Statement of the Problem

We are expected to solve the Hilbert matrix problem for a given size of matrix n and kmax.

Algorithm

Gauss Elimination with pivoting:-

- Matrix A from the form Ax = b, needs to be factorised into an upper triangular matrix.
- We can use pivoting to achieve the above stated solution as it has higher accuracy
- We then solve for the vector x using back substitution starting from the last independent variable x_n.

Conjugate Gradient Method:-

- It is an iterative process where we start with an initial guess of x vector
- We go along the direction of steepest descent to find a newer approximation for x
- Repeat this iteratively to get to the solution

Results

The results presented by the program are as follows:

For n = 5 and k = 20

X_Gauss *= 1.0e+05 **

{*0.043496666666650*

-0.822779999999827

3.576799999999591

-5.432733333333061

*2.669099999999988****}***

X_CGM *= 1.0e+05 **

{*0.034050000549298*

-0.634799999664007

2.736300000237130

-4.132799999792216

*2.022300000157464****}***

X_Actual *=*

{*3405*

-63480

273630

-413280

*202230****}***

For n = 20 and k = 20

X_Gauss *= 1.0e+24 **

{*0.000000015608047*

-0.000002527660093

0.000100549566910

-0.001709488698979

0.015320287608136

-0.079548787590692

0.246264352192398

-0.435646684980905

0.353706994291999

0.003073304987272

0.112394576481425

-0.996318985464918

1.378297052579915

-0.467797642960715

-0.576119702585127

1.055105161568082

-1.437722458385108

1.441387294768335

-0.777345404103919

*0.166562106568218****}***

X_CGM *= 1.0e+10 **

{ *0.000286085777338*

-0.015811831090364

0.209926487018946

-1.116129509832501

2.779172467586507

-3.366074215132986

2.388270435877810

-2.590658215423280

3.082971704737785

-2.396554782782170

2.863799566751452

-2.798012643815333

2.481816546773234

-3.035859338787802

2.441601104148840

-2.874827809535726

2.778420936728237

-2.401160475323960

3.346654422948601

*-1.778124527081477****}***

X__Actual *= 1.0e+28 **

{*0.000000000000215*

-0.000000000079965

0.000000007423761

-0.000000303655625

0.000006885570366

-0.000097888398924

0.000940674301899

-0.006420179317279

0.032187499282604

-0.121302081376523

0.348884667831600

-0.772578029890824

1.321147011366073

-1.739581395590645

1.746535403650232

-1.312018310550693

0.713641936083402

-0.265338754441208

0.060313042553530

-0.006320184762151}

Comments

1. ***Conjugate Gradient Method converges almost exactly in the n = 5 and k = 5 condition, however the Gauss elimination with pivoting has quite significant error. Also the time taken for CGM was almost half of the time taken for gauss elimination which was roughly 0.001112 seconds.***

2. ***For n = 20 and k = 20, we can see that there is significant error in both the CGM and gauss elimination method. However the error in Gauss elimination is significantly less than the error in CGM. The time taken in Gauss elimination remains almost the same for n = 20 as it was for n = 5, however for CGM as we increase the value of k, the time can greatly increase without much improvement in the accuracy.***