Name: Naman Goyal
Roll No: 180123029

Sol^n (2) BVP::
$$-\frac{d^2u}{dx^2} = f(x) \qquad u(0) = u(2\pi) = 0$$

from Taylor's expansion:

$$u(x_{i+1}) = u(x_i+h) = u(x_i) + u'(x_i)h + \frac{1}{\lfloor 2}u''(x_i)h^2 + \frac{1}{\lfloor 3}u'''(x_i)h^3 + \frac{1}{\lfloor 4}u''''(x_i)h^4 + O(h^6) \longrightarrow (A)$$

and we also have

$$u(x_{i-1}) = u(x_i-h) = u(x_i) + u'(x_i)(-h) + \frac{1}{\lfloor 2}u''(x_i)(-h)^2 + \frac{1}{\lfloor 3}u'''(x_i)(-h)^3 + \frac{1}{\lfloor 4}u''''(x_i)(-h)^4 + O((-h)^6) \longrightarrow (B)$$

(A) + (B) $\Rightarrow$

$$\frac{u(x_{i+1}) + u(x_{i-1}) - 2u(x_i)}{h^2} = u''(x_i) + \frac{u''''(x_i)h^2}{12} + O(h^4)$$
$$= D_x^+ D_x^- u(x_i)$$

differentiating the given BVP 2 times:

$$-\frac{d^4u}{dx^4} = f''(x) \longrightarrow (C)$$

from (C) we have:

$$-D_x^+ D_x^- U(x_i) = f_{x_i} + \frac{h^2}{12} f''(x_i)$$

$$\therefore \quad f''(x_i) = \frac{f(x_{i+1}) + f(x_{i-1}) - 2f(x_i)}{h^2} + O(h^2)$$

$$\Rightarrow \quad -D_x^+ D_x^- U(x_i) = f_i + \frac{h^2}{12} D_x^+ D_x^- f_i + O(h^4)$$

$\Rightarrow$ Truncation Errors $= O(h^4)$.