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Q:4: $X_t = Z_t - \theta Z_{t-1}$

Here, $Z_t \sim N(0, \sigma^2)$

Take prediction of $\hat{X}_{n+1} = \sum_{j=1}^{n} \phi_j X_{n-j+1}$

If $\hat{X}_{n+1}$ is the best linear predictor of X_{n+1}

$MSE = E((\hat{X}_{n+1} - X_{n+1})^2)$ is $\rightarrow$ minimised.

The squared loss error is minimised.

$$MSE = E\left(\sum_{j=1}^{n} \phi_j (Z_{n-j+1} - \theta Z_{n-j}) - (Z_{n+1} - \theta Z_n)\right)^2)$$

$$= E\left((\theta_1 + \phi) Z_n + \sum_{j=1}^{n} (\phi_{j+1} - \theta\phi_j) Z_{n-j} - \theta\phi_n Z_0 - Z_{n+1}\right)^2)$$

As the expectation's square is taken, when square is opened.

$E(Z_i Z_j) = 0$, $E(Z_i^2) = \sigma^2$.

So, expectation of cross terms $= 0$,

$$\text{So } MSE = \sigma^2\left((\phi_1 + \theta)^2 + \sum_{j=1}^{n-1} (\phi_{j+1} - \theta\phi_j)^2 + \theta^2\phi_n^2 + 1\right)$$

$\frac{\partial MSE}{\partial \phi_i} = 0 \quad \forall\, i = 1, 2, \ldots n.$

$\frac{\partial MSE}{\partial \phi_1} = 0 \Rightarrow \boxed{(1+\theta^2)\phi_1 - \theta\phi_2 = 0}$

$\frac{\partial MSE}{\partial \phi_j} = 0 \Rightarrow \boxed{\theta\phi_{j-1} + (1+\theta^2)\phi_j - \theta\phi_{j+1} = 0.}$ Ans

for $\theta_n \Rightarrow \boxed{(1+\theta^2)\phi_n - \theta\phi_{n-1} = 0}$

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