Jansson 2006
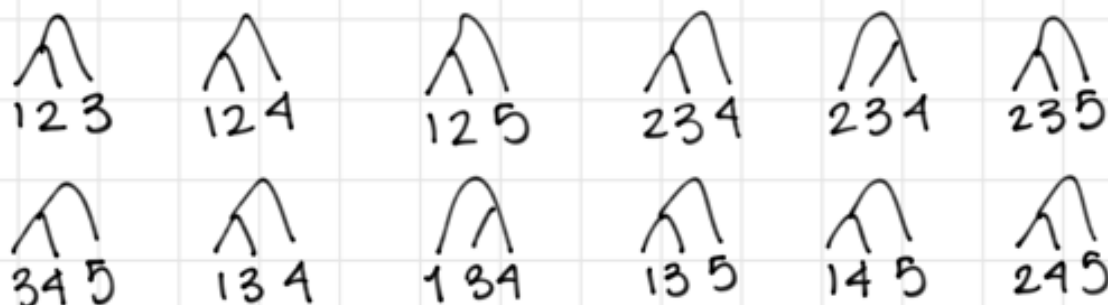
Output:

Input: leaf set $L = \{1, 2, 3, 4, 5\}$
Triplet set:

level One (1) $\forall x_1 \in L, x_2 \in L$ (even $x_1 = x_2$) compute $SN(\{x_1, x_2\})$

$\boxed{x_1 = 1, x_2 = 2}$

$X = \{1\}$ $Z = \{2\}$
- $z = 2 \rightarrow a = 1$ $L \setminus (X \cup Z) = \{3, 4, 5\}$

$SN\{1, 2\} = \{1, 2\}$

$\Rightarrow X = \{1, 2\}$ $Z = \phi$

Note $SN\{x,x\} = \{x\}$ (Proof of lemma 7)

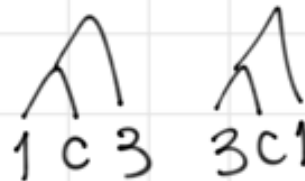$SN(1,1) = 1$

$SN(2,2) = 2$

$SN(3,3) = 3$

$SN(4,4) = 4$

$SN(5,5) = 5$

$\rightarrow SN(1,2) = \{1,2\}$

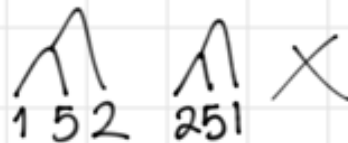$$\boxed{X_1 = 1, \quad X_2 = 3}$$

$X = \{1\}$ $Z = \{3\}$

• $Z = 3 \longrightarrow a = 1$ 1 c 3   3 c 1

$c = 2, 4$

$X = \{1,3\}$ $Z = \{2,4\}$

• $Z = 2 \longrightarrow a = 1$ 1 5 2   2 5 1   ✗

           $\longrightarrow a = 3$ 1 5 3   3 5 1   ✗

$X = \{1,2,3\}$ $Z = \{4\}$

• $Z = 4$ $a = 1$ 1 5 4   4 5 1 $a = 3$

$X = \{1,2,3,4\}$ $a = 2$ 2 5 4   4 5 2   4 5 3   3 5 4

$x_1 = 1, \quad x_2 = 4$

$X = \{1\} \quad Z = \{4\}$

- $z = 4 \longrightarrow a = 1 \implies$

$1 \ C \ 4$

$c = 2, 3$

$X = \{1, 4\} \quad Z = \{2, 3\}$

- $z = 2 \longrightarrow a = 1$

$\longrightarrow a = 4$

$1 \quad 5 \quad 2 \qquad 2 \quad 5 \quad 1$

$2 \quad 5 \quad 4 \qquad 4 \quad 5 \quad 2$

$X = \{1, 2, 4\} \quad Z = \{3\} \qquad c \in \{5\}$

- $z = 3 \longrightarrow a = 1$
- $\longrightarrow a = 2$
- $\longrightarrow a = 4$

$1 \quad 5 \quad 3 \qquad 3 \quad 5 \quad 1$

$2 \quad 5 \quad 3 \qquad 3 \quad 5 \quad 2$

$3 \quad 5 \quad 4 \qquad 4 \quad 5 \quad 3$
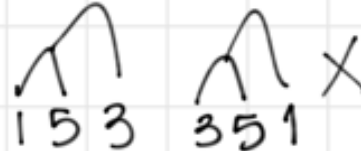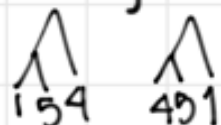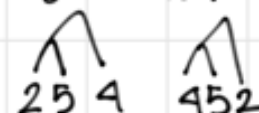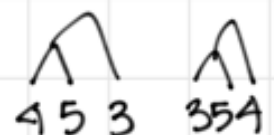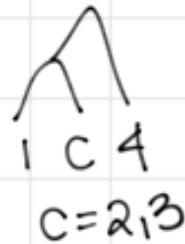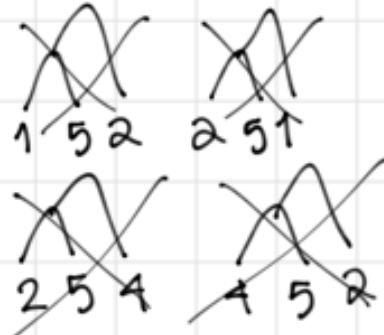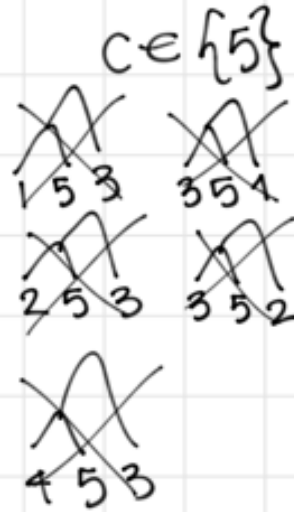
$\implies X = \{1, 2, 3, 4\}$

$\boxed{x_1 = 1 \quad x_2 = 5}$

$X = \{1\} \quad Z = \{5\}$

- $z = 5 \longrightarrow a = 1$

1 c 5   5 c 1

$c = 2 \checkmark \implies Z = \{2, 5\}$

$X = \{1, 5\} \quad Z = \{2\}$

- $z = 2 \longrightarrow a = 1$
  $\downarrow$
  $a = 5$

2 c 1   1 c 2

2 c 5   5 c 2

$c = 3 \implies X = \{1, 2, 5\} \quad Z = \{3\}$

- $z = 3 \longrightarrow a = 1$

  $\longrightarrow a = 2$

  $\longrightarrow a = 5$

1 4 3   3 4 1

2 4 3   3 4 2 $\checkmark$ $c = 4$

3 4 5   5 4 3
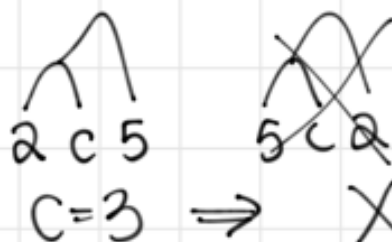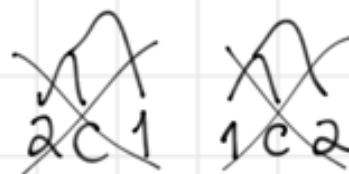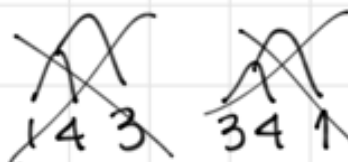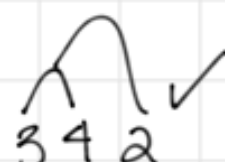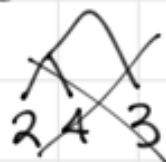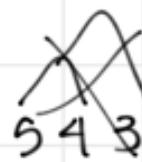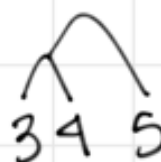
$c = 4$

$X = \{1, 2, 3, 5\} \quad Z = \{4\} \implies X = \{1, 2, 3, 4, 5\}$

$$\boxed{x_1 = 2 \quad x_2 = 3}$$

$$X = \{2\} \quad Z = \{3\}$$

- $z = 3 \to a = 2$

$$\overbrace{\;3 \; c \; 2\;} \quad \overbrace{\;2 \; c \; 3\;}$$

$$C = 1, 4$$
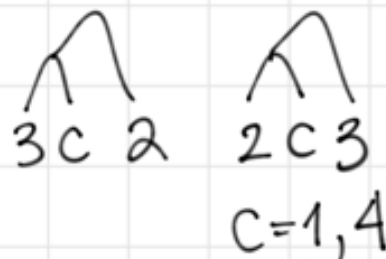
$$X = \{2, 3\} \quad Z = \{1, 4\}$$

- $z = 1 \qquad a = 2 \to \nexists \, c \in \{5\}$
$$a = 3 \to \nexists \, c \in \{5\}$$

$$X = \{1, 2, 3\} \quad Z = 4$$

$\times$ 5 never in the middle

$$\Downarrow$$

$$X = \{1, 2, 3, 4\}$$

$$\boxed{x_1 = 2 \qquad x_2 = 4}$$

$$X = \{2\} \qquad Z = \{4\}$$

$\bullet \; z = 4 \qquad a = 2$

$2 \, c \, 4 \qquad 4 \, c \, 2$

$c = 1, 3$

$$X = \{2, 4\} \qquad Z = \{1, 3\}$$

$*$ 5 never in the middle

$$\Rightarrow X = \{1, 2, 3, 4\}$$

$$\boxed{x_1 = 2 \quad x_2 = 5}$$

$$X = \{2\} \quad Z = \{5\}$$

- $z = 5 \quad c = 2$

$$2c \; 5 \qquad 5c \; 2$$

$$c = 1, 3, 4$$

$$X = \{2, 5\} \quad Z = \{1, 3, 4\}$$

* nothing on $L \setminus (X \cup Z)$

$$\Rightarrow X = \{1, 2, 3, 4, 5\}$$

$$\boxed{x_1 = 3 \quad x_2 = 4}$$

$$X = \{3\} \quad Z = \{4\}$$

- $z = 4 \longrightarrow a = 3$

$$3 \, c \, 4 \qquad 4 \, c \, 3$$
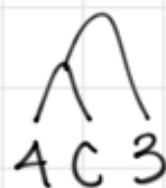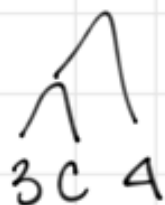
$$c = 2, 1$$

$$\Longrightarrow X = \{1, 2, 3, 4\}$$

$$\boxed{x_1 = 3 \quad x_2 = 5}$$

$$X = \{3\} \quad Z = \{5\}$$

- $z = 5 \quad a = 3$

$$3 \, c \, 5 \quad 5 \, c \, 3$$
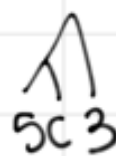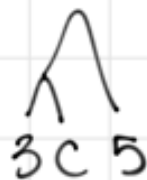
$$c = 2, 4, 1$$

$$X = \{3, 5\} \quad Z = \{2, 4, 1\}$$

* nothing on $L \setminus (X \cup Z)$

$$\Longrightarrow X = \{1, 2, 3, 4, 5\}$$

$$\boxed{x_1 = 4 \quad x_2 = 5}$$

$$X = \{4\} \quad Z = \{5\}$$

- $z=5 \longrightarrow a=4$

$$4 \subset 5 \qquad 5 \subset 4$$
$$c = 1$$

$$X = \{4,5\} \quad Z = \{1\}$$

$\Longrightarrow c = 2$

$$4 2 5$$

$$X = \{4,5\} \quad Z = \{1,2\}$$

- $z=1 \longrightarrow a=4$

$\Rightarrow c = 3$

$$1 \subset 4$$

$$Z = \{1,2,3\}$$

$$a=5 \Longrightarrow \nexists c$$
$$X = \{1,4,5\} \quad Z = \{2,3\}$$

- $z = 2 \begin{cases} a=1 \\ a=4 \\ a=5 \end{cases} \nexists c$

$$X = \{1,2,4,5\} \quad Z = \{3\}$$

- $z = 3 \begin{cases} a=1 \\ a=2 \\ a=4 \\ a=5 \end{cases} \nexists c \Rightarrow X = \{1,2,3,4,5\}$
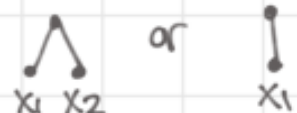
Level One (2) $SN = SN_{11}, SN_{22}, SN_{33}, SN_{44}, SN_{55},$
                 $SN_{12}, SN_{13}, SN_{14}, SN_{15}, SN_{23},$
                 $SN_{24}, SN_{25}, SN_{34}, SN_{35}, SN_{45}$
                 $q = 15$

not all !
Need to be
the set of
"maximal SN"

- $SN(x,y) = \bot \implies$ trivial
- $SN(x,y)$ maximal if
  - nontrivial
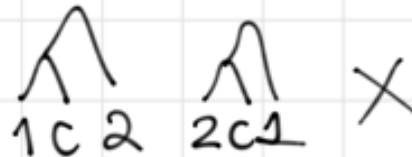 - not a proper subset of any nontrivial $SN(y_1, y_2)$

trivial
or
proper subset
of a nontrivial SN $\implies$ <u>not</u> maximal

∗ Maximal Subsets $\{1,2,3,4\}$, $\{5\}$ $\implies$

1234 5

Need to get SN

Level One (3) $\forall SN_i ;$ if $|SN_i| \geq 3 \implies N_i = $ Level One $(T | SN_i)$
               if $|SN_i| < 3 \implies N_i = $ or
               $x_1 \; x_2$ x_1

- $X = \{1\}$ $Z = \{2\}$

  $z=2 \longrightarrow a=1$ 1 c 2   2 c 1 $\times$

  $\implies X = \{1,2\}$

- $X = \{1\}$ $Z = \{3\}$

  $z=3 \longrightarrow a=1$ 1 c 3   3 c 1

  $c = 2, 4$

  $\implies X = \{1,2,3,4\}$

- $X = \{1\}$ $Z = \{4\}$

  $z=4 \longrightarrow a=1$ 1 c 4   4 c 1
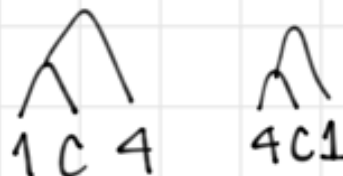
  $c = 2, 3$

  $\implies X = \{1,2,3,4\}$

- $X = \{2\}$ $Z = \{3\}$

  $$z = 3 \longrightarrow a = 2$$

  $2 \subset 3 \quad 3 \subset 2$
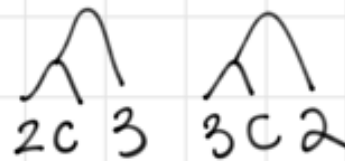
  $C = 1, 4$

  $$\Longrightarrow X = \{1, 2, 3, 4\}$$

- $X = \{2\}$ $Z = \{4\}$

  $$z = 4 \longrightarrow a = 2$$

  $2 \subset 4 \quad 4 \subset 2$
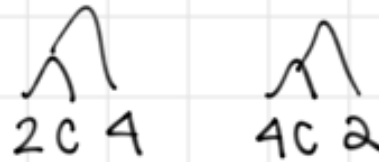
  $C = 1, 3$

  $$\Longrightarrow X = \{1, 2, 3, 4\}$$

- $X = \{3\}$ $Z = \{4\}$

  $$z = 4 \longrightarrow a = 3$$

  $3 \subset 4 \quad 4 \subset 3$

  $C = 1, 2$

  $$\Longrightarrow X = \{1, 2, 3, 4\}$$

Maximal SN: $\{1,2\}$, $\{3\}$, $\{4\}$ $q=3$
$\quad\quad\quad\quad\quad\quad \alpha_1 \quad\quad \alpha_2 \quad\quad \alpha_3$

Level One (5) $q \geq 3$: Build J' from J, and compute
$\quad\quad\quad\quad\quad\quad\quad\quad N =$ one Hybrid leaf (J')

$*$ $x \in L$ $\quad f(x) = \alpha_i \quad$ if $x \in SN_i$

$$J' = \left\{ \left((f(x), f(y)), f(z) \right) : ((x,y),z) \in J \text{ & } \begin{matrix} f(x) \\ f(y) \\ f(z) \end{matrix} \neq \right\}$$

$\{1,2\} \longrightarrow \alpha_1 \quad\quad\quad f(1) = \alpha_1$
$\{3\} \longrightarrow \alpha_2 \quad \Longrightarrow \quad f(2) = \alpha_1$
$\{4\} \longrightarrow \alpha_3 \quad\quad\quad f(3) = \alpha_2$
$\quad\quad\quad\quad\quad\quad\quad\quad f(4) = \alpha_3$

$J' \quad\quad$ leaf set $= \{\alpha_1, \alpha_2, \alpha_3\}$
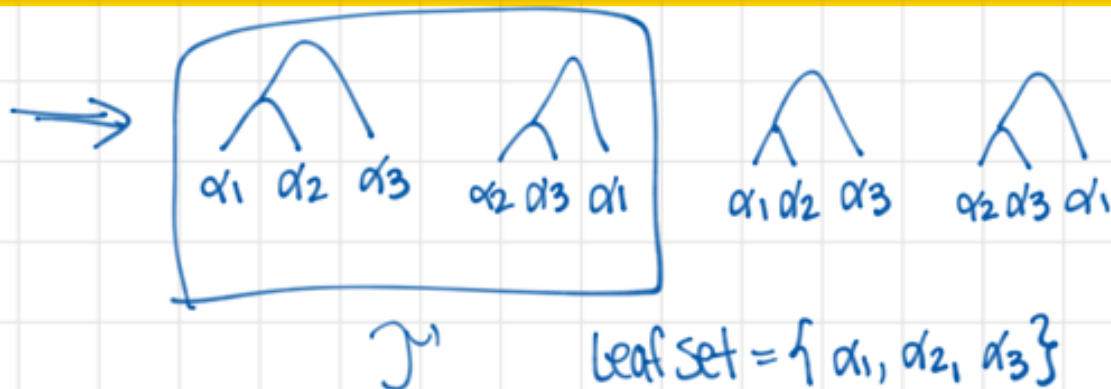
Need to call Build Tree

$J' =$ $\alpha_1 \ \alpha_2 \ \alpha_3 \qquad \alpha_2 \ \alpha_3 \ \alpha_1$
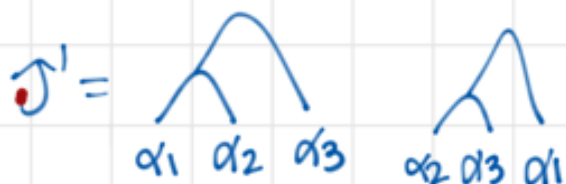
$L = \{\alpha_1, \alpha_2, \alpha_3\}$

$C = \alpha_1 \implies R =$ $\alpha_2 \ \alpha_3$ (simple case when $|L| = 3$)
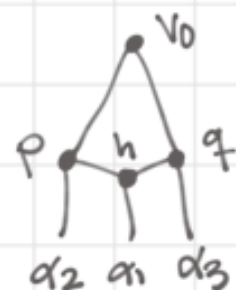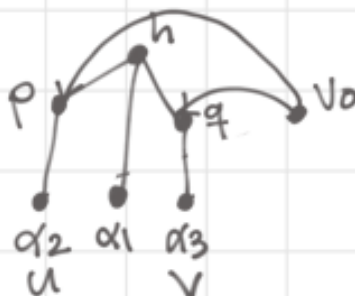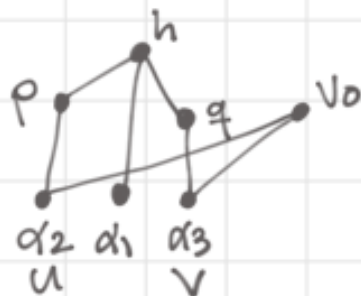
Not consistent with all triplets
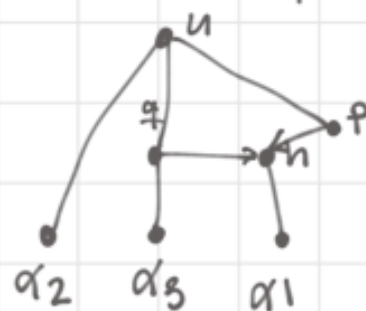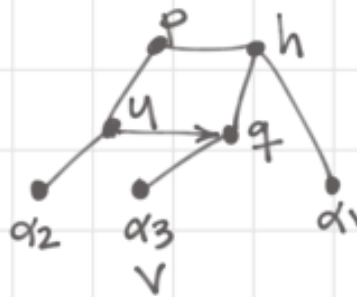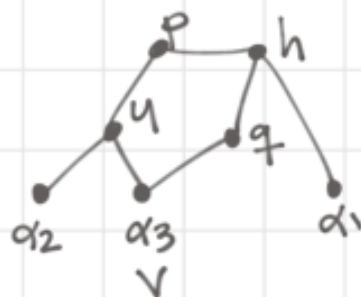
Not consistent with triplets

$C = \alpha_2 \implies R =$

$C = \alpha_3 \implies R =$

$\implies \mathcal{N} =$