Lab 5: MA 322

Date: 23/02/2021

Submission date: By 5 pm on 01/03/2021

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1. Approximate the following integrals using Gaussian quadrature with $n = 2$, and compare your results to the exact values of the integrals
   (a) $\int_1^{1.5} x^2 \ln x \mathrm{dx}$
   (b) $\int_0^{0.35} \frac{2}{x^2-4}\mathrm{dx}$
   (c) $\int_0^{\pi/4} x^2 \sin x \mathrm{dx}$

2. Let $x_1, \ldots, x_n \in [a, b]$, $w_1, \ldots, w_n \in \mathbb{R}$ be the nodes and the weight of a quadrature formula. Assume that $w_j < 0$ for some $j \in 1, \ldots, n$. Construct a continuous function $f : [a, b] \to \mathbb{R}$ such that $f(x) \geq 0$, $x \in [a, b]$, i.e.,
$$\int_a^b \mathrm{f(x)dx} > 0,$$
   but
$$\sum_{i=1}^n w_i f(x_i) < 0.$$

3. Use the two-point Gaussian quadrature rule to approximate
$$\int_{-1}^1 \frac{1}{x+2}\mathrm{dx}$$
   and compare the result with the trapezoidal and Simpson's rules.

4. Use the three-point Gaussian quadrature formula to evaluate
$$\int_0^1 \frac{1}{1+x}\mathrm{dx}.$$
   Compare this result with that obtained by Simpson's $\frac{1}{3}$ rule with $h = 0.125$.

5. There are two Newton-Cotes formulas for $n = 2$; namely,
$$\int_0^1 f(x)dx \approx \mathrm{a}f(0) + \mathrm{b}f(\frac{1}{2}) + \mathrm{c}f(1),$$
$$\int_0^1 f(x)dx \approx \alpha f(\frac{1}{4}) + \beta f(\frac{1}{2}) + \gamma f(\frac{3}{4}),$$
   Which is better?

6. Use the $n = 1, 2, 3, 4, 5$ point Gaussian quadrature formula to evaluate

$$\int_0^{\pi/2} \frac{\cos x \log(\sin x)}{1 + \sin^2 x} \mathrm{dx}$$

to 2 correct decimal places.

END