

Lab Quiz 1: MA 322

Date: 07/04/2021

Submission time: 5-7 pm

1. **(2 points)** Apply bisection method to find the root of the function

$$f(x) = \sqrt{x} - 1.1$$

Starting from the interval $[0, 2]$, with $\text{altol} = 10^{-8}$ (absolute error tolerance).

- (a) How many iterations are required? Does the iteration count match the expectations, based on our convergence analysis?
 - (b) Convert this into a fixed point iteration and find the approximated value of the root of f with $\text{altol} = 10^{-8}$.
2. **(3 points)** The function $f(x) = \tan(\pi x) - 6$ has a zero at $(1/\pi) \arctan 6 \approx 0.447431543$. Let $x_0 = 0$ and $x_1 = 0.48$, and use ten iterations for each of the following methods to approximate this root. Which method is most successful and why?
- (a) Bisection method.
 - (b) Secant method.
3. **(2 points)** Draw the graph of a function having the following properties:
- (a) The function has exactly two fixed points.
 - (b) Give two choices of the initial guess x_0 and y_0 such that the corresponding sequences $\{x_n\}$ and $\{y_n\}$ have the properties that $\{x_n\}$ converges to one of the fixed point and the sequence $\{y_n\}$ goes away and diverges. Point out the first three terms of both the sequences on the graph.

4. **(3 points)** Use the Euler's methods and Runge-Kutta methods of order 2 and 4 to solve the IVP

$$\frac{dy}{dx} = 0.5(x - y) \quad \forall x \in [0, 3]$$

with initial condition $y(0) = 1$. Compare the solutions for $h = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ along with the exact solution

$$y(x) = 3 \exp(-x/2) + x - 2$$

End