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Sol^n ①: Considering single-server system type:
Arrival time will be Markovian.
and service times are exponentially distributed.

The service rates: $\mu = \frac{1}{E[T_S]} = \frac{1}{\cancel{0.8/60}} = \cancel{120 h}$

$= \frac{1}{(30/60)/60} = 120 h^{-1}$.

As we're not given the no. of customers per grp is an iid process, and also distribution not given.
Hence
let $q_1\ q_2\ q_3\ q_4$ denotes prob of randomly arriving grp containing 1,2,3,4 customers.

Let λ_G denote the Poisson grp arrival rate. thus individual rates are also poisson processes, based on poisson split property

$$\lambda_G q_1 \quad \lambda_G q_2 \quad \lambda_G q_3 \quad \lambda_G q_4$$

$\forall i \in \{1,2,3,4\}$

$\Rightarrow$ avg rate of arrivals of grp's of type i $= \lambda_G q_i$

$\Rightarrow$ avg rate of arrival of customers (type i) $= \lambda_G q_i i$

Using the data given in the problem:

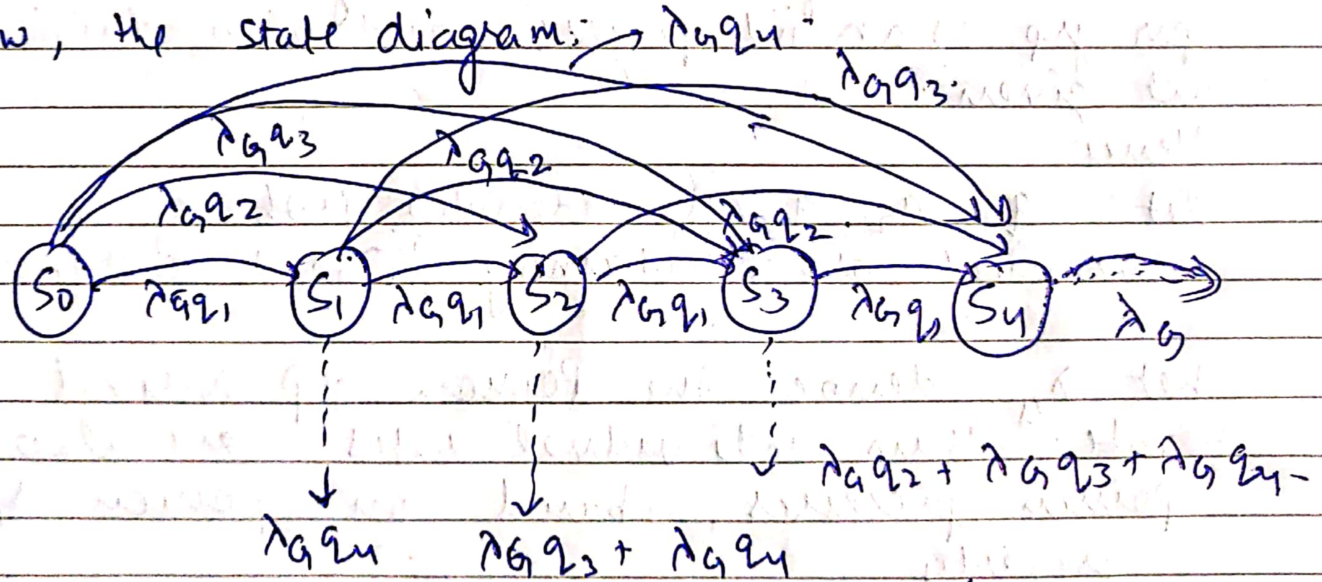
$$\lambda_G q_1(1) = 10\% \text{ of } 75 = 7.5$$
$$\lambda_G q_2(2) = 20\% \text{ of } 75 = 15$$
$$\lambda_G q_3(3) = 30\% \text{ of } 75 = 0.3 \times 75$$
$$\lambda_G q_4(4) = 40\% \text{ of } 75 = 0.4 \times 75$$

as we can see that $q_1 = q_2 = q_3 = q_4 \Rightarrow q = 1/4$.

$\Downarrow$

$$\boxed{\lambda_G = 30 \text{ grps/hr}}$$

Now, the state diagram:



avg. no. of customers in system and in queue.

$$\bar{N}_{queue} = 1 \cdot P_2 + 2 \cdot P_3 + 3 \cdot P_4$$

$$\bar{N} = 1 \cdot P_1 + 2 \cdot P_2 + 3 \cdot P_3 + 4 \cdot P_4.$$

Scanned by CamScanner

∴ System is homogenous.

$$\lambda_{eff} = P_0 \lambda_G \cdot (q_1 + 2q_2 + 3q_3 + 4q_4) + P_1 \lambda_G (q_1 + 2q_2 + 3q_3) + P_2 \lambda_G (q_1 + 2q_2) + P_3 \lambda_G (q_1)$$

system time $\Rightarrow \overline{N} = \lambda_{eff} E(T)$

avg. waiting time $= \overline{W} = E[T] - E[T_s]$

Prob(system is full) $= P_4$.
(seen by observer independently)

$\Rightarrow$ Prob(customer of grp k doesn't join queue) = P(whole particular grp doesn't join queue)

Pr(a random grp 1 is blocked) = P_4.
Pr(a random grp 2 is blocked) = $P_4 + P_3$
Pr(a random grp 3 is blocked) = $P_4 + P_3 + P_2$
Pr(a random grp 4 is blocked) = $P_4 + P_3 + P_2 + P_1$.

from using given data:

$\Rightarrow$

Pr(a random customer blocked) =
$40\% (P_1 + P_2 + P_3 + P_4) + 30\% (P_2 + P_3 + P_4) + 20\% (P_3 + P_4) + 10\% \, P_4$.

Pr{ a random grp is blocked) =

$\sum_{i=1}^{4}$ Pr{a random grp has i customers}. Pr{ a random grp i is blocked}

$= P_4 + P_3\, 3/4 + P_2\, 1/2 + P_1\, 1/4.$

∴ the arrivals are homogeneous, grps see state 0,1 with prob P_0, P_1.

$$\overline{W_3} = \frac{P_0}{P_0+P_1} W_3^0 + \frac{P_1}{P_0+P_1} W_3^1$$

where

$$W_3^0 = \frac{1}{3}(0.5 + 1 + 0)$$

$$W_3^1 = \frac{1}{3}(0.5 + 1 + 1.5)$$