows:

$$\begin{cases} \boldsymbol{x}^{(k+1)} &= \operatorname{argmin}_{\boldsymbol{x} \in X} \phi(\boldsymbol{x}) + \langle K\boldsymbol{x}, \boldsymbol{z} \rangle + \frac{1}{2s} \left\| \boldsymbol{x} - \boldsymbol{x}^{(k)} \right\|_2^2, \\ \widehat{\boldsymbol{x}}^{(k+1)} &= \boldsymbol{x}^{(k+1)} + \theta(\boldsymbol{x}^{(k+1)} - \boldsymbol{x}^{(k)}), \\ \boldsymbol{z}^{(k+1)} &= \operatorname{argmax}_{\boldsymbol{z} \in Z} \langle K\widehat{\boldsymbol{x}}^{(k+1)}, \boldsymbol{z} \rangle - \psi(\boldsymbol{z}) - \frac{1}{2t} \left\| \boldsymbol{z} - \boldsymbol{z}^{(k)} \right\|_2^2. \end{cases}$$

←approximate extragradient step [C]

Advantages

- converges fast
- matrix inversion-free. Very useful when the matrix is huge and difficult to invert

(See next slide)

A. Chambolle and T. Pock. A first-order primal-dual algorithm for convex problems with applications to imaging. Journal of Mathematical Imaging and Vision, 40(1):120–145, 2011.

B. M. Zhu, and T. F. Chan, An Efficient Primal-Dual Hybrid Gradient Algorithm for Total Variation Image Restoration, UCLA CAM Report [08-34], May 2008. (ref A minus extragradient step)

C. G.M. Korpelevich, "The extragradient method for finding saddle points and other problems." *Ekonomika i Matematicheskie Metody* **12** (1976): 747-756.

TV-L1 model

- TV-L1 formulation

$$\min_u \{ \int_\Omega |\nabla u|\, d\Omega + \lambda \int_\Omega |f - u| d\Omega \}$$

- Implementation 1): Euler-Lagrange equation

$$-\nabla \cdot \left(\frac{\nabla u}{|\nabla u|} \right) + \lambda \frac{(u-f)}{|u-f|} = 0 \quad \Rightarrow \quad -\nabla \cdot \left(\frac{\nabla u^{n+1}}{|\nabla u^n|_\varepsilon} \right) + \lambda \frac{(u^{n+1}-f)}{|u^n - f|_\delta} = 0$$

TV-L1 model

- Aujol (06) Aujol, J.-F., G. G. C. T. and S. Osher, “Structure-texture image decomposition-modeling, algorithms, and parameter Selection,” 2006

$$\min_u \left\{ \int_\Omega |\nabla u| d\Omega + \lambda \int_\Omega |u - f| d\Omega \right\}$$ ← not strictly convex

- Implementation 2): Strictly convex model

$$\min_{u,v} \left\{ \int_\Omega |\nabla u| d\Omega + \frac{1}{2\theta} \int_\Omega (u-v)^2 d\Omega + \lambda \int_\Omega |v - f| d\Omega \right\}$$

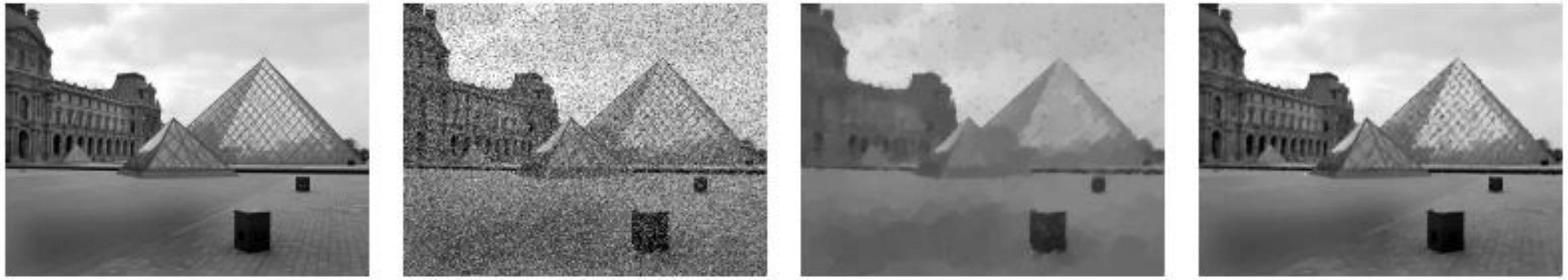
– For fixed v, $\min_u \left\{ \int_\Omega |\nabla u|\, d\Omega + \frac{1}{2\theta} \int_\Omega (u-v)^2 d\Omega \right\}$ ⇒ Chambolle method,,,

– For fixed u, $\min_v \left\{ \frac{1}{2\theta} \int_\Omega (u-v)^2 d\Omega + \lambda \int_\Omega |v - f| d\Omega \right\}$

$$v = \begin{cases} u - \lambda\theta & \text{if} \quad u - f > \lambda\theta \\ u + \lambda\theta & \text{if} \quad u - f < -\lambda\theta \\ f & \text{if} \quad |u - f| \leq \lambda\theta \end{cases}$$

TV-L1 model

- Test results by recent algorithms



(a) Clean image (b) Noisy image (c) ROF (d) TV-L^1

	$\lambda = 1.5$	
	$\varepsilon = 10^{-4}$	$\varepsilon = 10^{-5}$
PD	187 (15.81s)	421 (36.02s)
ADMM	385 (33.26s)	916 (79.98s)
EGRAD	2462 (371.13s)	8736 (1360.00s)
NEST	2406 (213.41s)	15538 (1386.95s)

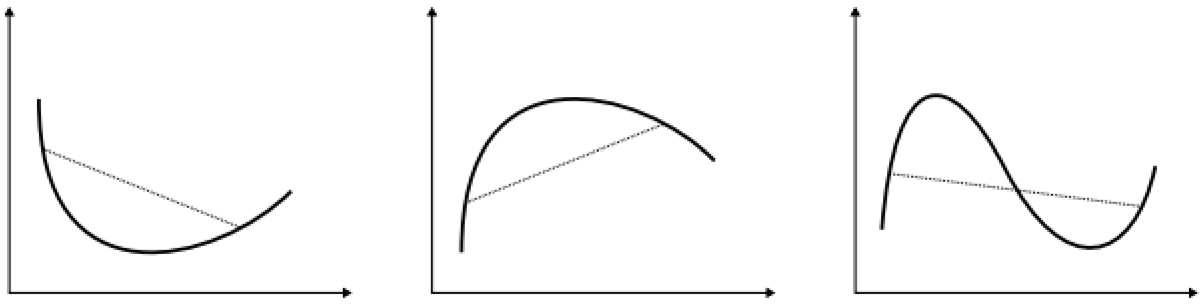
Primal-dual performs best, ADMM reasonable well

results by Pock

ref

TV-L1 model

- Convex function



Convex　　　Concave　　　Neither

– It is a well-known fact that if the second derivative $f''(x)$ is ≥ 0 for all x in an interval *I*, then *f* is convex on *I*. (Note that it is sufficient condition.)

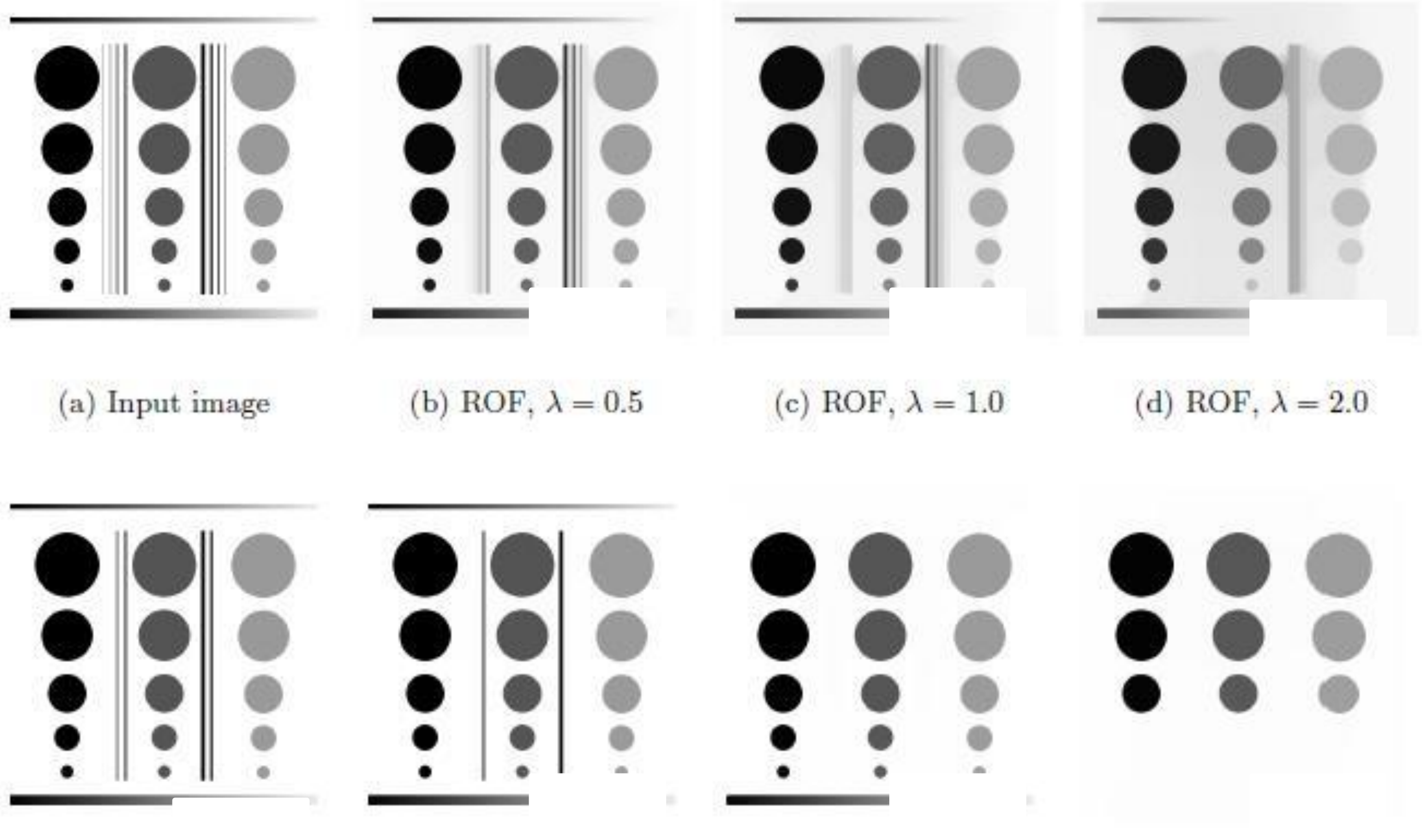
Definition: let X be a convex set. A function $f : X \to \mathbb{R}$ is said to be convex if for all $x, y \in X$ and $\alpha \in [0, 1]$,

$$f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha) f(y).$$

With a strict inequality, *f* is said to be strictly convex.

TV-L1 model

- ROF Vs TV-L1



(a) Input image (b) ROF, $\lambda = 0.5$ (c) ROF, $\lambda = 1.0$ (d) ROF, $\lambda = 2.0$

(e) TV-L^1, $\lambda = 1.0$ (f) TV-L^1, $\lambda = 0.5$ (g) TV-L^1, $\lambda = 0.2$ (h) TV-L^1, $\lambda = 0.1$

* Smoothness term increases ----------- >

Splitting techniques

- The energy function

$$E = \sum_p \left\{ \left(S_p - I_p\right)^2 + \lambda \cdot \left\| \partial S_p \right\|_1 \right\}$$

$$\partial S_p = (\partial_x S_p, \partial_y S_p)^T$$

  - Keep the result similar to the original input
  - Minimize l_1 norm of gradients
  - Constrain the number of non-zero gradients

[4] Yilun Wang et al., “A New Alternating Minimization Algorithm for Total Variation Image Reconstruction,” *SIAM Journal on Imaging Sciences,* 2008

Splitting techniques

- Easy subproblems

Easy problems	$\arg\min_u \|Au - f\|_2^2$ (a)	Differentiable
	$\arg\min_u \|u\|_1 + \|u - f\|_2^2$ (b)	Solvable by shrinkage formula
Hard problems	$\arg\min_u \|\phi u\|_1 + \|u - f\|_2^2$ (c)	L1 and L2 terms are coupled
	$\arg\min_u \|u\|_1 + \|Au - f\|_2^2$ (d)	

Splitting techniques

- Optimization of the energy function

$$\min_{S} \sum_{p} \left\{ \left(S_p - I_p\right)^2 + \lambda \cdot \left\| \partial S_p \right\|_1 \right\}$$

$$\min_{S,h,v} \sum_{p} \left\{ (S_p - I_p)^2 + \lambda\left(\left|h_p\right| + \left|v_p\right|\right) + \beta\left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2 \right) \right\}$$

  - Special alternating optimization strategy
  - Auxiliary variables h_p and v_p are substituting ∂S_p
  - Two equations are equivalent when β is large enough

Splitting techniques

- Alternating optimization strategy

$$\min_{S,h,v} \sum_{p} \left\{ (S_p - I_p)^2 + \lambda(|h_p| + |v_p|) + \beta\left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2\right) \right\}$$

  - Two subproblems

$$\min_{S} \sum_{p} \left\{ (S_p - I_p)^2 + \beta\left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2\right) \right\}$$

$$\min_{h,v} \sum_{p} \left\{ \left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2\right) + \frac{\lambda}{\beta}(|h_p| + |v_p|) \right\}$$

Splitting techniques

- Fixing h and v, optimizing S

$$\min_{S} \sum_{p} \left\{ (S_p - I_p)^2 + \beta\left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2 \right) \right\}$$

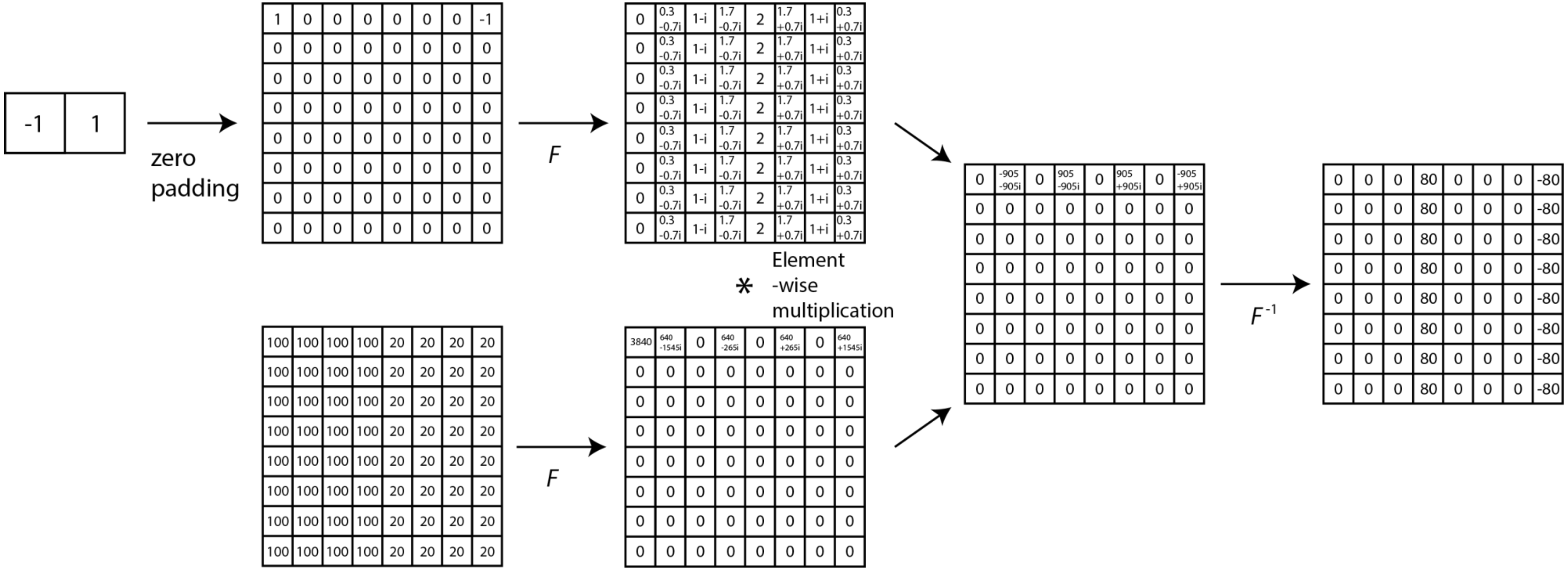
$$\Rightarrow \quad S = F^{-1}\left(\frac{F(I) + \beta(F(\partial_x)^* \circ F(h) + F(\partial_y)^* \circ F(v))}{F(1) + \beta(F(\partial_x)^* \circ F(\partial_x) + F(\partial_y)^* \circ F(\partial_y))} \right)$$

$\mathcal{F}(\quad)$: *Fourier transform*
$\mathcal{F}(\quad)^*$: *Complex conjugate*

- The function is quadratic
- Using Fast Fourier Transform (FFT) for speed up
- "∘" denotes component-wise multiplication
- and the division is component-wise as well.

Splitting techniques

- Derivatives in Fourier domain



Splitting techniques

- Fixing S, optimizing h and v

$$\min_{h,v} \sum_p \left\{ \left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2 \right) + \frac{\lambda}{\beta} \left(|h_p| + |v_p| \right) \right\}$$

$$\Rightarrow \quad (h_p, v_p) = \max(\|\partial S_p\|_1 - \frac{1}{\beta}, 0) \cdot \frac{\partial S_p}{\|\partial S_p\|_1},$$

$$where\, \partial S_p = (\partial_x S_p, \partial_y S_p)$$

  - Closed form solution
  - by using soft thresholding algorithm

Splitting techniques

- Soft threshoding algorithm

$$\min_{\mathbf{x}} \|\mathbf{x}-\mathbf{y}\|_2^2 + \lambda\|\mathbf{x}\|_1 \longrightarrow \min_{x_i} \left((x_i - y_i)^2 + \lambda|x_i|\right) \text{ for all } i$$

1) $x_i > 0$ 때, $\min\limits_{x_i} (x_i - y_i)^2 + \lambda x_i$,

2) $x_i < 0$ 때, $\min\limits_{x_i} (x_i - y_i)^2 - \lambda x_i$,

$$\frac{\partial}{\partial x_i}\left((x_i - y_i)^2 + \lambda x_i\right) = 0$$

$$2x_i - 2y_i + \lambda = 0$$

$$\therefore x_i = y_i - \frac{\lambda}{2} > 0.$$

$$\frac{\partial}{\partial x_i}\left((x_i - y_i)^2 - \lambda x_i\right) = 0$$

$$2x_i - 2y_i - \lambda = 0$$

$$\therefore x_i = y_i + \frac{\lambda}{2} < 0.$$

3) otherwise

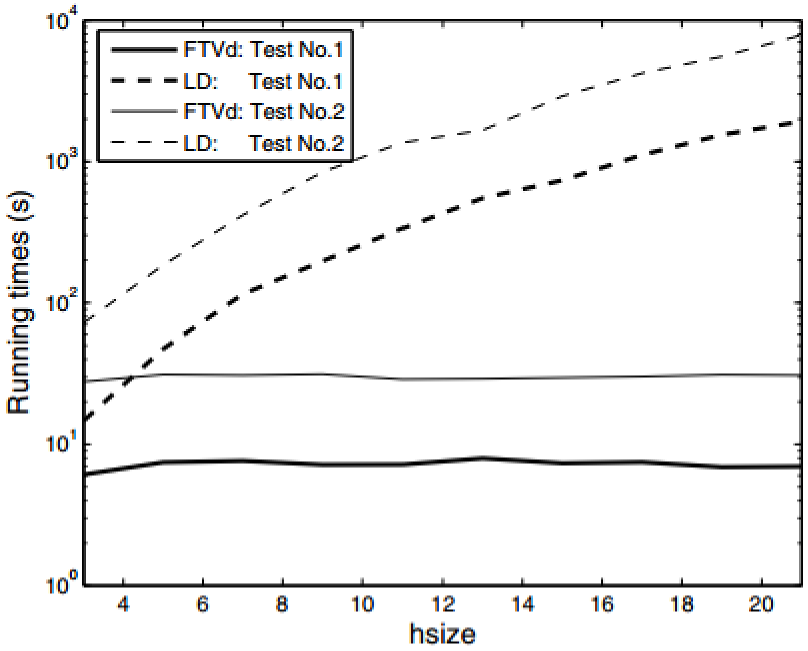
$$x_i = 0.$$

$$x_i = \max\left(\|y_i\| - \frac{\lambda}{2}, 0\right)\frac{y_i}{\|y_i\|}, \text{여기서 } i = 1,\ldots,n^2 \text{ 이고 } 0\cdot\left(\frac{0}{0}\right) = 0.$$

Splitting techniques (2008)

Processing time compared with LD

Blurry&Noisy. SNR: 5.19dB



FTVd: $\beta = 2^7$, SNR: 13.11dB, t = 14.10s

[FTVd] Yilun Wang et al., “A New Alternating Minimization Algorithm for Total Variation Image Reconstruction,” *SIAM Journal on Imaging Sciences,* 2008

[LD] Vogel and Oman, “Iterative methods for total vartiation denoising,” SIAM J. Sci.Computing, 17, 1996

Splitting techniques

[1] Y. Wang, J. Yang, W. Yin, and Y. Zhang. A new alternating minimization algorithm for total variation image reconstruction. SIAM Journal on Imaging Sciences, 1(3):248–272, 2008

[2] Stanley Osher, et al, An iterative regularization method for total variation-based image restoration, SIAM, 2005

[3] T. Goldstein and S.Osher, "The split Bregman method for L1 regularized problems," SIAM Journal on Imaging Science, 2009

[4] M. Afonso, J. Bioucas-Dias, and M. Figueiredo. Fast image recovery using variable splitting and constrained optimization. Image Processing, IEEE Transactions on, 19(9):2345–2356, 2010.

Applied [1] (Variable-splitting combined with a penalty function) to [2]] (Bregman iterative regularization method) to get [3](split Bregman method), which is shown equivalent to ADMM for denoising [4].

ADMM applied to

- alternating direction method of multipliers
- In image processing tasks, image inpainting and deblurring, motion segmentation and reconstruction in addition to denoising.
- In signal processing/reconstruction, ADMM has been applied to sp arse and low-rank recovery, where nuclear norm minimization is in volved, and to the l_1-regularized problems in compressed sensing.
- In machine learning, ADMM has been successfully applied to struct ured-sparsity estimation problems as well as many others.

ADMM of TV regularization

$$\underset{f}{\textbf{minimize}} \ \frac{\mu}{2}\|\boldsymbol{Hf}-\boldsymbol{g}\|^2+\|D\boldsymbol{f}\|_1$$

ADMM form

$$\underset{f,z}{\textbf{minimize}} \ \frac{\mu}{2}\|\boldsymbol{Hf}-\boldsymbol{g}\|^2+\|\boldsymbol{z}\|_1 \ , \quad s.t. \ \ \boldsymbol{z}=D\boldsymbol{f}$$

Augmented Lagrangian

$$L_\rho(\boldsymbol{f},\boldsymbol{z},\boldsymbol{u})=\frac{\mu}{2}\|\boldsymbol{Hf}-\boldsymbol{g}\|^2+\|\boldsymbol{z}\|_1+\frac{\rho}{2}\|\boldsymbol{z}-D\boldsymbol{f}+\boldsymbol{u}\|^2 \ ,$$

ADMM of TV regularization (TV/L2)

- ADMM update rule
  1. f-update

$$\frac{\partial L}{\partial f} = \mu \boldsymbol{H}^T(\boldsymbol{Hf} - \boldsymbol{g}) - \rho \mathbf{D}^T(\mathbf{z} - \boldsymbol{Df} + \boldsymbol{u}) = 0$$

$$\left(\mu \boldsymbol{H}^T \boldsymbol{H} + \rho \mathbf{D}^T \mathbf{D}\right) f = \mu \boldsymbol{H}^T \boldsymbol{g} + \rho \mathbf{D}^T (\mathbf{z} + \boldsymbol{u})$$

$$\boldsymbol{f} = F^{-1}\left(\frac{F[(\mu \boldsymbol{H}^T \boldsymbol{g}) + \rho \mathbf{D}^T (\mathbf{z} + \boldsymbol{u})]}{\mu \left| F[\boldsymbol{H}] \right|^2 + \rho \left(\left| F(\boldsymbol{D}_x) \right|^2 + \left| F(\boldsymbol{D}_y) \right|^2 \right)} \right)$$

ADMM of TV regularization

- ADMM update rule
  1. f-update
  2. z-update

Note that $\boldsymbol{z} = \begin{bmatrix} \boldsymbol{z}_x \\ \boldsymbol{z}_y \end{bmatrix}.$

Let $\boldsymbol{v} = \boldsymbol{Df} - \boldsymbol{u},$

$$\boldsymbol{z}_x = \max\left(\left|\boldsymbol{v}_x\right| - \frac{1}{\rho}, 0\right) \cdot sign(\boldsymbol{v}_x)$$

$$\boldsymbol{z}_y = \max\left(\left|\boldsymbol{v}_y\right| - \frac{1}{\rho}, 0\right) \cdot sign(\boldsymbol{v}_y)$$

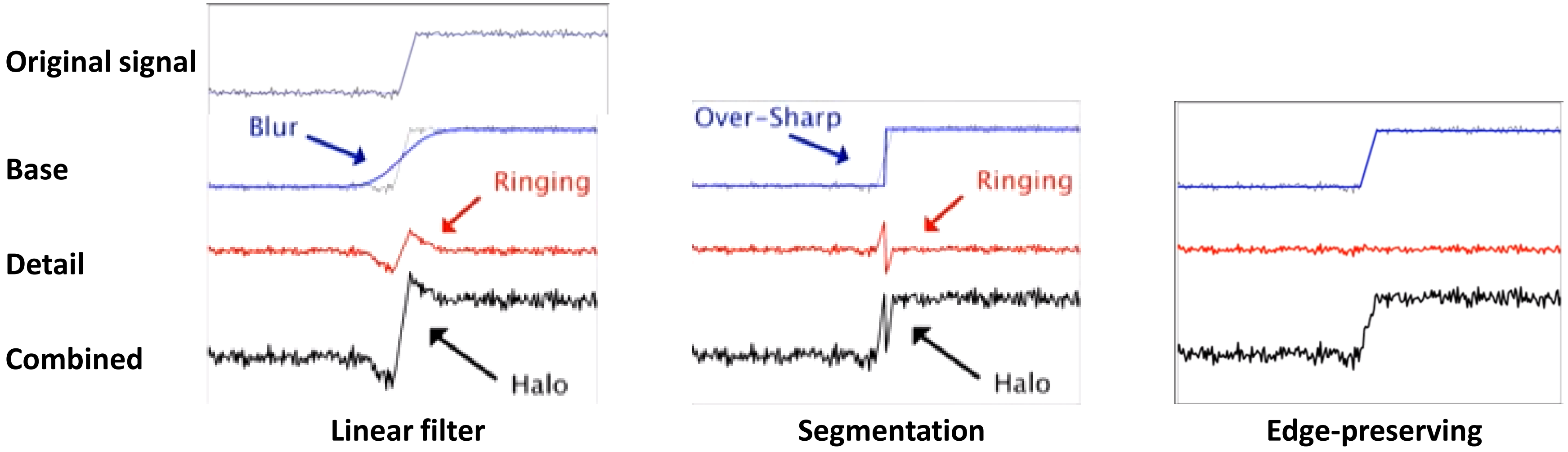
  3. u-update

$$\boldsymbol{u}^{k+1} = \boldsymbol{u}^k + \boldsymbol{z}^{k+1} - \boldsymbol{v}^{k+1}.$$

Weighted Least Squares (WLS) and L0 Smoothing

Weighted Least Squares (WLS,08)

- Edge-preserving multi-scale image decompositions
  - Decompose an image into a piecewise smooth base layer and a detail layer
  - Advocate an alternative edge-preserving operator
    - Based on the weighted least squares framework
    - Smooth out small gradients from the image while keeping strong gradients



Weighted Least Squares (WLS)

- Objective Function

$$\sum_p \left(\left(u_p - g_p\right)^2 + \lambda \left(a_{x,p}(g) \left(\frac{\partial u}{\partial x} \right)_p^2 + a_{y,p}(g) \left(\frac{\partial u}{\partial y} \right)_p^2 \right) \right)$$

  - g : input image, u : new image
  - a_x, a_y determines whether a gradient is small or strong (smoothness weights)
    - $a_{x,p}(g) = \left(\left| \frac{\partial l}{\partial x}(p) \right|^\alpha + \varepsilon \right)^{-1}$ $a_{y,p}(g) = \left(\left| \frac{\partial l}{\partial y}(p) \right|^\alpha + \varepsilon \right)^{-1}$
    - l is the log-luminance channel of g
  - λ determines the amount small gradients to be removed

Farbman et al., "Edge-preserving decompositions for multi-scale tone and detail manipulation, ACM Tr. On Graph. 27(3), 2008

Weighted Least Squares (WLS)

- Optimizing the Objective Function
  - Using matrix notation,

$$(u-g)^T(u-g)+\lambda\left(u^T D_x^T A_x D_x u + u^T D_y^T A_y D_y u\right)$$

  - A_x, A_y : diagonal matrices containing the smoothness weights $a_x(g), a_y(g)$
  - D_x, D_y : discrete differentiation operators

  - Can be minimized by solving

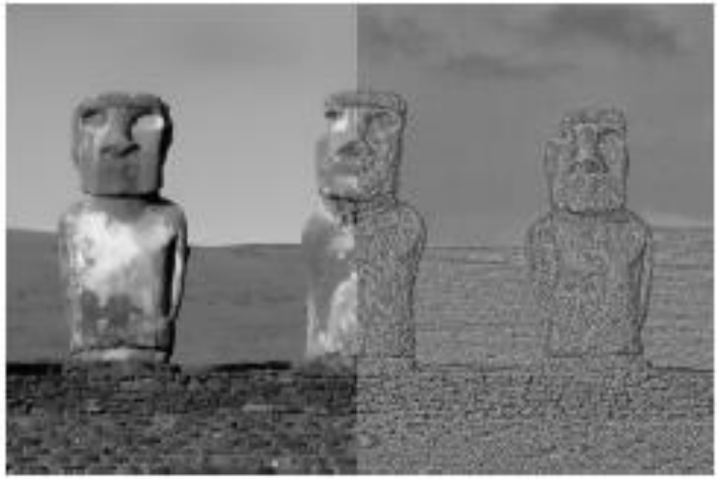
$$(I+\lambda L_g)u = g$$

  - $L_g = D_x^T A_x D_x + D_y^T A_y D_y$

Weighted Least Squares (WLS)

Input Image

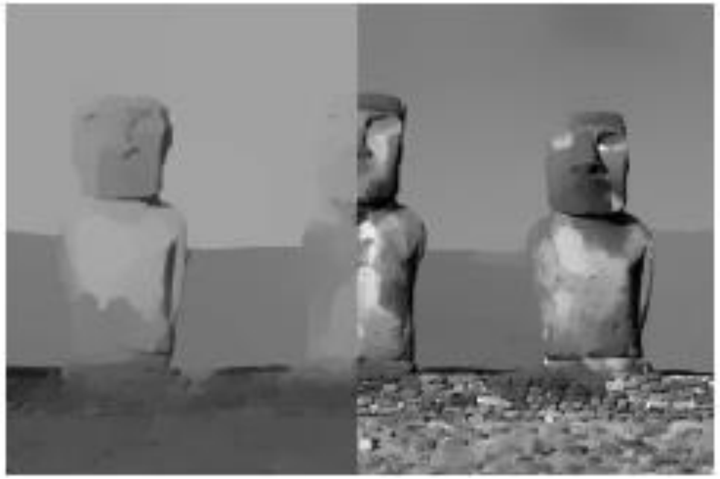
$\lambda = 0.2$



$\lambda = 0.8$

structure texture

$\lambda = 3.2$



Weighted Least Squares (WLS)

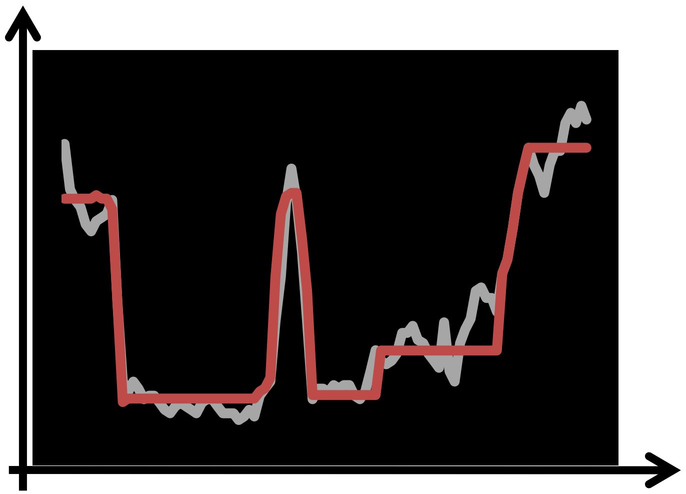
- Pros
  - (Almost) the first gradient preserving global images smoothing algorithm introduced in the image/graphics field
    - High quality multiscale image decomposition
  - Enables various applications
- Cons
  - Unable to handle high contrast textures
  - Global optimization is not fast
- TV Vs WLS
  - WLS gives more control to users
  - WLS better keeps strong gradients, enabling variety of graphics applications
  - TV smooths textures better, compared to WLS

L_0 smoothing (2011)

- The energy function

$$E = \sum_p \left\{ (S_p - I_p)^2 + \lambda \cdot C(S) \right\}$$

$$C(S) = \# \left\{ p \,\middle\|\, \partial_x S_p \right| + \left| \partial_y S_p \right| \neq 0 \right\}$$

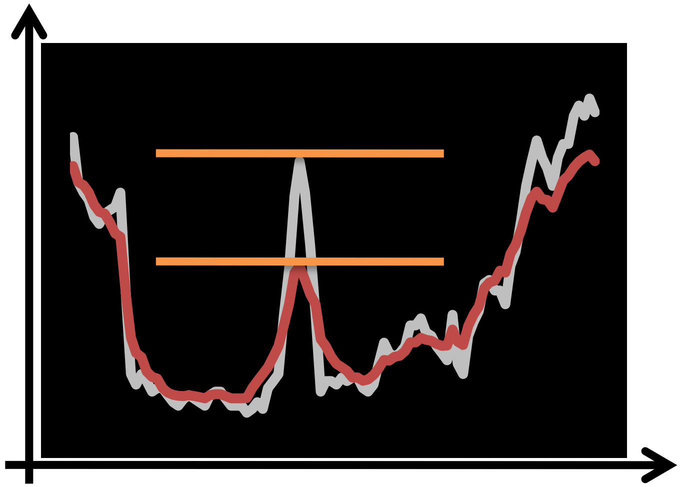


  - Keep the result similar to the original input
  - Minimize l_0 norm of gradients
  - Constrain the number of non-zero gradients

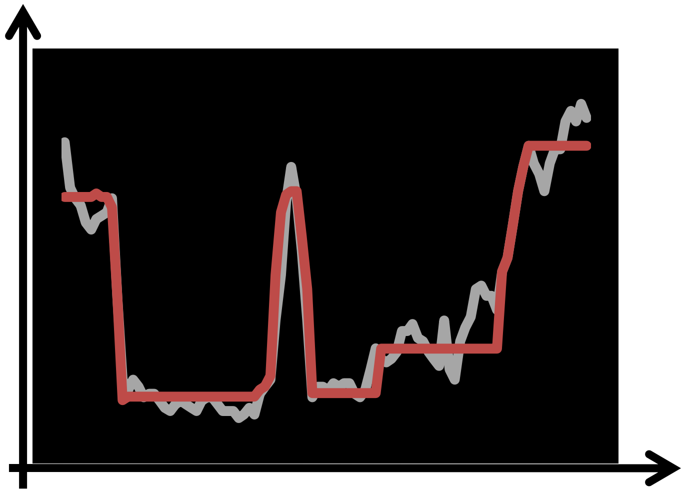
[5] Li Xu et al., "Image Smoothing via L0 Gradient Minimization," *ACM TOG,* 2011

L_0 smoothing

- L_1 gradient vs. L_0 gradient



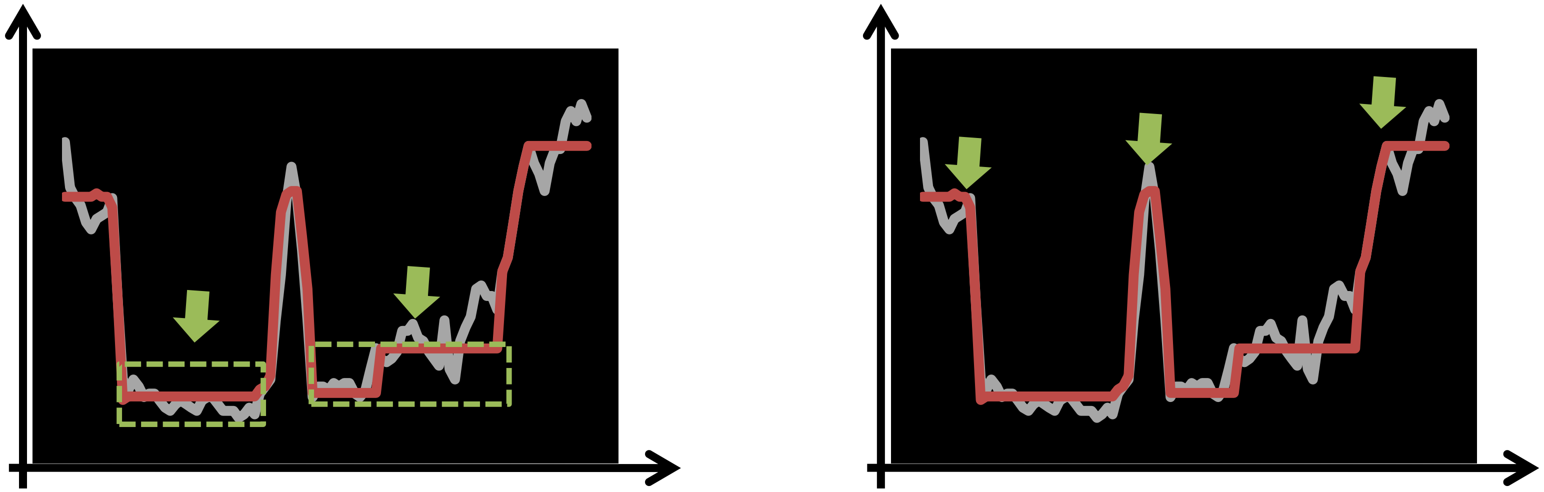
L_1 gradient



L_0 gradient

L_0 smoothing

- L_1 gradient vs. L_0 gradient



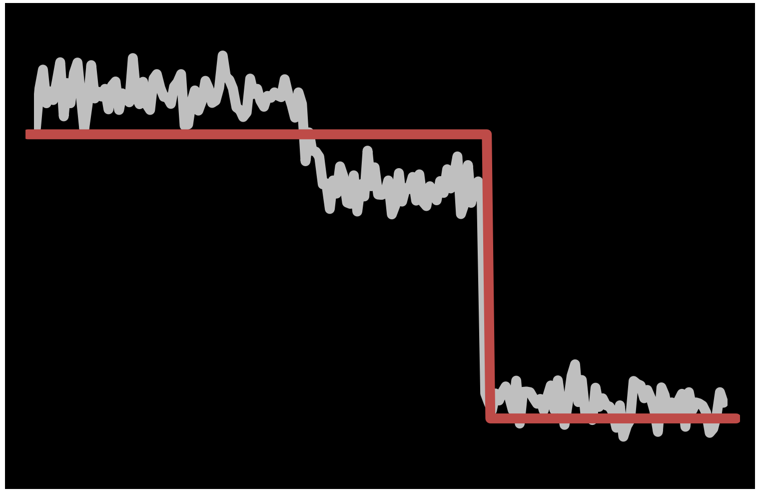
(a)   (b)

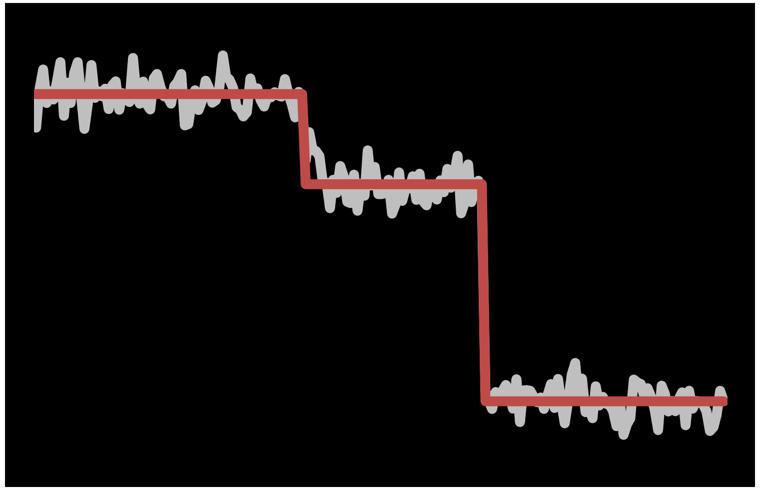
(a) Flattening insignificant details by removing small non-zero gradients

(b) Enhancing prominent edges due to large and small gradients have the same penalty

L_0 smoothing

- Framework in 1D





$$\min_s \sum_p (s_p - i_p)^2, \quad s.t.\ \ c(s) = 1 \qquad \min_s \sum_p (s_p - i_p)^2, \quad s.t.\ \ c(s) = 2$$

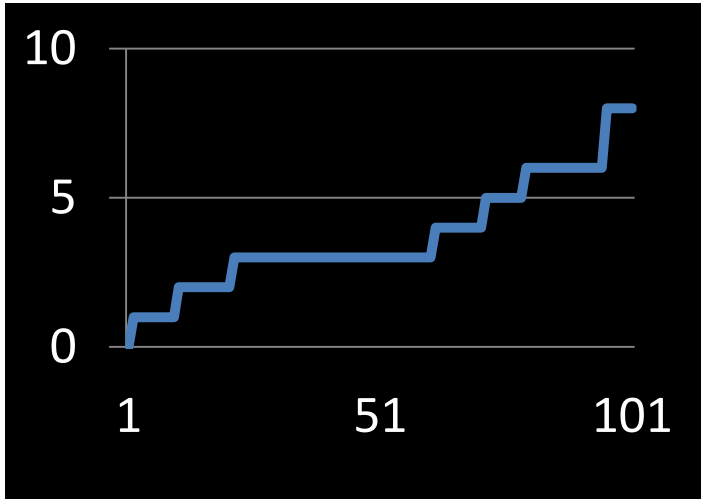
L_0 smoothing

- In practice,

$$\min_s \sum_p \left(s_p - i_p\right)^2, \quad s.t. \;\; c(s) = \boldsymbol{k}$$

$$\Rightarrow \min_s \sum_p \left(s_p - i_p\right)^2 + \boldsymbol{\lambda} \cdot c(s)$$

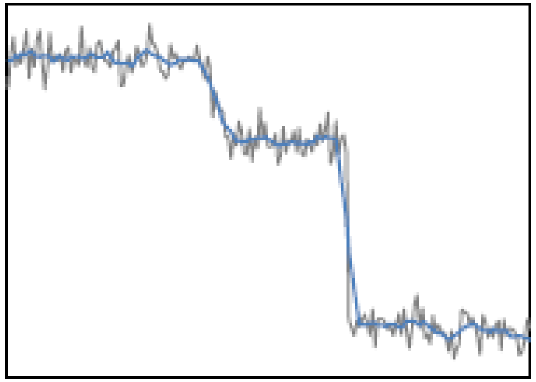
k



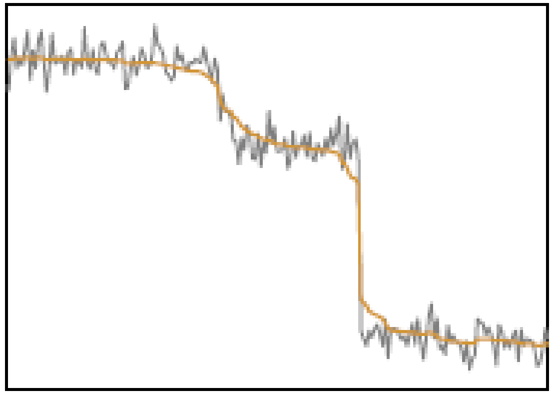
$1/\lambda$

L_0 smoothing

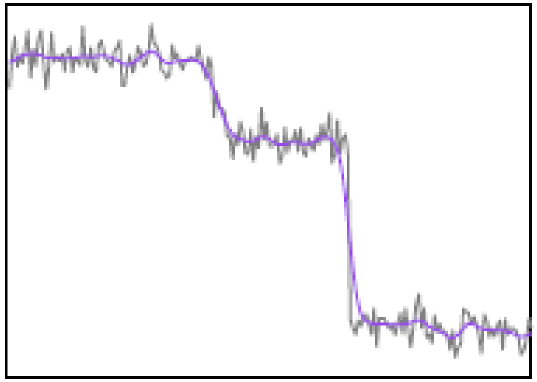
- Comparisons [Xu]



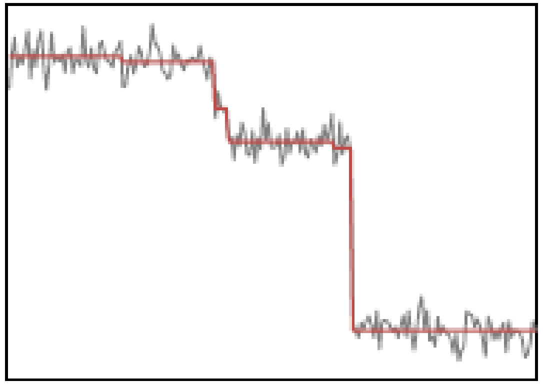
BLF 1998



WLS 2008



TV 1992



L0 smoothing 2011

[Xu] Li Xu et al., “Image Smoothing via L0 Gradient Minimization,” *ACM TOG,* 2011

L_0 smoothing

- Optimization of the energy function

$$\min_S \sum_p \left\{ (S_p - I_p)^2 + \lambda \cdot C(S) \right\},$$

$$where\, C(S) = \#\{ p \,\|\, \partial_x S_p | + | \partial_y S_p | \neq 0 \}$$

$$\min_{S,h,v} \sum_p \left\{ (S_p - I_p)^2 + \lambda \cdot C(h,v) + \beta\left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2 \right) \right\},$$

$$where\ C(h,v) = \#\{ p \mid |h_p| + |v_p| \neq 0 \}$$

  - Special alternating optimization strategy
  - Auxiliary variables h_p and v_p are substituting $\partial_x S_p$ and $\partial_y S_p$
  - Two equations are equivalent when β is large enough

L_0 smoothing

- Alternating optimization strategy (splitting technique)

$$\min_{S,h,v} \sum_p \left\{ (S_p - I_p)^2 + \lambda \cdot C(h,v) + \beta\left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2 \right) \right\}$$

  - Two subproblems

$$\min_{S} \sum_p \left\{ (S_p - I_p)^2 + \beta\left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2 \right) \right\}$$

$$\min_{h,v} \sum_p \left\{ \left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2 \right) + \frac{\lambda}{\beta} \cdot C(h,v) \right\}$$

L_0 smoothing

- Fixing h and v, optimizing S

$$\min_S \sum_p \left\{ (S_p - I_p)^2 + \beta\left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2 \right) \right\}$$

$$\Rightarrow \quad S = F^{-1}\left(\frac{F(I) + \beta(F(\partial_x)^* \circ F(h) + F(\partial_y)^* \circ F(v))}{F(1) + \beta(F(\partial_x)^* \circ F(\partial_x) + F(\partial_y)^* \circ F(\partial_y))} \right)$$

$\mathcal{F}(\ )$: *Fourier transform*
$\mathcal{F}(\ )^*$: *Complex conjugate*

- The function is quadratic
- Using Fast Fourier Transform (FFT) for speed up
- Same as total variation minimization

L_0 smoothing

- Fixing S, optimizing h and v

$$\min_{h,v} \sum_p \left\{ \left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2 \right) + \frac{\lambda}{\beta} \cdot C(h,v) \right\}$$

$$\Rightarrow \min_{h,v} \sum_p \left\{ \left((\partial_x S_p - h_p)^2 + (\partial_y S_p - v_p)^2 \right) + \frac{\lambda}{\beta} \cdot H\left(|h_p|, |v_p|\right) \right\}$$

$$\text{where } H(|h_p|, |v_p|) = \begin{cases} 1 & |h_p|, |v_p| \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow (h_p, v_p) = \begin{cases} (0,0) & (\partial_x S_p)^2 + (\partial_y S_p)^2 \le \lambda / \beta \\ (\partial_x S_p, \partial_y S_p) & \text{otherwise} \end{cases}$$

[Xu] Li Xu et al., “Image Smoothing via L0 Gradient Minimization,” *ACM TOG,* 2011

Experimental Results

Input Image

TV

Weighted Least Square

L0 Smoothing

Experimental Results

Input Image

TV

Weighted Least Square

L0 Smoothing

Experimental Results

Input Image

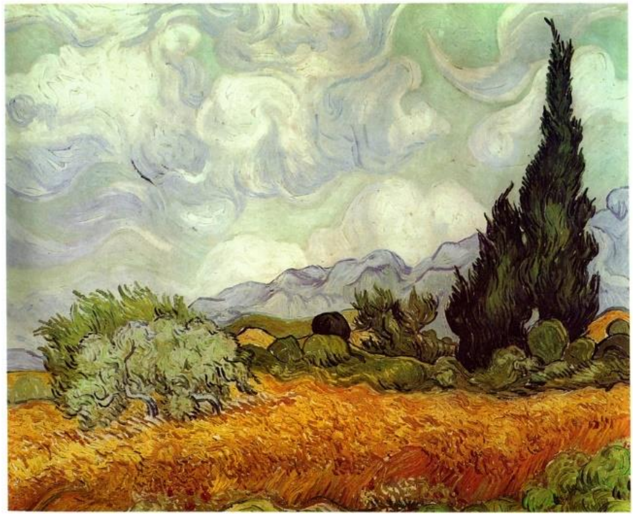
TV

Weighted Least Square

L0 Smoothing

Discussion (I)

- Image restoration
  - Edge-aware filtering
  - Scale-aware filtering
  - Content-based,,,,
- Requires better analysis of textures



Input Image

Relative Total Variation

Discussion (II-1)

- The TV model (i.e., ROF model) can be reused as a building block in total variation solutions for quite a variety of vision problems, such as:
  - Optical flow calculation
  - Deblurring
  - Multi-view stereo recognition
  - Segmentation/multi-class labeling
  - Globally consistent depth estimation from 4D light fields (CVPR, 2012)

Discussion (II-2)

- Computing optical flow using TV formulation

$$E_{\mathrm{TVL1}} = \lambda \cdot \int\int |I_0(\mathbf{x}) - I_1(\mathbf{x} + \mathbf{u}(\mathbf{x}))| d\mathbf{x} + \int\int |\nabla u_1(\mathbf{x})| + |\nabla u_2(\mathbf{x})| d\mathbf{x}$$

Approximation to strictly convex functional

$$E_{\mathrm{TVL1}} = \lambda \int\int |\rho(\mathbf{v})| d\mathbf{x} + \frac{1}{2\theta} \int\int (u_1 - v_1)^2 + (u_2 - v_2)^2 d\mathbf{x}$$
$$+ \int\int |\nabla u_1| + |\nabla u_2| d\mathbf{x}$$

$$\rho(\mathbf{v}) = I_0(\mathbf{x}) - I_1(\mathbf{x} + \mathbf{v}_0) - \langle \mathbf{v} - \mathbf{v}_0, \nabla I_1(\mathbf{x} + \mathbf{v}_0) \rangle$$

$$\min_{u_d} \left[\frac{1}{2\theta} \int\int (u_d - v_d)^2 d\mathbf{x} + \int\int |\nabla u_d| d\mathbf{x} \right] \qquad \min_{\mathbf{v}} \left[\lambda \cdot \int\int |\rho(\mathbf{v})| d\mathbf{x} + \frac{1}{2\theta} \int\int (u_1 - v_1)^2 + (u_2 - v_2)^2 d\mathbf{x} \right]$$

Reference

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  - Wang et al., A New Alternating Minimization Algorithm for Total Variation Image Reconstruction, SIAM Journal on Imaging Sciences, Volume 1, Issue 3, 248-272, July 2008.
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  - Farbman et al., Edge-preserving decompositions for multi-scale tone and detail manipulation, ACM Trans. Graphics, Volume 27, Issue 3, Article No. 67, August 2008.
- L_0 gradient minimization
  - Xu et al., Image smoothing via L0 gradient minimization, ACM Trans. Graphics, Volume 30, Issue 6, Article No. 174, December 2011.
- Relative total variation
  - Xu et al., Structure extraction from texture via relative total variation , ACM Trans. Graphics, Volume 31 Issue 6, Article No. 139, November 2012.
- Rolling guidance filter (← not variational method)
  - Zhang et al., ECCV 2014