1. λ - calculus.

1. a) $(((\lambda x.\lambda y.\lambda z.((xy)z)(\lambda u.\lambda v.u))A)B)$

$= \lambda x.\lambda y.\lambda z.((xy)z)(\lambda u.\lambda v.u)\ A\ B$

Replacing x with $(\lambda u.\lambda v.u)$.

$= \lambda y.\lambda z.(((\lambda u.\lambda v.u)y)z)\ A\ B$

Replacing y with A

$= \lambda z.(((\lambda u.\lambda v.u)A)z)\ B$

Replacing z with B

$= (\lambda u.\lambda v.u)\ A\ B$

Replacing u with A

$= (\lambda v.A)\ B$

$= A$ // (since if condition is true, output should be first argument A)

b) $(((\lambda x.\lambda y.\lambda z.((xy)z)(\lambda u.\lambda v.v))A)B)$

$= \lambda x.\lambda y.\lambda z.((xy)z)(\lambda u.\lambda v.v)\ A\ B$

Replacing x with $(\lambda u.\lambda v.v)$

$= \lambda y.\lambda z.((\lambda u.\lambda v.v)yz)\ A\ B$

Replacing y with A

$= \lambda z.((\lambda u.\lambda v.v)Az)B$

Replacing z with B

$= (\lambda u.\lambda v.v)\ A\ B$

Replacing u with A

$= (\lambda v.v)\ B$

$= B$ //.

Since if condition is false, output should be second parameter B.

2. a) $(\lambda x.xx)\ (\lambda y.yx)\ z$

Replacing x with $\lambda y.yx$

$= (\lambda y.yx)\ (\lambda y.yx)\ z$

Replacing y with $\lambda y.yx$

$= (\lambda y.yx)\ x\ z$

Replacing y with x

$= xxz$ //.

b) $(((\lambda x.\lambda y.(xy))\ (\lambda y.y))\ w)$

$(\lambda x.\lambda y.(xy))\ (\lambda y.y)\ (w)$

Replacing ~~x with $\lambda y.y$~~ variable name change

$= (\lambda x.\lambda y.(xy))\ (\lambda t.t)\ (w)$

Replacing x with $\lambda t.t$

$= \lambda y.((\lambda t.t)y)\ (w)$

Replacing y with w

$= (\lambda t.t)\ w$

Replacing t with w

$= w$ //.

3. a) NOT (NOT TRUE)

$= \lambda x.((x\ \text{false})\ \text{true})$ (NOT TRUE)

Replacing x with NOT TRUE

$=$ ((NOT TRUE) FALSE) TRUE

$=$ (((λx.((x false true)) TRUE) FALSE) TRUE

Replacing x with TRUE

((TRUE FALSE TRUE) FALSE) TRUE.

$=$ ((((λx.λy.x) FALSE) TRUE) FALSE) TRUE.

Replacing x with FALSE

$=$ (((λy. FALSE) TRUE) FALSE) TRUE.

Replacing y with TRUE.

$=$ (FALSE FALSE) TRUE.

$=$ ((λx.λy.y) FALSE) TRUE.

Replacing x with FALSE

$=$ (λy.y) TRUE

$=$ **TRUE**

b) OR FALSE TRUE

$= (\lambda x.\, \lambda y.\, ((x\ \text{true})\, y)\ \text{FALSE})\ \text{TRUE}$

Replacing x with FALSE

$= \lambda y.\, ((\text{FALSE}\ \text{TRUE})\, y)\ \text{TRUE}$

Replacing y with TRUE

$= (\text{FALSE}\ \text{TRUE})\ \text{TRUE}$

$= ((\lambda x.\, \lambda y.\, y)\ \text{TRUE})\ \text{TRUE}$

Replacing x with TRUE

$= (\lambda y.\, y)\ \text{TRUE}$

= Replacing y with TRUE

= TRUE //.