Lab: MA 322, Date: 11/1/21

1. The iteration
$$x_{n+1} = \frac{x_n(x_n^2 + 3a)}{3x_n^2 + a},\ a > 0,$$
converges to $\sqrt{a}$. For $a = 2$, determine
   (a) Number of iterations n such that $|x_{n+1} - x_n| \leq 10^{-5}$.
   (b) Determine the order of convergence assuming $\sqrt{2} = 1.4143$.

2. Let $f(x) = \tan(\pi - x) - x$ and consider the equation $f(x) = 0$. Now, we wish to determine the approximate root for the equation in $[1.6, 3]$ using the following algorithm.
Step 1: Divide the interval into n equal parts by the points
$$x_0 = 1.6,\ x_1 = x_0 + h, \ldots, x_n = x_{n-1} + h = 3.$$
Step 2: Then determine the values of $f(x_k),\ k = 0, 1, \ldots, n$ and set that value of x_k to be the root for which $|f(x_k) - 0|$ is minimum.

3. Consider the equation
$$\frac{x}{2} - \sin x = 0.$$
Use bisection method to find an approximate root in the interval $[\pi/2,\ \pi]$. Then modify the approximation using Newton's method which is correct up to seven decimal places.

4. Consider the equation
$$\frac{x}{2} - \sin x = 0.$$
Use bisection method to find an approximate root in the interval $[\pi/2,\ \pi]$. Then modify the approximation using fix point iteration and calculate the order of convergence.

5. Consider $f(x) = 0,\ f(x) = e^{-x}(x^2 + 5x + 2) + 1$. Find an approximate root using secant method with $x_0 = -1$ and the stopping criterion $|x_{n+1} - x_n| \leq 10^{-5}|x_{n+1}|$.

6. Consider $f(x) = 0,\ f(x) = e^{-x}(x^2 + 5x + 2) + 1$. Use Bisection method to find an approximation of actual root. Then modify the root using following iterative scheme
$$x_{n+1} = \frac{(x_0 f(x_k) - x_k f(x_0))}{f(x_k) - f(x_0)}.$$
Determine the order of convergence.

7. Consider the equation
$$\frac{x}{2} - \sin x = 0.$$
Use bisection method to find an approximate root in the interval $[\pi/2,\ \pi]$. Then modify the root using following iterative scheme
$$x_{n+1} = \frac{(x_0 f(x_k) - x_k f(x_0))}{f(x_k) - f(x_0)}.$$
Determine the order of convergence.

8. Consider $f(x) = 0,\ f(x) = e^{-x}(x^2 + 5x + 2) + 1$. Use Bisection method to find an approximation of actual root. Then modify the root using following iterative scheme

$$x_{n+1} = x_n - \frac{f(x_k)^2}{(f(x_k) - f(x_k - f(x_k)))}.$$

Determine the order of convergence.