ME 620: Fundamentals of Artificial Intelligence

Lecture 15: First Order Logic – Part I

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Language to formulate Knowledge

- A system aspiring to be intelligent, need to be able to formulate knowledge of the world!

  Propositional Logic is a weak Language!

  - Language of our choice is the First-order Logic
    - Simple and Convenient to begin with.
- Three things of a *language* that are of our concern
  - **Syntax**
    - Specify which group of symbols, arranged in what way, are to be considered properly formed.
  - **Semantics**

    In English - There is someone behind you; Warning! Or Request
    - Specify what the well-formed expressions are supposed to mean.
  - **Pragmatics**

    In KR &R - How to use meaningful sentences as part of a KB from which inferences are drawn.
    - Specify how the meaningful expression are to be used.

Propositional Logic

Commits only to the existence of facts that may not be the case in the world being represented.

- **Logical constants**: true, false
- **Propositional symbols**: P, Q, S, ... (atomic sentences)
- Wrapping **parentheses**: (…)
- Sentences are combined by **propositional connectives**:

$\wedge$	and	[conjunction]
$\vee$	or	[disjunction]
$\rightarrow$	implies	[implication / conditional]
$\leftrightarrow$	is equivalent	[biconditional]
$\neg$	not	[negation]

It has a simple syntax and simple semantics. It suffices to illustrate the process of inference. Propositional logic quickly becomes impractical, even for very small worlds.

Weak Language

Propositional Logic is a **weak Language.**

- Consider the problem of representing the following information:
  - Every person is mortal.
  - Socrates is a person.
  - Socrates is mortal.

Although the third sentence is entailed by the first two, an explicit symbol, to represent an individual was required.

- How can these sentences be represented so that we can infer the third sentence from the first two?
  - Create propositional symbols.
    P = He is a Person; M = He is Mortal; S = Socrates
  - $P \rightarrow M$; $S \rightarrow P$; Therefore $S \rightarrow M$

To represent other individuals we need separate symbols for each one; some way to represent the fact that all individuals who are "people" are also "mortal".

First-Order Logic

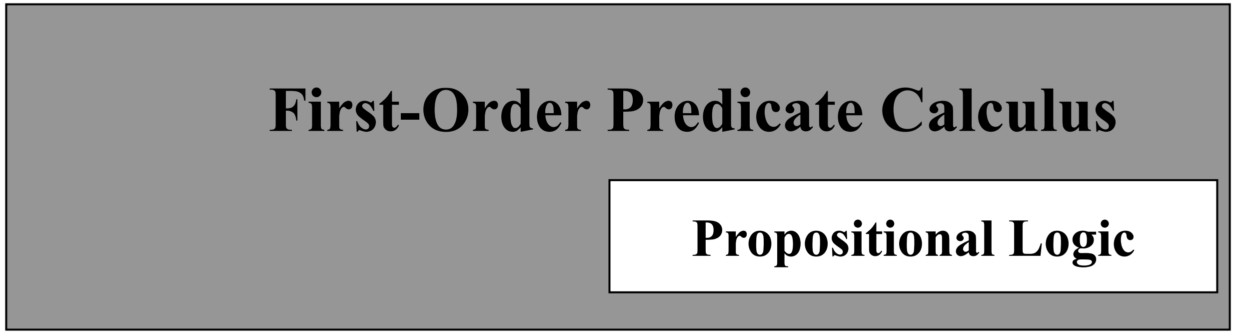
- Propositional Logic
  - Hard to identify "individuals"
    - E.g., Mary, 3
  - Can't directly talk about properties of individuals or relations between individuals
    - E.g., Ben is fat.
  - Generalizations, patterns, regularities can't easily be represented
    - E.g., All triangles have 3 sides.
- First-Order Logic
  - FOL or FOPC is expressive enough to concisely represent this kind of information
  - FOL adds relations, variables, and quantifiers, e.g.,
    - Every elephant is gray. : $\forall$ x (elephant(x) → gray(x))
    - There is a white alligator.: $\exists$ x (alligator(X) ^ white(X))

First-order logic allows us to get at the internal structure of certain propositions in a way that is not possible with propositional logic.

First-Order Logic

- **Propositional Logic.**
    - Have drawbacks so we will consider the more general
- **First-Order Predicate Calculus.**

First-order logic is symbolized reasoning in which each sentence, or statement, is broken down into a subject and a predicate. The predicate modifies or defines the properties of the subject. In first-order logic, a predicate can only refer to a single subject. First-order logic is also known as first-order predicate calculus or first-order functional calculus.



First-Order Logic

- First-order logic is a logical system for reasoning about properties of objects.
- Augments the logical connectives from propositional logic.
  - **predicates** that describe properties of objects.
  - **functions** that map objects to one another.
  - **quantifiers** to reason about multiple objects simultaneously.

First-Order Logic

- First-order logic models the world in terms of
  - **Objects**
    - Things with individual identities
  - **Properties**
    - Distinguish objects from other objects.
  - **Relations**
    - Hold among sets of objects.
  - **Functions**
    - subset of Relations; one value for a given input.

The notion of an *object* is quite broad. Objects can be concrete or abstract; Objects can be primitive or composite.

A relation takes objects as arguments and generates a truth value. Functions applied to arguments name things.

First-Order Logic

- Each variable refers to some object in a set called the **domain of discourse**.
- First-order variables refer to arbitrary objects, it does not make sense to directly apply connectives to them:
- To reason about objects, first-order logic uses **predicates.**
  - In English, the predicate is the part of the sentence that tells you something about the subject.

Predicate

Definition: A **predicate** is a property that a variable or a finite collection of variables can have.

- Predicates can take any number of arguments, but each predicate has a fixed number of arguments called its arity.
  - $P(x_1, x_2, .., x_n)$ is a predicate of n variables or n arguments.
- A predicate becomes a proposition when specific values are assigned to the variables.
  - Applying a predicate to arguments produces a proposition, which is either true or false.

Predicate

- Example
  - She is a student at IIT Guwahati.

    We could have a predicate

    P(x, IIT) - `x’ is a student at IIT Guwahati.

    OR

    P(x, y) - `x’ is a student at `y’.
  - He lives in the city.

    We could have a predicate

    P(x, y) - `x’ lives in `y’.

    Mohan lives in Guwahati.

    Note that P(Mohan, Guwahati) is a proposition!

Domain and Truth Sets

Definition: The **domain** or **universe** or **universe of discourse** for a predicate variable is the set of values that may be assigned to the variable.

Definition: If P(x) is a predicate and x has domain U, the **truth set** of P(x) is the set of all elements t of U such that P(t) is true, i.e., $\{t \in U | P(t) \text{ is true}\}$.

- Example
  - U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
  - P(x): `x' is even.
  - The truth set is: {2, 4, 6, 8, 10}

Functions

Definition: A **function** return objects associated with other objects.

- Functions can take any number of arguments, but each function has a fixed number of arguments called its arity.
  - $F(x_1, x_2, \ldots, x_n)$ is a function of n variables or n arguments.
- Functions evaluate to objects, not propositions when specific values are assigned to the variables.
  - MotherOf(x): a function that returns the mother of `x'.
    MotherOf(Jesus) would return `Mary'.

Syntax of First-Order Logic

Two types of symbols

- Variables
  - A variable is any sequence of *lowercase* alphabet and numeric characters in which the first character is lowercase alphabet.
- Constants
  - Object Constants
    - An object constant is used to name a specific element of a universe of discourse.
  - Function Constants
    - A function constant is used to designate a function on members of the universe of discourse.
  - Relation Constants
    - A relation constant is used to name a relation on the universe of discourse.

Syntax of First-Order Logic

FOL Provides

- Variable symbols
  - E.g., x, y, foo
- Connectives
  - Same as in PL:
  - $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$
- Quantifiers
  - Universal $\forall x$
  - Existential $\exists x$

User Provides

- Constant symbols
  - Mary
  - Green
- Function symbols
  - father-of(Mary) = John
  - color-of(Sky) = Blue
- Predicate symbols
  - greater(5,3)
  - color(Grass, Green)

Syntax of First-Order Logic

- A **term** (denoting a real-world individual) is a constant symbol, a variable symbol, or an n-place function of n terms.

  x and $f(x_1, \ldots, x_n)$ are terms, where each x_i is a term.

  A term with no variables is a **ground term.**

In FOL, facts are stated in the form of expressions called sentences or well-formed formulas.

- An **atomic sentence** (which has value true or false) is an n-place predicate of n terms.

Syntax of First-Order Logic

- A **complex sentence** is formed from atomic sentences connected by the logical connectives:
  $\neg P$, $P \vee Q$, $P \wedge Q$, $P \rightarrow Q$, $P \leftrightarrow Q$ where P and Q are sentences
- A **quantified sentence** adds quantifiers $\forall$ and $\exists$
  - Universally quantified
  - Existentially quantified

  Quantified sentences provide a more flexible way of talking about objects in the universe of discourse.
- A **well-formed formula** (wff) is a sentence containing no "free" variables. That is, all variables are "bound" by universal or existential quantifiers.

Equality

- First-order logic includes a special predicate =
  - States whether two objects are equal to one another.
  - Example
    - Two = 2

Equality is a part of first-order logic

First Order Logic without equality is a weaker version of FOL that has no distinguished equality symbol.

  - **Equality** symbol (=) is a logical constant and can be best understood as the identity relation.
- Equality can only be applied to object.
  - Biconditional $\leftrightarrow$ is used to see if propositions are equal.
- Define $\neq$ as $x \neq y \equiv \neg(x = y)$