CS528 Task Scheduling

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Scheduling Problems

Ref: "Scheduling Algorithm" Book by P. Brucker

Google "Scheduling Algorithm Brucker pdf" to get a PDF copy of the Book Soft copy uploaded to Course Website

Parallel Machine Problems

- P: We have jobs j as before and m identical machines M₁, ..., M_m.
- The processing time for j is the same on each machine.
- One has to assign the jobs to the machines and to schedule them on the assigned machines.
- This problem corresponds to an RCPSP with r
 = 1, R₁ = m, and r_{i1} = 1 for all jobs j.

Parallel Machine Problems

- **Q:** The machines are called **uniform** if $p_{jk} = p_j/r_k$.
- **R**: For **unrelated machines** the processing time p_{jk} depends on the machine M_k on which j is processed.
- MPM: In a problem with multi-purpose machines a set of machines μ_j is is associated with each job j indicating that j can be processed on one machine in μ_j only.

Parallel Machines

Ti	P1	P2	Р3	P4
T1	10	10	10	10
T2	12	12	12	12
Т3	16	16	16	16
T4	20	20	20	20

P: Identical

Ti	P1	P2	P3	P4
T1	10	15	20	25
T2	12	18	24	30
Т3	16	24	32	40
T4	20	30	40	50

P1 P2 P3 P4 12 10 8 2 **T1 T2** 12 28 **25** 13 **T3** 16 **32** 14 4 42 20 38 22 **T4**

Q: Uniform : with speed difference $(S_1=1, S_2=2/3, S_3=1/2, S_4=2/5)$

R: Unrelated: heterogeneous

Classification of Scheduling Problems

Classes of scheduling problems can be specified in terms of the three-field classification

where

- α specifies the machine environment,
- β specifies the **job characteristics**, and
- γ describes the **objective function(s)**.

Machine Environment: α

Symbol	Meaning	
1	Single Machine	
P	Parallel Identical Machine	
Q	Uniform Machine	
R	Unrelated Machine	
MPM	Multipurpose Machine	
J	Job Shop	
F	Flow Shop	

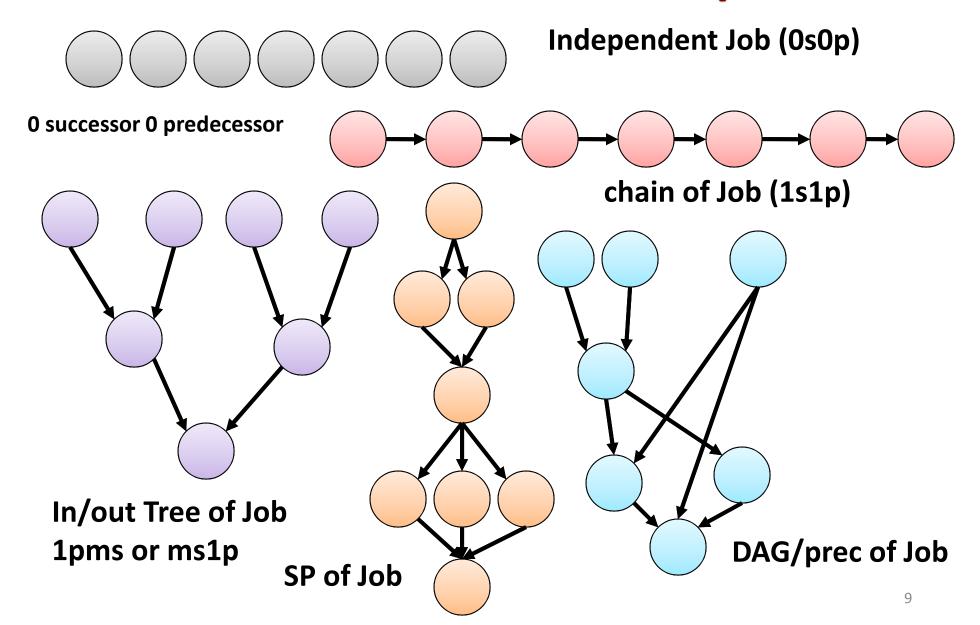
If the number of machines is fixed to m we write

Pm, Qm, Rm, MPMm, Jm, Fm, Om.

Job Characteristics : β

Symbol	meaning
pmtn	preemption
r_j	release times
d_{j}	deadlines
$p_j = 1 \text{ or } p_j = p \text{ or } p_j \in \{1,2\}$	restricted processing times
prec	arbitrary precedence constraints
intree (outtree)	intree (or outtree) precedence
chains	chain precedence
series-parallel	a series-parallel precedence graph

Job Precedence Examples



Objective Functions: γ

Two types of objective functions are most common:

- bottleneck objective functions
 max {f_i(C_i) | j= 1, ..., n}, and
- sum objective functions $\Sigma f_j(C_j) = f_1(C_1) + f_2(C_2) + ... + f_n(C_n)$.

C_i is completion time of task j

Objective Functions: γ

- C_{max} and L_{max} symbolize the bottleneck objective
 - $-\mathbf{C}_{max}$ objective functions with $f_j(C_j) = C_j$ (makespan)
 - L_{max} objective functions $f_j(C_j) = C_j d_j$ (maximum Lateness)

- Common sum objective functions are:
 - $-\Sigma C_i$ (mean flow-time)
 - $-\Sigma \omega_{i} C_{i}$ (weighted flow-time)

Objective Functions: γ

• Σ **U**_j (number of late jobs) and Σ ω_j **U**_j (weighted number of late jobs) where U_j = 1 if $C_j > d_j$ and $U_j = 0$ otherwise.

• Σ T_j (sum of tardiness) and Σ ω_j T_j (weighted sum of tardiness/lateness) where the tardiness of job j is given by

$$T_j = \max \{ 0, C_j - d_j \}.$$

Examples of Scheduling Problem

- 1 | $prec; p_j = 1 | \Sigma \omega_j C_j$
- P2 | | C_{max}
- P | $p_j = 1$; $r_j | \sum \omega_j U_j$
- R2 | chains; pmtn | C_{max}
- R | n = 3 | C_{max}
- P | $p_{ij} = 1$; outtree; $r_j \mid \sum C_j$
- Q | $p_j = 1 | \Sigma T_j$

Scheduling of Independent Tasks

Polynomial algorithms

 A problem is called polynomially solvable if it can be solved by a polynomial algorithm.

Example

 $\begin{array}{c|c} 1 \mid \sum \omega_{j} C_{j} \text{ can be solved by} \\ \text{Scheduling the jobs in an ordering of non-increasing } \omega_{i}/p_{i} \text{ - values.} \end{array}$

Complexity: O(n log n)

Polynomial algorithms for $1 \mid \Sigma C_j$

Example

```
1 | \Sigma C<sub>j</sub> can be solved by
Scheduling the jobs in an ordering of non-
increasing 1/p_j - values. == > SJF
C<sub>i</sub> = Q<sub>i</sub>+P<sub>i</sub>: Waiting time + Processing time
(SJF is optimal)
```

Complexity: O(n log n)

Polynomial algorithms: P|p_i=1|Cmax

 A problem is called polynomially solvable if it can be solved by a polynomial algorithm.

Example

P|p_i=1|Cmax can be solved by
Scheduling the jobs in phase wise, P jobs in one phase, require ceil(n/P) phases.

Complexity: O(n)

P2 | | C_{max}

- n tasks, 2 processors
- ET: t₁, t₂, t₃,...., t_n
- Subset Sum problem: 1+e APPROX
 - Ref: CLR Book Chapter 37 Section 4
- Divide the tasks in two sets such that
 - Difference of Sum of ETs of both the set is minimized
 - Min (Max(Sum(Set₁), Sum(Set₂)))

P_m | | C_{max}

- n tasks, m processors
- ET: t₁, t₂, t₃,...., t_n
- m-Subset Sum problem
- INDEP(m) Problem: NPC in strong sense
- Divide the tasks in m sets such that
 - Difference of Sum of ETs of all the set is minimized: does not exceed a value K
 - Min (Max(Sum(Set₁), Sum(Set₂), ...Sum(Set_m)))

- n tasks, m processors, infinite pre-emption allowed
- ET: t₁, t₂, t₃,...., t_n
- Divide all the work among all the cores equally
 - $Avg = (\Sigma t_i)/m$ work to each cores
 - If $max(t_i)$ >Avg, C_{max} = $max(t_i)$, Task need to execute serially
- Handle the boundary cases

- n tasks, m uniform processors, infinite preemption allowed
- Longer task executed on high speed processor till it execution is long enough as compared to others
 - Sort the tasks based on LPT
 - Allocate long task to higher speed processors one by one
 - When execution time of longer task is no longer long as compared to other then co-execute

Algorithm level

Assign (t)

```
J := \{i \mid pi(t) > 0\};

M := \{M1, ..., Mm\};

WHILE J = \emptyset and M = \emptyset {

Find the set I \subseteq J of jobs with highest level;

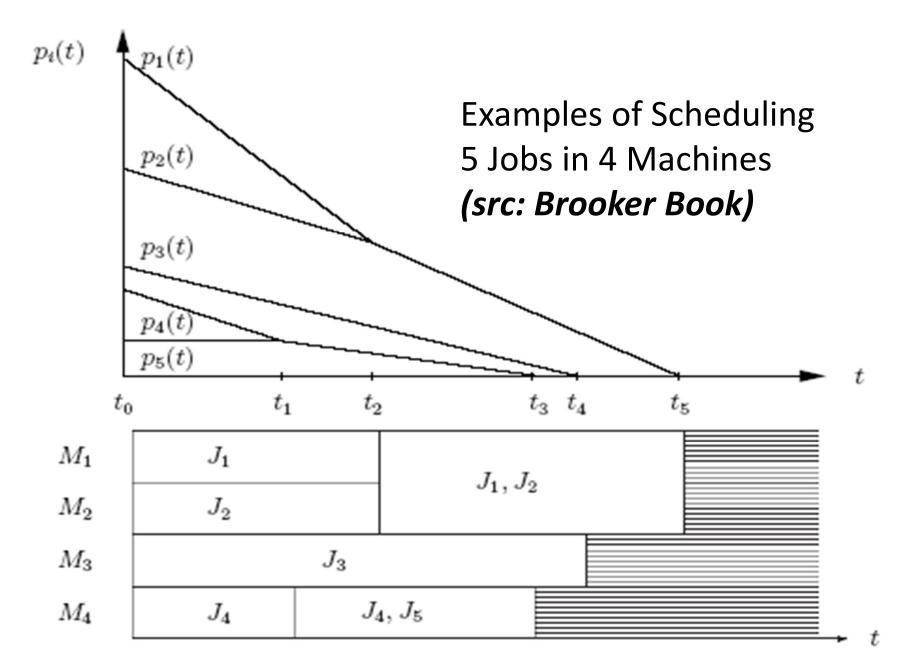
r := \min\{|M|, |I|\};

Assign jobs in I to be processed jointly on the r

fastest machines in M;

J := J \setminus I;

Eliminate the r fastest machines in M from M
```



$Q|ptmn|\Sigma C_{j}$

- LPT on High speed is good to optimize Σe_j the sum of task execution time but not ΣC_i
- Modified version of SPT (shortest remaining time) rule. As ΣC_j include waiting time of all the tasks
- Order the tasks according to non-decreasing processing time.
- Schedule task 1 on available highest speed machine up to time $t_1=p_1/s_1$.
- Schedule 2^{nd} task on M2 for t_1 time and then on M_1 from time t1 to time $t_2 \ge t_1$ until it is completed and same process continues

$Q|ptmn|\Sigma C_{j}$

- Example m=3, s1=3, s2=2, s3=1 and n=4, p1=10, p2=8, p3=8, p4=3
- SRT Job J₄ get scheduled on M1 with speed s1 for 1 time unit. Job 3 get scheduled on M₂ upto time 1 and then shifted to M1. Gant chat is given bellow with $\Sigma C_i = 14$

