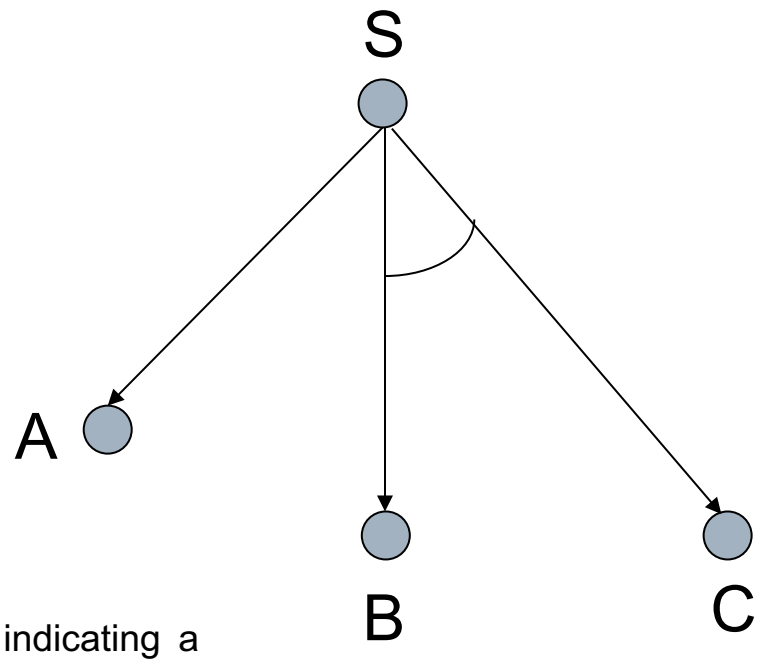
ME 620: Fundamentals of Artificial Intelligence

Lecture 10: Searching AND/OR Graphs

Shyamanta M Hazarika
Biomimetic Robotics and Artificial Intelligence Lab
Mechanical Engineering and M F School of Data Sc. & AI
IIT Guwahati

AND-OR Graph

- Represent the solution of problems that can be solved by decomposing them into a set of smaller problems, all of which must then be solved.
  - AND arc may point to any number of successor node, all of which must be solved in order for the arc to point to a solution.
  - AND arcs are indicated with a line connecting all the components.

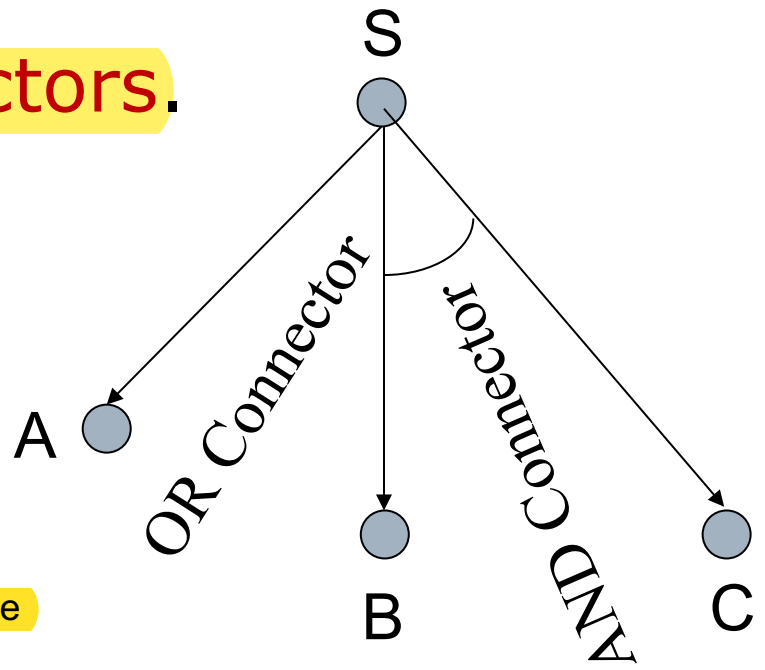


Several arcs may emerge from a single node, indicating a variety of ways in which the original problem might be solved.

Hypergraph

- AND-OR Graphs are defined as hypergraphs.
  - Instead of arcs connecting pairs of nodes, there are *hyperarcs* connecting a parent node with a set of successor nodes.
  - These hyperarcs are called connectors.

Each k-connector is directed from a parent node to a set of k successor node.

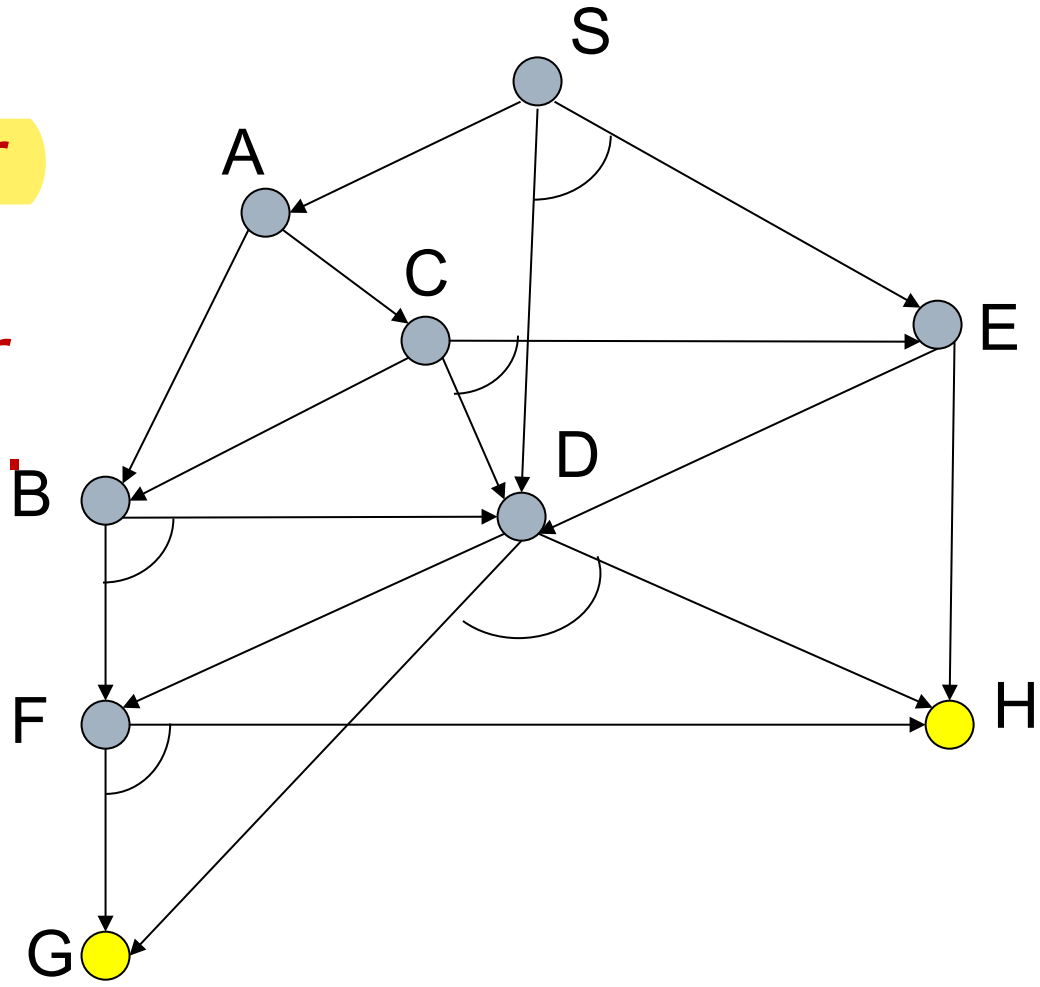


If all of the connectors are 1-connectors, we have the special case of an ordinary graph.

AND-OR Graph

- Figure below shows an example AND-OR Graph.

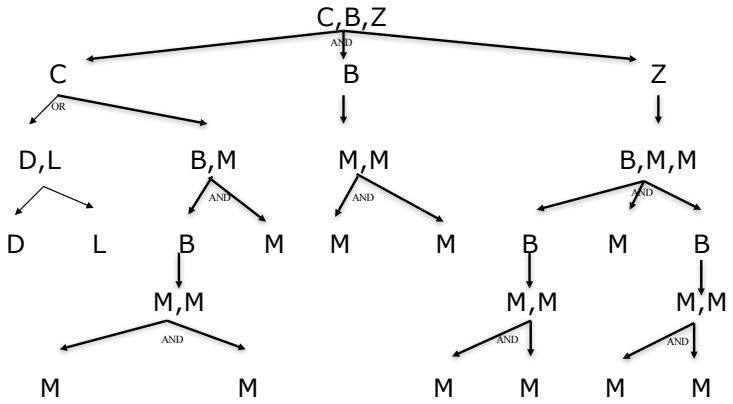
  - Node S has a *one-connector* directed to successor A
  - Node S has a *two-connector* directed to successors (D, E).

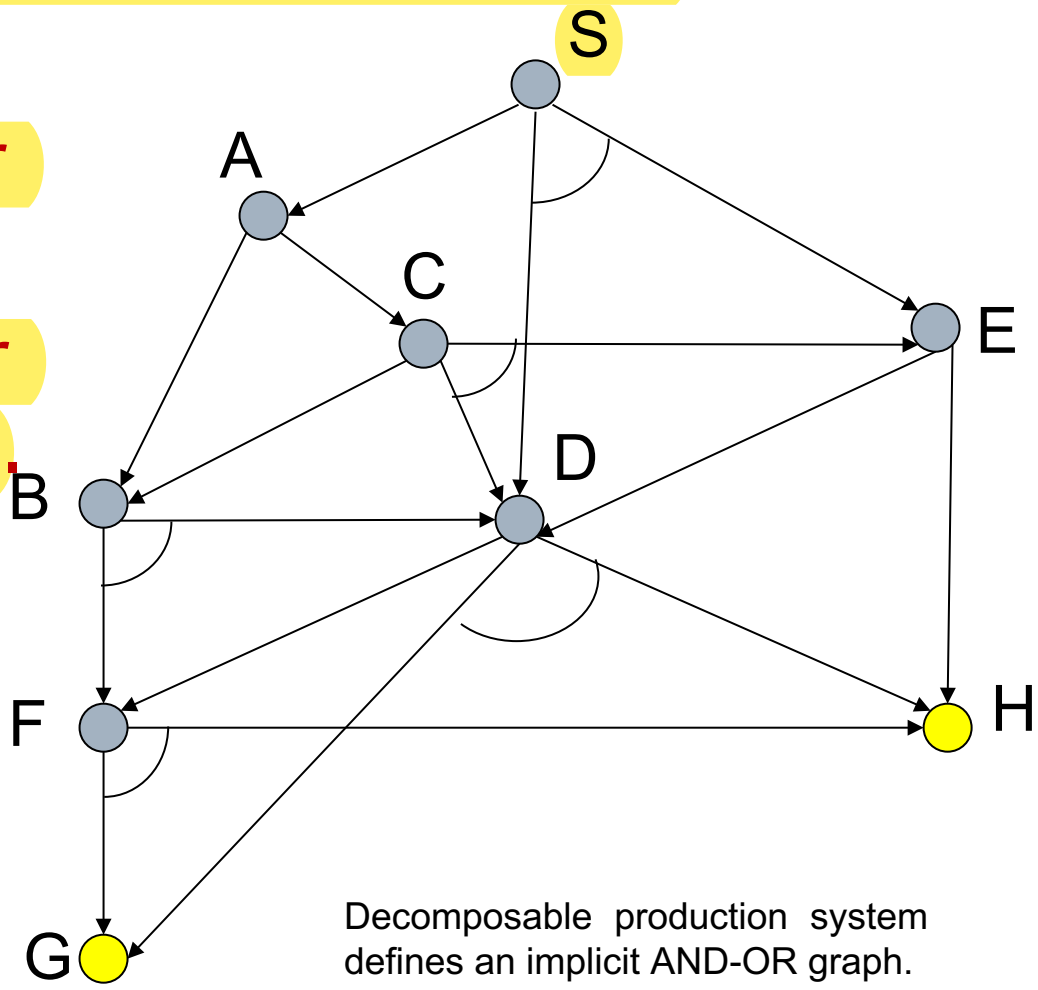


For k> 1; each k-connector is denoted by a curved line joining the arcs from parent to elements of the successor set.

AND-OR Graph

- Initial database corresponds to start node S.
  - Node S has a *one-connector* directed to successor A
  - Node S has a *two-connector* directed to successors (D, E).

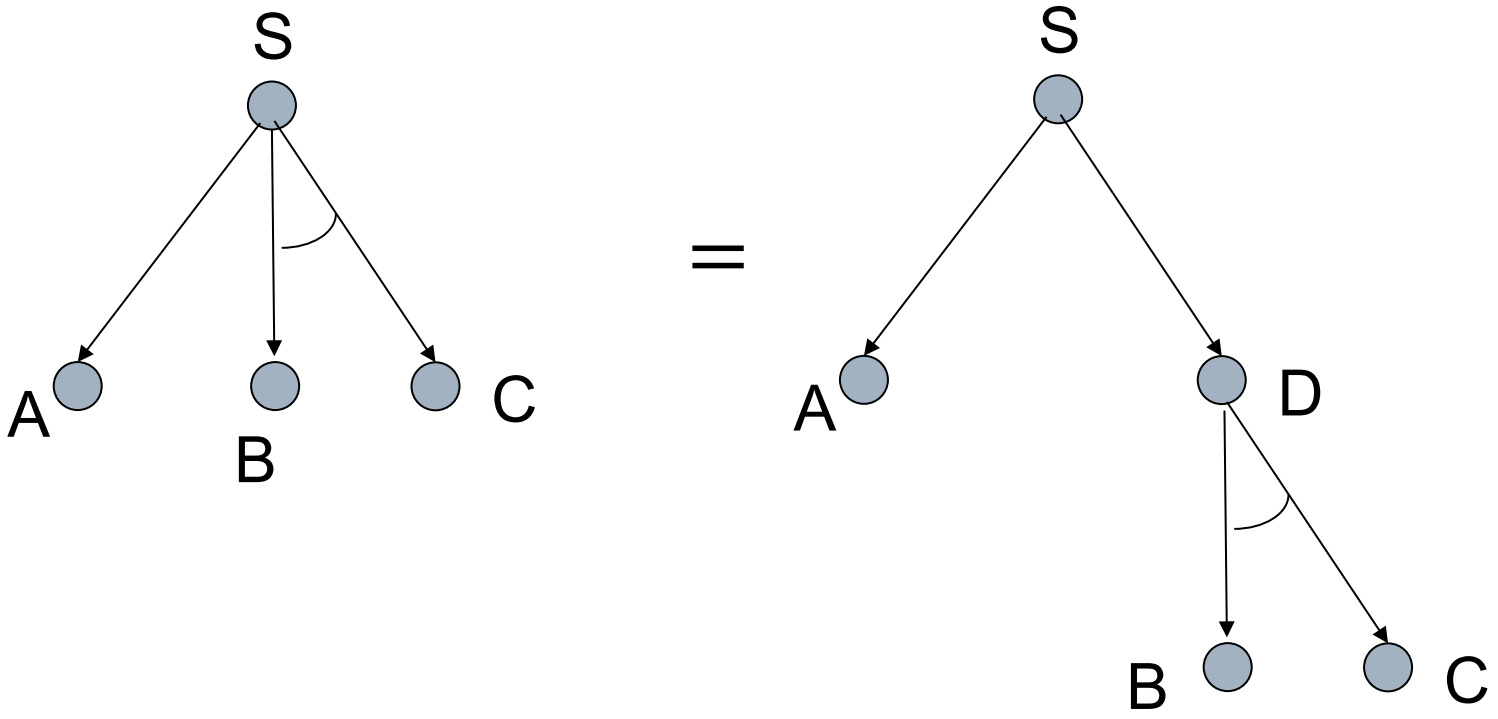




Decomposable production system defines an implicit AND-OR graph.

AND-OR Graph

If required one can always transform the graph on the left to one of the right; of course, keeping track of the cost of the edges.

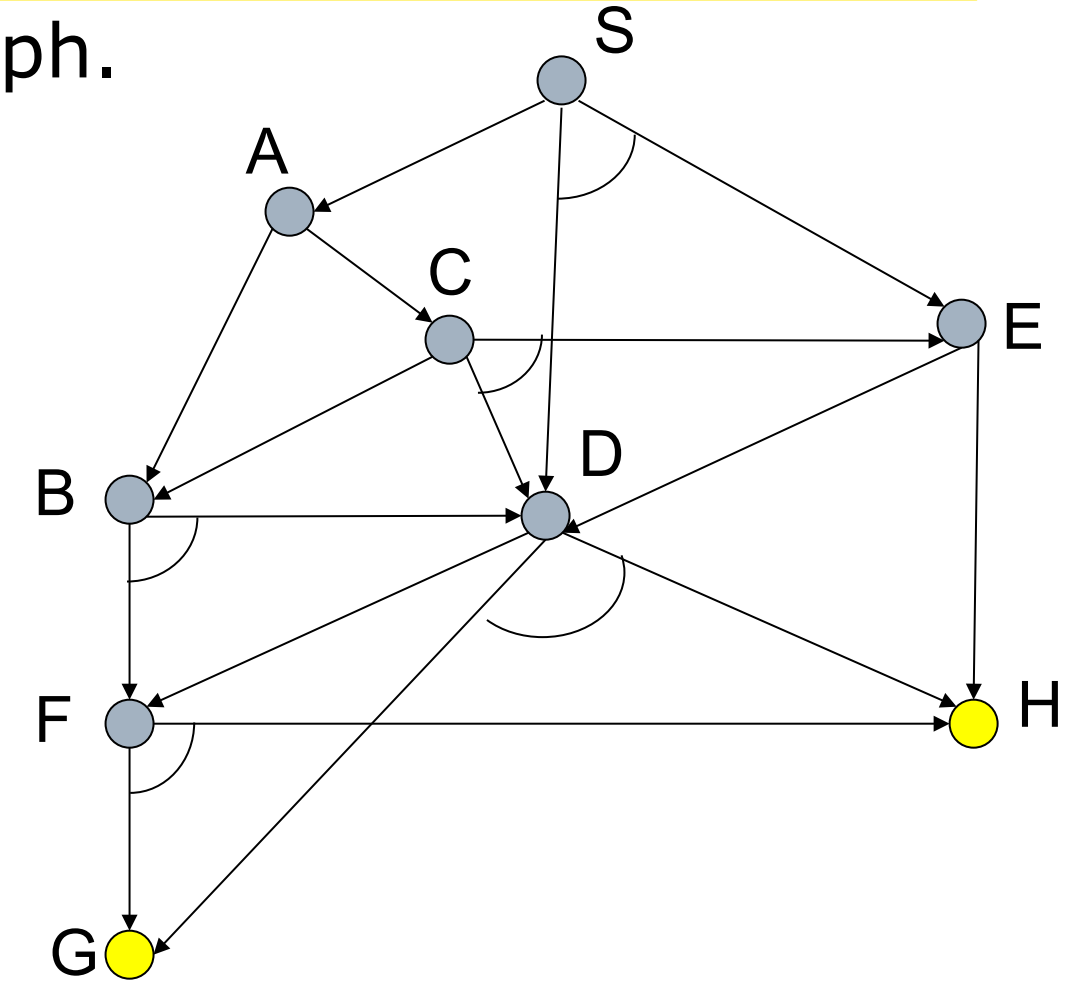


Recall that the AND-OR graphs used in our discussion of decomposable production systems had alternate AND and OR nodes.

Solution Graph

- A solution graph of an AND-OR graph is analogous to a path in an ordinary graph.
  - Solution graph from *n* to *N*

    n = S; *N* = {G, H}
  - Starting from node S; select exactly one connector.
  - From each successor node to which the connector is directed, select an outgoing connector and so on.
  - Eventually every successor is an element of N



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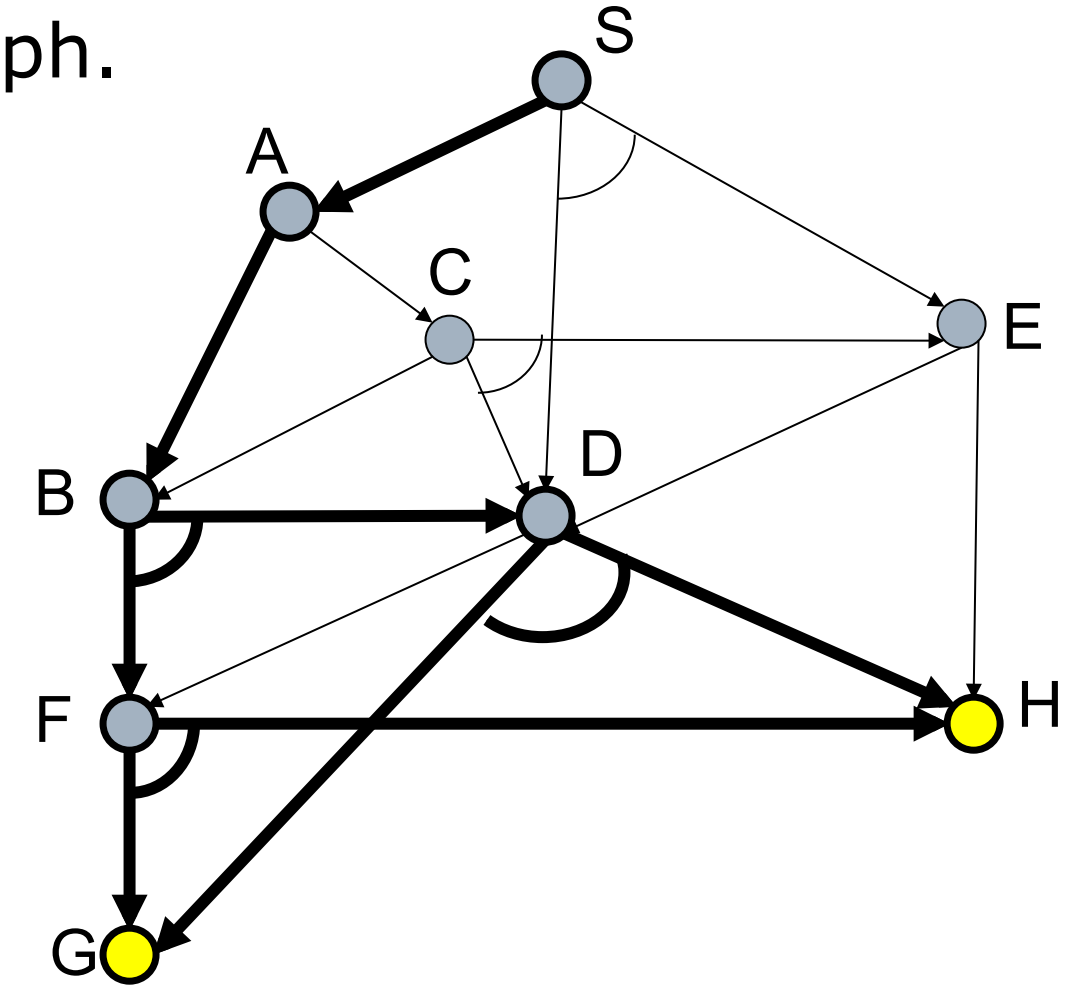

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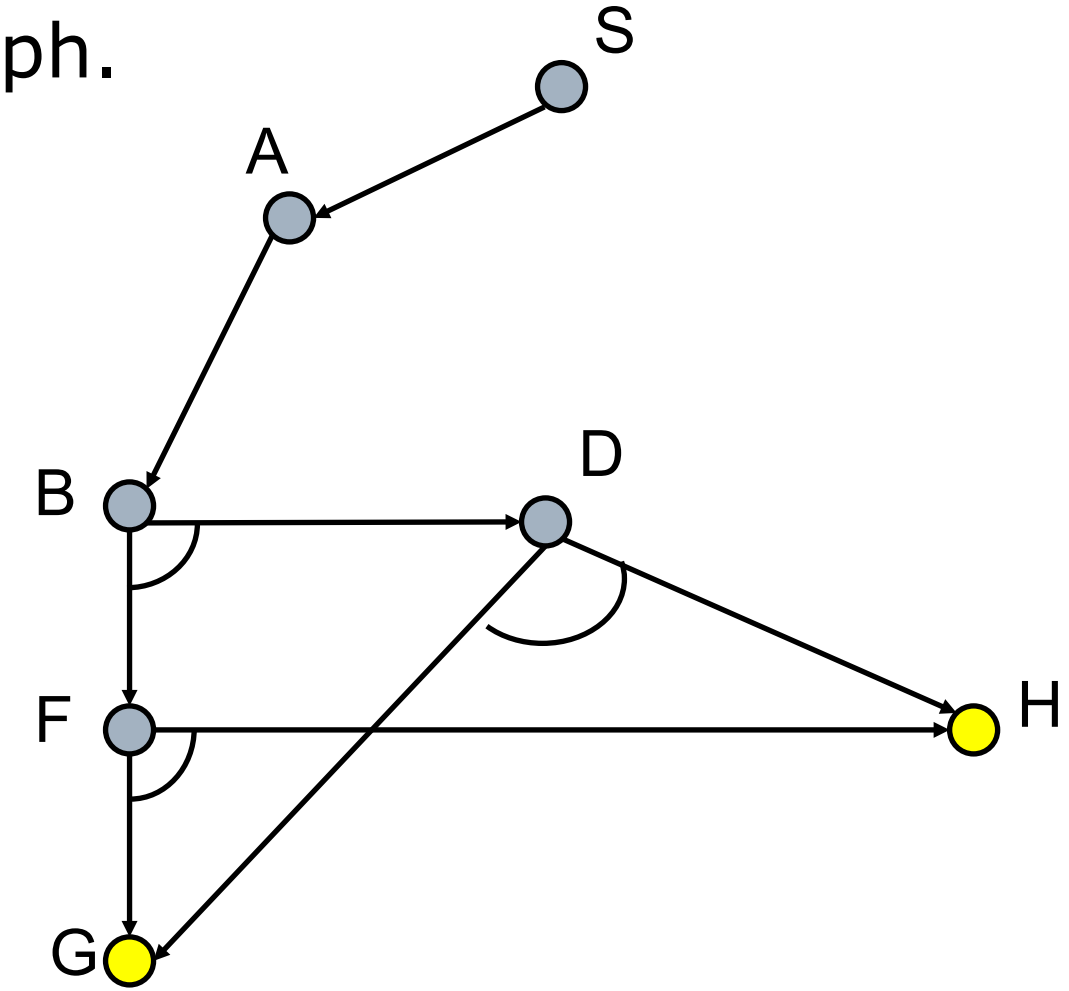

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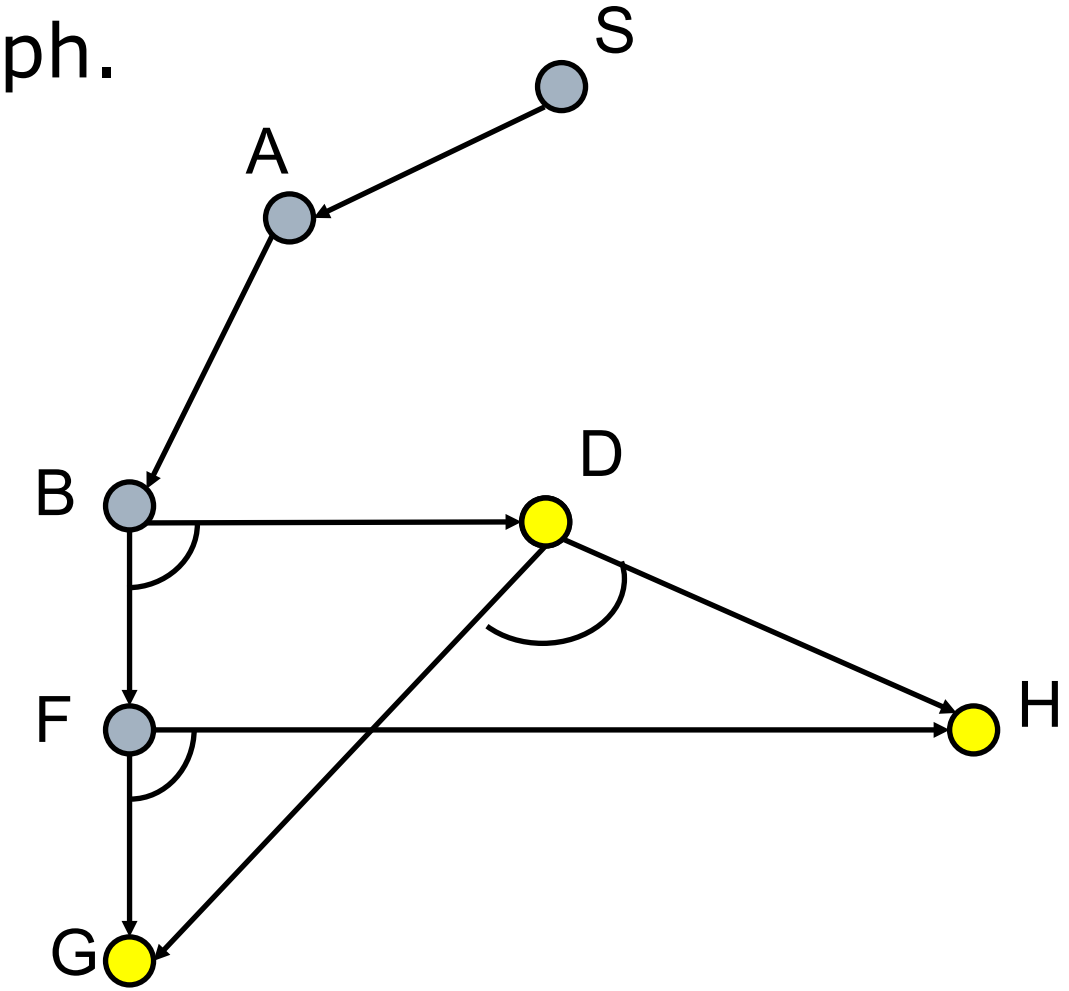

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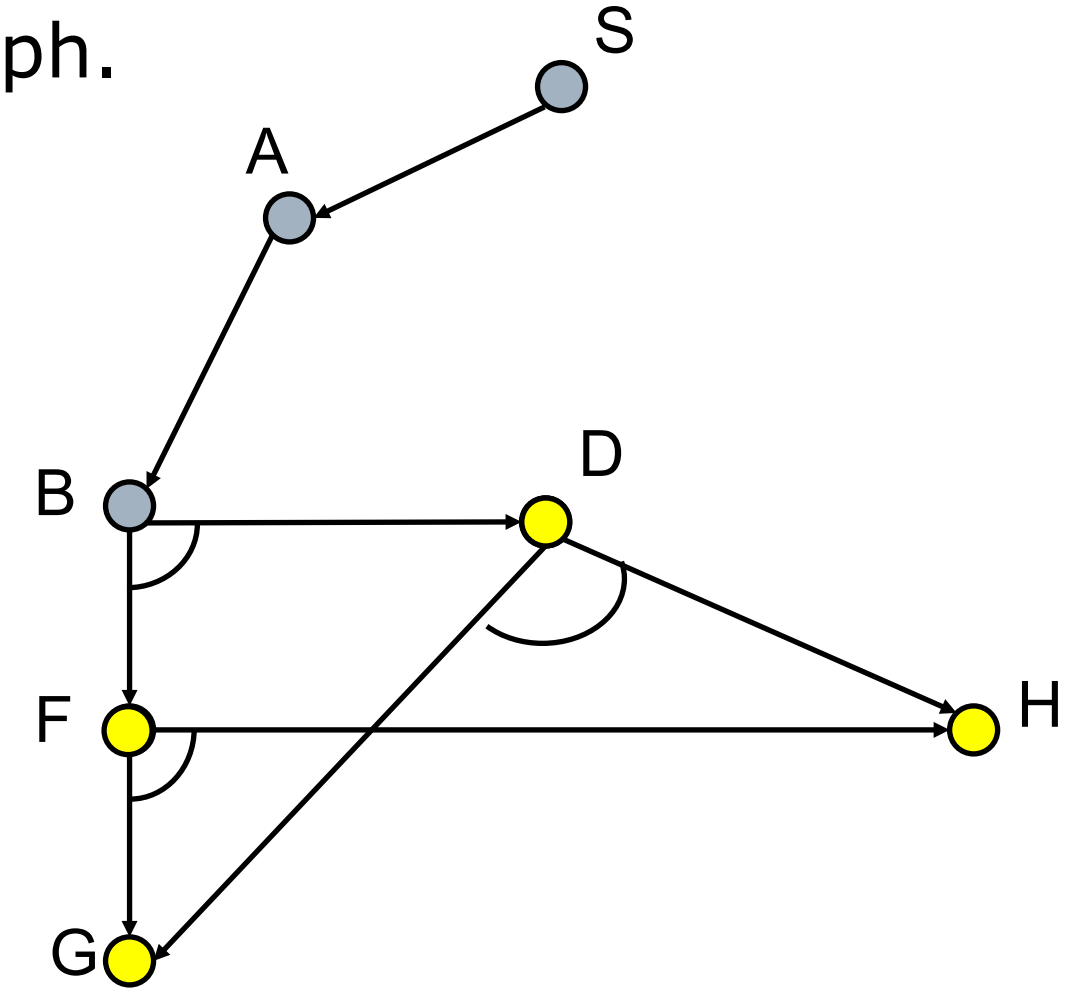

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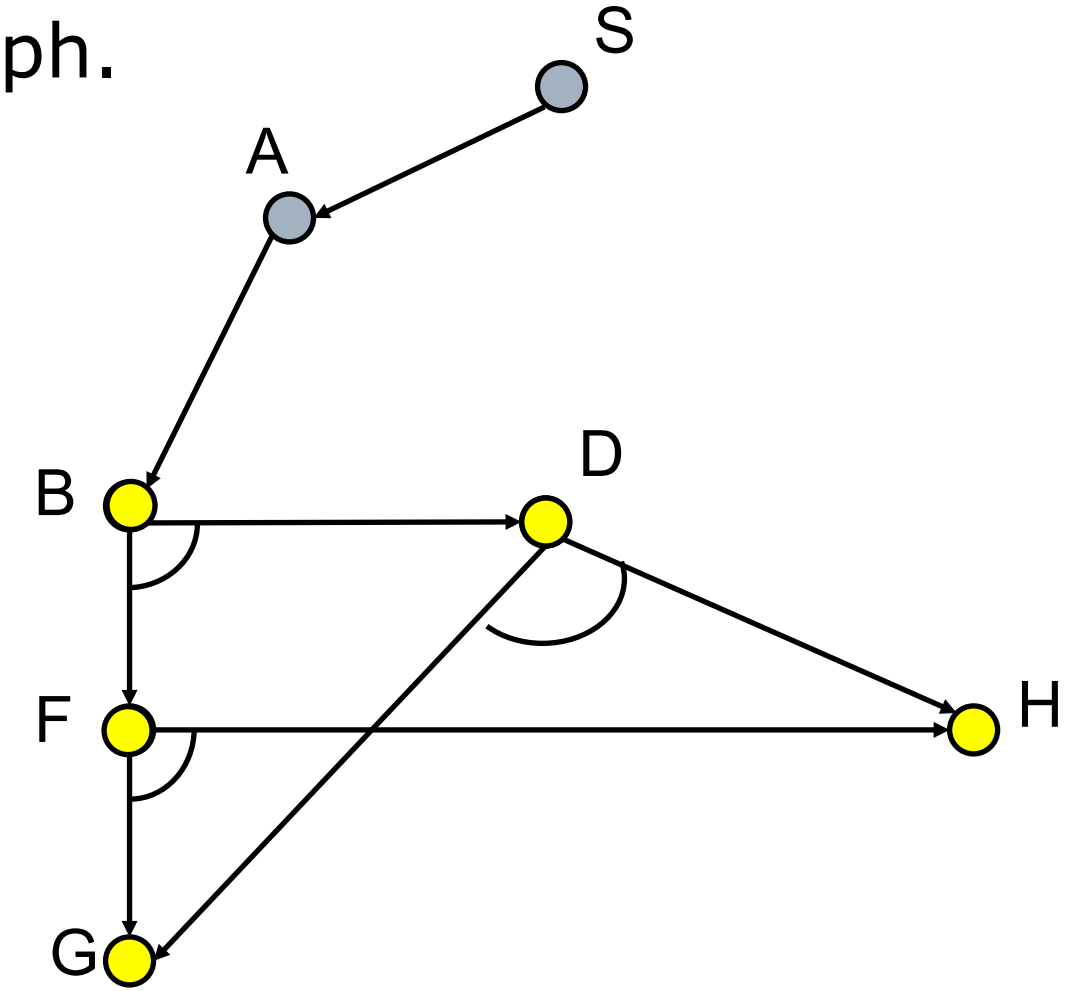

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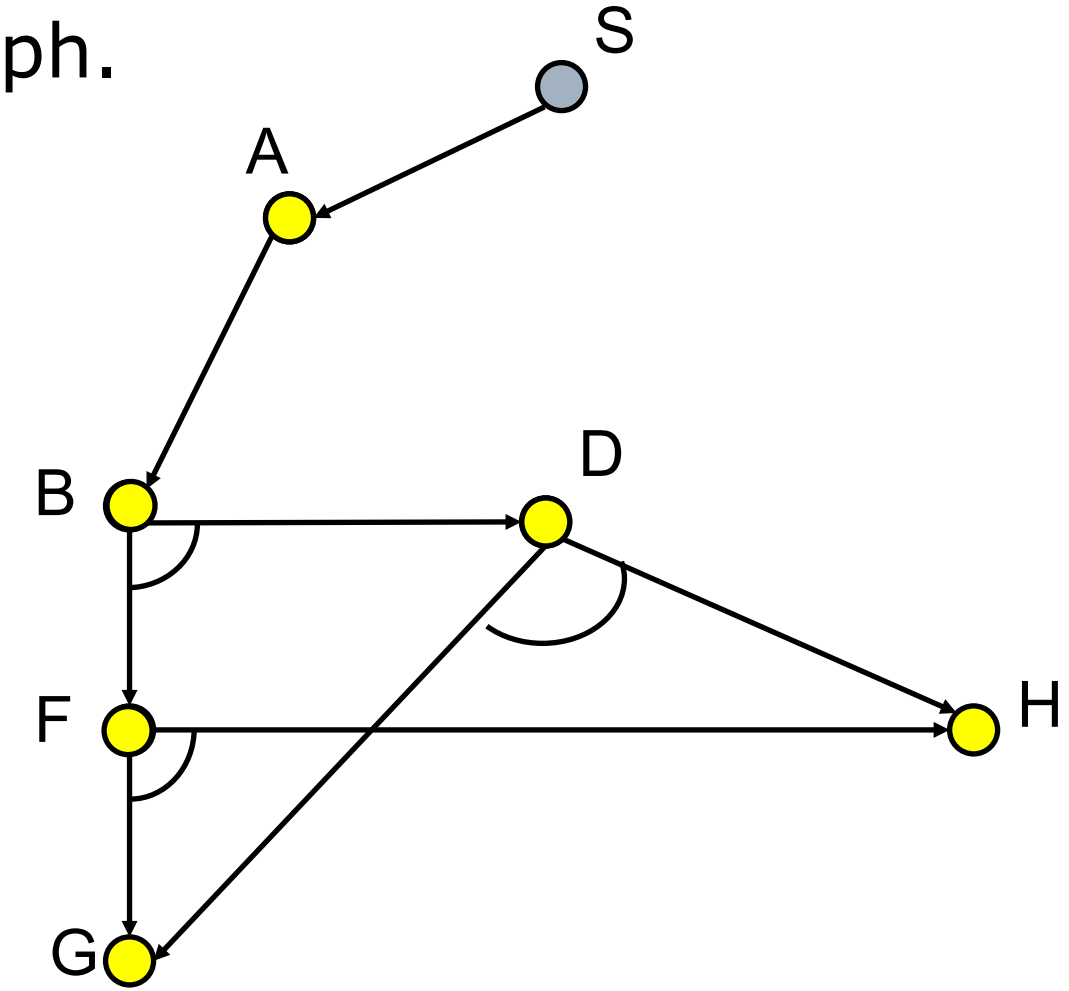

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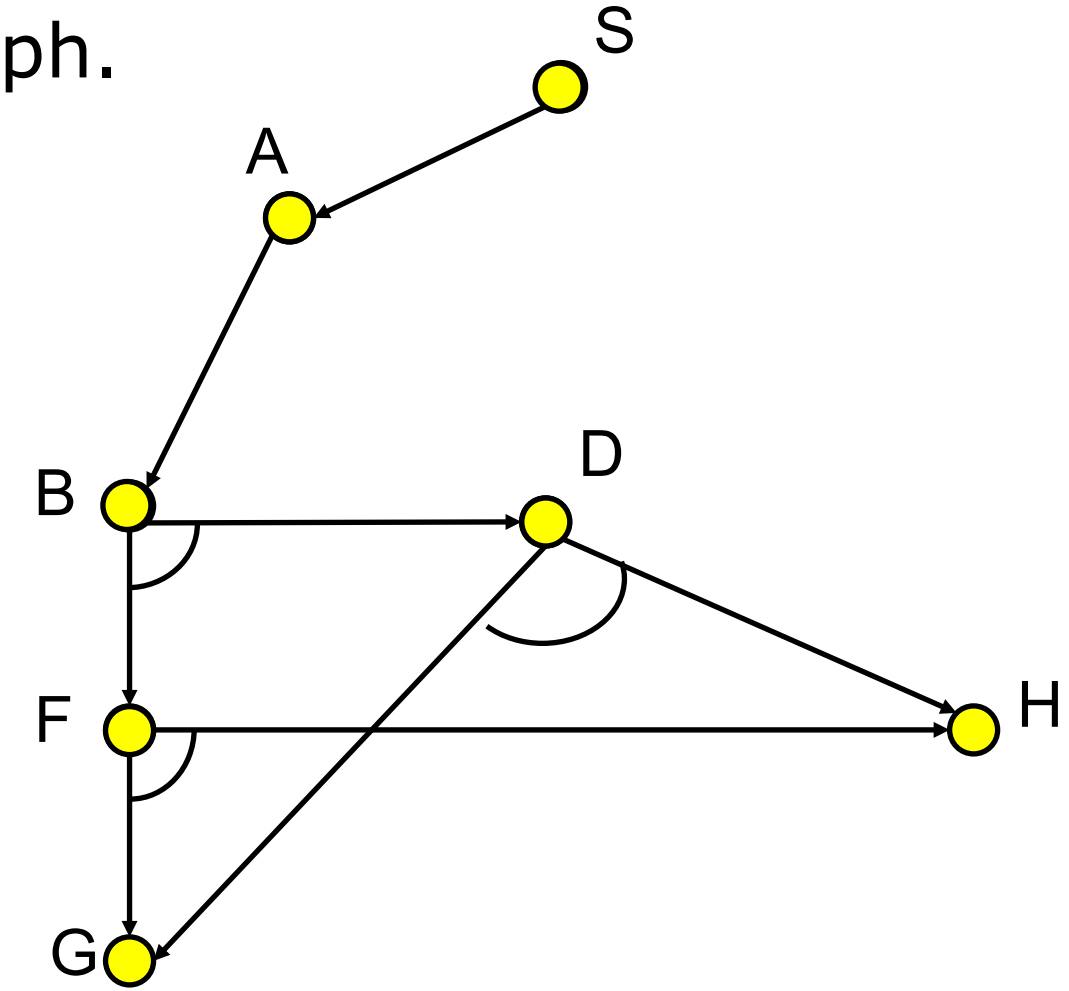

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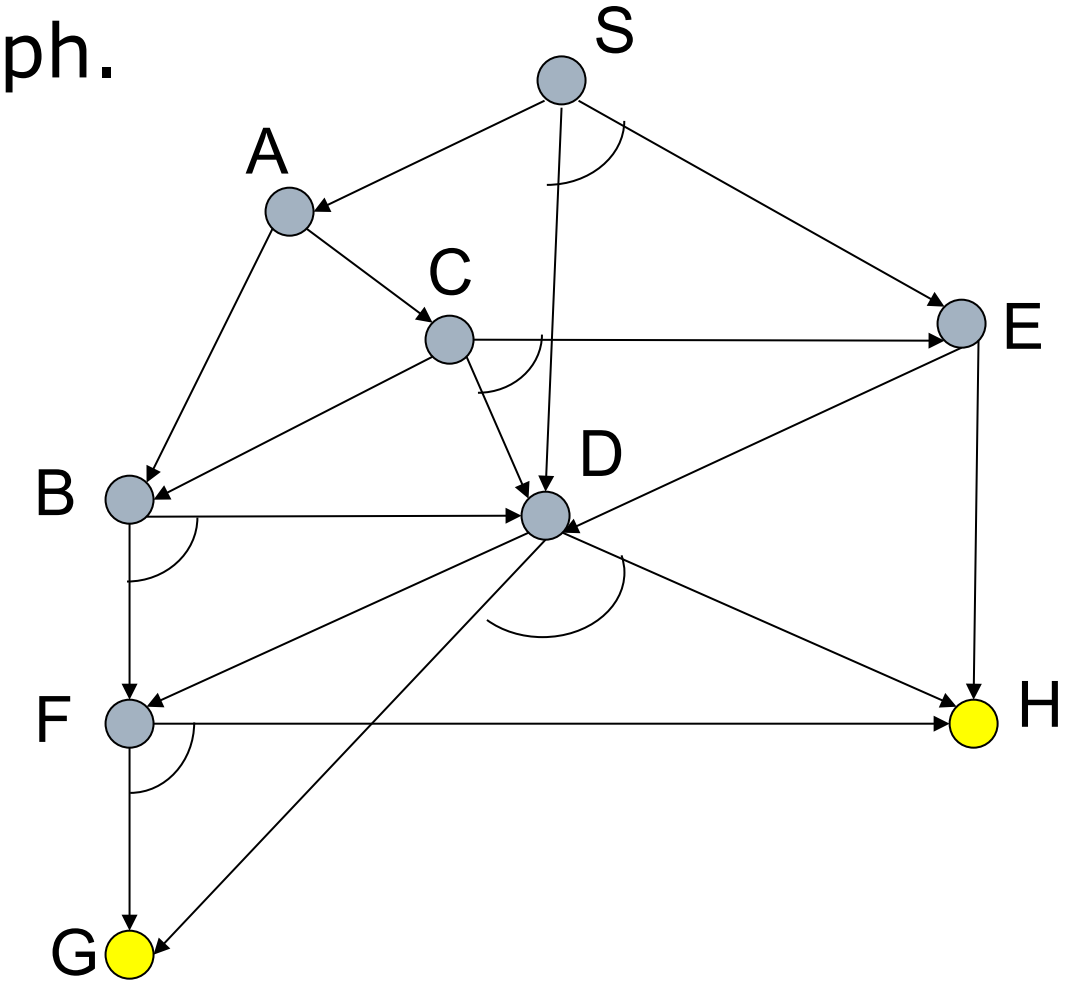

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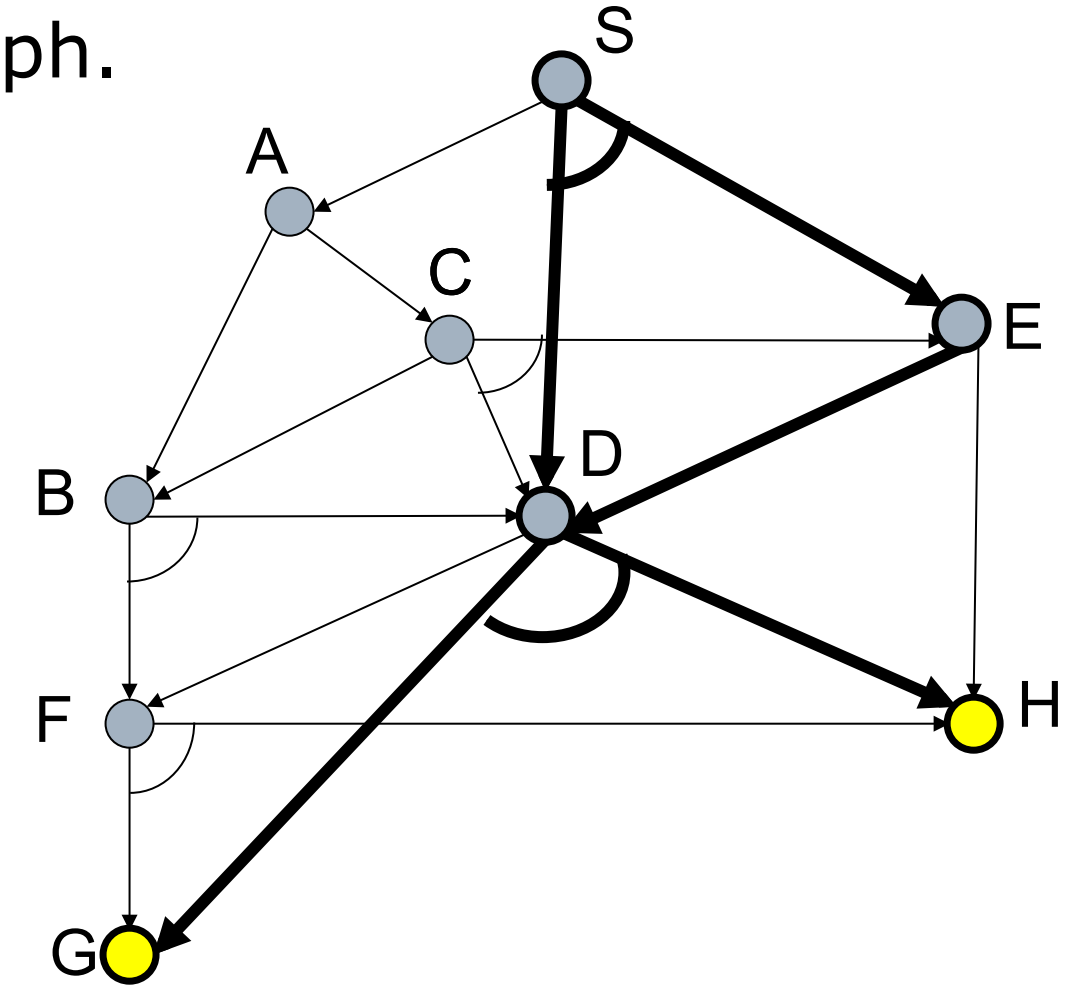

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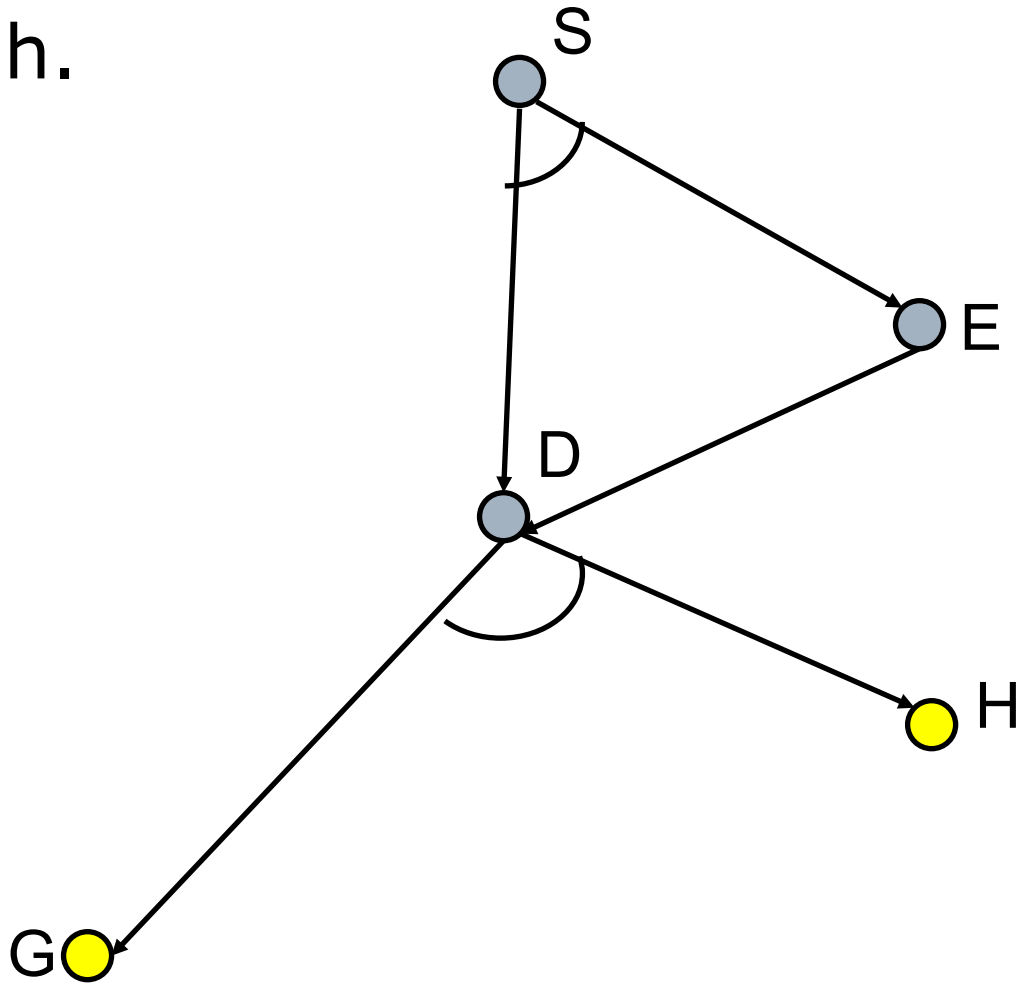

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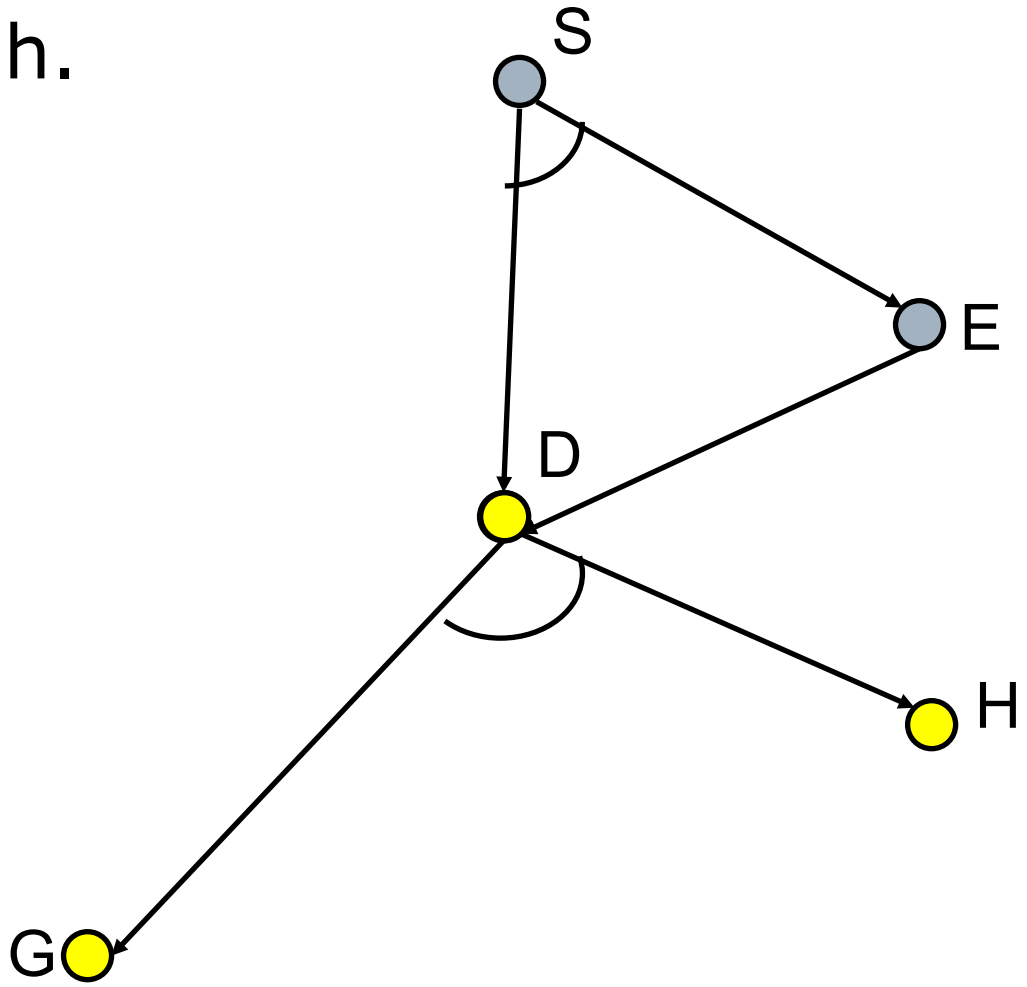

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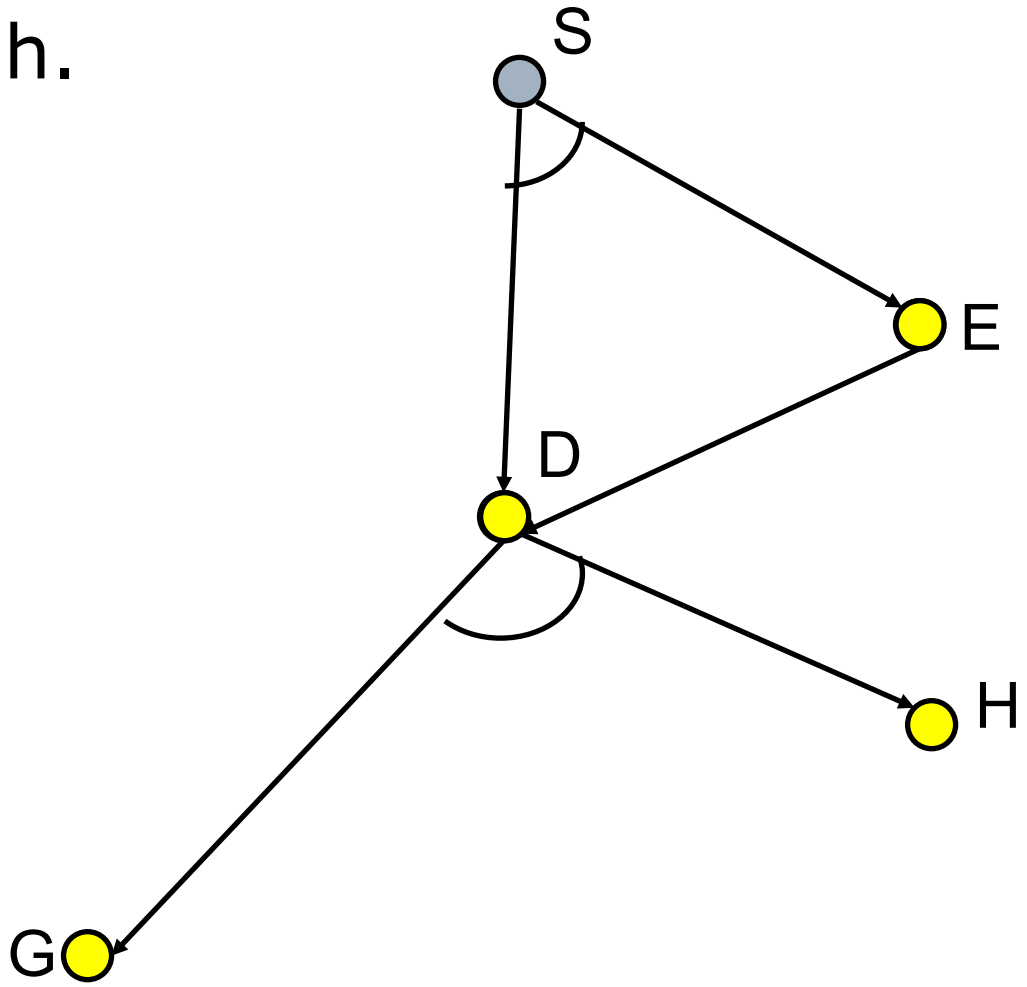

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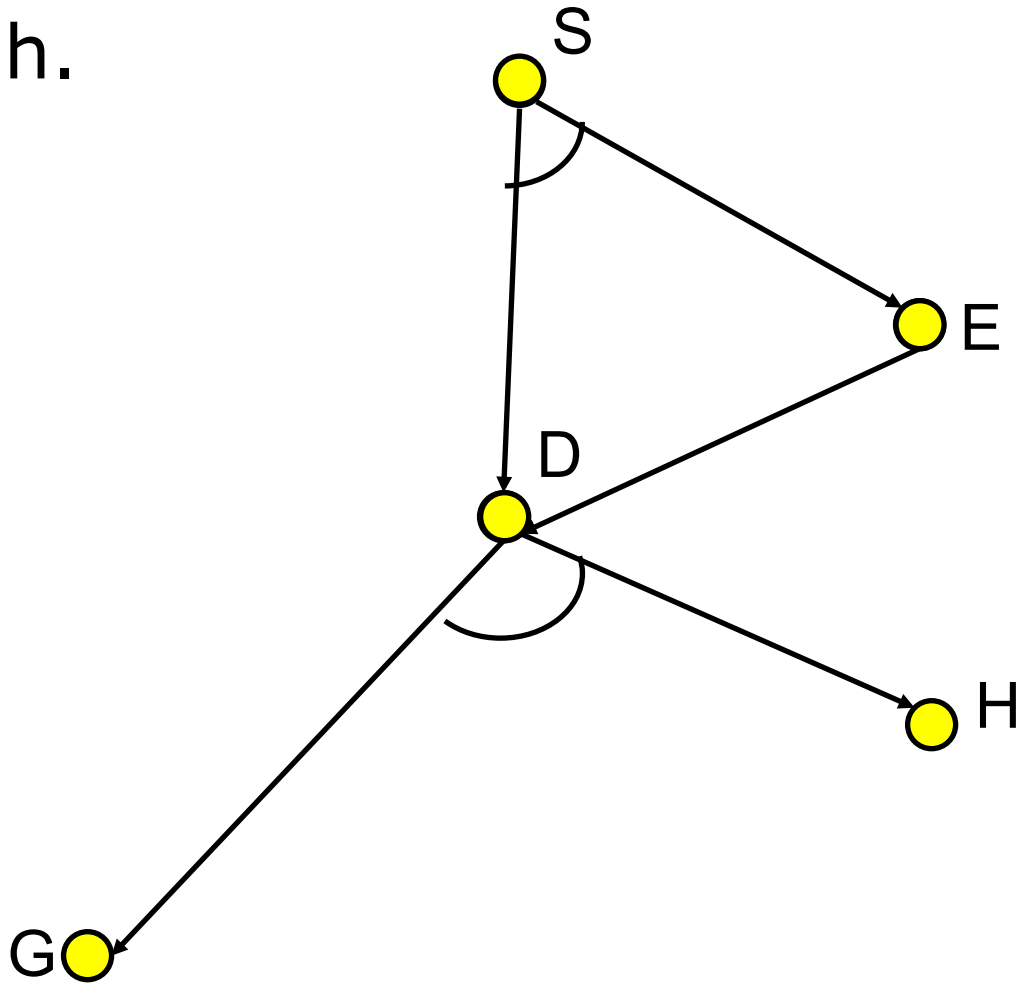
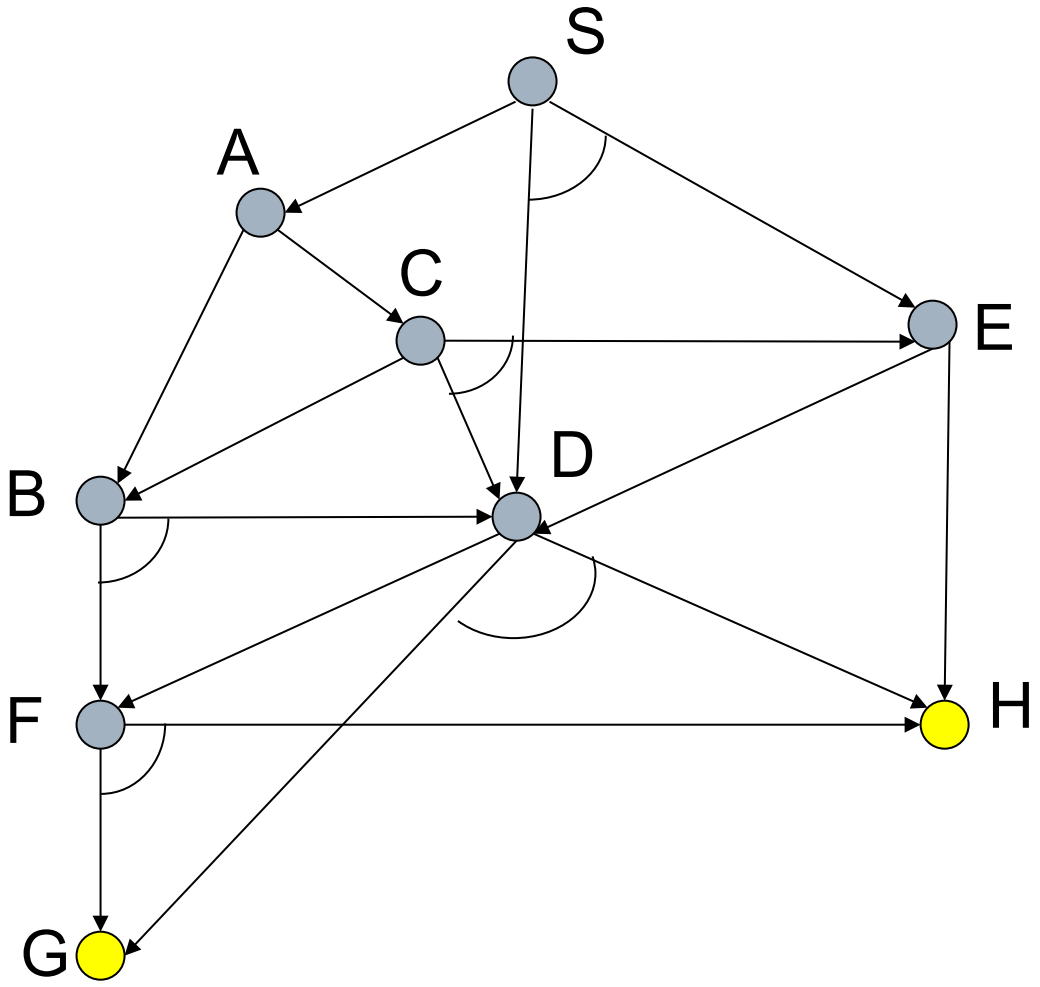


Searching AND-OR Graph

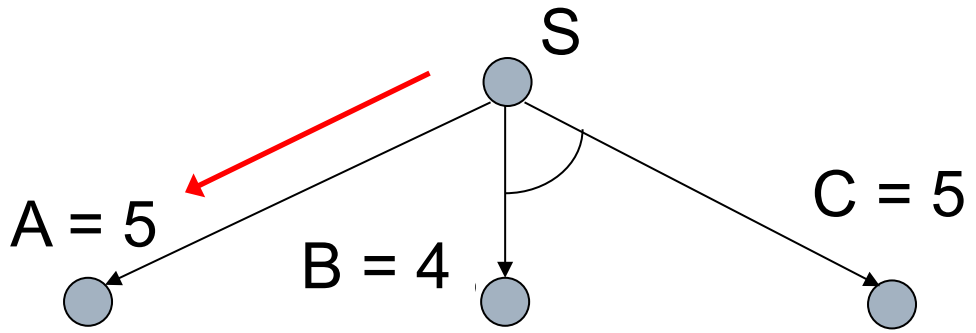
- To find solutions, we need an algorithm similar to Best-First Search
  - But with the ability to handle the AND arcs appropriately.
- Algorithm should find a path from the start node to a set of nodes representing the solution states.
  - May be required to reach to more than one solution state; Each arm of an AND arc must lead to a solution node.



Best-First Algorithm ?

- To find solution in an AND-OR graph we need an algorithm similar to best-first search; but with the ability to handle the AND arc appropriately.

Edge cost = 1 unit.
Best-first Search ? Node B



Involving B = 4 + 5 + 2 = 11
Path S-A = 5 +1 = 6 ; Better

B is part of an AND arc;
If we choose to use B, we must also use C.

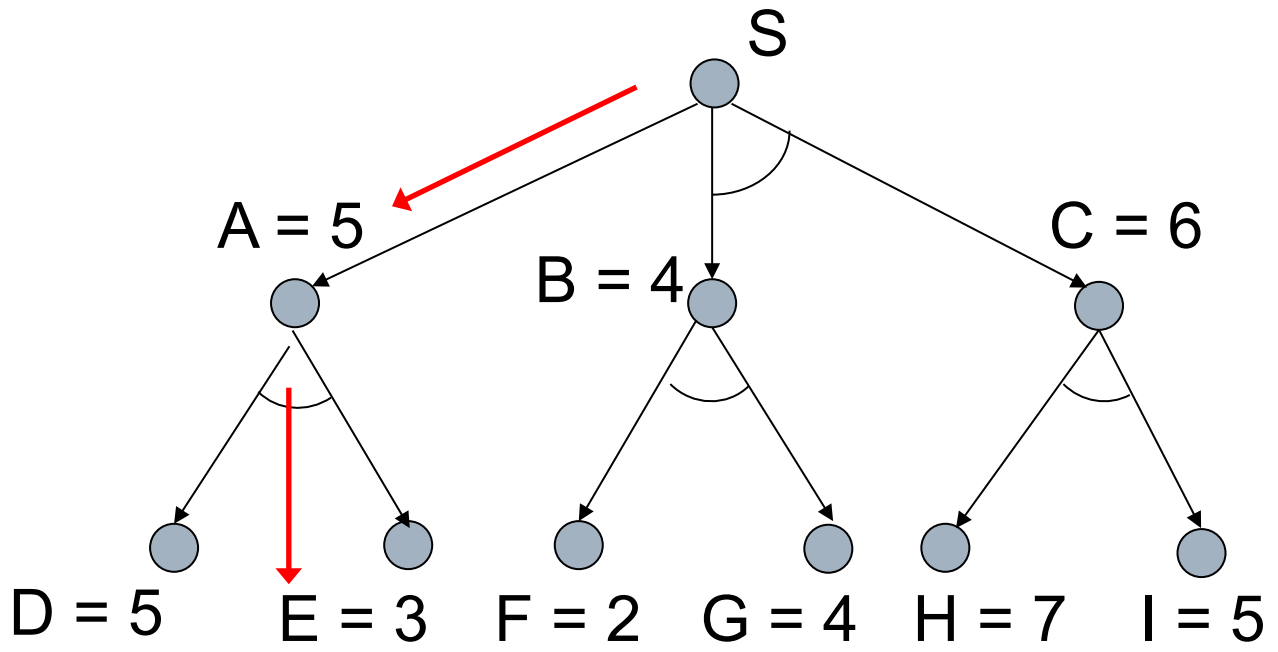
Assumption: Each k-connector has a cost of 1 for each of its k successors.
AND arc with TWO successors have a cost of 2.

Best-First Algorithm ?

- To find solution in an AND-OR graph we need an algorithm similar to best-first search; but with the ability to handle the AND arc appropriately.

Edge cost = 1 unit.
Best-first Search ? Edge B-FG

Involving B-FG; Also C-HI
Path S-A; A-DE Better



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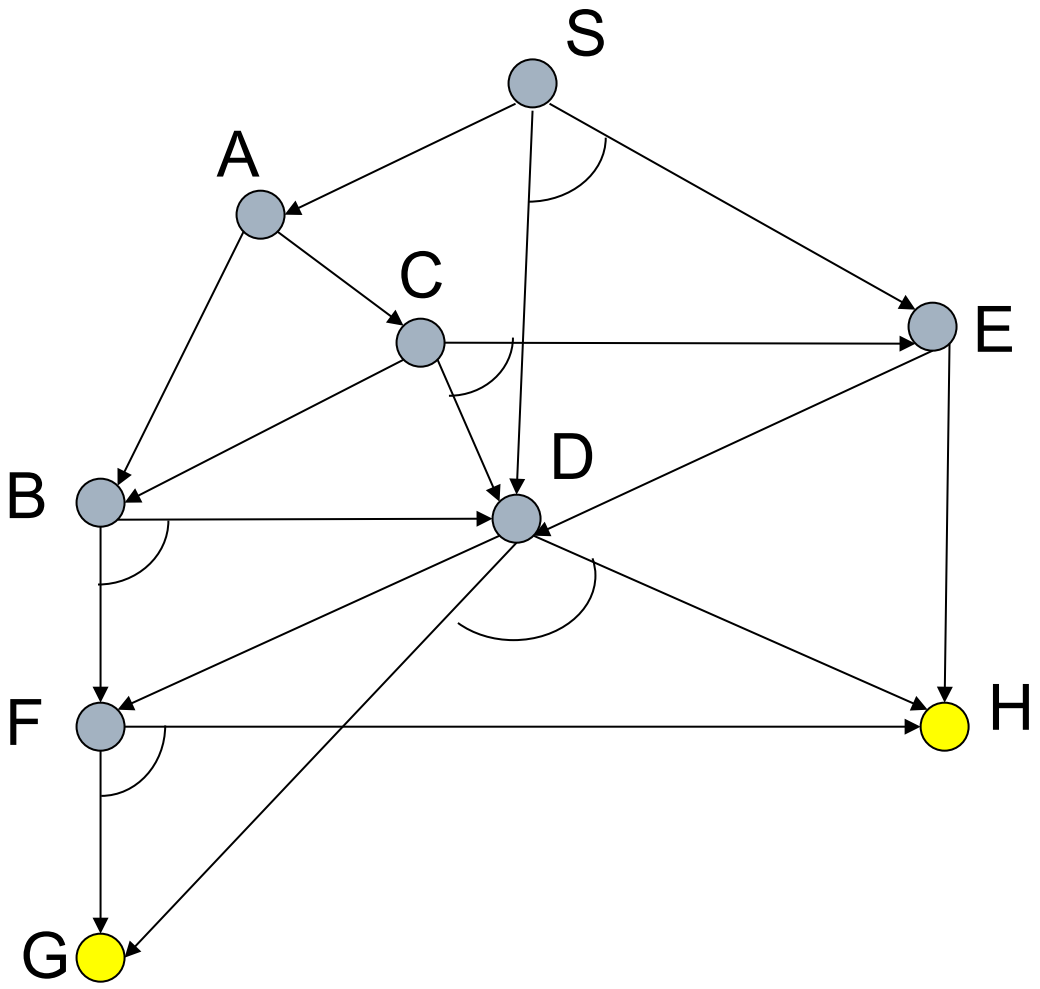


AO*: A Heuristic Search Procedure

Use a heuristic function h(n); estimate of the cost of an optimal solution from node n to set of terminal nodes.

Each node will have an associated h value; serve as measure of goodness of the node.

Rather than two lists – OPEN and CLOSED - used for A* algorithm, the AO* algorithm use a single structure - a graph - representing part of the search graph that has been explicitly generated so far.

Two major operations:

1. Top-down, graph-growing
2. Bottom-up, cost-revising, connnector-marking, SOLVE-labelling.

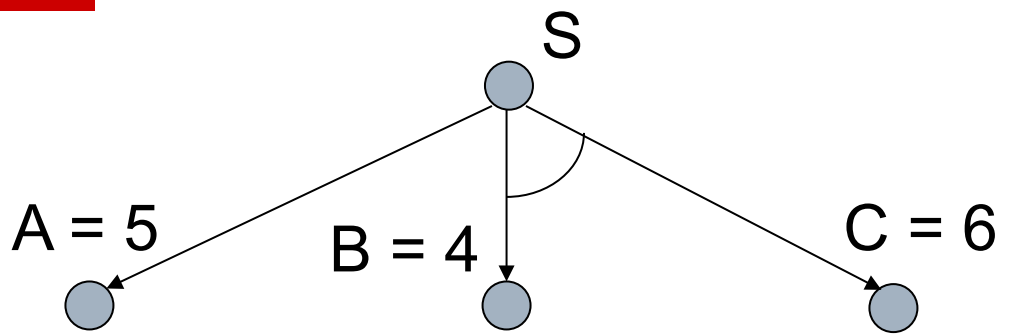
Not store a value for the cost to the current node; such a value is not necessary because of the top down traversal of the best-known path.

AO*: A Heuristic Search Procedure

After graph G is initialized to S

Follow which hyperedge?

Alternatives – Path S-A; Path S-BC



Edge cost = 1 unit.

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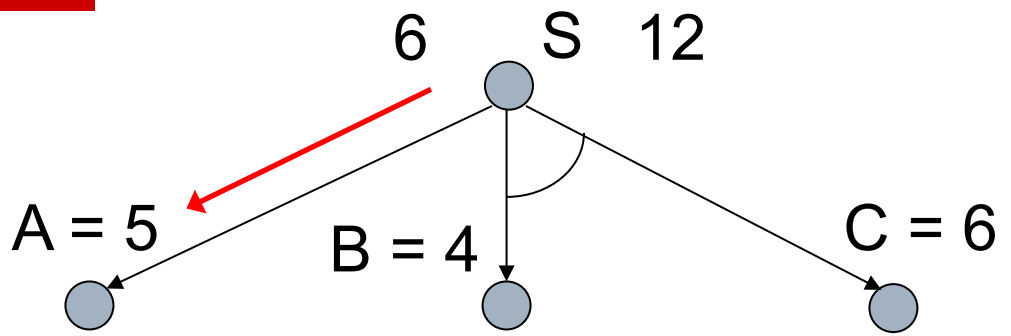


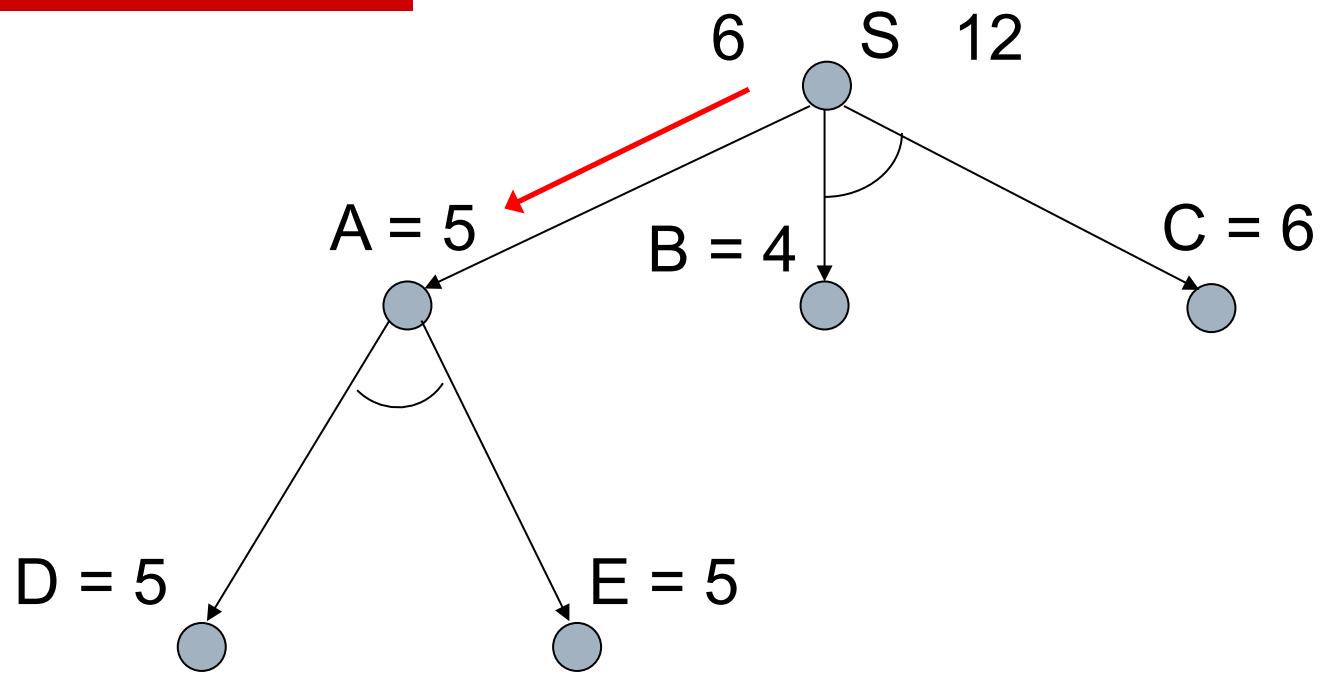
Edge cost = 1 unit.

AO*: A Heuristic Search Procedure

After graph A is expanded;
Nodes – D and E

How promising?

Evaluate h values – back-up



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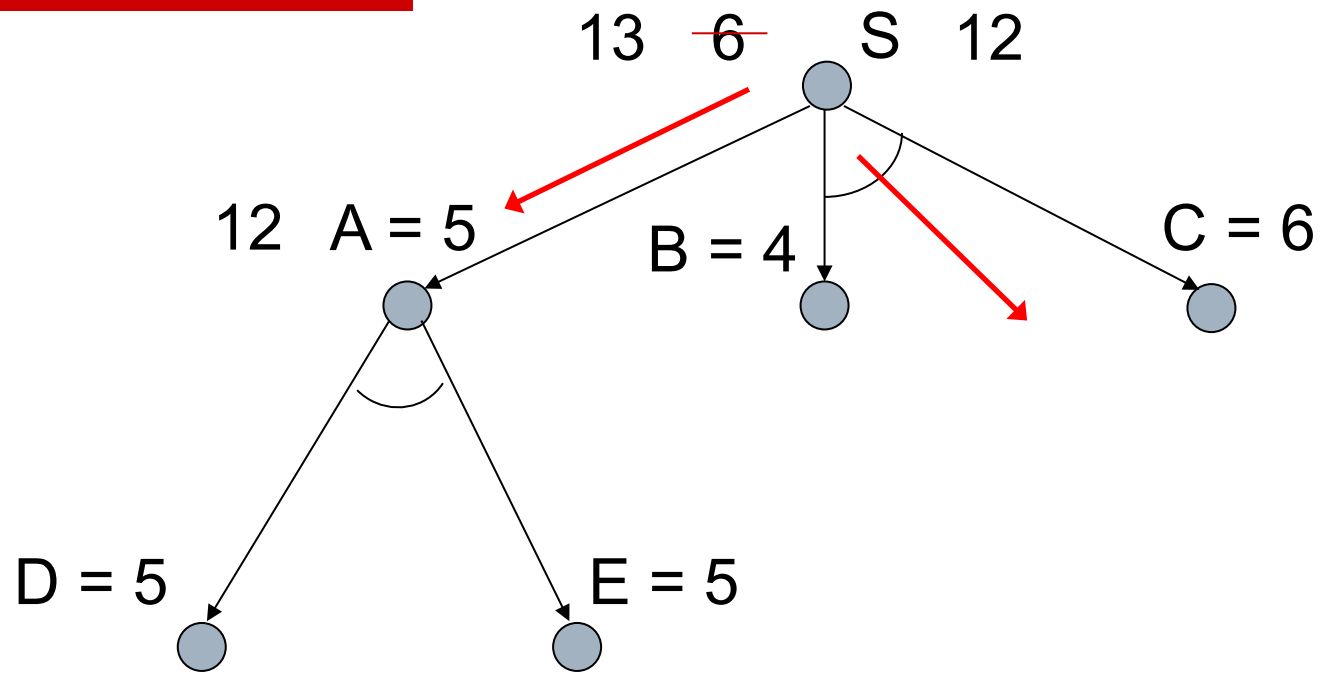


Edge cost = 1 unit.

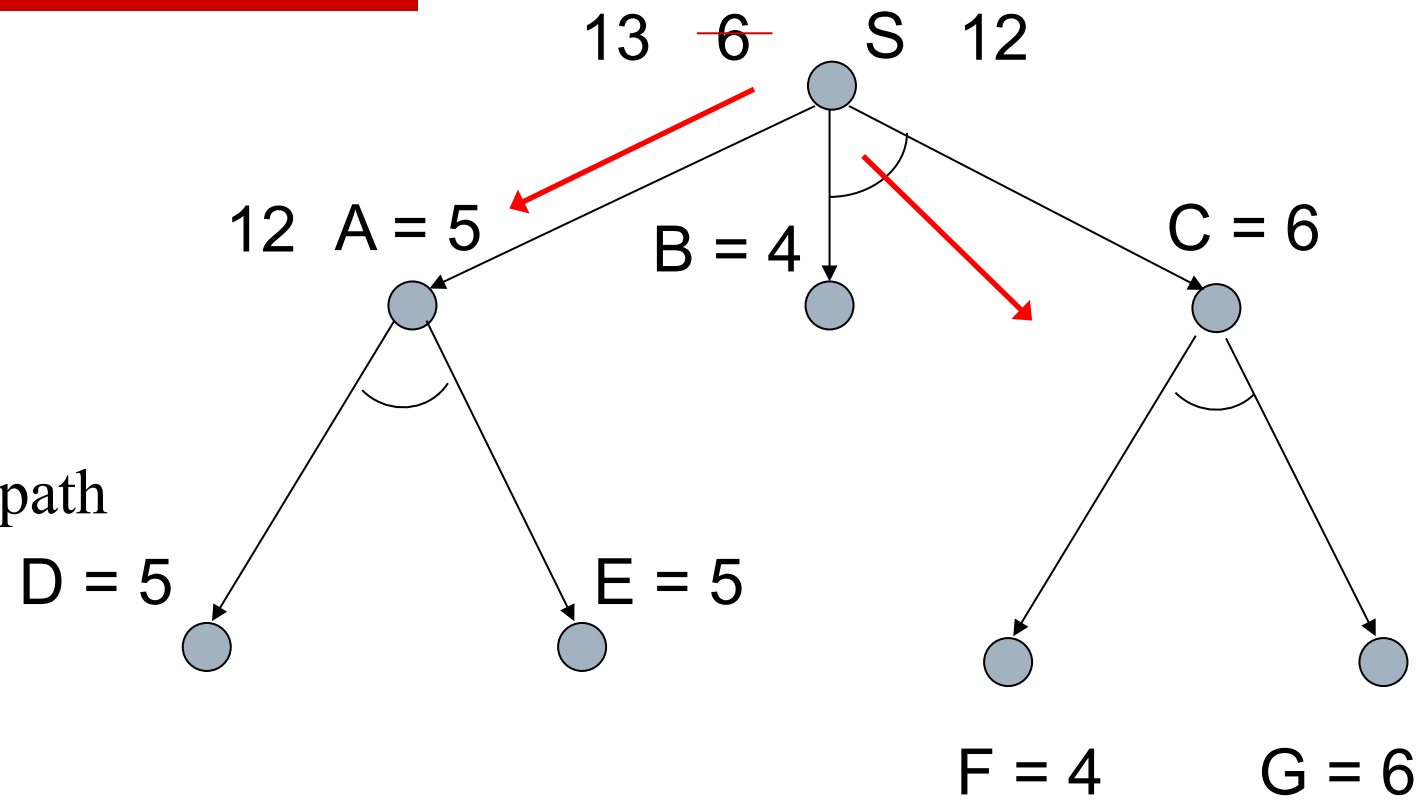
AO*: A Heuristic Search Procedure

More promising?

After node C is expanded;
Nodes – F and G

How promising?

Evaluate h values – revise on path



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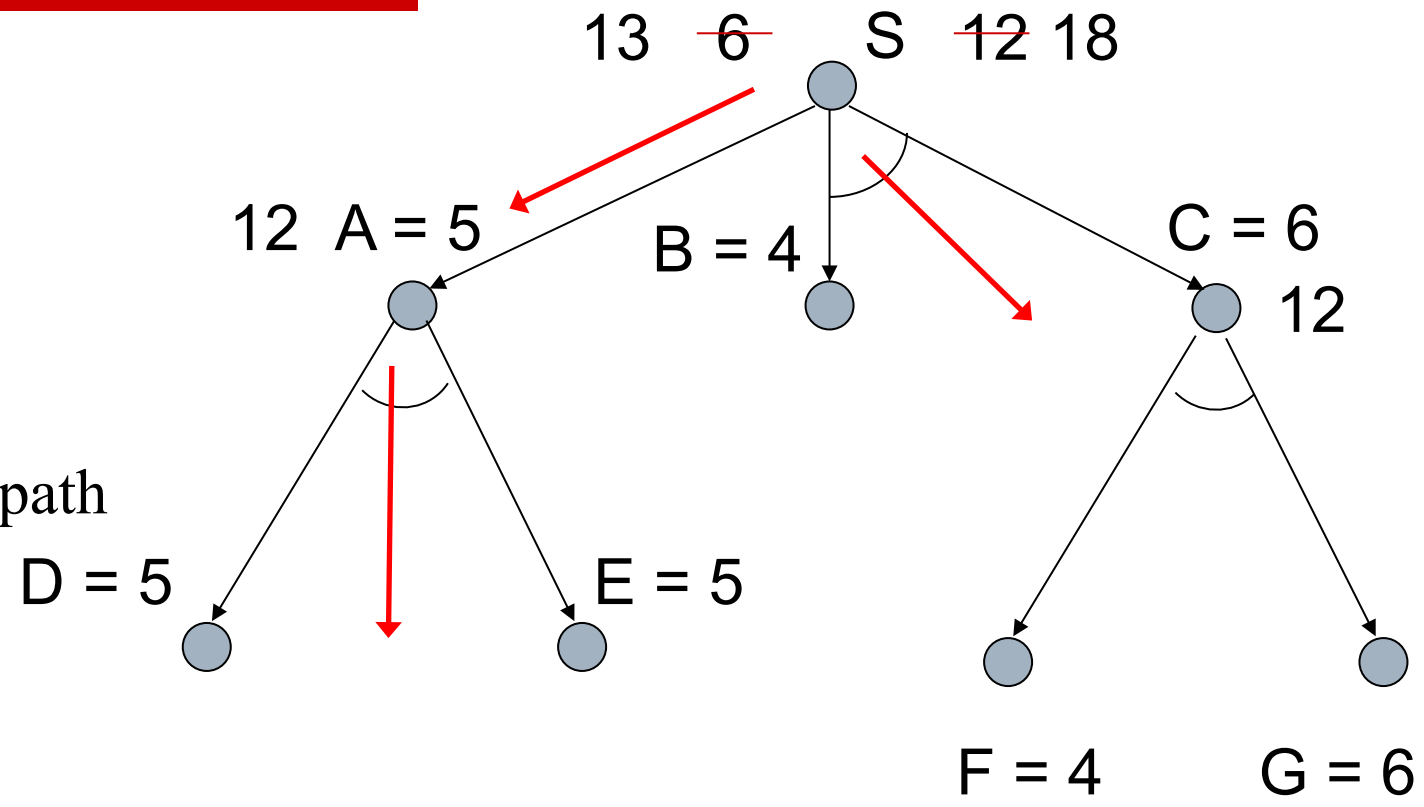


Nodes – D and E

How promising?

Evaluate h values – back-up

Edge cost = 1 unit.

Basic Idea of AO*

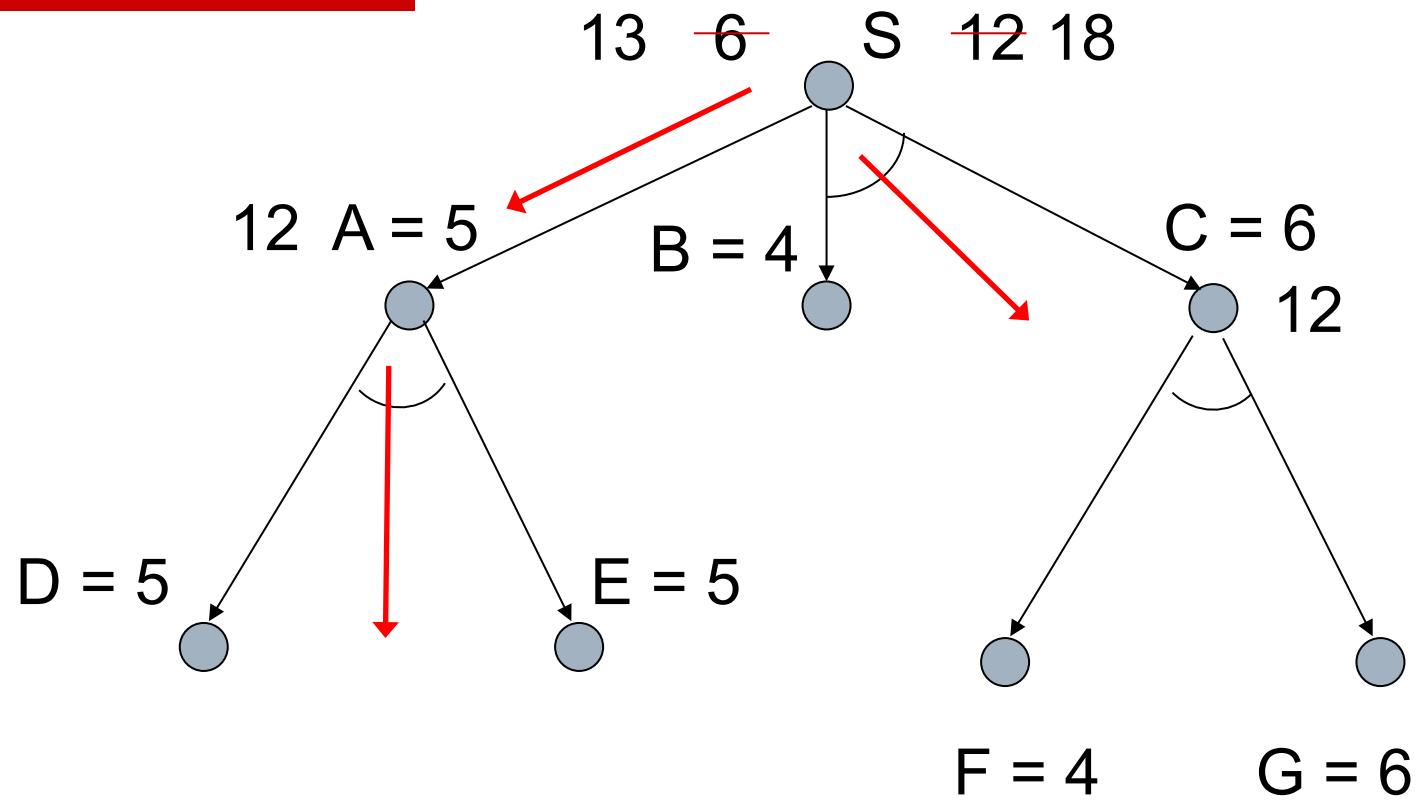
- Top-down graph growing
  - Picks out best available partial solution sub-graph from explicit graph by tracing down *marked* connectors.
    - Here *marked* – indicates the current best partial solution graph from each node in the search graph.
  - One of the nonterminal leaf nodes of this best partial solution graph is expanded.
    - cost is assigned to its successors.

AO*: A Heuristic Search Procedure

Node E is expanded;
Nodes – D and E

How promising?

Evaluate h values – back-up



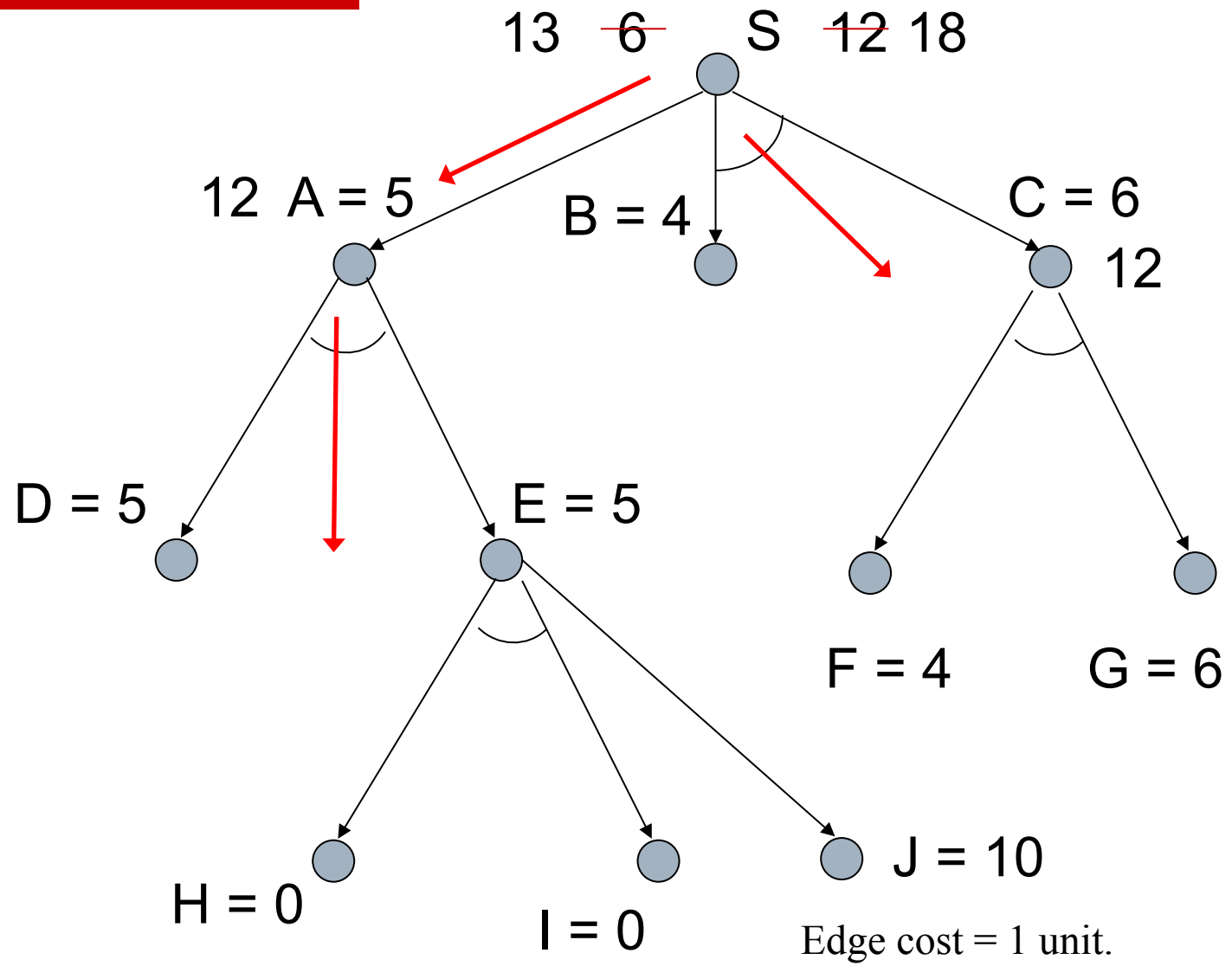
Edge cost = 1 unit.

AO*: A Heuristic Search Procedure

Node E is expanded;
Nodes – H-I; J

How promising?

Evaluate h values – back-up



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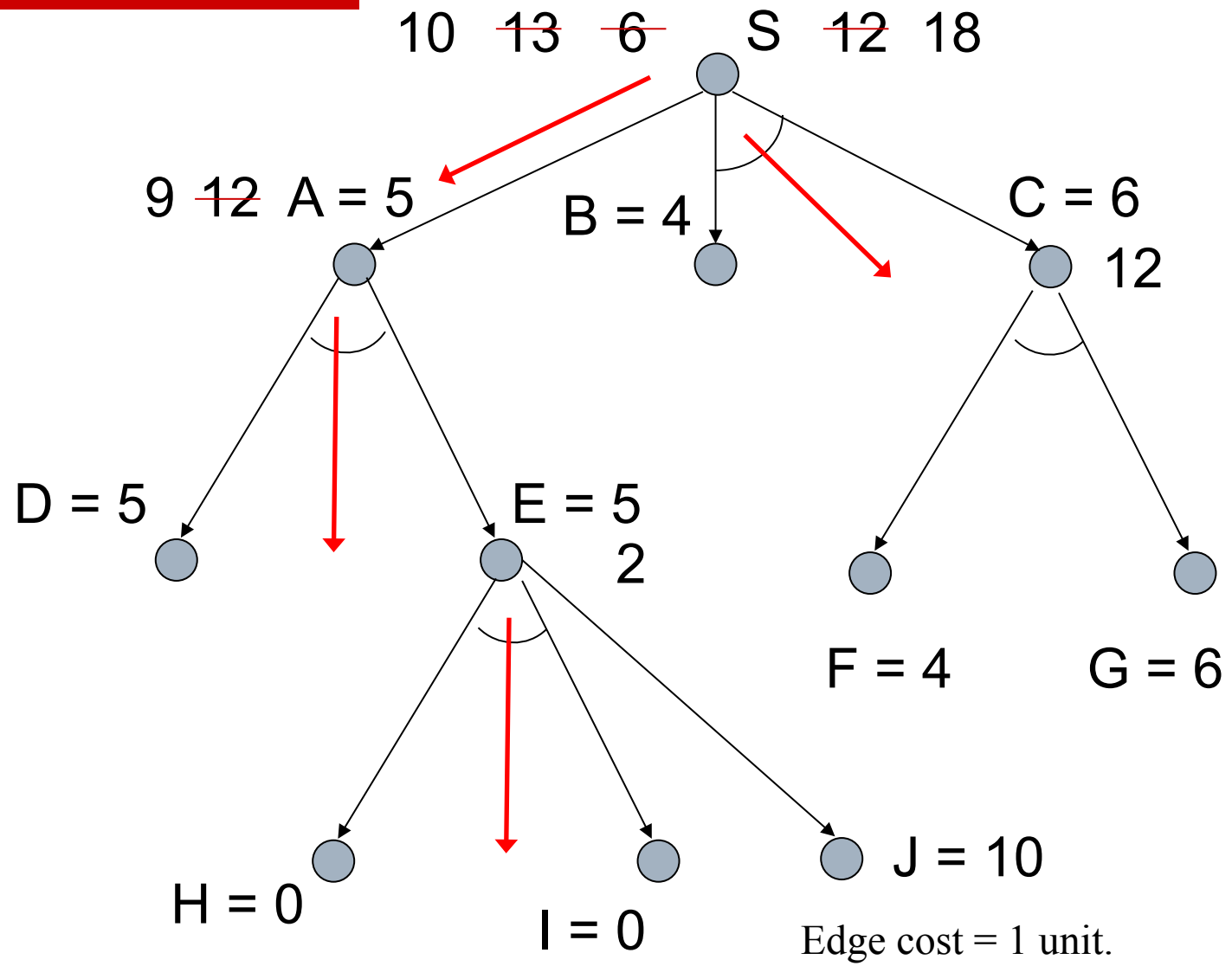

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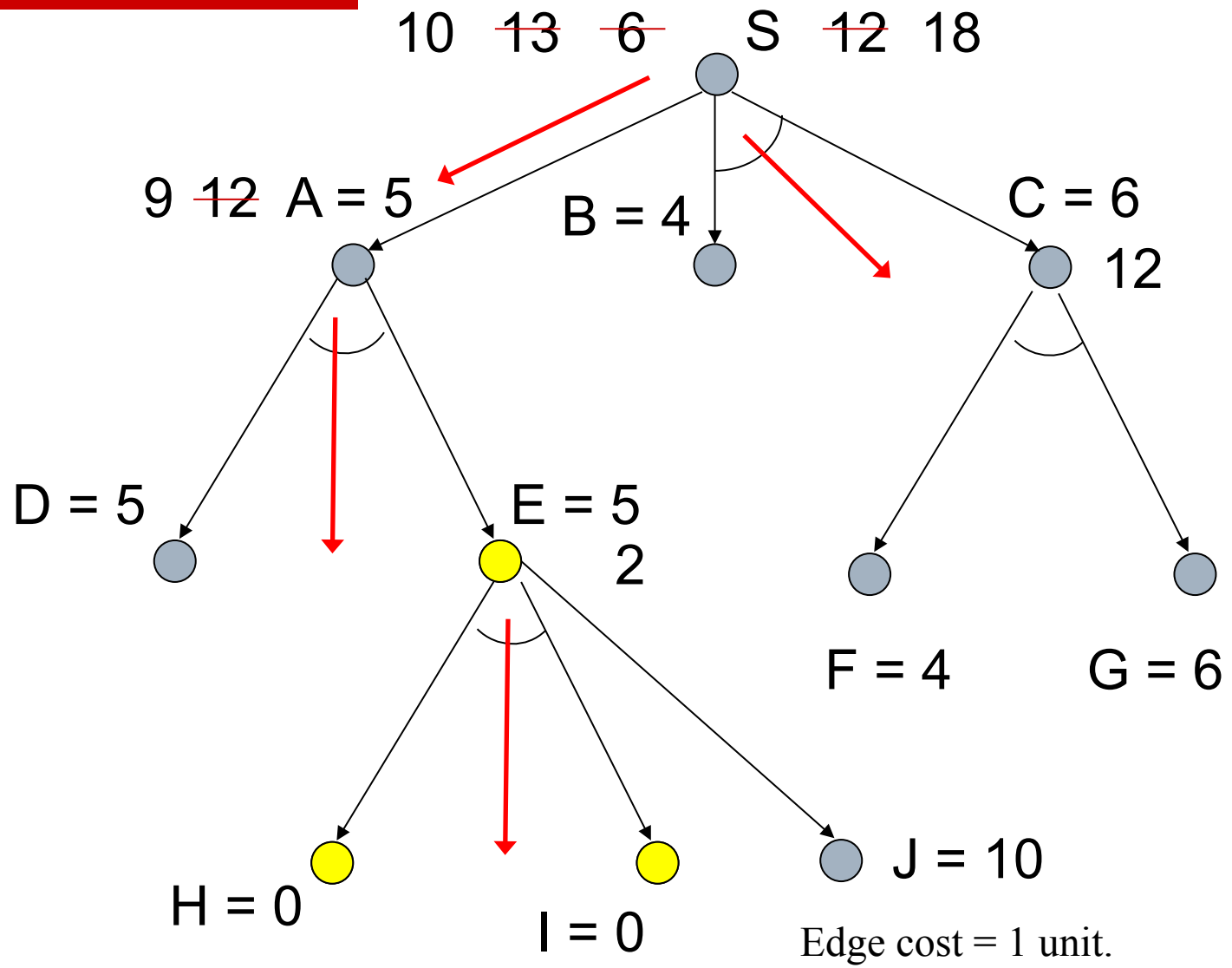
SOLVE - Labelling



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Node E is expanded;
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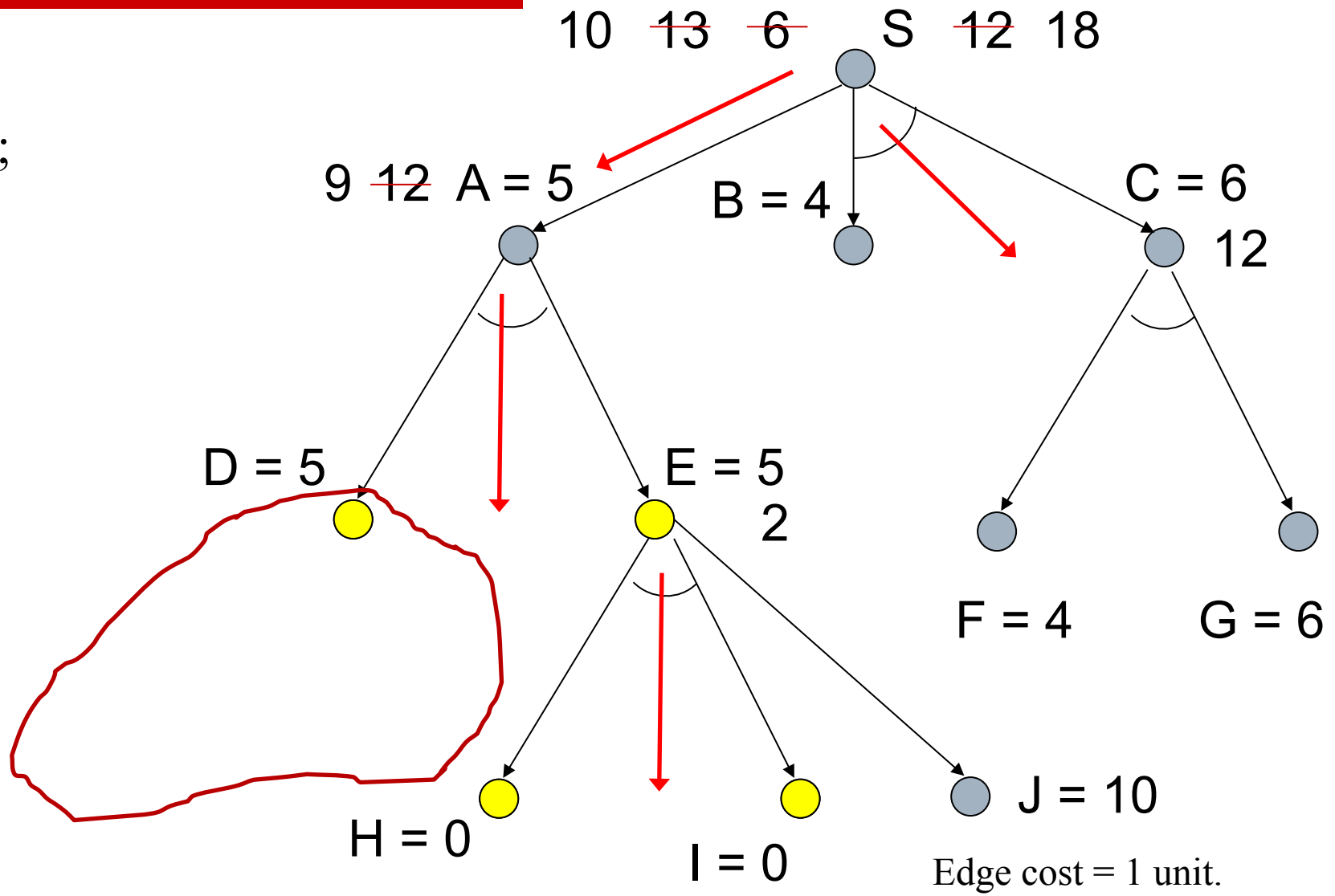
SOLVE - Labelling

Sub-tree at Node D
SOLVED!



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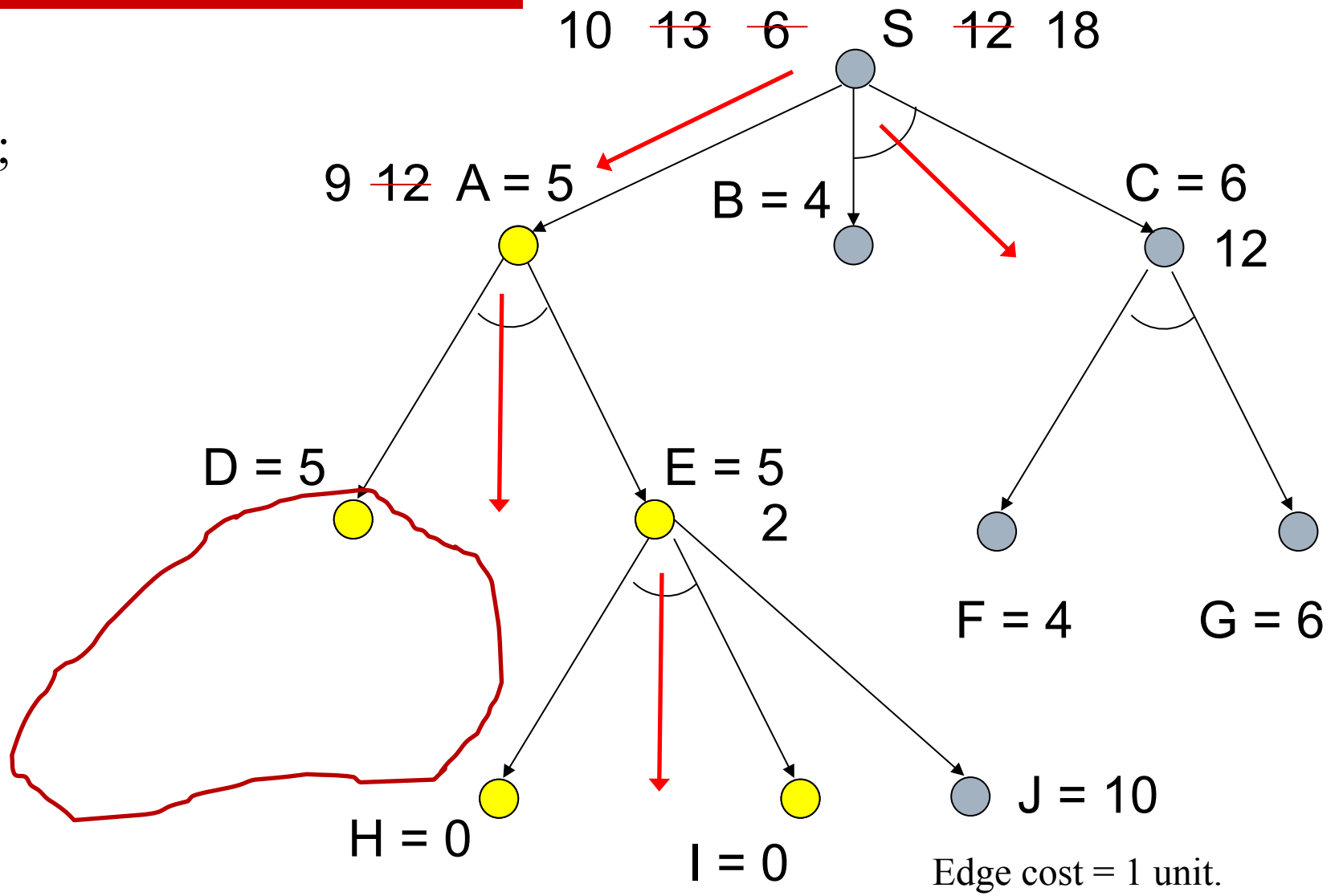

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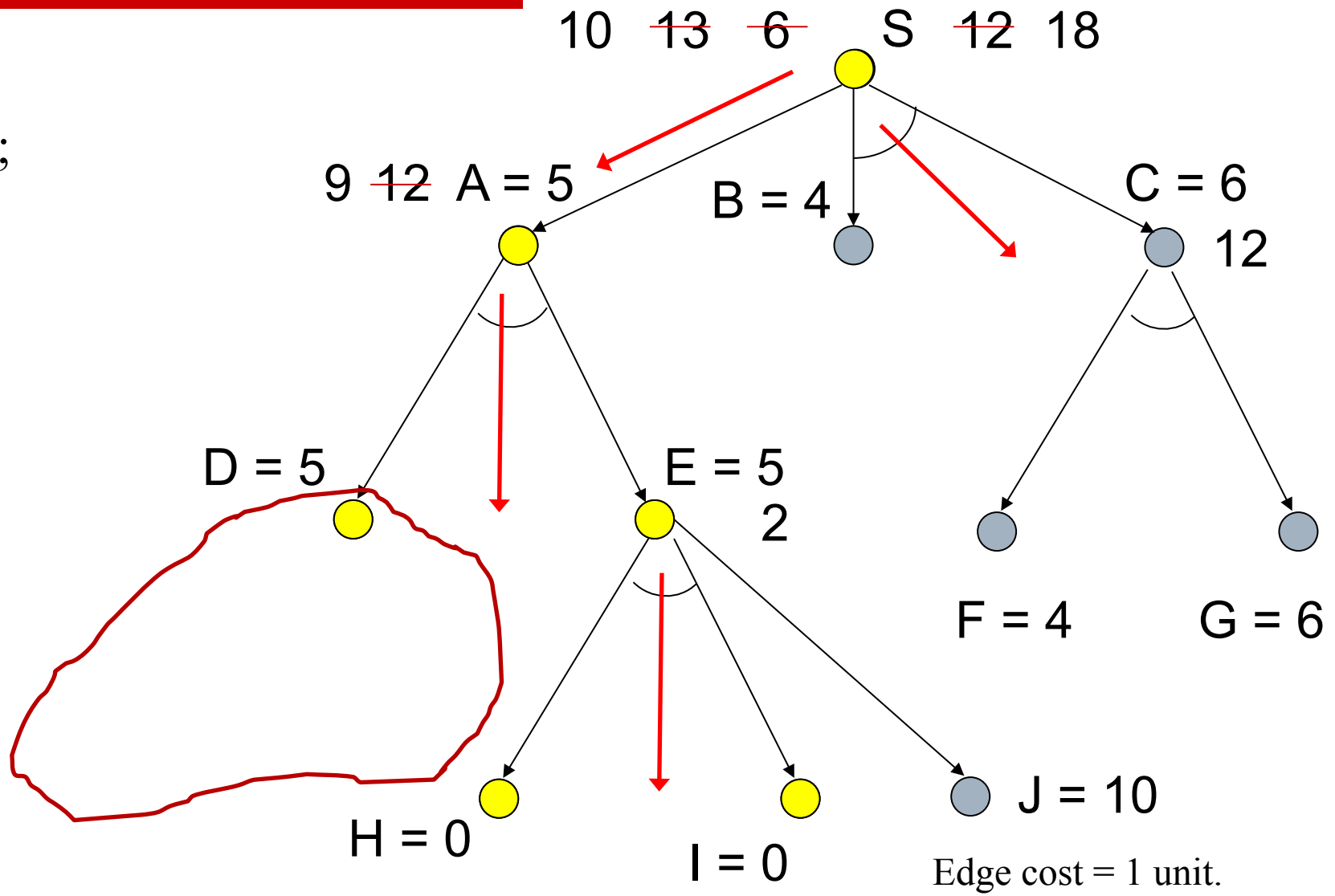


Basic Idea of AO*

- Bottom-up cost-revising, connector-marking
  - SOLVE-labeling.
  - If a direction has all successors SOLVED then n is marked SOLVED.
  - Starting with the node just expanded, the procedure revises its cost (using the newly computed cost of its successors).
    - Marks the outgoing connector on the estimated best path to terminal nodes.
    - Propagated upward in the graph.

AO* Algorithm

Create a search graph G = <s> ; q(s) = h (s) ; If s is a terminal node, mark s SOLVED

Until s labeled SOLVED do:

Top-down graph growing - picks out best available partial solution sub-graph

begin

a. **Compute G/ partial solution graph in G** by tracing down marked connectors in G from s.

b. **Select any nonterminal leaf node n of G/.**

c. **Expand n**, place successors in G, For each successor not already in G let q(successor) = h(successor). Label SOLVED all successors that are terminal nodes.

d. Let S := {n}.

Until S is empty do :

begin

a. Remove a node, m, from S which has no descendent in G also in S (minimal node).

b. Revise cost for m

q(m)=min [c +q(n1)+…+q(nk)].

Mark chosen connector.

If **all successors through the connectors are SOLVED** then mark m SOLVED.

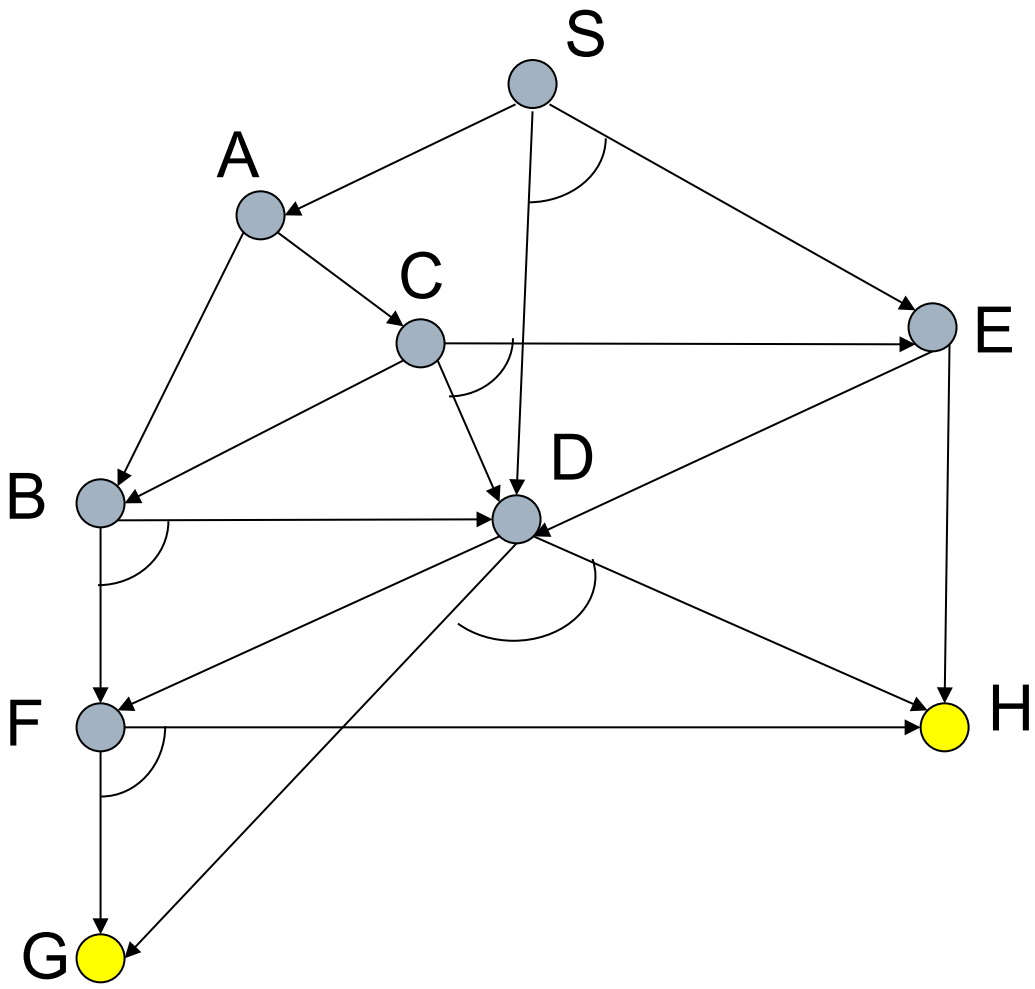
c. **If m SOLVED or changed q(m) then add to S all "preferred" parents of m.**

End.

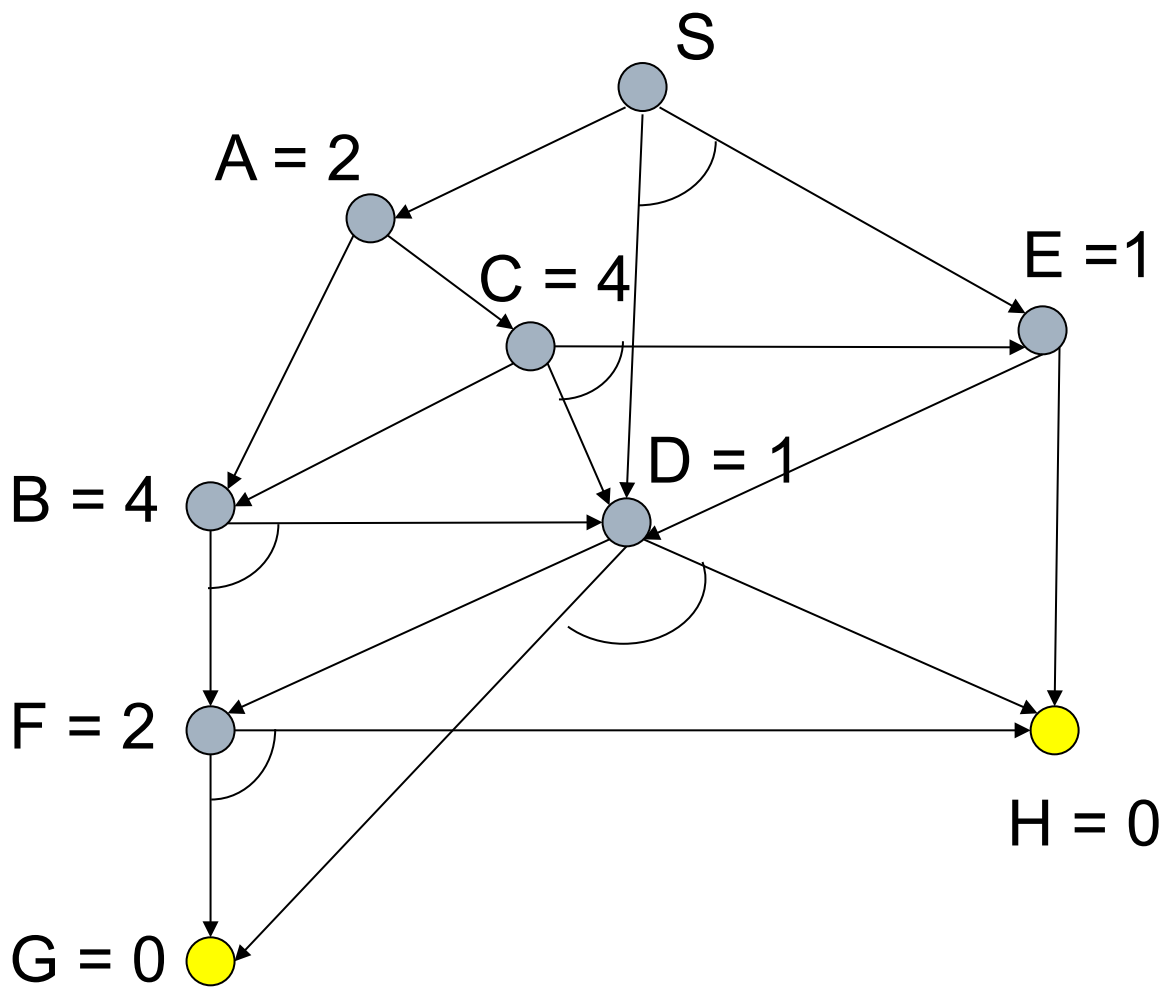
End.

Bottom-up, cost-revising, connector marking; SOLVED labelling procedure

Tracing AO*

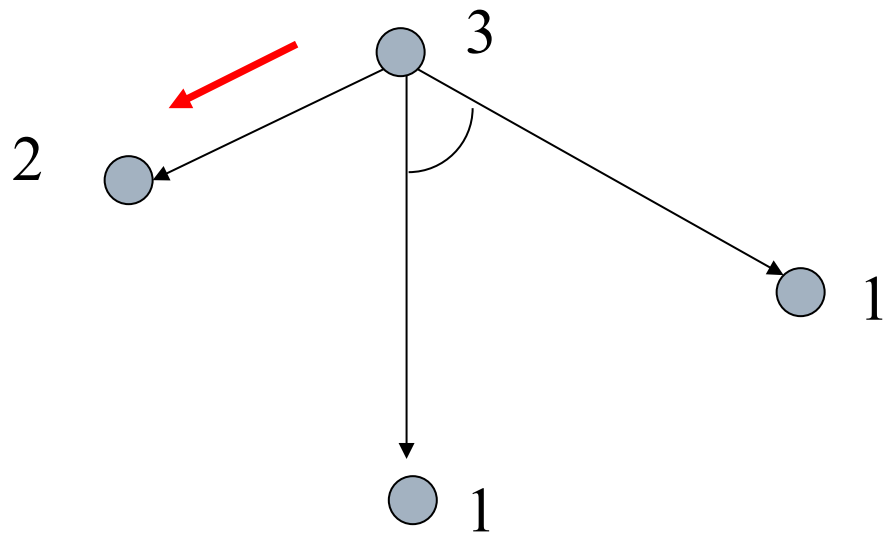


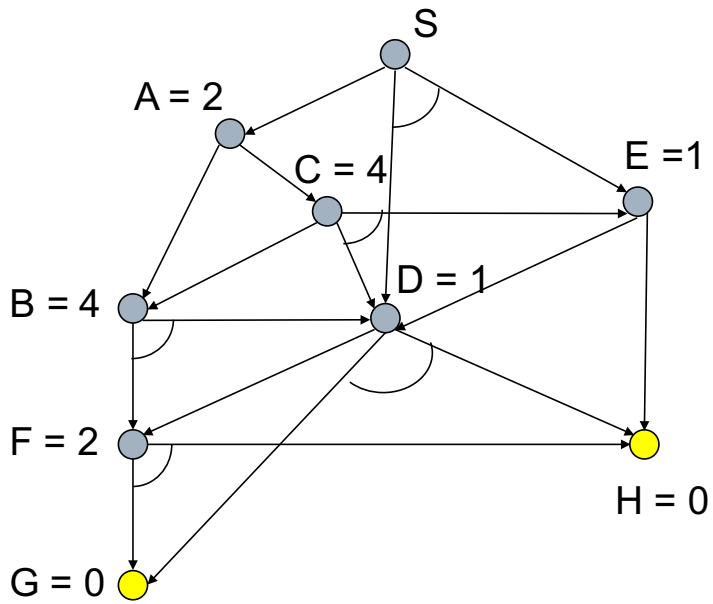
Tracing AO*



Nils J. Nilsson, Principles of Artificial Intelligence, Narosa.
Chapter 3, Pages 107-108.

Tracing AO*





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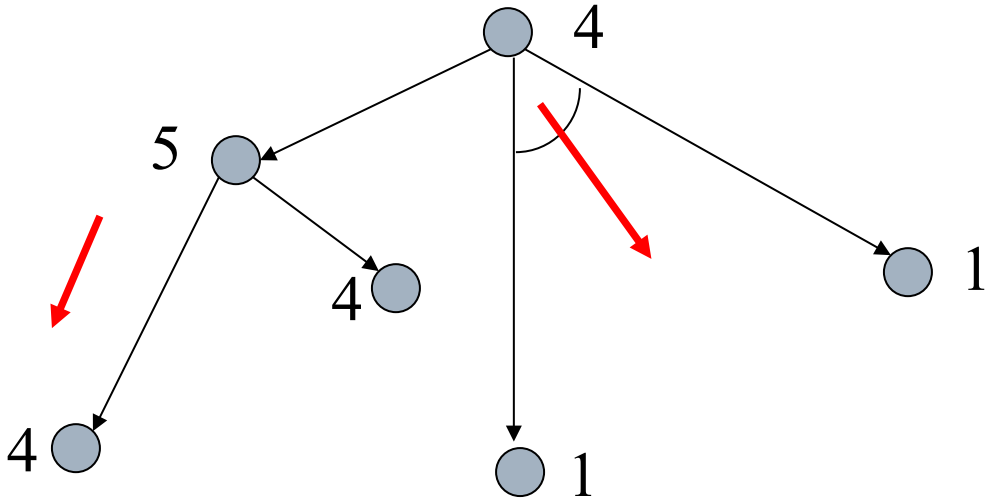

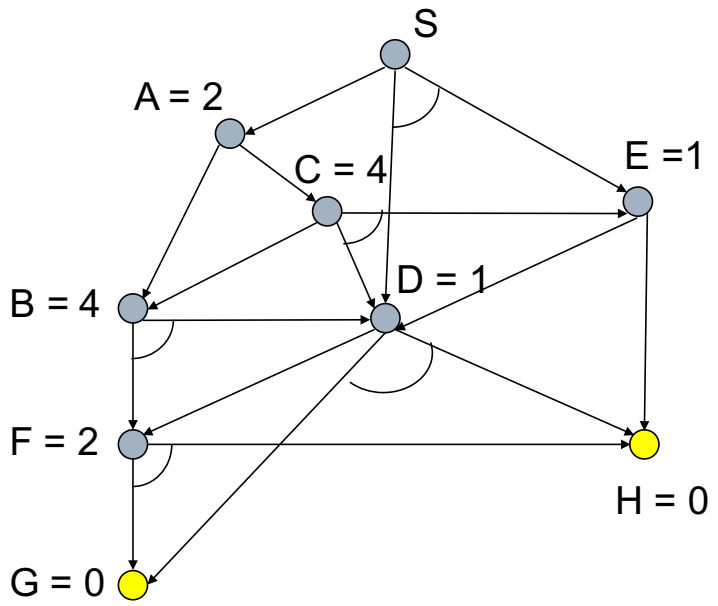

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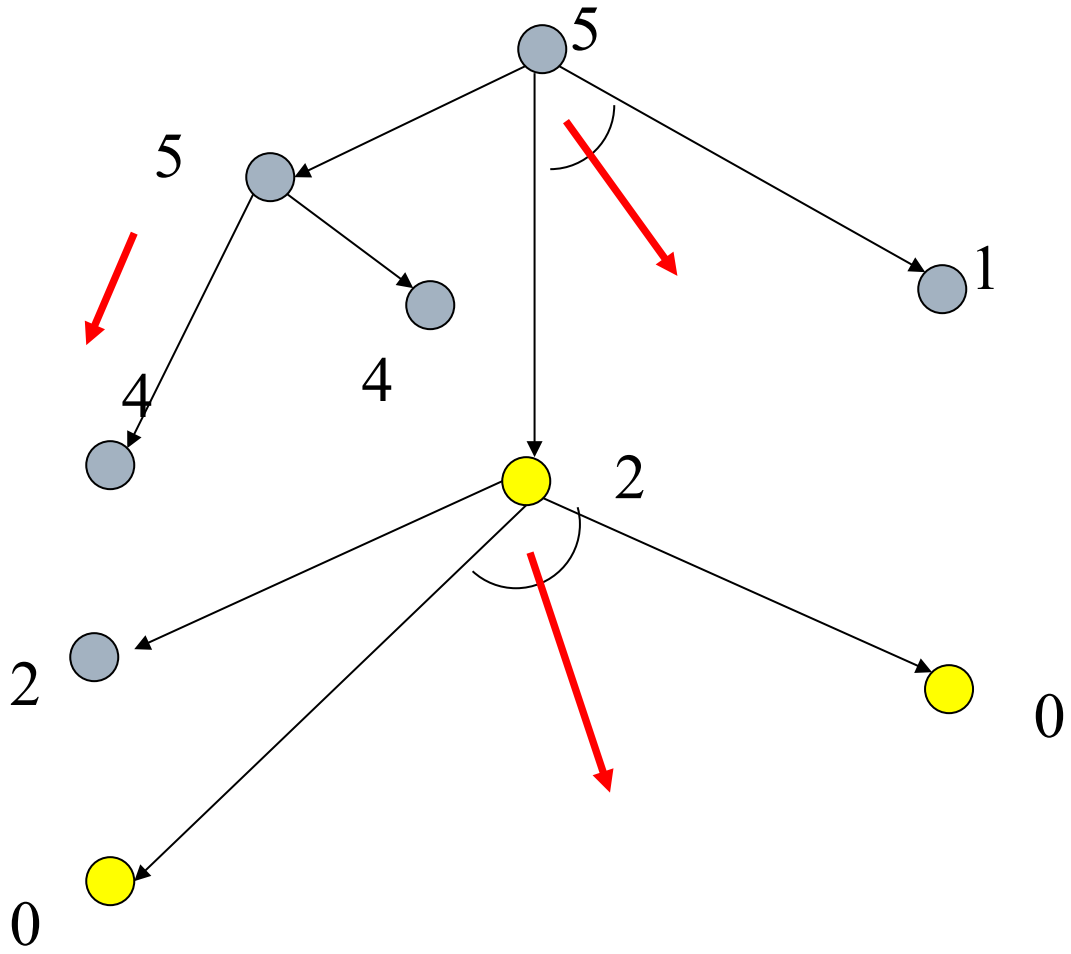

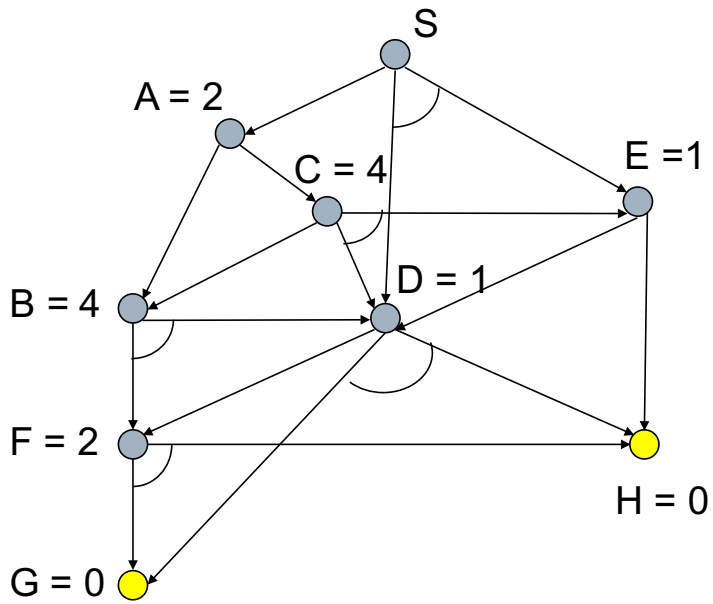

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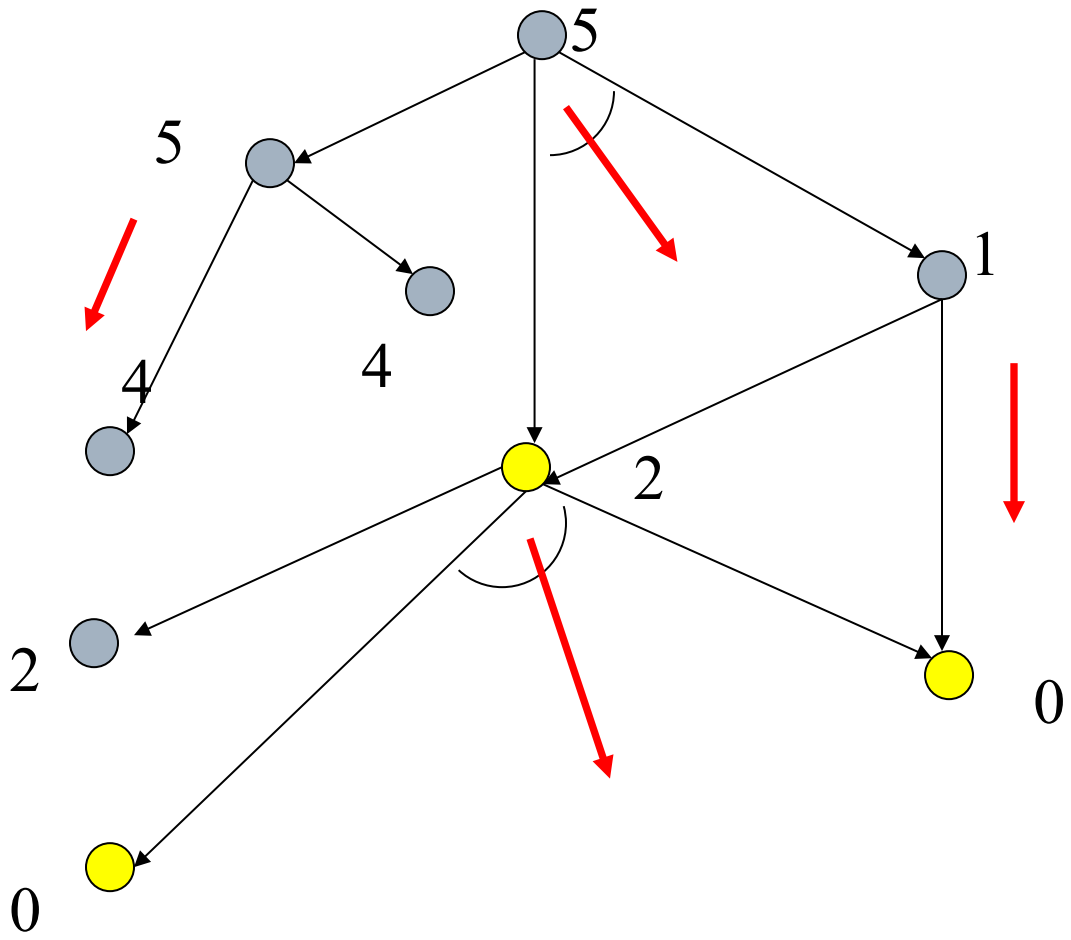

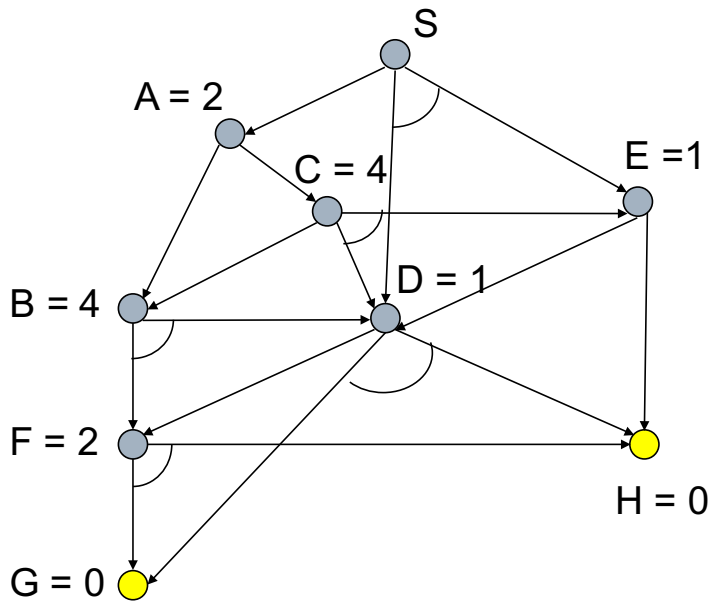

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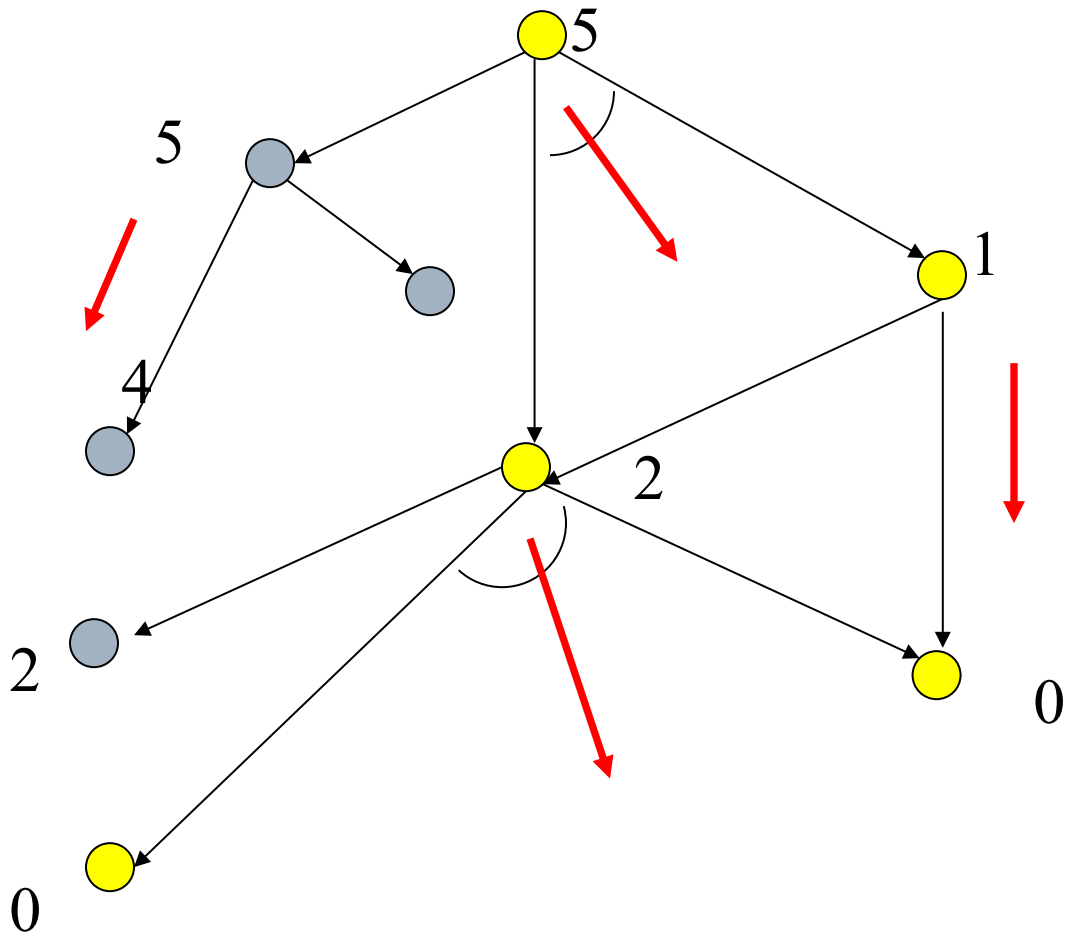

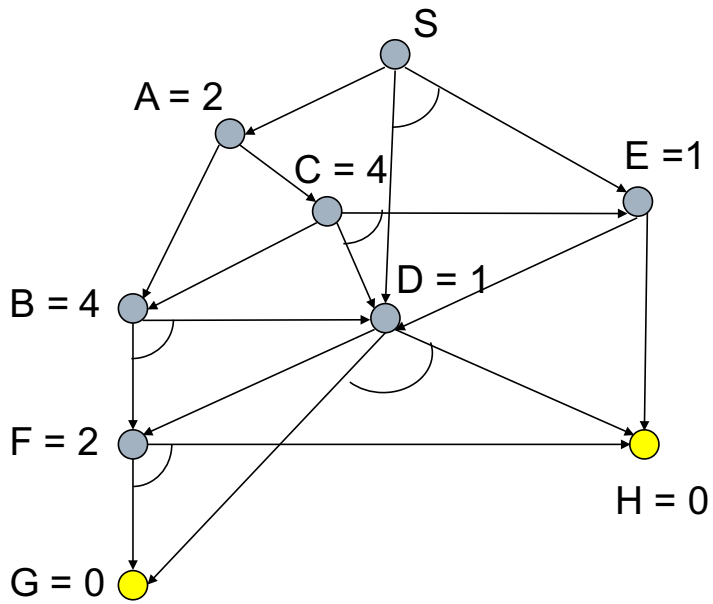

AO* Modifications

- AO* may be modified in a variety of ways to render it more practical.
  - Rather than recompute a new estimated partial solution graph after every node expansion; expand one or more leaf nodes and some number of their descendants all at once. Recompute an estimated best partial solution graph.
    - This would reduce the overhead of frequent bottom-up operations.
  - Staged-search strategy may be used for AND-OR graphs
    - Identify few partial solution graphs having largest estimated cost; these can be discarded periodically.
      - Risk of discarding one that might turn out to the top of an optimal solution graph.