MA 322: Scientific Computing

Lecture - 4

Motivation to Finite Differences

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which yields relation (b).

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(Take other relations as Home Work.)

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x	$f(x)$	$\Delta f(x)$
x_0	$f(x_0)$	
	$\longrightarrow$	$\Delta f(x_0) = f(x_1) - f(x_0)$
x_1	$f(x_1)$	
	$\longrightarrow$	$\Delta f(x_1) = f(x_2) - f(x_1)$
x_2	$f(x_2)$	
$\vdots$	$\vdots$	$\vdots$
x_{n-1}	$f(x_{n-1})$	
	$\longrightarrow$	$\Delta f(x_{n-1}) = f(x_n) - f(x_{n-1})$
x_n	$f(x_n)$	

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- Using the fact $\Delta^2 f(x_i) = \Delta f(x_{i+1}) - \Delta f(x_i)$, we have following tabular representation for first and second order finite differences

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
x_0	$f(x_0)$		
	$\longrightarrow$	$\Delta f(x_0)$	
x_1	$f(x_1)$	$\longrightarrow$	$\Delta^2 f(x_0) = \Delta f(x_1) - \Delta f(x_0)$
	$\longrightarrow$	$\Delta f(x_1)$	
x_2	$f(x_2)$	$\longrightarrow$	$\Delta^2 f(x_1) = \Delta f(x_2) - \Delta f(x_1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{n-1}	$f(x_{n-1})$	$\longrightarrow$	$\Delta^2 f(x_{n-2}) = \Delta f(x_{n-1}) - \Delta f(x_{n-2})$
	$\longrightarrow$	$\Delta f(x_{n-1})$	
x_n	$f(x_n)$		

Tabular Representation of Forward Differences Contd..

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Example If $f(0) = 3,\ f(1) = 12,\ f(2) = 81,\ f(3) = 200$ and $f(4) = 100$, find $\Delta^4 f(0)$.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	3				
	$\longrightarrow$	9			
1	12	$\longrightarrow$	60		
				-10	
					-259
	$\longrightarrow$	69			
2	81	$\longrightarrow$	50		
				-269	
	$\longrightarrow$	119			
3	200	$\longrightarrow$	-219		
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4	100				

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The answer is given by *Fundamental Theorem of Finite Difference*.

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