FAC Summer Project ‘24

Deciphering Decisions

Game Theory

What did we discuss in the intro?

The Grade Game

Consider you have 2 choices to make α(alpha) and β(beta). I randomly pair 2 responses and the following happens -

1. if u choose α and pair chooses α, both get B
2. if u choose α and pair chooses β, you get A, they get C.
3. if u choose β and pair chooses β, both get B+
4. if u choose β and pair chooses α, you get C, they get A.

After seeing the 'Nash Equilibrium', what should be your choice?

		Pair	
		α	β
Me	α	(8,8)	(10, 6)
	β	(6, 10)	(9,9)

Now what is a...

‘Game’

1. **Players**
2. **Actions**
3. **Information**
4. **Outcomes**
5. **Preferences**

what else can you think of?

Before that
Consider these...

- In a perfectly competitive market of tomatoes, you jump in either as a buyer or a seller
- On a beach, there are 2 ice-cream shops, how should they place each other on a beach from x=0 to x=L
- Let's say there are only 2 companies, Apple and Samsung, then when will Apple launch its new phone relative to a new launch by Samsung?

Thus in each case, there is a different

Stategic Interaction

Strategic interaction in game theory refers to the decision-making process where each player's optimal choice depends on the anticipated choices of other players.

A Strategy is a **complete contingent plan** for that planer in the game.

Anticipation
Best Response
Interdependence

Think of CHESS to understand these 3 features!

As an example, in Prisoner's Dilemma

		Player2	
		Confess	Silent
Player1	Confess	(5,5)	(0, 10)
	Silent	(10,0)	(2,2)

As an example, in Prisoner’s Dilemma

		Player2	
		Confess	Silent
Player1	Confess	(5,5)	(0, 10)
	Silent	(10,0)	(2,2)

Strategic Interaction: Each player must consider the possibility of the other defecting and how their own decision will influence the other's choice.

Representations

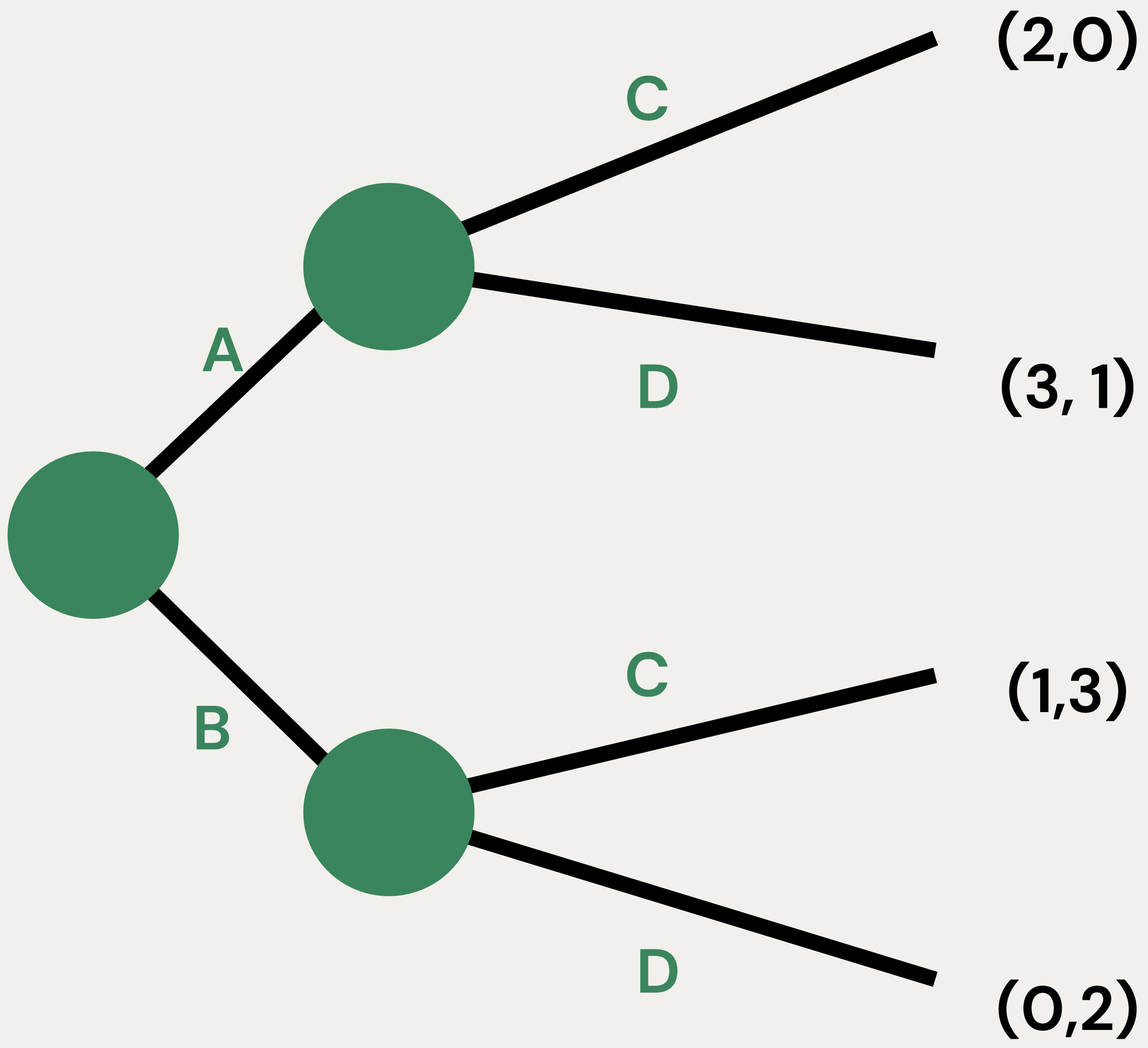
Lets say **Player 1 –> (A, B)** and **Player 2 –> (C, D)** both have 2 possible moves.

	C	D
A	(2,0)	(3, 1)
B	(1,3)	(0,2)

(Player1, Player2)

Representations

Lets say **Player 1 –> (A, B)** and **Player 2 –> (C, D)** both have 2 possible moves.



Decision Nodes /
Decision Tree

	C	D
A	(2,0)	(3, 1)
B	(1,3)	(0,2)

(Player1, Player2)

To find the most dominant strategy?

Iterative Elimination of Dominant Strategy

It involves a process where players sequentially eliminate strategies that are dominated by others until a unique solution is reached.

Example

	Left	Centre	Right
Up	13, 3	1, 4	7, 3
Middle	4, 1	3, 3	6, 2
Down	-1, 9	2, 8	8, -1

Example

	Left	Centre	Right
Up	13, 3	1, 4	[redacted]
Middle	4, 1	3, 3	[redacted]
Down	-1, 9	2, 8	[redacted]

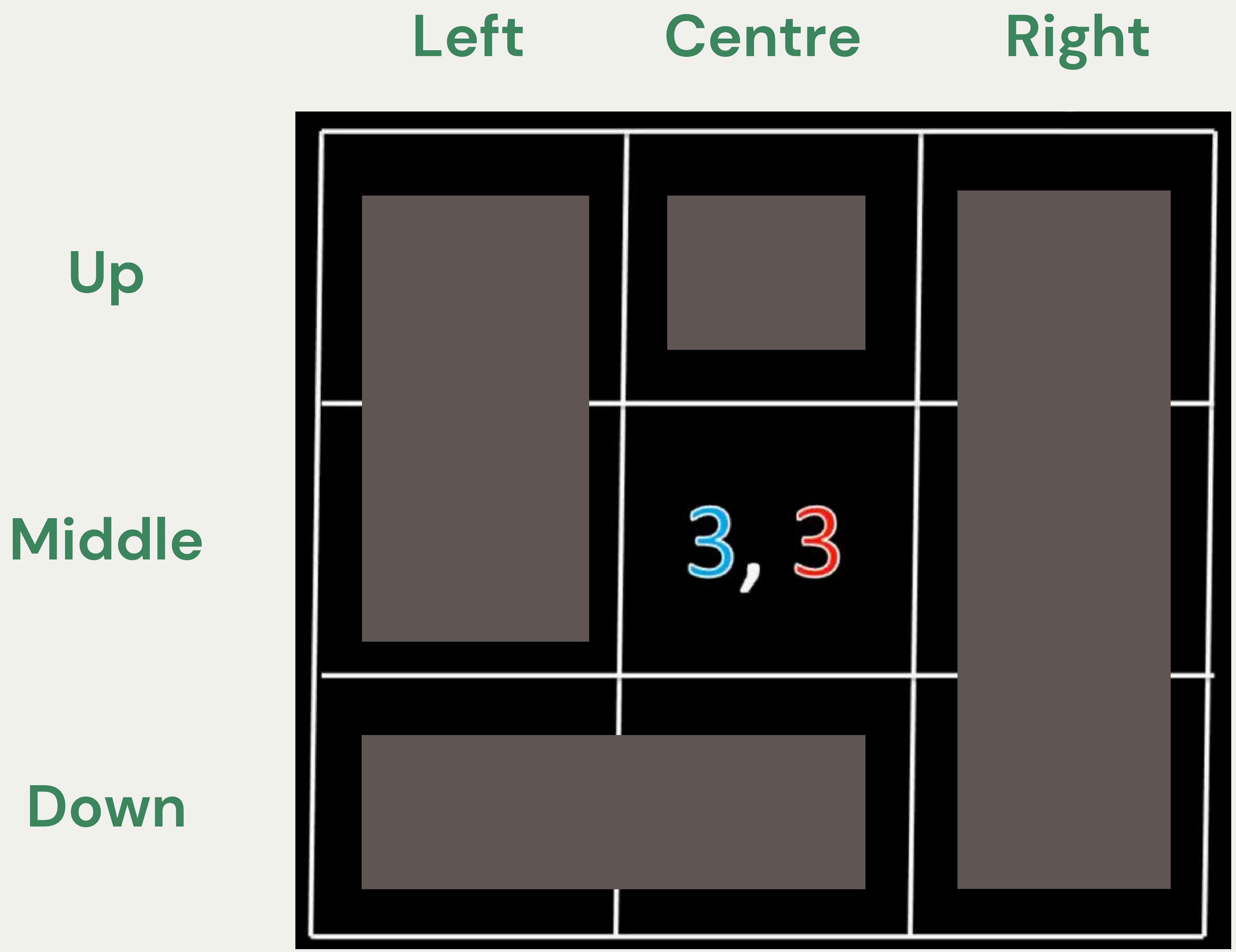
Example

	Left	Centre	Right
Up	13, 3	1, 4	
Middle	4, 1	3, 3	
Down			

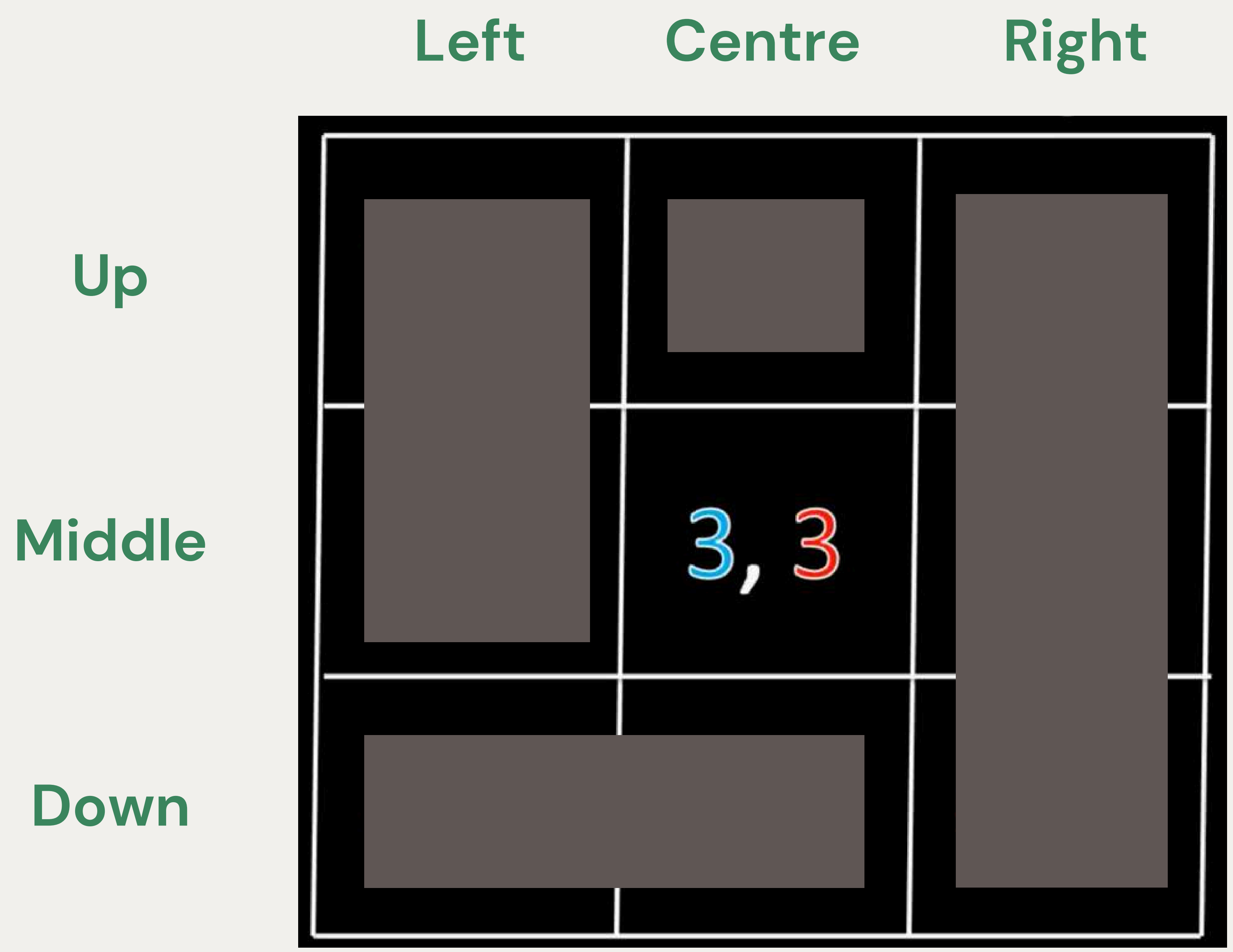
Example

	Left	Centre	Right
Up		1, 4	
Middle		3, 3	
Down			

Example



Example



So, we get to a single dominant strategy!

**But this method might not help you to simplify further in some cases!

Types of Games

- **Cooperative vs. Non-Cooperative**

Cooperative games allow for binding agreements between players, while non-cooperative games do not.

- **Zero-Sum vs. Non-Zero-Sum**

In zero-sum games, one player's gain is exactly another's loss. In non-zero-sum games, the total gain or loss can vary.

- **Simultaneous vs. Sequential**

Simultaneous games involve players making decisions at the same time, while sequential games involve players making decisions one after another.

Static v/s Dynamic Games

In **static games**, players make their decisions (or choose their strategies) simultaneously, or if they move sequentially, they do so without knowledge of the other players' choices.

In **dynamic games**, players make decisions at different points in time, allowing them to observe the choices of others before making their own.

Static v/s Dynamic Games

Feature	Static Games	Dynamic Games
Timing of Decisions	Simultaneous or without observation	Sequential, with observation
Structure	Single stage	Multiple stages
Representation	Payoff matrix	Game tree
Examples	Prisoner's Dilemma, Battle of the Sexes	Chess, Stackelberg Competition
Solution Concepts	Nash Equilibrium	Subgame Perfect Equilibrium, Backward Induction
Strategic Complexity	Simpler, focusing on immediate decisions	More complex, involving planning and adaptation over time

Complete v/s Incomplete Info

A **game of complete information** is one in which all players know the structure of the game, including the payoffs, strategies, and types of all other players.

A **game of incomplete information** is one in which some aspects of the game or the characteristics of the players are unknown to some or all players.

Complete v/s Incomplete Info

Feature	Games of Complete Information	Games of Incomplete Information
Knowledge	Full knowledge of game structure and players	Partial or hidden knowledge about some aspects
Strategic Complexity	Simpler due to transparency	More complex due to uncertainty
Beliefs	Not required	Required for decision-making
Examples	Chess, perfect market competition	Poker, auctions, diplomatic negotiations
Solution Concepts	Nash Equilibrium	Bayesian Nash Equilibrium

Pure v/s Mixed Strategies

A **pure strategy** is a specific, deterministic course of action that a player follows in a game. It involves choosing one particular action from the set of all possible actions available to the player.

A **mixed strategy** is a probabilistic approach to decision-making where a player chooses among available actions according to a specific probability distribution.

Pure v/s Mixed Strategies

Feature	Pure Strategies	Mixed Strategies
Nature of Choice	Deterministic (specific action)	Probabilistic (distribution over actions)
Predictability	Predictable	Unpredictable
Complexity	Simpler to understand and implement	More complex and flexible
Adaptability	Less adaptable	Highly adaptable
Common Use Cases	Simple games, dominant strategy scenarios	Complex games, multiple equilibria, need for unpredictability

Beliefs

In game theory, beliefs refer to a **player's subjective probability assessments regarding the unknown factors in the game**, including the strategies and types of other players.

- **Prior Beliefs:** The initial probability distribution over possible types or actions.
- **Likelihood:** The probability of observing certain actions or signals given the possible types.
- **Posterior Beliefs:** Updated beliefs after observing actions, calculated using Bayes' rule.

$$\text{Posterior} \propto \text{Prior} \times \text{Likelihood}$$

Why beliefs are imp?

- Strategic Decision-Making
- Updating Information
- Equilibrium Concepts

Repeated Games

When games played over multiple rounds, where players can condition their strategies on past actions. **Cooperation can emerge in repeated games through strategies like "tit-for-tat."**

Evolutionary Game Theory

Focuses on strategies that evolve over time based on their success. Players adapt their strategies based on the success of others in the population.

Bayesian Game

Games where players have incomplete information about others and must use beliefs to make decisions. Strategic interaction involves updating beliefs based on observed actions and choosing optimal strategies accordingly.

So, today all the examples we saw were of **static games with complete information**, but in real-life, or atleast in financial markets, most of the games are of incomplete information.

Bayesian Games

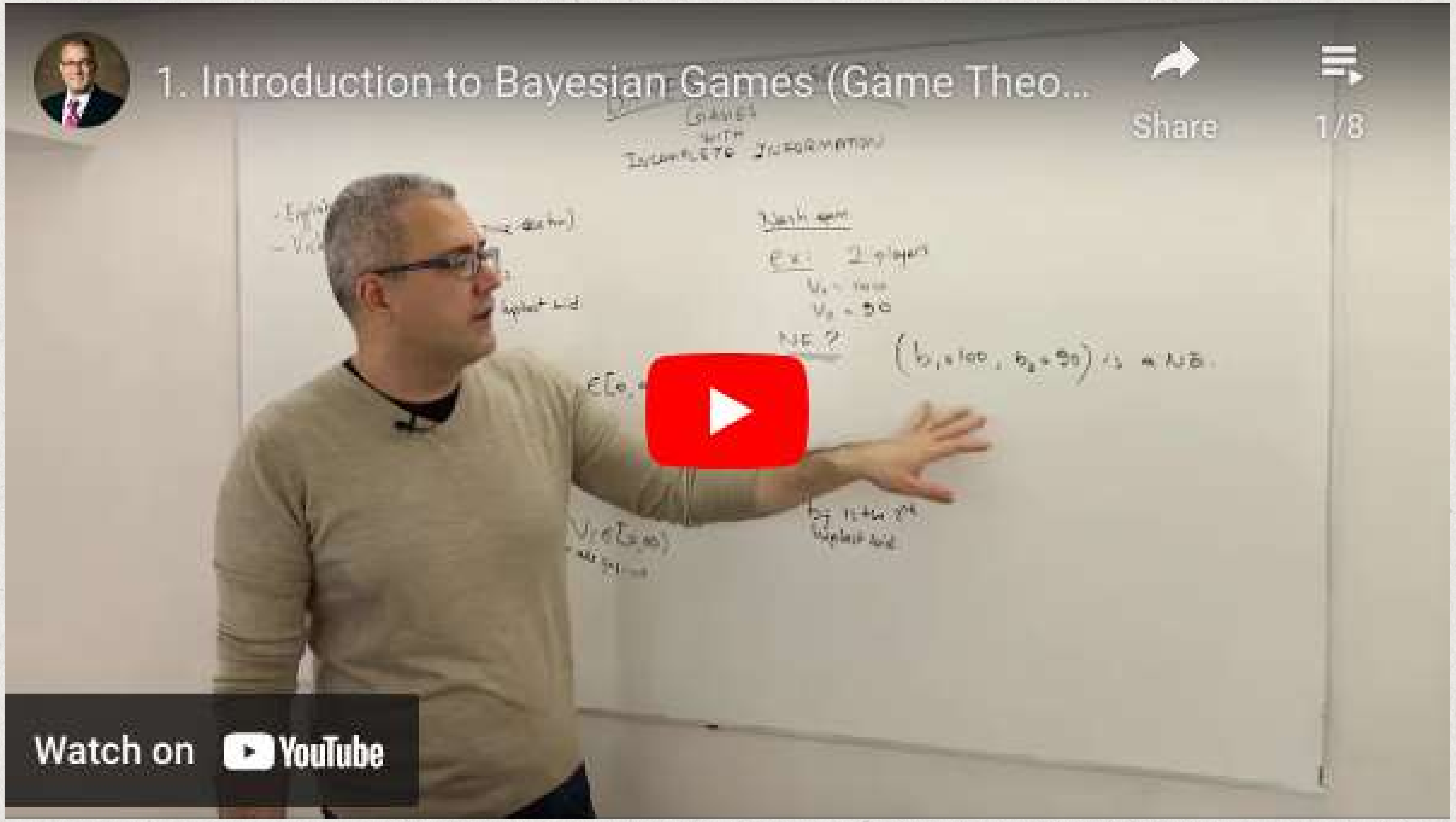
A game where players have **incomplete information** about the other players, but they have **beliefs** (represented by probability distributions) about the unknowns.

Introduction to Game theory

A very well-explained set of short videos!

Bayesian Game Theory

Games with incomplete information.



That’s all !!

THANK YOU