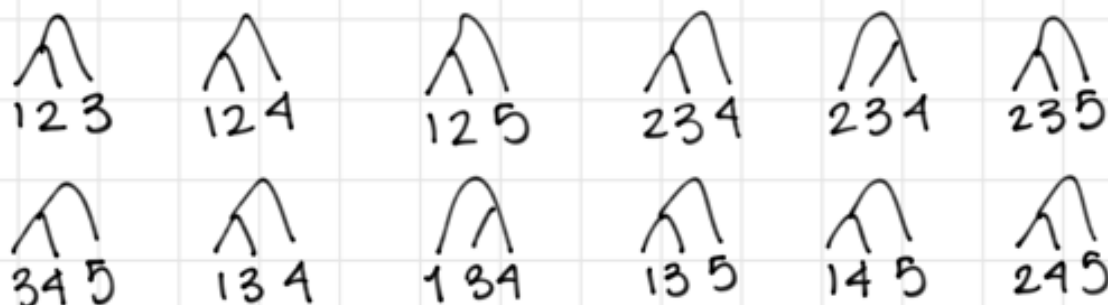


Output:



Input: leaf set $L = \{1, 2, 3, 4, 5\}$
Triplet set:



level One (1) $\forall x_1 \in L, x_2 \in L$ (even $x_1 = x_2$) compute $SN(\{x_1, x_2\})$

$$\boxed{x_1=1, x_2=2}$$

$$X = \{1\} \quad Z = \{2\}$$

$$\bullet z=2 \rightarrow a=1 \quad L \setminus (X \cup Z) = \{3, 4, 5\}$$

$$SN(\{1, 2\}) = \{1, 2\}$$

$$\begin{array}{c} \cancel{1 \in 2} \quad \cancel{2 \in 1} \end{array} \Rightarrow X = \{1, 2\} \quad Z = \emptyset$$

Note $SN(\{x_1, x\}) = \{x\}$ (Proof of lemma 7)

$$SN(1,1) = 1$$

$$SN(2,2) = 2$$

$$SN(3,3) = 3$$

$$SN(4,4) = 4$$

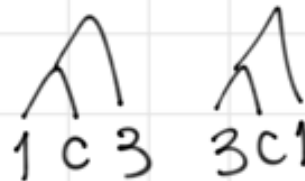
$$SN(5,5) = 5$$

$$\rightarrow SN(1,2) = \{1,2\}$$

$$\boxed{X_1=1, X_2=3}$$

$$X = \{1\} \quad Z = \{3\}$$

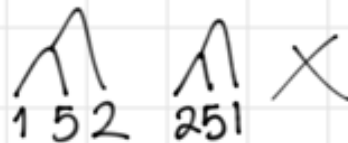
$$\bullet Z=3 \rightarrow a=1$$



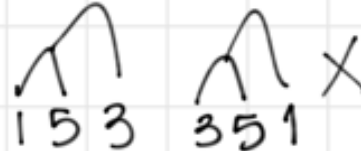
$$\swarrow c=2,4$$

$$X = \{1,3\} \quad Z = \{2,4\}$$

$$\bullet Z=2 \rightarrow a=1$$



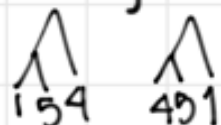
$$\searrow a=3$$



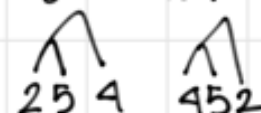
$$X = \{1,2,3\} \quad Z = \{4\}$$

$$\bullet Z=4$$

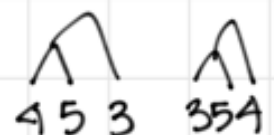
$$a=1$$



$$a=2$$



$$a=3$$

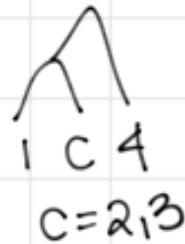


$$X = \{1,2,3,4\}$$

$$X_1=1, X_2=4$$

$$X=\{1\} \quad Z=\{4\}$$

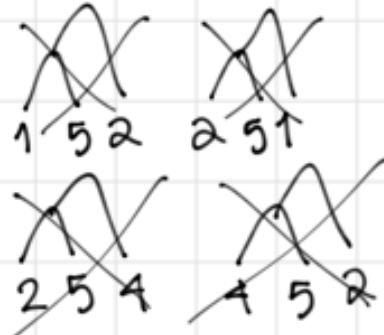
$$\bullet z=4 \rightarrow a=1 \Rightarrow$$



$$X=\{1,4\} \quad Z=\{2,3\}$$

$$\bullet z=2 \rightarrow a=1$$

$$\rightarrow a=4$$

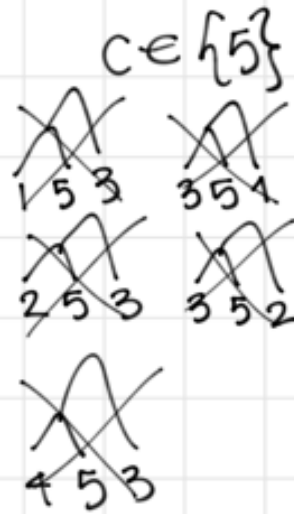


$$X=\{1,2,4\} \quad Z=\{3\}$$

$$\bullet z=3 \rightarrow a=1$$

$$\rightarrow a=2$$

$$\rightarrow a=4$$

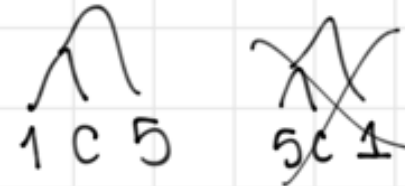


$$\Rightarrow X=\{1,2,3,4\}$$

$$x_1=1 \quad x_2=5$$

$$X=\{1\} \quad Z=\{5\}$$

$$\bullet \quad z=5 \rightarrow a=1$$

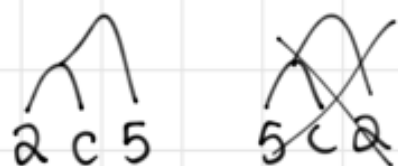
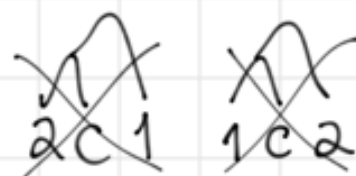


$$C=2 \checkmark \Rightarrow Z=\{2,5\}$$

$$X=\{1,5\} \quad Z=\{2\}$$

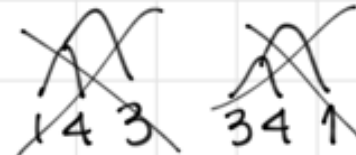
$$\bullet \quad z=2 \rightarrow a=1$$

$$\downarrow \\ a=5$$

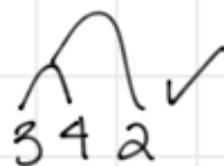
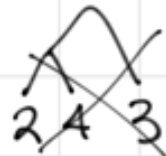


$$C=3 \Rightarrow X=\{1,2,5\} \quad Z=\{3\}$$

$$\bullet \quad z=3 \rightarrow a=1$$

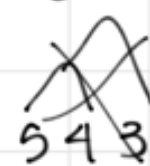
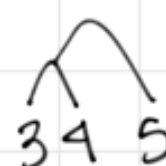


$$\rightarrow a=2$$



$$C=4$$

$$\rightarrow a=5$$



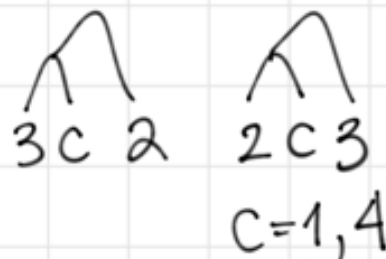
$$C=4$$

$$X=\{1,2,3,5\} \quad Z=\{4\} \Rightarrow X=\{1,2,3,4,5\}$$

$$\boxed{x_1=2 \quad x_2=3}$$

$$X=\{2\} \quad Z=\{3\}$$

$$\bullet z=3 \rightarrow a=2$$



$$X=\{2,3\} \quad Z=\{1,4\}$$

$$\bullet z=1 \quad a=2 \rightarrow \nexists c \in \{5\}$$

$$a=3 \rightarrow \nexists c \in \{5\}$$

$$X=\{1,2,3\} \quad Z=4$$

\times 5 never in the middle

$$\Downarrow$$

$$X=\{1,2,3,4\}$$

$$\boxed{x_1=2 \quad x_2=4}$$

$$X=\{2\} \quad Z=\{4\}$$

$$\bullet z=4 \quad a=2$$

$$\begin{array}{cc} \wedge & \wedge \\ 2c4 & 4c2 \\ c=1,3 \end{array}$$

$$X=\{2,4\} \quad Z=\{1,3\}$$

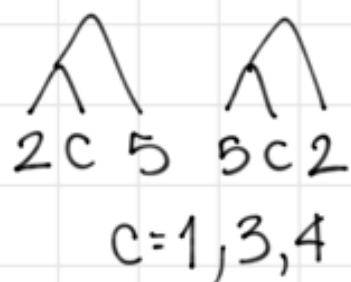
* 5 never in the middle

$$\Rightarrow X=\{1,2,3,4\}$$

$$\boxed{x_1=2 \quad x_2=5}$$

$$X=\{2\} \quad Z=\{5\}$$

$$\bullet \quad z=5 \quad c=2$$



$$X=\{2,5\} \quad Z=\{1,3,4\}$$

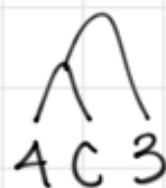
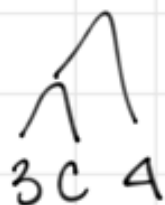
* nothing on $L \setminus (X \cup Z)$

$$\Rightarrow X = \{1, 2, 3, 4, 5\}$$

$$\boxed{x_1=3 \quad x_2=4}$$

$$X=\{3\} \quad Z=\{4\}$$

$$\bullet \quad z=4 \rightarrow a=3$$



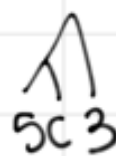
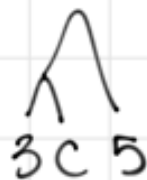
$$C=2, 1$$

$$\Rightarrow X=\{1, 2, 3, 4\}$$

$$\boxed{x_1=3 \quad x_2=5}$$

$$X=\{3\} \quad Z=\{5\}$$

$$\bullet \quad z=5 \quad a=3$$



$$C=2, 4, 1$$

$$X=\{3, 5\} \quad Z=\{2, 4, 1\}$$

* nothing on $L \setminus (X \cup Z)$

$$\Rightarrow X=\{1, 2, 3, 4, 5\}$$

$$\boxed{x_1=4 \quad x_2=5}$$

$$X=\{4\} \quad Z=\{5\}$$

$$\bullet z=5 \rightarrow a=4$$

$$\begin{array}{c} \wedge \\ 4 \quad 5 \\ c=1 \end{array}$$

$$\begin{array}{c} \wedge \\ 5 \quad 4 \\ \text{crossed out} \end{array}$$

$$X=\{4,5\} \quad Z=\{1\}$$

$$\begin{array}{c} \wedge \\ 4 \quad 5 \\ \Rightarrow c=2 \end{array}$$

$$X=\{4,5\} \quad Z=\{1,2\}$$

$$\bullet z=1 \rightarrow a=4$$

$$\begin{array}{c} \wedge \\ 1 \quad 4 \\ \Rightarrow c=3 \end{array}$$

$$Z=\{1,2,3\}$$

$$a=5 \Rightarrow \nexists c$$

$$X=\{1,4,5\} \quad Z=\{2,3\}$$

$$\bullet z=2 \left\{ \begin{array}{l} \rightarrow a=1 \\ \rightarrow a=4 \\ \rightarrow a=5 \end{array} \right\} \nexists c$$

$$X=\{1,2,4,5\} \quad Z=\{3\}$$

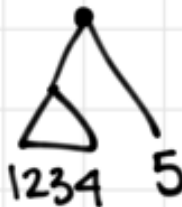
$$\bullet z=3 \left\{ \begin{array}{l} \rightarrow a=1 \\ \rightarrow a=2 \\ \rightarrow a=4 \\ \rightarrow a=5 \end{array} \right\} \nexists c \Rightarrow X=\{1,2,3,4,5\}$$

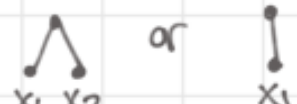

levelOne (2) $SN = SN_{11}, SN_{22}, SN_{33}, SN_{44}, SN_{55},$
 $SN_{12}, SN_{13}, SN_{14}, SN_{15}, SN_{23},$
 $SN_{24}, SN_{25}, SN_{34}, SN_{35}, SN_{45}$
 $q=15$

not all!
 Need to be
 the set of
 "maximal SN"

- $SN(x,y) = L \Rightarrow$ trivial
- $SN(x,y)$ maximal if
 - nontrivial
 - not a proper subset of any nontrivial $SN(y_1, y_2)$

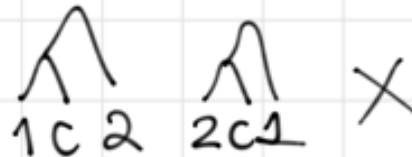
trivial
 or
 proper subset
 of a nontrivial SN \Rightarrow not maximal

* Maximal Subsets $\{1,2,3,4\}, \{5\} \Rightarrow$ 
 need to get SN

levelOne (3) $\forall SN_i; \text{ if } |SN_i| \geq 3 \Rightarrow N_i = \text{levelOne}(\mathcal{T} | SN_i)$
 $\text{ if } |SN_i| < 3 \Rightarrow N_i =$  or 



- $X = \{1\}$ $Z = \{2\}$
 $z=2 \rightarrow a=1$



$$\Rightarrow X = \{1, 2\}$$

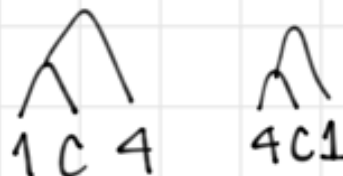
- $X = \{1\}$ $Z = \{3\}$
 $z=3 \rightarrow a=1$



$$C = 2, 4$$

$$\Rightarrow X = \{1, 2, 3, 4\}$$

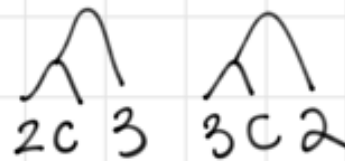
- $X = \{1\}$ $Z = \{4\}$
 $z=4 \rightarrow a=1$



$$C = 2, 3$$

$$\Rightarrow X = \{1, 2, 3, 4\}$$

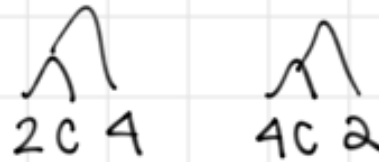
- $X = \{2\}$ $Z = \{3\}$
 $z = 3 \rightarrow a = 2$



$$C = 1, 4$$

$$\Rightarrow X = \{1, 2, 3, 4\}$$

- $X = \{2\}$ $Z = \{4\}$
 $z = 4 \rightarrow a = 2$



$$C = 1, 3$$

$$\Rightarrow X = \{1, 2, 3, 4\}$$

- $X = \{3\}$ $Z = \{4\}$
 $z = 4 \rightarrow a = 3$



$$C = 1, 2$$

$$\Rightarrow X = \{1, 2, 3, 4\}$$

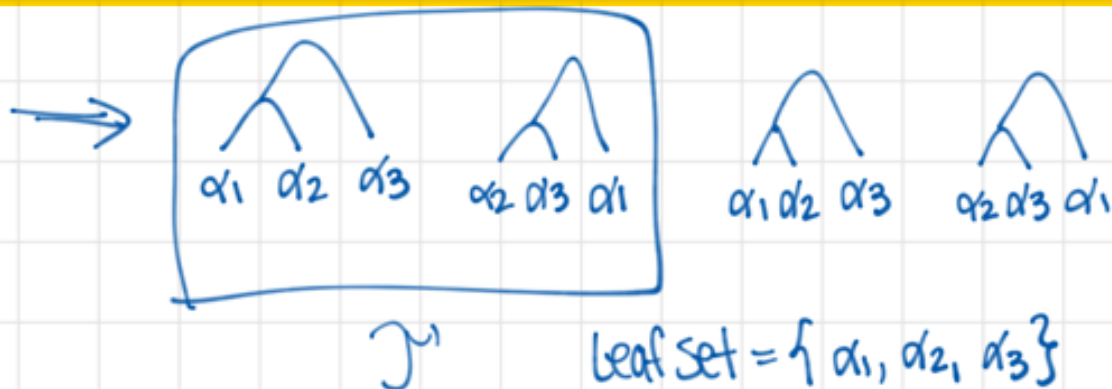
Maximal SN: $\underbrace{\{1,2\}}_{\alpha_1}, \underbrace{\{3\}}_{\alpha_2}, \underbrace{\{4\}}_{\alpha_3} \quad q=3$

level One (5) $q \geq 3$: Build \mathcal{T}' from \mathcal{T} , and compute $N = \text{one hybrid leaf}(\mathcal{T}')$

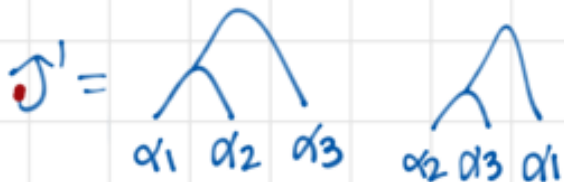
* $x \in L \quad f(x) = \alpha_i \quad \text{if } x \in \text{SN}_i$

$$\mathcal{T}' = \left\{ \left((f(x), f(y)), f(z) \right) : (x, y, z) \in \mathcal{T} \text{ and } \left. \begin{matrix} f(x) \\ f(y) \\ f(z) \end{matrix} \right\} \neq \right\}$$

$$\begin{array}{lcl} \{1,2\} \rightarrow \alpha_1 & & f(1) = \alpha_1 \\ \{3\} \rightarrow \alpha_2 & \Rightarrow & f(2) = \alpha_1 \\ \{4\} \rightarrow \alpha_3 & & f(3) = \alpha_2 \\ & & f(4) = \alpha_3 \end{array}$$

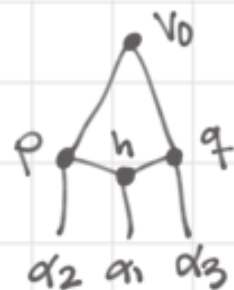
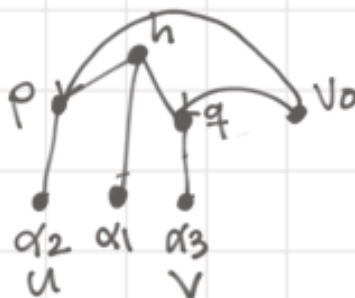
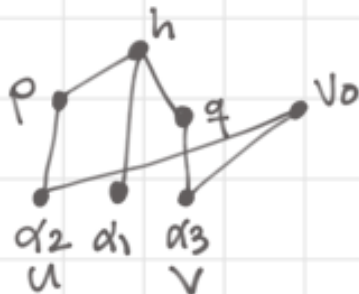


Need to call BuildTree

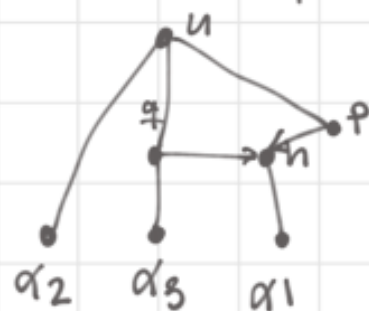
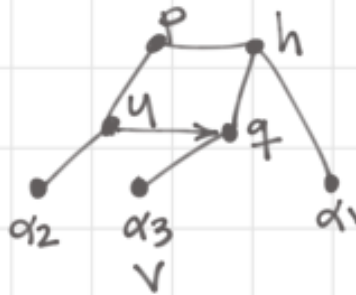
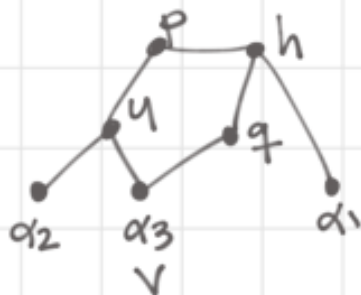


$$L = \{ \alpha_1, \alpha_2, \alpha_3 \}$$

$C = \alpha_1 \Rightarrow R =$ (simple case when $|L| = 3$)



Not consistent
with all triplets



Not consistent
with triplets

$C = \alpha_2 \Rightarrow R =$ \Rightarrow ✓

