Sol^n 2 given $x_1, x_2 - - x_n \in [a,b]$ and
$\omega_1, \omega_2 - - \omega_n \in R$

Let $f[a,b] \to R$ a continous fx^n
such that $f(x) \geq 0 \;\; \forall x \in [a,b]$
and
$$\int_a^b f(x)\,dx > 0$$

So let's consider $k_1, k_2 - - k_m \in 1 - - n, m < n$
such that $\omega_{k_i} \geq 0 \;\; \forall \; 1 \leq i \leq m$.
means $k_1, k_2 - - k_m$ are those indices
where $\omega_i \geq 0$ and rest other are where
$\omega_i < 0$.

Consider a set $S = \{k_1, k_2 - -, k_m\}$

$$f(x) = \prod_{i \in S} (|x - x_i|)^2 \Rightarrow f(x) \geq 0 \;\; \forall x \in [a,b]$$
as $f(x)$ is modulus quadratic fxn.

a polynomial fx^n and continous.
$f(x) = 0 \;\; \forall i \in S. \longrightarrow (1)$
as $f(x) > 0, \; \forall i \notin S$ hence the integral
value > 0.
$$\int_a^b f(x)\,dx > 0 \rightarrow (i).$$

Consider $\sum_{i=1}^{n} \omega_i f(x_i) = \sum_{i \in S}^{n} \omega_i \underbrace{f(x_i)}_{=0} + \sum_{i \notin S} \omega_i f(x_i)$
(because of (1))

$$= \sum_{i \notin S} w_i f(x_i)$$

$\therefore$ $f(x_i) > 0 \;\forall\; i \notin S$.
and $w_i < 0 \;\forall\; i \notin S$.
hence each $w_i f(x_i) < 0 \;\forall\; i \notin S$.

$$\Rightarrow \sum_{i \notin S} w_i f(x_i) < 0 \;\forall\; i \notin S.$$

hence $\sum_{i=1}^{n} w_i f(x_i) < 0$.

Thus, there exists a continous fxn $f(x)$ & $x \in [a, b)$ such that $\int_a^b f(x)\,dx > 0$ but $\sum_{i=1}^{n} w_i f(x_i) < 0$.