Matrix Computations MA423 Lab 03

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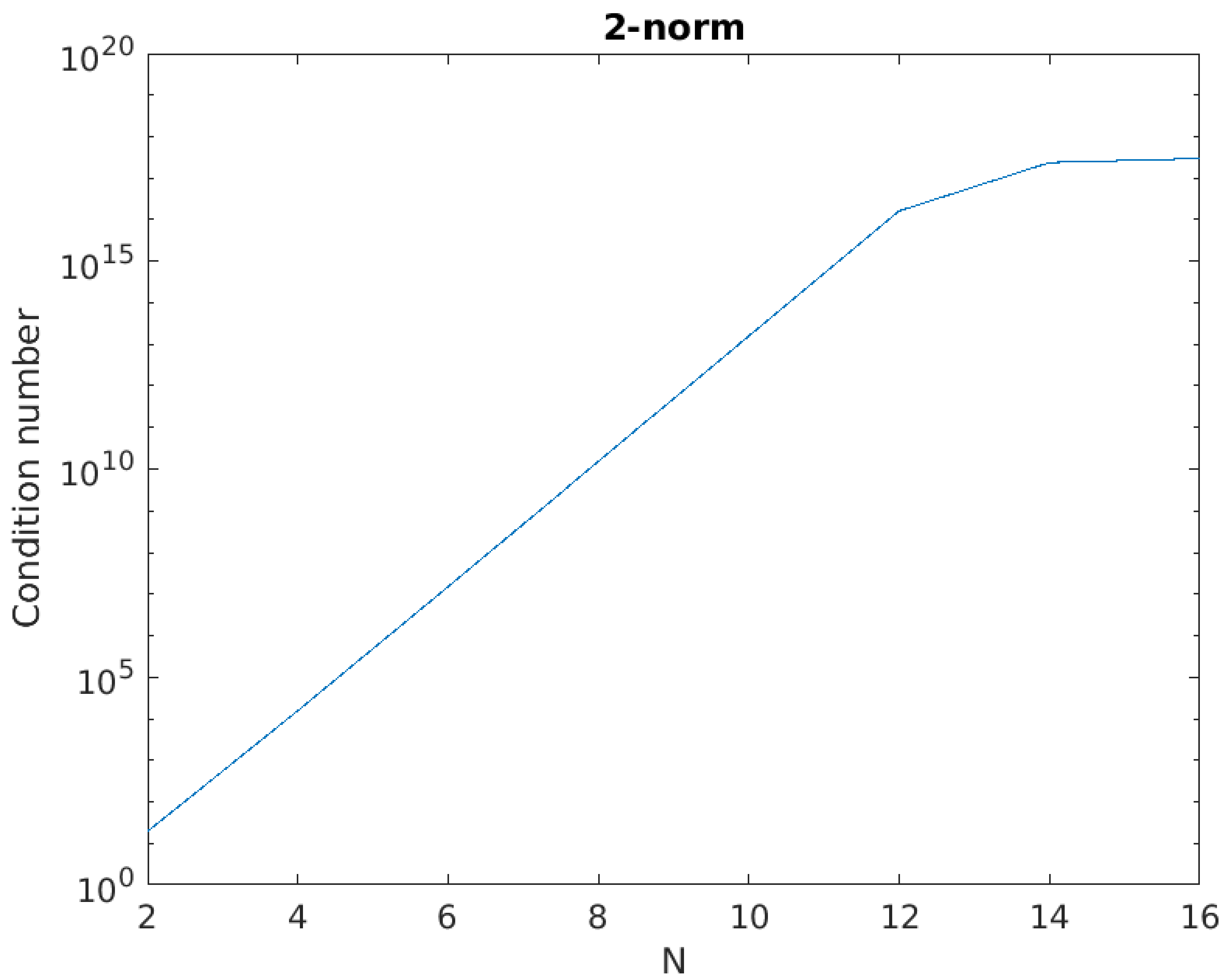
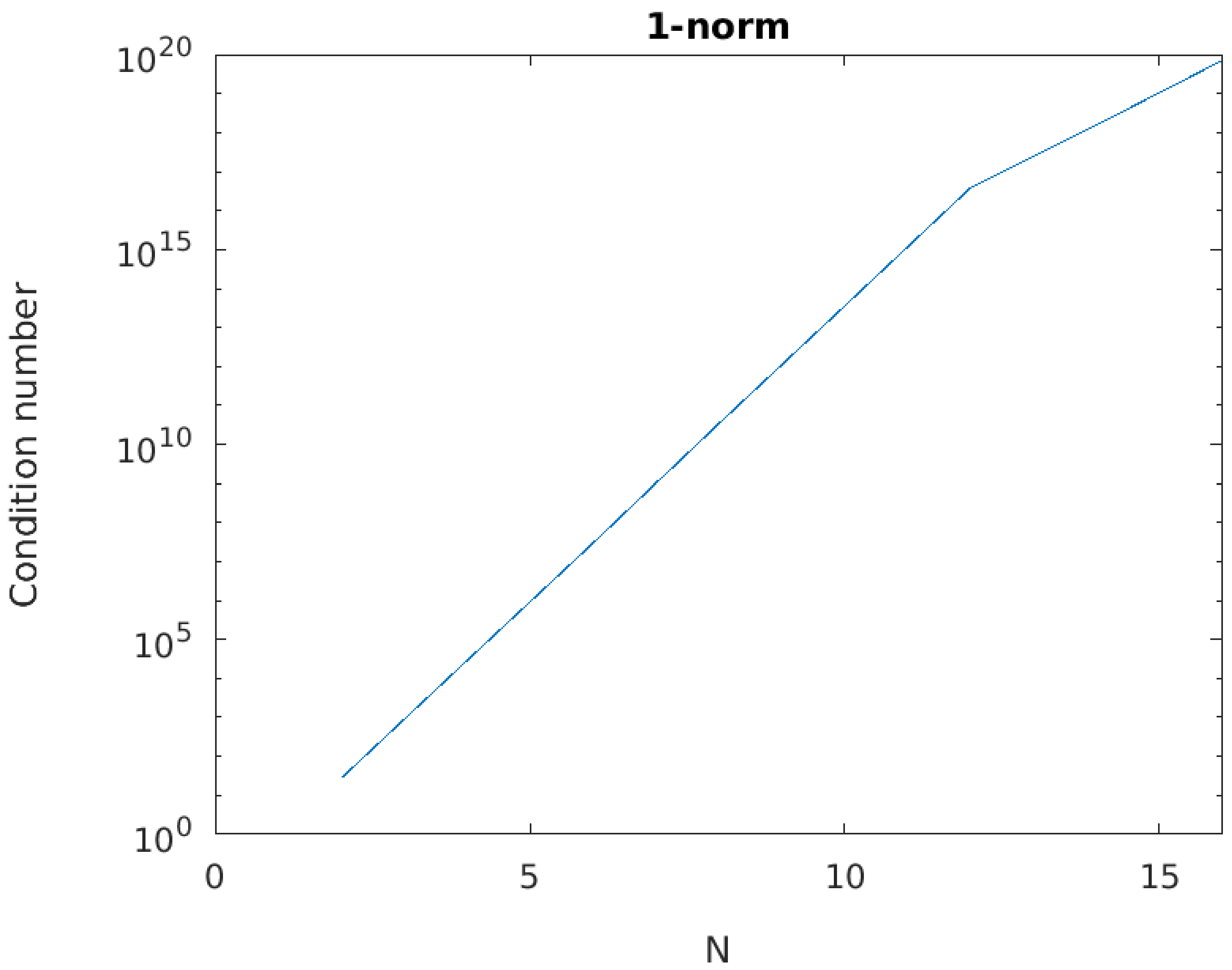
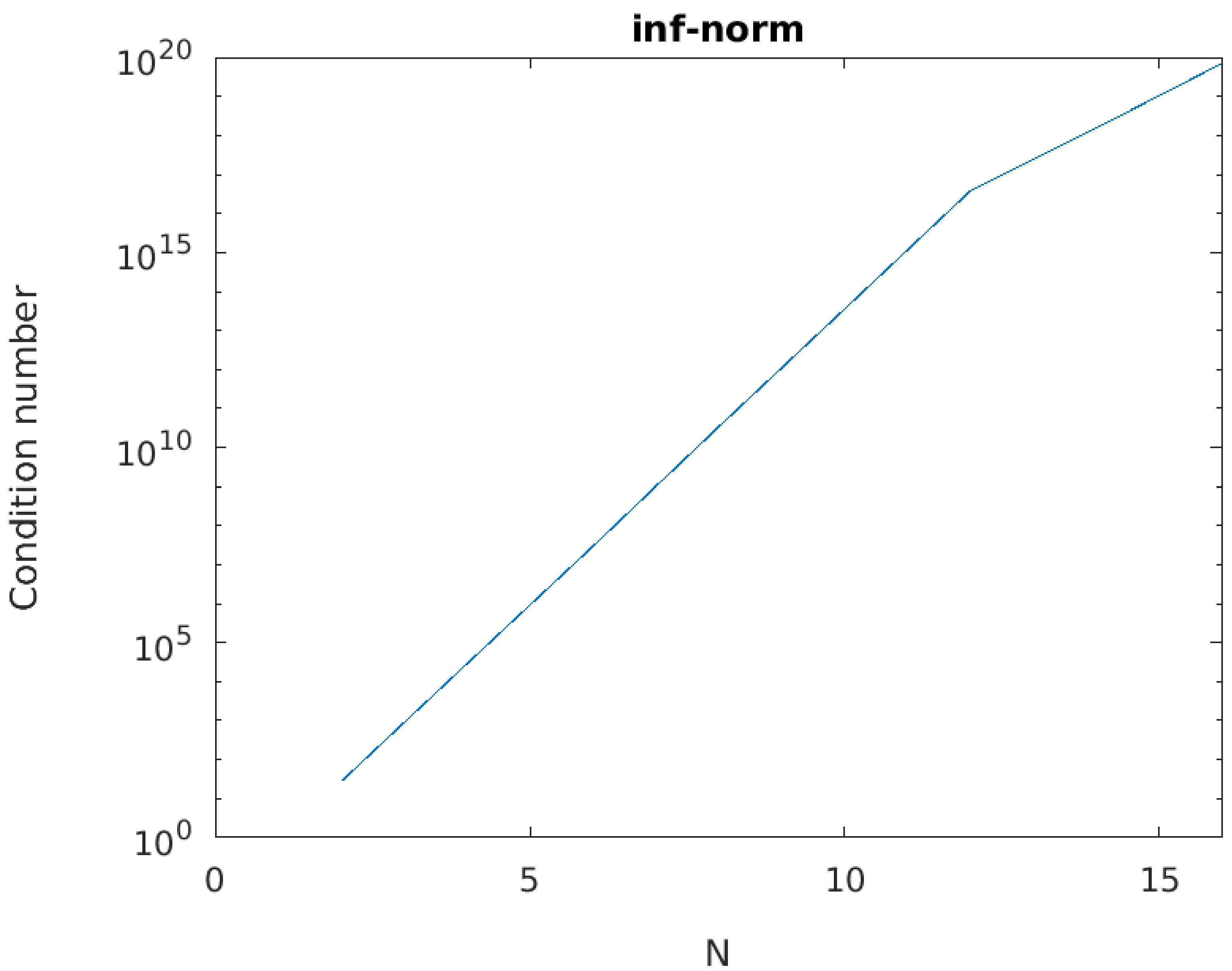
Ques.**1**

- The condition number of a Hilbert matrix grows exponentially with size. The sizes for which the condition number is 10^17 or more are extremely ill-conditioned and behave like singular matrices numerically,and hence the graph is not linear.




1-norm
Condition number
N
10^{20}
10^{15}
10^{10}
10^{5}
10^{0}
0
5
10
15


inf-norm
Condition number
N
10^{20}
10^{15}
10^{10}
10^{5}
10^{0}
0
5
10
15

Ques.**2**

For n = 8

x	x1	x2	x3
0.814723686393179	0.814723686422228	0.814723686442449	0.814723686426444
0.905791937075619	0.905791935561067	0.905791935510933	0.905791935345122
0.126986816293506	0.126986835712850	0.126986829563975	0.126986838436625
0.913375856139019	0.913375752416219	0.913375854492188	0.913375738066455
0.632359246225410	0.632359522708011	0.632359415292740	0.632359560526186
0.097540404999410	0.097540016840601	0.097540318965912	0.097539964247204
0.278498218867048	0.278498493353370	0.278498440980911	0.278498530249743
0.546881519204984	0.546881442165746	0.546881422400475	0.546881431880088

cond(H)	$\frac{\|\|x-x_2\|\|}{\|\|x\|\|}$	$\frac{\|\|x-x_2\|\|}{\|\|x\|\|}$	$\frac{\|\|x-x_3\|\|}{\|\|x\|\|}$
1.0e+10 *			
1.525757556903566	0.000000000000000	0.000000000000000	0.000000000000000

For n = 10

x	x1	x2	x3
0.957506835434298	0.957506834813467	0.957506834995002	0.957506835436017
0.964888535199277	0.964888589270883	0.964888669550419	0.964888535934643
0.157613081677548	0.157611921436303	0.157611250877380	0.157613051112744
0.970592781760616	0.970603400599128	0.970598220825195	0.970593170056121
0.957166948242946	0.957115991081083	0.957046508789062	0.957164657357426
0.485375648722841	0.485516492437505	0.485809326171875	0.485382968292224
0.800280468888800	0.800048258276981	0.800048828125000	0.800267019682035
0.141886338627215	0.142111729059102	0.142364501953125	0.141900544579410
0.421761282626275	0.421642485881836	0.421798706054688	0.421753267985452
0.915735525189067	0.915761744001462	0.915794372558594	0.915737395996441

cond(H)	$\frac{\|\|x-x_2\|\|}{\|\|x\|\|}$	$\frac{\|\|x-x_2\|\|}{\|\|x\|\|}$	$\frac{\|\|x-x_3\|\|}{\|\|x\|\|}$
1.0e+13 *			
1.602491697370065	0.000000000000000	0.000000000000000	0.000000000000000

For n = 12

x	x1	x2	x3
0.792207329559554	0.792207243483492	0.792207262478769	0.792207253674061
0.959492426392903	0.959503421835453	0.959502577781677	0.959502107900038
0.655740699156587	0.655392822358455	0.655445098876953	0.655434728361295
0.035711678574190	0.040473789450707	0.040222167968750	0.039896110490523
0.849129305868777	0.814088142132945	0.816406250000000	0.818365034103620
0.933993247757551	1.088423191048246	1.082031250000000	1.069472149161244
0.678735154857773	0.247383165527530	0.296875000000000	0.300572454638449
0.757740130578333	1.540154098353333	1.515625000000000	1.443259662512051
0.743132468124916	-0.175734893194733	-0.031250000000000	-0.061500601212604
0.392227019534168	1.066170568902395	0.945312500000000	0.982091720284890
0.655477890177557	0.374922817576178	0.445312500000000	0.410035254896954
0.171186687811562	0.221789732210819	0.214843750000000	0.215438216690404

cond(H)	$\frac{\|\|x-x_2\|\|}{\|\|x\|\|}$	$\frac{\|\|x-x_2\|\|}{\|\|x\|\|}$	$\frac{\|\|x-x_3\|\|}{\|\|x\|\|}$
1.0e+16 *			
1.628373038825766	0.000000000000000	0.000000000000000	0.000000000000000

a. In some cases x1, x2, x3 are not even accurate up to 1 significant digit. So in total 16 digits are lost.
b. Results are almost the same because the methods used to generate them is same.
c. Given s = 16. For n = 8,t = 10,For n = 10,t = 13, n=12, t=16. The rule of thumb analysis is followed at s-t has the same number of significant digits.

Ques.**3**

$$\frac{||r||}{||b||} = \quad 1.738970e-16$$

$$\frac{||x-\hat{x}||}{||x||} = \quad 9.185740e-05$$

- This shows that 1st Norm (||r||/||b||) being small doesn't imply the perturbation in the solution of the 2nd norm will be small.

Ques.**4**

For n = 32

GEPP Method

cond(A)	$\frac{\|\|r\|\|}{\|\|b\|\|}$	$\frac{\|\|x-\hat{x}\|\|_\infty}{\|\|x\|\|_\infty}$
1.421555e+01	1.123154e-08	1.151686e-08

QR method

1.421555e+01	4.514876e-16	4.133005e-16

For n = 64

GEPP Method

cond(A)	$\frac{\|\|r\|\|}{\|\|b\|\|}$	$\frac{\|\|x - \hat{x}\|\|_\infty}{\|\|x\|\|_\infty}$
2.860298e+01	7.724501e-01	5.423152e-01

QR method

2.860298e+01	8.570133e-16	1.107166e-15

a) For both n, QR seems to give lower forward error.
b) The rule of thumb is predicting well for the QR method as s = 16, t = 1 and we have s - t = 15. and our is correct upto 15 significant digits as seen in forward error.
c) for both n, QR has lower
d) Given that QR method has lower forward error compared to GEPP. It is a more stable algorithm.

Ques.**5**

N	GENP norm(L*U - A)	LU norm(L*U - P*A)
20	8.562682e-02	8.363228e+02
40	1.197000e-01	7.037776e+03
60	1.311398e-01	1.362659e+04
80	3.164670e+00	3.324881e+04
100	9.142869e-02	4.896241e+04

- For all n GENP produces larger norm.
