Lab 8: MA 322

Date: 23/03/2021

Submission date: By 5 pm on 31/03/2021

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1. Consider following BVP

$$\frac{dy}{dx} + y(x) = 1 + x, \quad y(0) = y(\pi/2) = 0$$

with exact solution

$$y(x) = 1 + x - \cos x - (1 + \pi/2)\sin x.$$

Use second order scheme to complete following table

h	$y(1/2)$	f.d. solution at 1/2	error	ratio of error
1/4				
1/8				
1/16				
1/32				
1/64				

Finally plot exact solution and finite difference solution for $h = 1/64$.

2. Consider the following BVP

$$-\frac{d^2u}{dx^2} = f(x) \quad u(0) = u(2\pi) = 0.$$

Consider the following finite difference scheme

$$-D_x^+ D_x^- U_j = f_j + \frac{h^2}{12} D_x^+ D_x^- f_j, \quad j = 2, 3, \ldots, N-1.$$

and $U_1 = U_N = 0$, where

$$D_x^+ D_x^- u(x_i) = \frac{u(x_{i-1}) - 2u(x_i) + u(x_{i+1})}{h^2})$$

Compute the local truncation error of the above scheme and show that it is $\mathrm{O}(h^4)$. Hence show that the scheme is fourth order accurate. Take $f(x) = \sin(x)$ so that the exact solution is $u(x) = \sin(x)$. Write a computer program to implement the above scheme. Solve the problem for $N = 10, 20, 40, 80, 160, 320$ grid points and compute error in maximum norm and discrete L^2 norm in each case. Plot the error versus N on a log-log plot and verify the fourth order accuracy in both the norms.

3. Consider following BVP

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + 1, \quad y(0) = 1, \;\; y(1) = 2(e-1)$$

with $h = 1/3$. If the exact solution is $y(x) = 2e^x - x - 1$, find the absolute errors at the nodal points using second order finite difference scheme.

4. Solve the boundary value problem

$$\frac{d^2y}{dx^2} - 10\frac{dy}{dx} = 0, \quad y(0) = 0, \;\; y(1) = 1$$

with $h = 0.25$, by using central difference approximation to $\frac{d^2y}{dx^2}$ and

   i. central difference approximation to $\frac{dy}{dx}$,
   ii. backward difference approximation to $\frac{dy}{dx}$,
   iii. forward difference approximation to $\frac{dy}{dx}$.

If the exact solution is $y(x) = (e^{10x} - 1)/(e^{10} - 1)$, compare the magnitudes of errors at the nodal points in the three methods.

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