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I-4

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Assignment-1

Ans1 $x_1 + 2x_2 + x_3 = 4$ — ①

$2x_1 + x_2 + 5x_3 = 5$ — ②

$m = 2$ $\quad$ $n = 3$

$$\underset{A}{\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 5 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 4 \\ 5 \end{bmatrix}}$$

Basic solution are obtained from following eq^n -

(1) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ — ① — ② $\qquad x_3 = 0$

(2) $\begin{pmatrix} 2 & 1 \\ 1 & 5 \end{pmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ — ② $\qquad x_1 = 0$

(3) $\begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ — ③ $\qquad x_2 = 0$

→ From eq^n ①

$x_1 + 2x_2 = 4 \quad \times 2$

$2x_1 + x_2 = 5$

$x_2 = 1$, $x_1 = 2$ when $x_3 = 0$ ← Ans

→ From eq^n ②

$2x_2 + x_3 = 4$

$x_2 + 5x_3 = 5 \quad \times 2$

$x_3 = \frac{2}{3}$, $x_2 = \frac{5}{3}$ when $x_1 = 0$ ← Ans

→ From eq^n ③

$x_1 + x_3 = 4 \quad \times 2$

$2x_1 + 5x_3 = 5$

$x_3 = -1$, $x_1 = 5$ when $x_2 = 0$ ← Ans

Ans: 2(a) Max $z = 3x_1 + 5x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$

such that,

$$2x_1 + 3x_2 + S_1 = 8$$

$$2x_2 + 5x_3 + S_2 = 10$$

$$3x_1 + 2x_2 + 4x_3 + S_3 = 15$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Iteration-1:

		C_j	3	5	4	0	0	0	
C_B	Basic variable	x_B	x_1	x_2	x_3	S_1	S_2	S_3	
0	S_1	8	2	3	0	1	0	0	$\frac{8}{3}$
0	S_2	10	0	2	5	0	1	0	$\frac{10}{2}$
0	S_3	15	3	2	4	0	0	1	$\frac{15}{2}$
$Z=0$		Z_j	0	0	0	0	0	0	
		$C_j - Z_j$	3	5↑	4	0	0	0	

Key element is 3.

Iteration-2:

		C_j	3	5	4	0	0	0	
C_B	Basic variable	x_B	x_1	x_2	x_3	S_1	S_2	S_3	
5	x_2	$8/3$	$2/3$	1	0	$1/3$	0	0	
0	S_2	$14/3$	$-4/3$	0	5	$-2/3$	1	0	0.93→
0	S_3	$29/3$	$5/3$	0	4	$-2/3$	0	1	2.42
$Z = \frac{40}{3}$		Z_j	$10/3$	5	0	$5/3$	0	0	
		$C_j - Z_j$	$-1/3$	0	4↑	$-5/3$	0	0	

Next key element is 5.

Iteration-3:

		C_j	3	5	4	0	0	0	
C_B	Basic variable	x_B	x_1	x_2	x_3	S_1	S_2	S_3	
5	x_2	$8/3$	$2/3$	1	0	$1/3$	0	0	4
4	x_3	$14/5$	$-4/15$	0	1	$-2/15$	$1/15$	0	
0	S_3	$89/15$	$41/15$	0	0	$-2/15$	$-4/15$	1	2.17→
$Z = \frac{256}{15}$		Z_j	$34/15$	5	4	$17/15$	$4/5$	0	
			$11/15$↑	0	0	$-17/15$	$-4/5$	0	

Now key element is 41.

Iteration-4:

C_B	Basic variable	C_j:	3	5	4	0	0	0
		x_B	x_1	x_2	x_3	S_1	S_2	S_3
5	x_2	$\frac{50}{41}$	0	1	0	$\frac{15}{41}$	$\frac{8}{41}$	$-\frac{10}{41}$
4	x_3	$\frac{62}{41}$	0	0	1	$-\frac{6}{41}$	$\frac{5}{41}$	$\frac{4}{41}$
3	x_1	$\frac{89}{41}$	1	0	0	$-\frac{2}{41}$	$-\frac{12}{41}$	$\frac{15}{41}$
$Z=\frac{765}{41}$		Z_j	3	5	4	$\frac{45}{41}$	$\frac{25}{41}$	$\frac{11}{41}$
		$C_j - Z_j$	0	0	0	$-\frac{45}{41}$	$-\frac{24}{41}$	$-\frac{11}{41}$

Optimal solution is arrived with values of variables are:

$x_1 = \frac{89}{41}$ $x_2 = \frac{50}{41}$ $x_3 = \frac{52}{41}$

$\text{Max } Z = \frac{765}{41}$. ← Ans

Ans 2(b) $\text{Max } Z = 107x_1 + x_2 + 2x_3$

such that

$14x_1 + x_2 - 6x_3 + 3x_4 = 7$

$16x_1 + \frac{x_2}{2} - 6x_3 + S_1 = 5$

$3x_1 - x_2 - x_3 + S_2 = 0$

C_B	B	C_j:	107	1	1	0	0	0	
		x_B	x_1	x_2	x_3	x_4	S_1	S_2	Min ratio
0	x_4	7	14	1	-6	3	0	0	$\frac{1}{2}$
0	S_1	5	16	$\frac{1}{2}$	-6	0	1	0	$5/16$
0	S_2	0	3	-1	-1	0	0	1	0
	Z_j	0	0	0	0	0	0	0	
	$C_j - Z_j$		107	1	1	0	0	0	

	C_j		107	1	1	0	0	0	
C_B	B	x_B	x_1	x_2	x_3	x_4	S_1	S_2	Min ratio
0	x_4	$21/8$	0	$9/16$	$-3/4$	3	$-7/48$	0	$-7/2$
107	x_1	$5/16$	1	$1/32$	$-3/8$	0	$1/16$	0	$-5/6$
0	S_2	$-5/16$	0	$-35/32$	$1/8$	0	$-3/16$	1	$-15/2$
	Z_j	$\frac{535}{16}$	107	$\frac{107}{32}$	$-\frac{321}{8}$	0	$\frac{107}{16}$	0	
			0	-234	41.8	0	668	0	

Since all the min ratio are -ve, therefore our solution is unbounded. ← Ans

Ans 3 Max $Z = x_1 + 2x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$

such that; $x_1 + 2x_2 + 3x_3 + A_1 = 15$

$2x_1 + x_2 + 5x_3 + A_2 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

Initial solⁿ given by -

$x_1 = x_2 = x_3 = 0$, $A_1 = 15$, $A_2 = 20$, $x_4 = 10$

	C_j		1	2	3	-1	-M	-M	
C_B	B	x_B	x_1	x_2	x_3	x_4	A_1	A_2	Min ratio
-M	A_1	15	1	2	3	0	1	0	$15/3$
-M	A_2	20	2	1	5	0	0	1	$20/5$ →
-1	x_4	10	1	2	1	1	0	0	$10/1$
	Z_j	-35M-10	-3M-1	-3M-2	-8M-1 ↑	-1	-M	-M	

Key element is 5.

	C_j		1	2	3	-1	-M	
C_B	B	x_B	x_1	x_2	x_3	x_4	A_1	Min ratio
-M	A_1	3	$-1/2$	$7/5$	0	0	1	$15/7$ →
3	x_3	4	$2/5$	$1/5$	1	0	0	$20/1$
-1	x_4	6	$3/5$	$9/5$	0	1	0	$30/9$
	Z_j	-3M+6	$\frac{1}{3}M + \frac{3}{5}$	$-\frac{7}{5}M - \frac{6}{5}$ ↑	3	-1	-M	

Key element is $\frac{7}{5}$.

		C_j	1	2	3	-1	
C_B	B	x_B	x_1	x_2	x_3	x_4	Min ratio
2	x_2	15/7	-1/7	1	0	0	-15
3	x_3	25/7	3/7	0	1	0	25/3
-1	x_4	15/7	6/7	0	0	1	5/2 $\rightarrow$
	Z_j	90/7	1/7	2	3	-1	
		$Z_j - C_j$	-6/7 $\uparrow$	0	0	0	

		C_j	1	2	3	-1	
C_B	B	x_B	x_1	x_2	x_3	x_4	Min ratio
2	x_2	5/2	0	1	0	1/6	
3	x_3	5/2	0	0	1	3/6	
1	x_4	5/2	1	0	0	7/6	
	Z_j	15	1	2	3	3	
		$Z_j - C_j$	0	0	0	4	

since all values of $Z_j - C_j \geq 0$, hence optimality conditions are satisfied.

$x_1 = x_2 = x_3 = \frac{15}{6}$, $x_4 = 0$

$\leftarrow$ Ans

Max $Z = 15$

Ans: 4(a) Dual:

Min $Z_w = 6w_1 - 6w_2 + 4w_3 - 4w_4$

subject to, $4w_1 - 4w_2 + w_3 - w_4 \geq 2$

$3w_1 - 3w_2 + 2w_3 - 2w_4 \geq 3$

$w_1 - w_2 + 5w_3 - 5w_4 \geq 1$ $\leftarrow$ Ans

$w_1, w_2, w_3, w_4 \geq 0$

(b) Dual :

$\text{Min } Z_w = 5w_1 - 5w_2 + 3w_3 - 4w_4$

subject to, $w_1 - w_2 + w_3 \geq -1$

$-3w_1 + 3w_2 - 2w_3 - 2w_4 \geq -1$

$4w_1 - 4w_2 + w_4 \geq -1$

$-4w_1 + 4w_2 - w_4 \geq 1$

$w_1, w_2, w_3, w_4 \geq 0$ ← Ans

Ans 5 Introducing slack variables such that

$-2x_1 - 3x_2 - 5x_3 + S_1 = -2$

$3x_1 + x_2 + 7x_3 + S_2 = 3$

$x_1 + 4x_2 + 6x_3 + S_3 = 5$

Basic solution: $x_1 = x_2 = x_3 = 0$, $S_1 = -2$, $S_2 = 3$, $S_3 = 5$

	C_j		-2	-2	4	0	0	0
C_B	B	x_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	-2	-2	-3	-5	1	0	0
0	S_2	3	3	1	7	0	1	0
0	S_3	5	1	4	6	0	0	1
	Z_j	0	0	0	0	0	0	0
	$Z_j - C_j$		2	2	4	0	0	0
Max ratio			-1	$-2/3$	$-4/5$			

	C_j		-2	-2	-4	0	0	0
C_B	B	x_B	x_1	x_2	x_3	S_1	S_2	S_3
-2	x_2	$2/3$	$2/3$	1	$5/3$	$-1/3$	0	0
0	S_2	$7/3$	$7/3$	0	$16/3$	$1/3$	1	0
0	S_3	$7/3$	$-5/3$	0	$-2/3$	$4/3$	0	1
	Z_j	$-4/3$	$-4/3$	-2	$-10/3$	$2/3$	0	0
	$Z_j - C_j$	~~0.66~~	0.66	0	0.66	$2/3$	0	0

Since all $Z_j - C_j \geq 0$ & $x_B \geq 0$, therefore it satisfy optimally conditions.

$x_1 = 0$, $x_2 = \frac{2}{3}$, $x_3 = 0$

Min $Z = \frac{4}{3}$ ← Ans

Ans 6(i) NWCR Method:

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	2 (6)	3	11	7	6-6=0
S_2	1 (1)	0	6	1	1-1=0
S_3	5	8 (5)	15 (3)	9 (2)	10-5=5-3-2=0
Demand (b_j)	~~7~~ ~~1~~	~~5~~	~~3~~	~~2~~	

$m+n-1 = 3+4-1 = 6$

My solution is degenerated

$x_{11} = 6$, $x_{21} = 1$, $x_{32} = 5$, $x_{33} = 3$, $x_{34} = 2$

Total cost = $12+1+40+45+18 = 116$ ← Ans

(ii) LCM Method:

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	2 (6)	3	11	7	6-6=0
S_2	1	0 (1)	6	1	1-1=0
S_3	5 (1)	8 (4)	15 (3)	9 (2)	10-1=9-4=5-2 =3-3=0
Demand (b_j)	~~7~~ ~~1~~	~~5~~ ~~4~~	~~3~~	~~2~~	

Total cost = $12+5+32+45+18 = 112$ ← Ans

(iii) VAM Method:

	D_1	D_2	D_3	D_4	a_i	P_1	P_2	P_3	P_4
S_1	2 (1)	3 (5)	11	7	6-5=1	1	1	← 5	-
S_2	1	0	6	1 (1)	1-1=0	1	-	-	-
S_3	5 (6)	8	15 (3)	9 (1)	10-3=7	3	3	4	4
b_j	~~7~~ ~~6~~	~~5~~	~~3~~	~~2~~ ~~1~~					
P_1'	1	3↑	5	6↑					
P_2'	3	5	4	2					
P_3'	3	-	4	2					
P_4'	5	-	15↑	9					

Total cost = 2+15+30+45+1+9

= 102 ← Ans

Ans 7

		To			
		1	2	3	supply
From	1	2	7 (2)	4 (3)	5-3=2
	2	3	3	1	8-8=0
	3	5	4 (7)	7 (8)	7-7=0
	4	1 (7)	6	2 (7)	14-7=7-7=0
	Demand	~~7~~	~~9~~ ~~2~~	~~18~~ ~~10~~ ~~3~~	

Total cost = 7+14+28+12+8+14

= 83 ← Ans

Ans 8 1) Select smallest no. from every row and then subtract from all row digits respectively

	1	2	3	4	5
A	3	9	0	8	12
B	3	1	6	0	9
C	1	4	3	0	4
D	4	7	0	11	9
E	4	0	2	1	5

2) Select smallest no. from every column & subtract from respective column values.

	1	2	3	4	5
A	2	9	0	8	8
B	2	1	6	0	5
C	0	4	3	0	0
D	3	7	0	11	5
E	3	0	2	1	1

3) Number of lines marked not equal to $n=5$.
So, process is repeated till condition satisfy.

	1	2	3	4	5
A	1	9	0	8	7
B	1	1	6	0	4
C	0	5	4	1	0
D	2	7	0	11	4
E	2	0	2	1	0

4)

	1	2	3	4	5
A	0	8	0	8	6
B	0	1	6	0	3
C	0	5	5	2	0
D	1	6	0	11	3
E	2	0	3	2	0

Job Assigned:

$A \to 1$, $B \to 4$, $C \to 5$, $D \to 3$, $E \to 2$

M. cost $= 11+10+17+6+16$

$= 60$ ← Ans