Lab 3: MA 322

Date: 09/02/2021

Submission date: By 5 pm on 15/02/2021

---

1. Use the Newton forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

   i. $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$

   ii. $f(0.18)$ if $f(0.1) = -0.29004986$, $f(0.2) = -0.56079734$, $f(0.3) = -0.81401972$, $f(0.4) = -1.0526302$

2. Use the Newton backward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

   i. $f(-1/3)$ if $f(-0.75) = -0.07181250$, $f(-0.5) = -0.02475000$, $f(-0.25) = 0.33493750$, $f(0) = 1.10100000$

   ii. $f(0.25)$ if $f(0.1) = -0.62049958$, $f(0.2) = -0.28398668$, $f(0.3) = 0.00660095$, $f(0.4) = 0.24842440$

3. A fourth-degree polynomial $P(x)$ satisfies $\Delta^4 P(0) = 24$, $\Delta^3 P(0) = 6$, and $\Delta^2 P(0) = 0$, where $\Delta P(x) = P(x+1) - P(x)$. Compute $\Delta^2 P(10)$.

4. The following data are part of a table for $g(x) = \frac{\sin x}{x^2}$.

x = 0.1	0.2	0.3	0.4	0.5
g(x) = 9.9833	4.9667	3.2836	2.4339	1.9177

   Calculate $g(0.25)$ as accurately as possible

   i. by forward difference interpolating directly in this table,

   ii. by first tabulating $xg(x)$ and then forward difference interpolating in that table,

   iii. explain the difference between the results in (i) and (ii) respectively.

5. i. Show that the cubic polynomials

$$P(x) = 3 - 2(x+1) + 0(x+1)(x) + (x+1)(x)(x-1)$$

   and

$$Q(x) = -1 + 4(x+2) - 3(x+2)(x+1) + (x+2)(x+1)(x)$$

   both interpolate the data

$$f(-2) = -1, f(-1) = 3, f(0) = 1, f(1) = -1, f(2) = 3$$

ii. Why does part (i) not violate the uniqueness property of interpolating polynomials?

6. The following data are given for a polynomial $P(x)$ of unknown degree.

$$P(0) = 4, P(1) = 9, P(2) = 15, P(3) = 18$$

Determine the coefficient of x^3 in $P(x)$, if all fourth-order forward differences are 1.

7. For a function f , the Newton divided-difference formula gives the interpolating polynomial

$$P(x) = 1 + 4x + 4x(x - 0.25) + \frac{16}{3}x(x - 0.25)(x - 0.5),$$

on the nodes $x_0 = 0, x_1 = 0.25, x_2 = 0.5$, and $x_3 = 0.75$. Find $f(0.75)$.

END