Department of Mathematics

Indian Institute of Technology Guwahati

End Semester Examination April 20, 2021

MA 473 Computational Finance

Time: **10:00 – 11:30 Hrs.** Marks: **30**

There are ***FOUR*** questions in this paper. Answer all questions.

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1. (i) Obtain the 1D heat-conduction equation $u_{\bar{\tau}} = u_{xx}$ from the Black-Scholes PDE by using a suitable transformation. (**4** marks)

   (ii) Obtain the analytical solution of the following transformed 1D parabolic PDE: (**6** marks)

$$\begin{cases} \dfrac{\partial u}{\partial \bar{\tau}} = \dfrac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \ \bar{\tau} > 0 \\ u(x, 0) = u_0(x). \end{cases}$$

2. Consider the American put option, by using the *finite element method*, obtain the minimization problem

$$\min_{v \in \mathcal{K}} I(y; v) = 0, \quad \text{where } I(y; v) = \int_{x_{\min}}^{x_{\max}} \left(\frac{\partial y}{\partial \tau} \cdot (v - y) + \frac{\partial y}{\partial x} \left(\frac{\partial v}{\partial x} - \frac{\partial y}{\partial x} \right) \right) dx,$$

   where $\mathcal{K}$ is the family of admissible functions. (**3** marks)

   (a) Further, obtain the following fully discrete version of the above minimization problem:

$$(v^{(n+1)} - w^{(n+1)}))^T \left(C w^{(n+1)} - r \right) \geq 0,$$

   where $C := B + \triangle\tau\,\theta\, A$, and $r := (B - \triangle\tau\,(1-\theta) A) w^{(n)}$, here A and B are tri-diagonal matrices obtained from the integrals involving the the basis functions and their derivatives by applying a numerical quadrature formula. Determine the matrices A and B. (**4** marks)

   (b) Also write down the fully discretized version of the American put option by the finite difference method. (**2** marks)

   (c) Prove that the discretized problems obtained in *(a)* and *(b)* are equivalent by showing that the solution of the finite difference method is the solution of the finite element method and vice-versa. (**3** marks)

3. Define the ***strong*** and ***weak-order of convergences*** of a numerical scheme for a SDE and provide some schemes with their order of convergence. (**2** marks)

4. Derive the second-order *Milstein scheme* for the following SDE, without omitting any intermediate steps: (**6** marks)

$$\begin{cases} dX(t) = a(X(t))dt + b(X(t))dW(t), \\ X(0) = X_0. \end{cases}$$
