Department of Mathematics

Indian Institute of Technology Guwahati

Mid Semester Examination September 22, 2021

MA 473 Computational Finance (Part – I)

Time: **09:00 – 10:00 Hrs.** Marks: **15**

There are ***TWO*** questions in this paper. Answer all questions.

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1. By using the transformation $x = \ln S$, $\tau = \sigma^2(T-t)/2$, and $V(S,t) = e^{\alpha x+\beta\tau}\,u(x,\tau)$, where $\alpha = -(2r/\sigma^2-1)/2$ and $\beta = -(2r/\sigma^2+1)^2/4$, convert the following Black-Scholes PDE:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 V}{\partial S^2} + rS\dfrac{\partial V}{\partial S} - rV = 0, \quad 0 \le S < \infty,\ t \le T, \\ V(S,T) = V_T(S), \quad 0 \le S < \infty, \end{cases}$$

into the one-dimensional heat-conduction parabolic PDE. (**7** marks)

2. Obtain the analytical solution of the following transformed 1D parabolic PDE:

$$\begin{cases} \dfrac{\partial u}{\partial \tau} = \dfrac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty,\ \tau > 0 \\ u(x,0) = u_0(x). \end{cases}$$

From the solution of the above PDE, obtain the solution of the Black-Scholes PDE for the European call option. (**8** marks)
