Department of Mathematics

Indian Institute of Technology Guwahati

Mid Semester Examination March 4, 2021

MA 473 Computational Finance

Time: **10:00 – 11:30 Hrs.** Marks: **30**

There are ***FOUR*** questions in this paper. Answer all questions.

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1. By using the transformation $\xi = \dfrac{S}{S+P_m}$, $\tau = T - t$ and $V(S,t) = (S+P_m)\overline{V}(\xi,\tau)$, where $P_m > 0$ is a constant, transform the following Black-Scholes PDE:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2(S)S^2\dfrac{\partial^2 V}{\partial S^2} + (r-\delta)S\dfrac{\partial V}{\partial S} - rV = 0, \quad 0 \le S,\ t \le T, \\ V(S,T) = V_T(S), \quad 0 \le S, \end{cases}$$

from infinite domain $0 \le S$ to finite domain $0 \le \xi \le 1$. Further, obtain the boundary conditions at $\xi = 0$ and $\xi = 1$. (**10** marks)

2. Show that the early-exercise curve $S_f(t)$ (for $t < T$) admits the following upper bound:

$$S_f(t) \ < \ \lim_{t\to T} S_f(t) \ = \ \min\left(K, \frac{r}{\delta}K\right). \qquad (\mathbf{5}\text{ marks})$$

3. Show that the following *Cryer* problem:

$$\begin{cases} \text{Find vectors } x \text{ and } y \text{ such that for } \hat{b} := b - Ag \\ Ax - y = \hat{b}, \quad y \ge 0, \quad x^T y = 0, \end{cases}$$

is equivalent to the minimization problem:

$$\min_{x\ge 0} G(x), \quad \text{where } G(x) = \frac{1}{2}(x^T Ax) - b^T x, \text{ is strictly convex.} \qquad (\mathbf{5}\text{ marks})$$

4. The PDE modelling the *Asian option* with arithmetic average (for $S > 0$, $A > 0$, $0 \le t \le T$) is given by

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} + S\frac{\partial V}{\partial A} - rV = 0.$$

By using the transformation $V(S,A,t) = \widetilde{V}(S,R,t) = SH(R,t)$, with $R = A/S$, show that the above PDE becomes

$$\frac{\partial H}{\partial t} + \frac{1}{2}\sigma^2 R^2 \frac{\partial^2 H}{\partial R^2} + (1 - rR)\frac{\partial H}{\partial R} = 0.$$

Also obtain the boundary and terminal conditions for the transformed PDE. (**10** marks)
