Name- Shrey Pachauri Roll No - 102103354

Q1) Normal Distribution

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$ Mean - Q_1 Variance $\to Q_2$

Joint Density of $(x_1, x_2 \dots x_n)$ is

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi Q_2}} e^{\frac{(x_i-\theta_1)^2}{(2\theta_2)}}$$

Take log on both sides

$$\ln(L(\theta_1, \theta_2)) = \ln\left((2\pi\theta_2)^{-n/2} \cdot e^{-\frac{\Sigma(x_i-\theta_1)^2}{2\theta_2}}\right)$$

$$\ln[L(\theta_1, \theta_2]) = -\frac{n}{2}\ln(2\pi\theta_2) - \frac{1}{2\theta_2}\Sigma(x_i-\theta_i)^2$$

① Differentiate w.r.t θ_1

$$\frac{\partial \ln L}{\partial \theta_1} = \frac{1}{\theta_2}\sum_1^n (x_i - \theta_1) = 0 \qquad \boxed{\theta_1 = \frac{\Sigma x_i^2}{n}}$$

② Differentiate w.r.t to θ_2

$$\frac{\partial \ln L}{\partial \theta_2} = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2}\Sigma(x_i-\theta_1)^2 = 0$$

$$\boxed{Q_2 = \frac{1}{n}\sum^{1}(x_i - \theta_1)^2} \leftarrow \text{Variance}$$

M.L.E of θ_1 is $\bar{x}$

M.L.E of θ_2 is var(x)

Name- Shrey Pachauri

Roll Nos - 102103354

Q2 $B(m, \theta) \rightarrow$ Binomial Distribution

$$f(x) = {}^{m}C_x \, p^x (1-p)^{m-x}$$

JPD ÷

$$L(\theta; x_1 x_2 x_n) = \prod_{i=1}^{m} P(x_i / m, p)$$

$$l(\theta) = \prod_{1}^{n} \left({}^{n}C_{x_i} \cdot \theta^{x_i} \cdot (1-\theta)^{m-x_i} \right)$$

Take log on Both sides

$$\ln(l(\theta)) = \sum_{1}^{n} \log({}^{n}C_{x_i}) + \sum_{1}^{n} x_i \log(\theta) + \sum_{1}^{n} (m-x_i) \log(1-\theta)$$

Diff w.r.t θ

$$\frac{d \ln(L)}{d(\theta)} = \frac{1}{\theta} \sum_{i=n}^{n} x_i + \frac{1}{1-\theta} \sum_{i=1}^{n} (m-x_i)(-1)$$

$$= \frac{1}{\theta} \sum_{1}^{n} x_i = \frac{1}{1-\theta} \sum (m-x_i)$$

$$\boxed{\theta = \frac{\sum x_i}{m}}$$ mean

M.L.E of θ for $B(m, \theta) \rightarrow \bar{x}$.