

(7) (a) Let estimate be E .

$$E = \left| \int_a^b f(x) dx - T(h) \right|$$

$T(h) \rightarrow$ step size $h = (b-a)/N$.

\therefore we know that

$$\begin{aligned} E_h &= I(f) - T_n(f) \\ &= -\frac{h^2}{12} (b-a) f''(\theta) \end{aligned}$$

Let $f(x) = f(x) + \delta(x)$

$$\begin{aligned} E'_h &= I(f) - T_n(\hat{f}) \\ &= I(f) - T_n(f) - \frac{h}{2} \sum_{i=0}^{n-1} (\delta(x_i) + \delta(x_{i+1})) \\ &= E_h(f) - \frac{h}{2} \sum_{i=0}^{n-1} (\delta(x_i) + \delta(x_{i+1})) \end{aligned}$$

$$\Rightarrow E'_h \equiv -\frac{h^2}{12} (b-a) f''(\theta) - \frac{h}{2} \sum_{i=0}^{n-1} (\delta(x_i) + \delta(x_{i+1}))$$

$$\Rightarrow |E'_h| \leq \frac{h^2}{12} (b-a) \|f''(\theta)\|_\infty + (b-a)\delta$$

$$\because |\delta(x_i)| \leq \delta \quad \forall x_i$$