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Sol^n ①: For the portfolio losses $X_A = 1_{D_A}$, $X_B = 1_{D_B}$ we have

~~Var~~ $VaR_{99\%}(X_A) = VaR_{99\%}(X_B) = 0$

Now consider a new portfolio C ~~with~~ as the union of portfolio's A, B. $\Rightarrow$ $X_C = X_A + X_B$, X_C being the corresponding portfolio loss.

Then $P[X_C = 0] = (1-p)^2 < 99\%$. $\therefore 0.006 \leq p \leq 0.01$

Hence $VaR_{99\%}(X_C) > 0$ $\Rightarrow$

$$VaR_{99\%}(X_A + X_B) > VaR_{99\%}(X_A) + VaR_{99\%}(X_B)$$

Hence, we can see that Value at Risk at confidence level 99% or 0.99 does not hold subadditive property.

Sol^n (2): $F(x) = 1 - \left(\frac{K}{K+x}\right)^{\alpha}$ $\{\alpha > 0, K > 0, x \geq 0\}$

$c_n = \frac{\alpha n^{-1/\alpha}}{K}$ and $d_n = K n^{1/\alpha} - K$.

hence:

$$y = c_n^{-1} x + d_n = \left(\frac{n^{1/\alpha} K}{\alpha}\right) x + \left(K n^{1/\alpha} - K\right)$$

$$F(y) = 1 - \left(\frac{K}{K + \left(\frac{n^{1/\alpha} K}{\alpha}\right) x + K n^{1/\alpha} - K}\right)^{\alpha}$$

$$= 1 - \left(\frac{\alpha}{n^{1/\alpha}(x+\alpha)}\right)^{\alpha} = 1 - \frac{\alpha^{\alpha}}{n(x+\alpha)^{\alpha}}$$

$$= 1 - \frac{1}{n\left(\frac{x}{\alpha}+1\right)^{\alpha}}.$$

we ~~cand~~ calculated the limiting distribution:

$$\lim_{n\to\infty} \left(F^n(c_n^{-1}x + d_n)\right) = \lim_{n\to\infty} \left(1 - \frac{1}{n\left(x/\alpha+1\right)^{\alpha}}\right)$$

$$= e^{-\left(x/\alpha+1\right)^{-\alpha}} = e^{-\left(\frac{x+\alpha}{\alpha}\right)^{-\alpha}}$$

Now define a new func^n.

$$H_{\varepsilon}(x) = \begin{cases} e^{-(1+\varepsilon x)^{-1/\varepsilon}} & \varepsilon \neq 0 \\ e^{-(e^{-x})} & \varepsilon = 0 \end{cases}$$

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$$\lim_{n\to\infty}\left(F^n(c_n^{-1}x+d_n)\right) = e^{-(x/\alpha+1)}$$ and

$e^{-(x/\alpha+1)}$ is $H_{1/\alpha}(x)$.

$\Rightarrow$ Hence $F \in MDA(H_{1/\alpha})$

where $(\alpha>0, \Rightarrow 1/\alpha>0, \varepsilon>0)$

and shape parameter is $1/\alpha$.

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Sol^n (4): Consider GARCH (1,1) model as

$a_t = \sigma_t \varepsilon_t$ and $\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

where $\alpha_0 > 0,\ \alpha_1 \geq 0,\ \beta_1 \geq 0,\ \alpha_1 + \beta_1 < 1$

and $\{\varepsilon_t\}$ is an iid sequence satisfying

$$E(\varepsilon_t) = 0, \quad Var(\varepsilon_t) = 1, \quad E(\varepsilon_t^4) = K_\varepsilon + 3$$

where $K_\varepsilon \rightarrow$ excess kurtosis of innovation ε_t

Then we have:

$$Var(a_t) = E(\sigma_t^2)$$

$$E(\sigma_t^2) = E(\alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2)$$

$$= \alpha_0 + \alpha_1 E(\sigma_{t-1}^2) E(\varepsilon_{t-1}^2) + \beta_1 E(\sigma_{t-1}^2)$$

Now σ_t is stationary. so $E(\sigma_t^2) = E(\sigma_{t-1}^2)$ and also $E(\varepsilon_{t-1}^2) = 1$.

Hence $\Rightarrow E(\sigma_t^2) = \dfrac{\alpha_0}{1-(\alpha_1+\beta_1)} = Var(a_t)$

Now $E(a_t^4) = E(\sigma_t^4 \varepsilon_t^4) = E(\sigma_t^4) E(\varepsilon_t^4)$

$$\Rightarrow E(a_t^4) = (K_\varepsilon + 3) E(\sigma_t^4)$$

we have

$$\Rightarrow \sigma_t^4 = (\alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2)^2$$

$$= \alpha_0^2 + \beta_1^2 \sigma_{t-1}^4 + \alpha_1^2 a_{t-1}^4 + 2\alpha_0\alpha_1 a_{t-1}^2 + 2\alpha_1\beta_1 \sigma_{t-1}^2 a_{t-1}^2 + 2\alpha_0 \beta_1 \sigma_{t-1}^2$$

using stationary property,

$$E(\sigma_t^4) = \frac{\alpha_0^2 (1+\alpha_1+\beta_1)}{(1-(\alpha_1+\beta_1))\left[1-\alpha_1^2(K_\epsilon t^2) - (\alpha_1+\beta_1)^2\right]}$$

given $0 \leq \alpha_1+\beta_1 < 1 \Rightarrow 1-\alpha_1^2(K_\epsilon t^2) - (\alpha_1+\beta_1)^2 > 0$

The excess kurtosis of a_t is $\Rightarrow$

$$K_a = \frac{E(a_t^4)}{(E(a_t^2))^2} - 3$$

$$= \frac{(K_\epsilon+3)(1-(\alpha_1+\beta_1)^2)}{1-2\alpha_1^2-(\alpha_1+\beta_1)^2-K_\epsilon\alpha_1^2} - 3$$

If $\epsilon_t \sim N(0,1)$ then $K_\epsilon = 0 \Rightarrow$

$$K_a = \frac{6\alpha_1^2}{1-2\alpha_1^2-(\alpha_1+\beta_1)^2}$$ given that $1-2\alpha_1^2-(\alpha_1+\beta_1)^2 > 0$

↳ Hence, we can see that excess kurtosis of residuals K_a is $\neq 0$. Hence distribution has outliers.