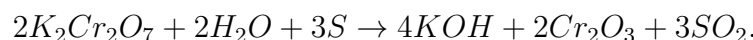


Lab 7: MA 322

Date: 16/03/2021

Submission date: By 5 pm on 22/03/2021

1. The irreversible chemical reaction in which two molecules of solid potassium dichromate ($K_2Cr_2O_7$), two molecules of water (H_2O), and three atoms of solid sulfur (S) combine to yield three molecules of the gas sulfur dioxide (SO_2), four molecules of solid potassium hydroxide (KOH), and two molecules of solid chromic oxide (Cr_2O_3) can be represented symbolically by the stoichiometric equation:



If n_1 molecules of $K_2Cr_2O_7$, n_2 molecules of H_2O , and n_3 molecules of S are originally available, the following differential equation describes the amount $x(t)$ of KOH after time t :

$$\frac{dy}{dx} = k \left(n_1 - \frac{x}{2} \right)^2 \left(n_2 - \frac{x}{2} \right)^2 \left(n_3 - \frac{3x}{4} \right)^3,$$

where k is the velocity constant of the reaction. If $k = 6.22 \times 10^{-19}$, $n_1 = n_2 = 2 \times 10^3$, and $n_3 = 3 \times 10^3$, use the Runge-Kutta method of order four to determine how many units of potassium hydroxide will have been formed after $0.2s$?

2. Show that the Runge-Kutta method of order second and the Modified Euler method give the same approximations to the initial-value problem

$$\frac{dy}{dt} = -y + t + 1, \quad 0 \leq t \leq 1, \quad y(0) = 1.$$

for any choice of h . Why is this true?

3. Use the Modified Euler method to approximate the solutions to each of the following initial-value problems, and compare the results to the actual values.

- (a) $\frac{dy}{dt} = \frac{1+t}{y+1}$, $1 \leq t \leq 2$, $y(0) = 1$, with $h = 0.5$; actual solution $y(t) = \sqrt{t^2 + 6 + 2t} - 1$.
- (b) $\frac{dy}{dt} = \frac{\sin 2t - 2ty}{4 + \cos 2 - \cos 2t}$, $1 \leq t \leq 2$, $y(1) = 2$, with $h = 0.25$; actual solution $y(t) = \frac{t^2}{2t^2}$.

4. Consider the problem

$$\frac{dy}{dt} = y - t^2 + 1, \quad 0 \leq t \leq 2 \quad y(0) = 0.5.$$

Use Euler's method with $h = 0.025$, the Runge-Kutta second-order method with $h = 0.05$, and the Runge-Kutta fourth-order method with $h = 0.1$ and compared at the common mesh points of these methods $0.1, 0.2, 0.3, 0.4$, and 0.5 .

-
5. Consider the initial-value problem

$$\frac{dy}{dt} = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5.$$

Use the exact values given from $y(t) = (t+1)^2 - 0.5e^t$ as starting values and $h = 0.2$ to compare the approximations from

- (a) by the explicit Adams-Bashforth four-step method.
 - (b) the implicit Adams-Moulton three-step method.
6. Use each of the Adams-Bashforth methods to approximate the solutions to the following initial-value problems. In each case use starting values obtained from the Runge-Kutta method of order four. Compare the results to the actual values.
- (a) $\frac{dy}{dt} = \frac{2-2ty}{t^2+1}$, $0 \leq t \leq 1$, $y(0) = 1$, with $h = 0.1$; actual solution $y(t) = \frac{2t+1}{t^2+1}$.
 - (b) $\frac{dy}{dt} = \frac{y^2}{t+1}$, $1 \leq t \leq 2$, $y(1) = \ln(2)^{-1}$, with $h = 0.1$; actual solution $y(t) = \frac{-1}{\ln(t+1)}$.
 - (c) $\frac{dy}{dt} = \frac{y^2+y}{t}$, $1 \leq t \leq 3$, $y(1) = -2$, with $h = 0.2$; actual solution $y(t) = \frac{2t}{1-t}$.
7. Apply the Adams fourth-order predictor-corrector method with $h = 0.2$ and starting values from the Runge-Kutta fourth order method to the initial-value problem

$$\frac{dy}{dt} = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5.$$

END
