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Rh Modi

Rohan Modi 180123039

Q1 $f(x,\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & \text{if } x>0 \\ 0 & \text{otherwise} \end{cases}$

$X_{(1)} = \min\{X_1, \dots, X_n\}$

$T = \sum_{i=1}^{n} X_i$

T is a complete sufficient statistic for θ

Let $W = E(X_{(1)} | T)$

By Lehman scheffe theorem

W is the UMVUE of $E_\theta(X_{(1)})$

Now $E_\theta(X_{(1)}) = \frac{\theta}{n}$

$\therefore$ W = UMVUE of $E_\theta(X_{(1)})$ = UMVUE of $\frac{\theta}{n}$

Consider the statistic $N = \frac{\sum X_i}{n^2}$

Now, $E\left(\frac{\sum X_i}{n^2}\right) = \frac{1}{n^2} \times n \times E(X_1) = \frac{\theta}{n}$

$$\boxed{\text{So, } W = \frac{\sum X_i}{n^2}}$$

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3) As pmf of Bernoulli dist = $p^x(1-p)^{1-x}$

So Likelihood function:

$$L(p) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i}$$

Log-Likelihood function:

$$LL(p) = \sum_{i=1}^{n} \left(X_i \ln p + (1-X_i)\ln(1-p)\right)$$

$$LL(\theta) = \sum_{i=1}^{n} \left(-X_i \ln(1+e^{\theta}) + (1-X_i)\ln(e^{\theta}) - (1-X_i)\ln(1+e^{\theta})\right)$$

$$= \sum_{i=1}^{n} \left[\theta(1-X_i) - \ln(1+e^{\theta})\right]$$

$$\frac{\delta}{\delta\theta} LL(\theta) = \sum_{i=1}^{n} \left[(1-X_i) - \frac{1}{1+e^{\theta}} \times e^{\theta}\right]$$

$$0 = n - \sum_{i=1}^{n} X_i - \frac{ne^{\theta}}{1+e^{\theta}}$$

$$\therefore \frac{e^{\theta}}{1+e^{\theta}} = \sum_{i=1}^{n} X_i$$

$$\therefore \hat{\theta} = \ln\left(\frac{n}{\sum_{i=1}^{n} X_i} - 1\right)$$

$$\boxed{\frac{\delta^2}{\delta\theta^2} LL(\theta) = \frac{-ne^{\theta}}{(1+e^{\theta})^2}}$$

But as It is decreasing funct
No MLE exist.