Lab 4: MA 322

Date: 16/02/2021

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1. Apply Mid-point, Trapezoidal and Simpson methods to evaluate
   (a) $\mathrm{I} = \int_0^{\pi/2} \frac{\cos(x)}{1+\cos^2(x)}\mathrm{dx}$ (exact value ≈ 0.623225)
   (b) $\mathrm{I} = \int_0^{\pi} \frac{1}{5+4\cos(x)}\mathrm{dx}$ (exact value ≈ 1.047198)
   (c) $\mathrm{I} = \int_0^{1} \exp(-x^2)\mathrm{dx}$ (exact value ≈ 0.746824)
2. A function f has the values shown below:

x	1	1.25	1.50	1.75	2
f(x)	10	8	7	6	5

   (a) Use trapezoidal rule to approximate $\int_1^2 f(x)\mathrm{dx}$.
   (b) Use Simpson's rule to approximate $\int_1^2 f(x)\mathrm{dx}$.
3. Use composite Simpson's and composite Trapezoidal rules to obtain an approx-mate value for the improper integral
$$\int_0^{\infty} \frac{1}{x^2+9}\mathrm{dx}, \quad \text{with } n = 4.$$
4. Compute the integral
$$\int_{-1}^{1} \frac{1}{x+2}\mathrm{dx}$$
using the trapezoidal and Simpson's rules and compare the results with help of plots.
5. Determine the values of n and h required to approximate
$$\int_0^{2} \frac{1}{x+4}\mathrm{dx}$$
to within 10^{-5} and compute the approximation. Use
   (a) Composite Trapezoidal rule.
   (b) Composite Simpson's rule.
   (c) Composite Midpoint rule.

6. Write a program that approximates $\int_a^b f(x)\mathrm{dx}$ by the composite trapezoidal rule $T(h)$. Starting with $h = (b-a)$ to compute

$$\frac{|T(h) - T(\frac{h}{2})|}{|T(\frac{h}{2})|}$$

and reduce h by a factor of 2 until $\frac{|T(h)-T(\frac{h}{2})|}{|T(\frac{h}{2})|} < 10^{-6}$.

Your program should return a table with $\frac{h}{2}$, $T(\frac{h}{2})$, $\frac{|T(h)-T(\frac{h}{2})|}{|T(\frac{h}{2})|}$ and it should return the total number of function evaluations $f(x)$. Your program should reuse computed function values as much as possible.

Apply your program to approximate the integrals

(a) $\int_0^3 \frac{x}{1+x^2}\mathrm{dx} = \frac{1}{2}\ln 10$,

(b) $\int_0^{0.95} \frac{1}{1-x}\mathrm{dx} = \ln 20$,

(c) $\int_0^{\pi/2} \frac{1}{\sqrt{1-m\sin^2 x}}\mathrm{dx}, \quad m = 0.5, 0.8, 0.95$

7. Suppose that the function $f : \mathbb{R} \to \mathbb{R}$ cannot be evaluated exactly, but instead of $f(x)$ a value $\hat{f}(x) = f(x) + \delta(x)$ is computed, where $|\delta(x) \leq \delta$ for all $x \in \mathbb{R}$. Suppose one is interested in the integral

$$\int_a^b f(x)\mathrm{dx}$$

and wants to approximate the integral by composite trapezoidal rule $T(h)$ with step size $h = (b-a)/N$.

(a) Derive an estimate for the error

$$\left| \int_a^b f(x)\mathrm{dx} - T(h) \right|$$

when inexact function evaluations are used.

(b) Given δ, what would the range of "reasonable" step sizes h be? Justify your answer. Demonstrate your findings by approximating

$$\int_0^1 x^3\mathrm{dx},$$

where instead of

$$f(x) = x^3,$$

the function

$$\hat{f}(x) = x^3 + 0.01 * rand$$

is used in the composite trapezoidal formula.

END