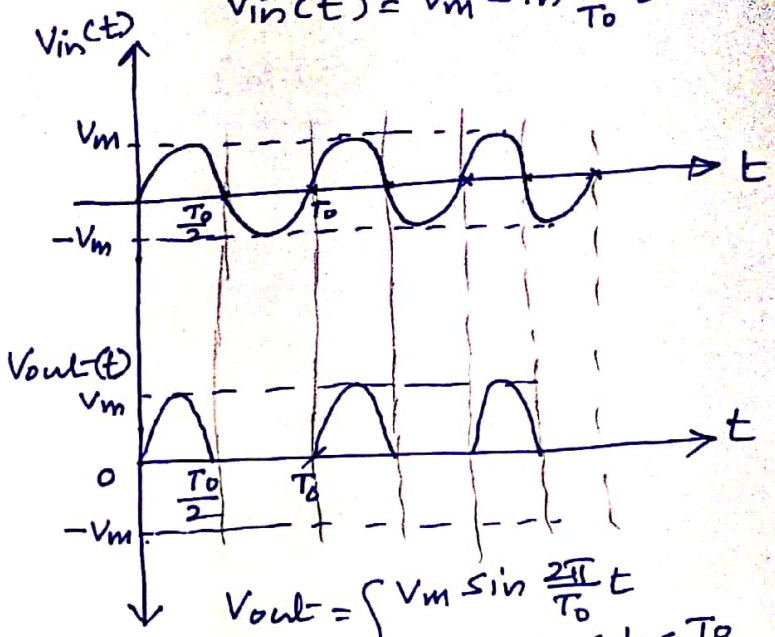
Half Wave Rectifier

1. Average Voltage

Indicates the dc content in the rectifier output

$$V_{avg} = \frac{1}{T_0}\int_0^{T_0} V_{out}(t)\,dt$$



$$V_{out} = \begin{cases} V_m \sin\frac{2\pi}{T_0}t & 0 \le t \le \frac{T_0}{2} \\ 0 \rightarrow & \frac{T_0}{2} \le t \le T_0 \end{cases}$$

$$V_{avg} = \frac{1}{T_0}\left\{\int_0^{\frac{T_0}{2}} V_m \sin\left(\frac{2\pi}{T_0}t\right)dt + \int_{\frac{T_0}{2}}^{T_0}(0)\,dt\right\}$$

$$V_{avg} = \frac{1}{T_0}\,\frac{T_0}{2\pi}\,V_m\left[-\cos\left(\frac{2\pi}{T_0}t\right)\right]_0^{T_0/2}$$

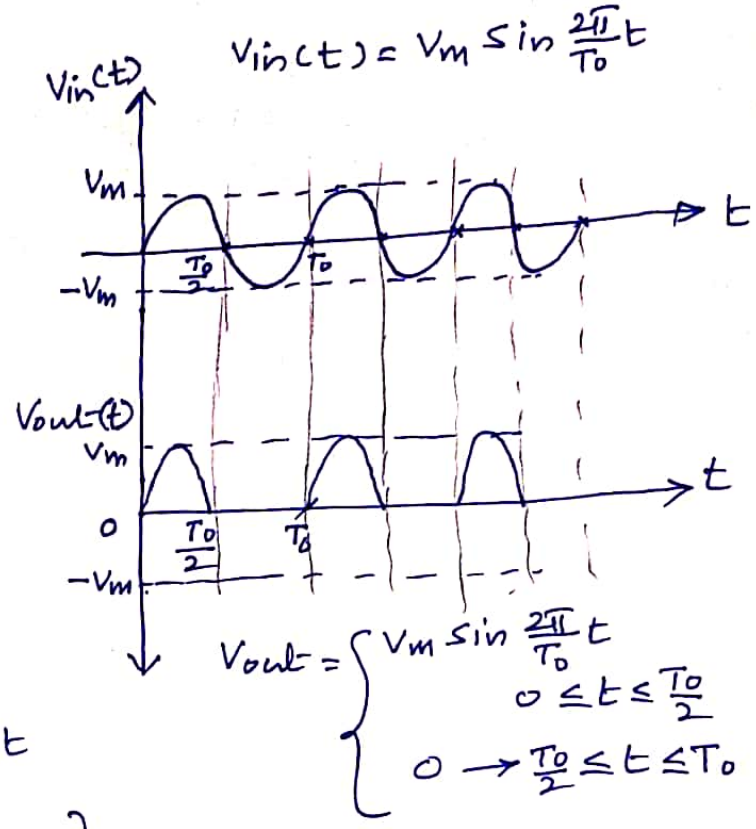
$$= \frac{V_m}{2\pi}\left[\left[-\cos\frac{2\pi}{T_0}\cdot\frac{T_0}{2}\right] - \left[-\cos\frac{2\pi}{T_0}(0)\right]\right]$$

$$= \frac{V_m}{2\pi}\left[-\cos\pi + \cos(0)\right] \qquad \cos\pi = -1$$

$$= \frac{V_m}{2\pi}\left[-(-1)+1\right] = \frac{V_m}{2\pi}\times 2 = \frac{V_m}{\pi}.$$

$$\boxed{V_{avg} = V_{dc} = \frac{V_m}{\pi}}$$

Half Wave Rectifier



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$$V_{avg} = \frac{1}{T_0} \frac{T_0}{2\pi} V_m \left[-\cos\left(\frac{2\pi}{T_0} t\right) \right]_0^{T_0/2}$$

$$= \frac{V_m}{2\pi} \left[\left[-\cos \frac{2\pi}{T_0} \cdot \frac{T_0}{2} \right] - \left[-\cos \frac{2\pi}{T_0}(0) \right] \right]$$

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$$\boxed{V_{avg} = V_{dc} = \frac{V_m}{\pi}}$$

HW RECTIFIER

2. ~~Room~~

2. Root Mean Square Voltage (RMS)

Indicates the AC component

$$V_{rms} = \sqrt{\frac{1}{T_0}\int_0^{T_0} V_{out}^2(t)\,dt}$$

$$V_{rms} = \sqrt{\frac{1}{T_0}\left[\int_0^{T_0/2} V_m^2 \sin^2\left(\frac{2\pi}{T_0}t\right)dt + \int_{\frac{T_0}{2}}^{T_0}(0)\,dt\right]}$$

(zero)

$$\int_0^{T_0/2} V_m^2\left(\frac{1-\cos\left[2\left(\frac{2\pi}{T_0}\right)t\right]}{2}\right)dt$$

$$\frac{V_m^2}{2}\left\{\left[t\right]_0^{T_0/2} + \left(\sin\frac{4\pi}{T_0}t\right)_0^{T_0/2} \times \frac{T_0}{4\pi}\right]$$

$$\frac{V_m^2}{2}\left[\frac{T_0}{2} - \underbrace{\sin\frac{4\pi}{T_0}\cdot\frac{T_0}{2}}_{\sin 2\pi = 0} + \underbrace{\sin(0)}_{0}\right]$$

$$\frac{V_m^2}{2}\,\frac{T_0}{2}$$

$$V_{rms} = \sqrt{\frac{1}{T_0}\cdot\frac{V_m^2}{2}\cdot\frac{T_0}{2}} = \frac{V_m}{\sqrt{2}}\,\frac{V_m}{2}$$

$$V_{rms} = V_{ac} = \frac{V_m}{2}$$

3. Ripple factor : Measure of Purity of DC output
Indicates the ● Ripple in output with reference to V_{dc}.

$$R.F = \sqrt{\frac{V_{ac}^2 - V_{dc}^2}{V_{dc}^2}}$$

$$= \sqrt{\frac{V_{rms}^2 - V_{avg}^2}{V_{avg}^2}}$$

$$= \sqrt{\frac{\frac{V_m^2}{4} - \frac{V_m^2}{\pi^2}}{\frac{V_m^2}{\pi^2}}}$$

$$= \sqrt{\pi^2\left(\frac{1}{4} - \frac{1}{\pi^2}\right)}$$

$$= \sqrt{\frac{2}{\pi} \times \frac{\pi^2 - 4}{4\pi^2}} = \sqrt{\frac{\pi^2 - 4}{4}} = 1.2114$$

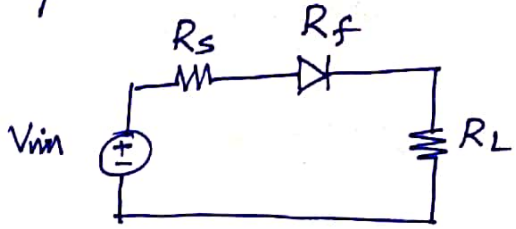
Ripple factor for Half wave Rectifier = 1.2114

4. Efficiency of Half wave Rectifier

$$\eta = \frac{\text{DC output Power}}{\text{Ac input Power}} = \frac{P_{DC}}{P_{AC}}.$$

$$P_{DC} = \frac{V_m^2}{\pi^2} \Big/ R_L = V_{dc}^2 / R_L$$

$$V_{dc} = \frac{V_m^2}{\pi^2}$$



$$P_{Ac} = V_{ac}^2 / R_L + R_f + R_s$$

$$P_{Ac} = \frac{V_{rms}^2}{R_L} = \frac{V_m^2}{4R_L}$$

[$R_f = R_s = 0$ for an ideal source & Diode]

$$\eta = \frac{\frac{V_m^2}{\pi^2} \times \frac{1}{R_L}}{\frac{V_m^2}{4R_L}} = \frac{4}{\pi^2} = 0.4053$$

Efficiency in Percentage = 40.53

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Halfwave Rectifier (5)

5. Transformer Utilization factor

$$TUF = \frac{\text{DC Power output}}{\text{Effective Voltage Ampere Rating of Transformer}}$$

Effective VA Rating of transformer is the average value of transformer primary and secondary VAs.

Rated voltage $= \frac{V_m}{\sqrt{2}}$

Rated current $= \frac{I_m}{2}$

$$TUF = \frac{V_{DC}^2/R_L}{\frac{V_m}{\sqrt{2}} \times \frac{I_m}{2}}$$

$$= \frac{\frac{V_m^2}{\pi^2 R_L}}{\frac{V_m}{2\sqrt{2}} \frac{V_m}{R_L + R_f + R_s}}$$

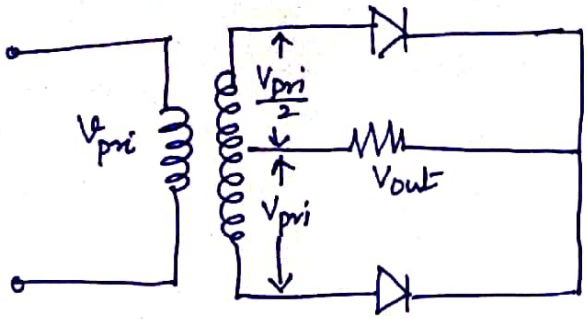
$R_f = R_s = 0$

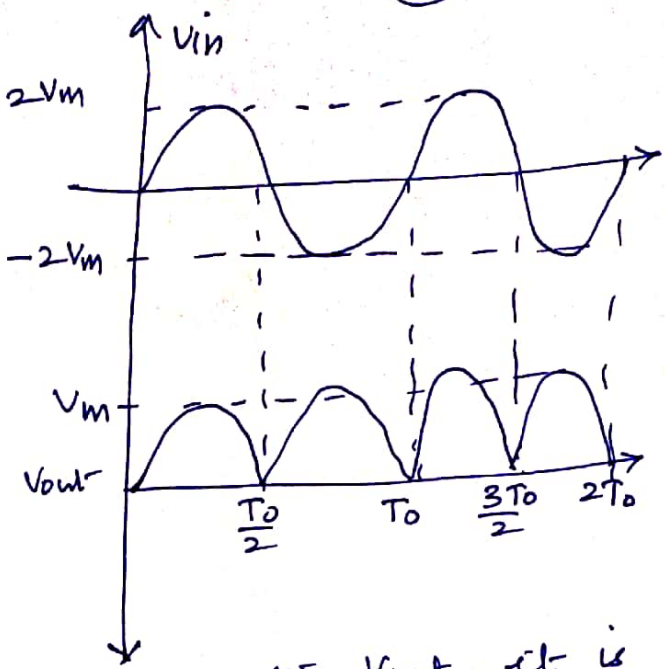
$$= \frac{V_m^2/\pi^2 R_L}{\frac{V_m^2}{2\sqrt{2}\, R_L}} = \frac{2\sqrt{2}}{\pi^2}$$

$$TUF = 0.286$$

$$TUF = 28.6\%$$

Full Wave Rectifier





Observe the V_{out}, it is same for every Half cycle (it is repeating for every $T_0/2$)

$$V_{pri} = 2V_m \sin\left(\frac{2\pi}{T_0}t\right)$$

$$V_{out} = V_m \sin\left(\frac{2\pi}{T_0}t\right) \qquad 0 \le t \le \frac{T_0}{2}$$

(1)

$$V_{avg} = \frac{1}{\frac{T_0}{2}} \int_0^{T_0/2} V_m \sin\left(\frac{2\pi}{T_0}t\right) dt$$

$$V_{avg} = \frac{2}{T_0} \cdot V_m \cdot \frac{T_0}{2\pi} \left[-\cos\frac{2\pi}{T_0}t\right]_0^{T_0/2}$$

$$= \frac{V_m}{\pi}\left[-\cos\frac{2\pi}{T_0}\cdot\frac{T_0}{2} - \left[-\cos(0)\right]\right]$$

$$= \frac{V_m}{\pi}\left[-(-1)+1\right] = \frac{2V_m}{\pi}$$

$$\boxed{V_{avg} = \frac{2V_m}{\pi}}$$

×100

RMS of FW

(2)

$$V_{rms} = \sqrt{\frac{1}{T_0/2}\int_0^{T_0/2} V_m^2 \sin^2\left(\frac{2\pi}{T_0}t\right) dt}$$

(7)

$$V_{rms} = \sqrt{\frac{2}{T_0}\cdot V_m^2 \cdot \left[\frac{1}{2}\int_0^{T_0/2} 1-\cos\left(\frac{4\pi}{T_0}t\right) dt\right]}$$

$$= \sqrt{\frac{2}{T_0} V_m^2 \frac{1}{2}\left(\frac{T_0}{2}-0\right)}$$

$$= \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

(3) Ripple factor

$$RF = \frac{\sqrt{V_{ac}^2 - V_{dc}^2}}{V_{dc}}$$

$$= \frac{\sqrt{\frac{V_m^2}{2} - \frac{4V_m^2}{\pi^2}}}{\frac{2V_m}{\pi}} = \frac{\sqrt{\frac{1}{2}-\frac{4}{\pi^2}}}{\frac{2}{\pi}} = 0.4834$$

(4) Rectifier Efficiency $= \frac{P_{dc}}{P_{AC}}$

$$P_{AC} = \frac{V_{ac(rms)}^2}{R_L} \qquad P_{dc} = \frac{V_{dc(avg)}^2}{R_L} = \frac{4V_m^2}{\pi^2 R_L}$$

$$= \frac{V_m^2}{2R_L}$$

$$\eta = \frac{\frac{4V_m^2}{\pi^2 R_L}}{\frac{V_m^2}{2R_L}} = \frac{8}{\pi^2} = 0.8106$$

η in Percentage $= 0.8106 \times 100$

$= 81.06\%$