Structured and Hybrid Products Final Project

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1 Problem Statement

Our goal is to find a pricing method for the exotic derivative whose payoff at expiration T is defined as below:

$$\Pi(T) = \max\left[0, \left(\frac{S(T)}{S(0)} - k\right) \cdot \left(\frac{L(T-\Delta, T-\Delta, T)}{L(0, T-\Delta, T)} - k'\right)\right] \tag{1}$$

Here, S is the STOXX50E spot price quantoed from EUR to USD, L is the LIBOR rate while Δ is 3 months. Finally, both k and k' are two different strike prices.

2 Assumptions

1. The STOXX50E index can be modeled using a GBM process.
2. The interest rate process in the EUR measure can be treated as a deterministic process.
3. The LIBOR rates can be modeled using the Vasicek short-term interest rate model.
4. The evolution of the EUR-USD FX rate can be modeled using a GBM process.
5. Volatility is assumed to be constant through the evolution of the stock price.
6. The correlation between the STOXX50E value and the interest rates does not change significantly through the lifetime of the option.

3 Methodology

1. Calibrate the parameters of the Vasicek Model: The Vasicek model for the evolution of interest rates is essentially a one factor short term model which can be represented using the following SDE:

$$dr = a(b - r(t))\,dt + \sigma\,dW_t \tag{2}$$

   where a and b are constants. In this step, we use gradient descent to find the optimal values of the parameters of the Vasicek model. We do this by minimizing the squared error between the observed rates and the predicted rates from the model.

2. Simulate the rates using the Vasicek Model: We simulate the progression of the interest rates using a Monte Carlo approach. We obtain the value for the spot and forward interest rates which will be used later to compute the price of the option.

3. Simulate the STOXX50E index simultaneously using a Geometric Brownian Motion model. This can be represented using the SDE:

$$\frac{dS}{S} = (r_f - \rho_{SX}\sigma_X\sigma_S)\,dt + \sigma_S dW^{Q^d} \tag{3}$$

   where:

   - r_f: Interest rate in foreign EUR measure
   - ρ_{SX}: Correlation between STOXX50E and FX rate
   - σ_X: Exchange rate volatility
   - σ_S: STOXX50E volatility

   The Brownian motions that are used to simulate the price processes of the rate process and the STOXX50E process are not independent, so we obtain the historical correlation between the two and use it in our simulations.

4. Calculate the zero coupon bond price $p(t,T)$ under the Vasicek interest rate model using:

$$p(t,T) = \exp\left(\left(b + \frac{\sigma\phi}{a} - \frac{\sigma^2}{a^2}\right)\Big(B(t,T) - T + t\Big) + \frac{B(t,T)^2\sigma^2}{4a}\right) \tag{4}$$

5. Compute the LIBOR rate from the zero coupon bond price using the closed form solution:

$$L(t,T) = \frac{100}{T-t}\left(\frac{1}{p(t,T)} - 1\right) \tag{5}$$

6. Calculate the contract payoff by substituting the values obtained above: Substitute the values of the index, the rates, k and k' in the below expression for the contract payoff, and compute it for this iteration of the simulation.

$$\Pi(T) = \max\left[0, \left(\frac{S(T)}{S(0)} - k\right)\cdot\left(\frac{L(T-\Delta, T-\Delta, T)}{L(0, T-\Delta, T)} - k'\right)\right] \tag{6}$$

7. Discount the expected payoff over the simulated paths to price the contract:

$$\pi(t) = \mathbb{E}\left[e^{-rT}\max\left\{0, \left(\frac{S(T)}{S(0)} - k\right)\cdot\left(\frac{L(T-\Delta, T-\Delta, T)}{L(0, T-\Delta, T)} - k'\right)\right\}\right] \tag{7}$$

4 Results

We ran Monte-Carlo simulations for different values of k and k' to price the option. The results were as follows:

$$k = 1\ k' = 1\ n = 10000$$

$$p = 0.989528$$

$$k = 0.1\ k' = 0.1\ n = 10000$$

$$p = 7.323559$$

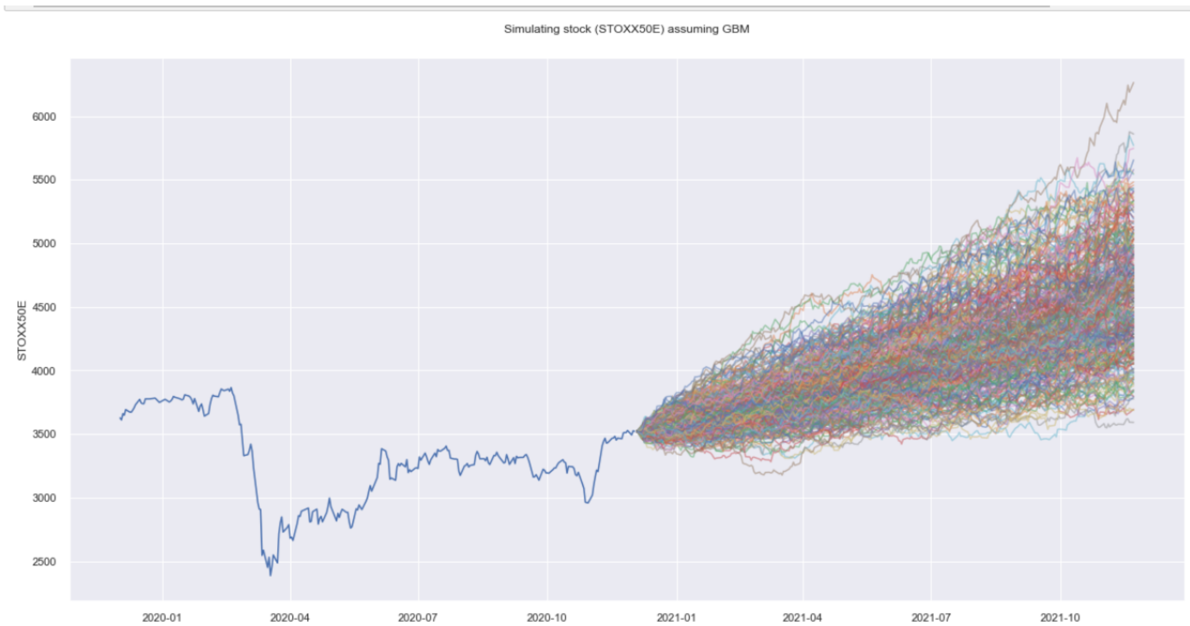

Figure 1: Simulated STOXX50E

Figure 2: Simulated Rates