MA374 Financial Engineering lab:10

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- **To execute my .py file**
  Run $python3 180123029_NamanGoyal.py on the terminal. The snapshot is shown below question-wise:

Ques.1

```
~/Desktop/IITG_SEMVI/FE_Labs/lab10
> python3 180123029_NamanGoyal.py
-------------------Q1-------------------

For K = 90
Call Price : 10.088167
Put Price  : 0.393561

For K = 105
Call Price : 1.692363
Put Price  : 7.112529

For K = 110
Call Price : 0.416840
Put Price  : 9.534677

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```
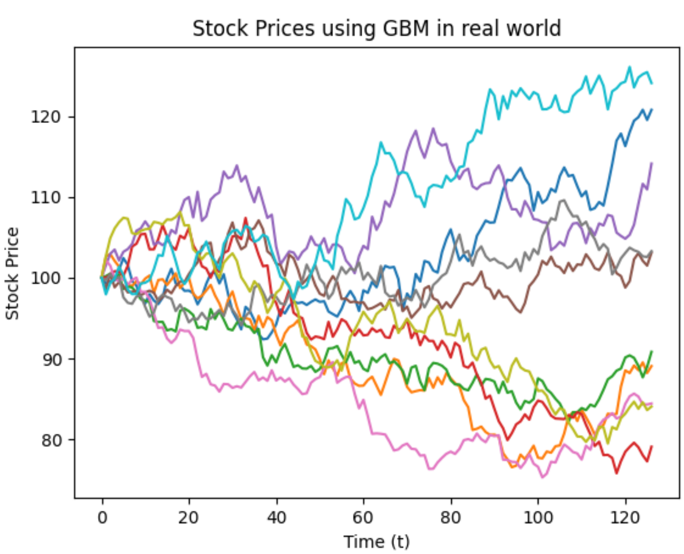
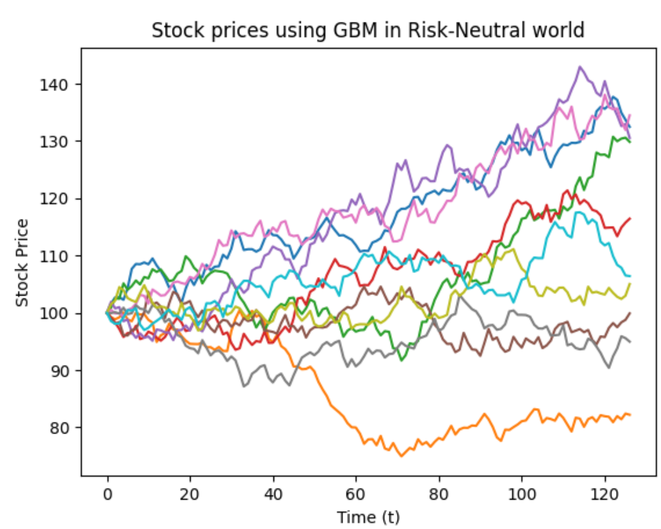

- In a **GBM Model** stock prices vary as:

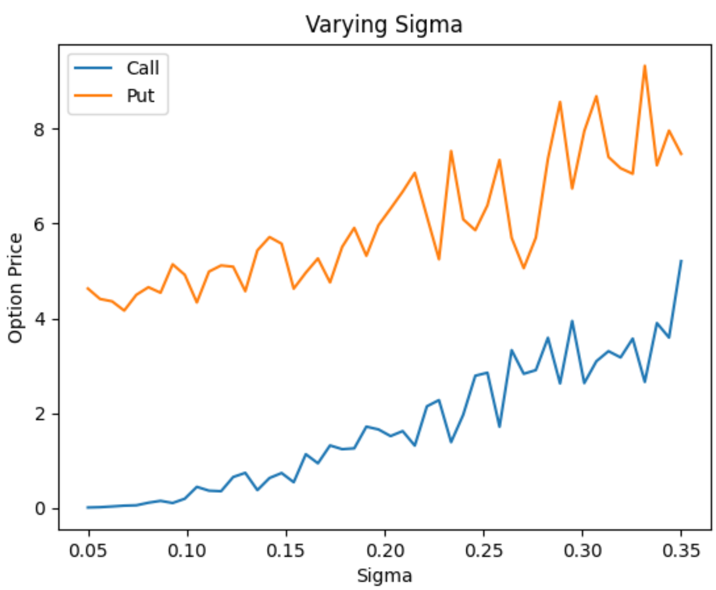
$$ds(t) = \mu s(t)dt + \sigma s(t)dw(t)$$

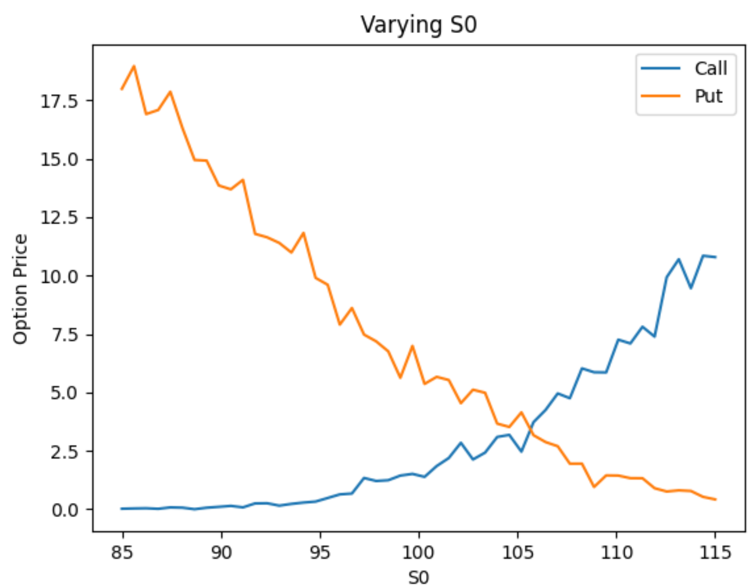
- Using the GBM Model, 10 different paths of an asset are simulated. The graphs obtained are been shown below:

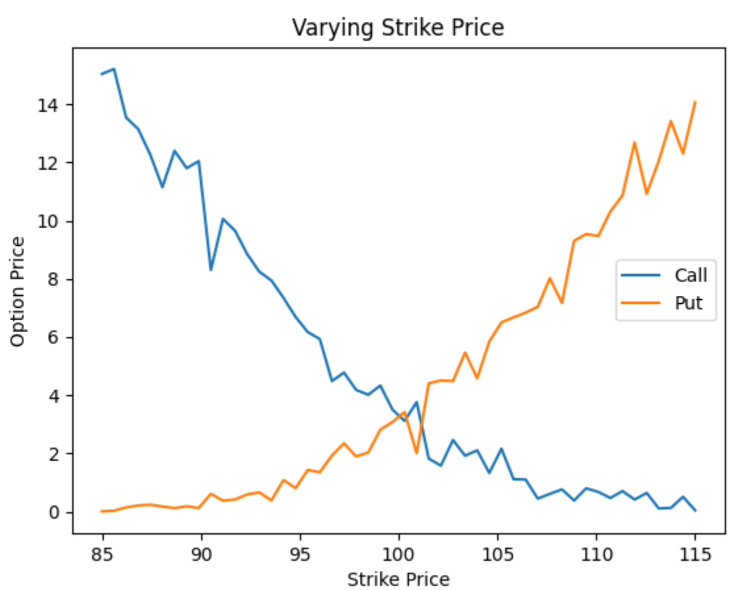




- The Price of the **Option (Asian)** is calculated using Monte-Carlo Concepts which are already shown in the ScreenShot above for Ques.1
- The Graphs for the **Sensitivity** of the Option Price against Strike Price(K), Sigma and S0 are shown:







Ques.2

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-------------------Q2-------------------

For K = 90
Without Variance Reduction:: Call Price : 10.750556 and Variance = 50.287846
With Variance Reduction:: Call Price : 10.595600 and Variance = 31.080284
Without Variance Reduction:: Put Price : 0.104086 and Variance = 0.478866
With Variance Reduction:: Put Price : 0.030916 and Variance = 0.052791

For K = 105
Without Variance Reduction:: Call Price : 1.320111 and Variance = 9.073656
With Variance Reduction:: Call Price : 0.710980 and Variance = 3.759720
Without Variance Reduction:: Put Price : 5.466336 and Variance = 35.450070
With Variance Reduction:: Put Price : 5.272613 and Variance = 18.591253

For K = 110
Without Variance Reduction:: Call Price : 0.794210 and Variance = 5.176378
With Variance Reduction:: Call Price : 0.176318 and Variance = 0.723981
Without Variance Reduction:: Put Price : 9.478480 and Variance = 59.886636
With Variance Reduction:: Put Price : 9.243705 and Variance = 27.130647

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- Here in this question, the **Method of Antithetic Variates** is used to reduce the variance. Antithetic method clearly reduces the variance. Here is the formulation for it.

$$\theta = E[Y] = E[g(X)]$$

where θ is the quantity we want to estimate ,

we can generate two sample Y_1 and Y_2 s.t. the new unbiased estimator of θ is

$$\hat{\theta} = \frac{Y_1 + Y_2}{2}$$

Hence we have

$$Var(\theta) = \frac{var(Y_1) + var(Y_2) + 2Cov(Y_1, Y_2)}{4}$$

It is obvious that we could get a variance reduction if we have the two samples negatively correlated.

If $X \sim \mathcal{N}(0,1)$ then we can apply the following algorithm

$$\hat{\theta} = \frac{1}{n} \sum_{i=1}^{N} \frac{g(X_i) + g(-X_i)}{2} \text{ with i.i.d. } X_i \sim \mathcal{N}(0,1)$$

- The **Variance before Antithetic method and after Antithetic method** && **Value of Option Prices** is shown in the **Screenshot** above for Ques.2
- The Graphs for the **Sensitivity** of the Option Price against Strike Price(K), Sigma and S0 are shown: