Probability Derivation

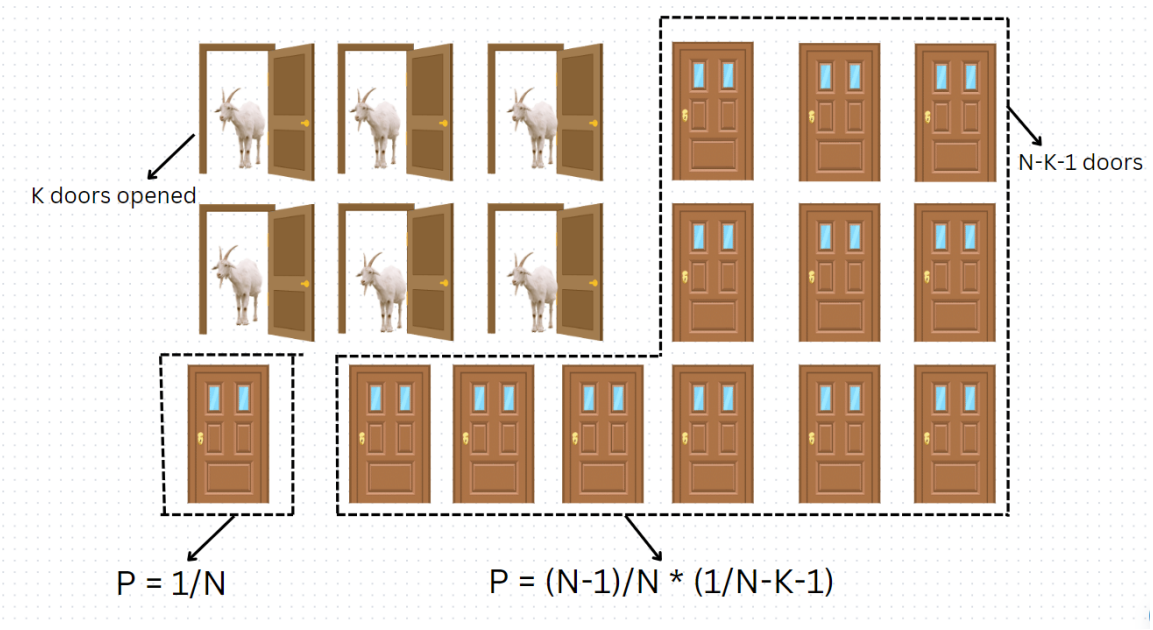

Figure 1: Monty Hall Problem

Let:

- n be the total number of doors,
- k be the number of doors opened by the host,
- $P(W_s)$ be the probability of winning by sticking with the initial choice, and
- $P(W_c)$ be the probability of winning by switching doors.

1. Sticking with Initial Choice ($P(W_s)$):

- When the player sticks with their initial choice, they win if and only if the prize is behind the initially chosen door. Since the probability of choosing the correct door initially is $\frac{1}{n}$, and assuming the host doesn't open any doors, $P(W_s) = \frac{1}{n}$.

2. Switching Doors ($P(W_c)$):

- When the player switches doors, they win if and only if they initially choose a door without the prize, and then switch to the door with the prize after the host opens k doors.
- The probability of initially choosing a door without the prize is $\frac{n-1}{n}$.
- After the host opens k doors, there are $n - k - 1$ doors left unopened, and among them, there's exactly one door with the prize.
- Therefore, the probability of switching to the door with the prize is $\frac{1}{n-k-1}$.
- Thus, $P(W_c) = \frac{n-1}{n} \times \frac{1}{n-k-1}$.

These expressions give us the probabilities of winning by sticking with the initial choice and by switching doors, respectively.

Plots

In this section, we present plots showing the switching probability versus the number of doors opened by the host for different values of N (total number of doors), both for theoretical and experimental scenarios.

Number of Simulations = 100

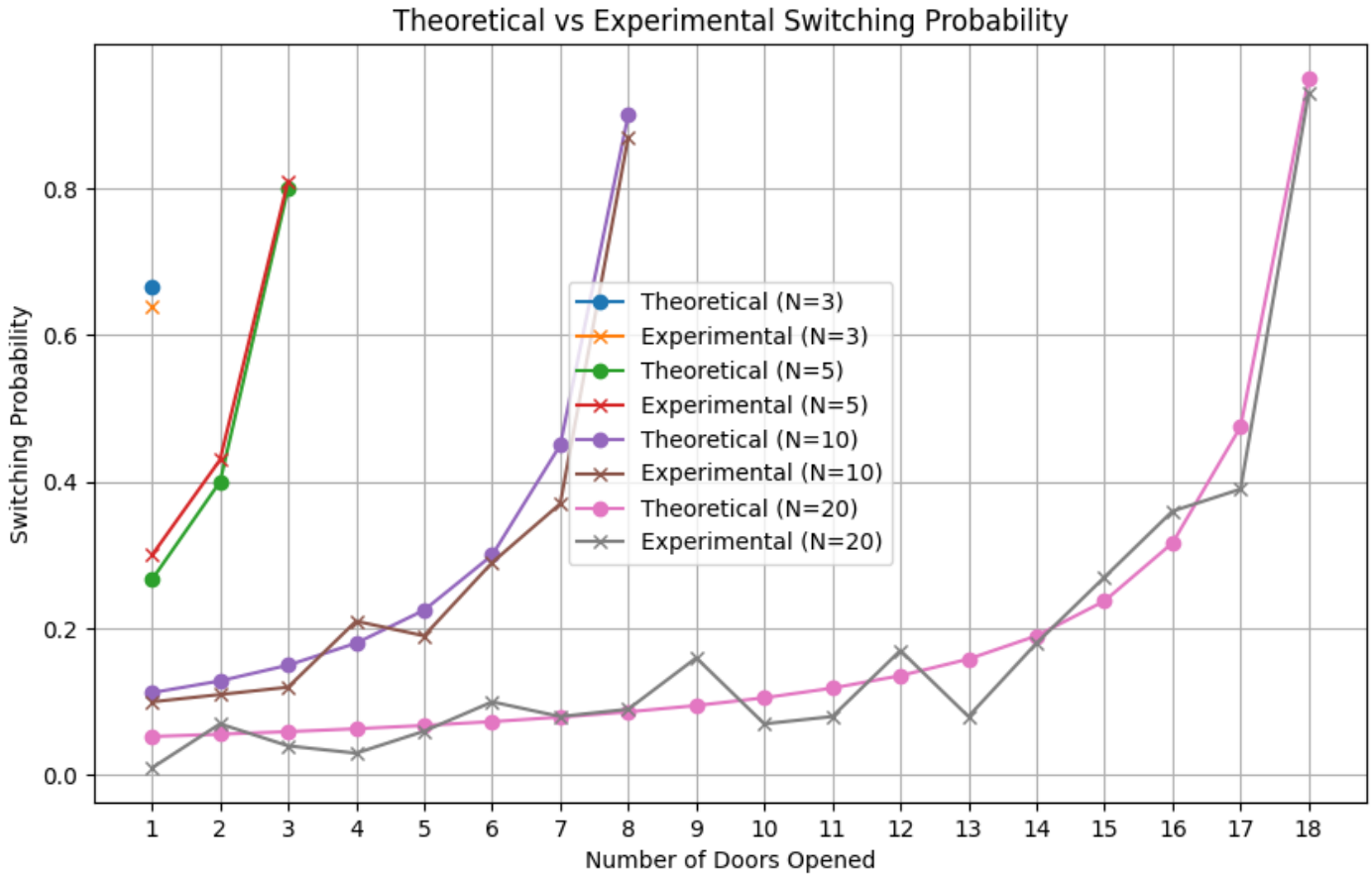

Figure 2: Switching Probability vs. Doors Opened (Simulations=100)

Number of Simulations = 1000

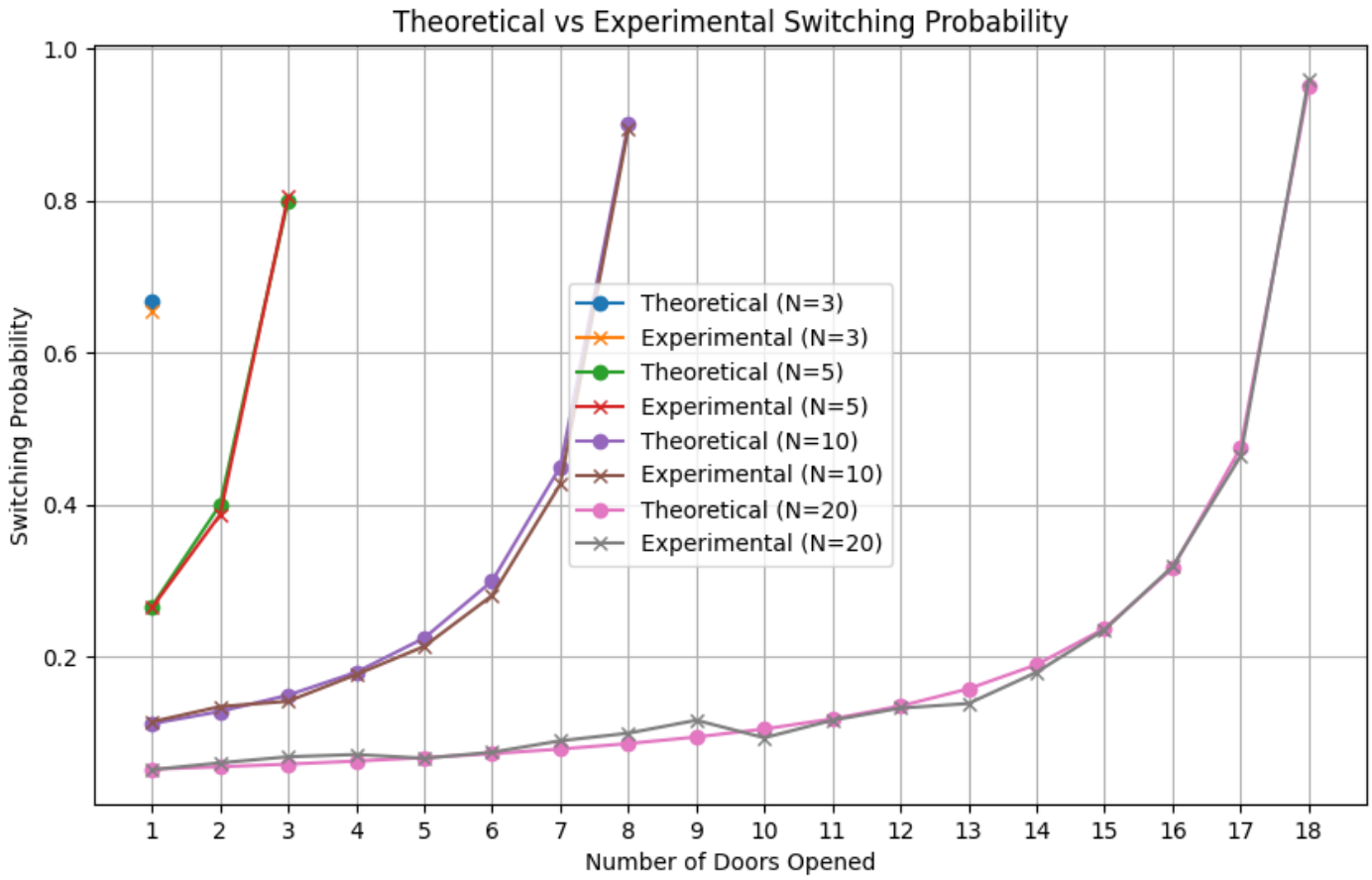

Figure 3: Switching Probability vs. Doors Opened (Simulations=1000)

Number of Simulations = 10000

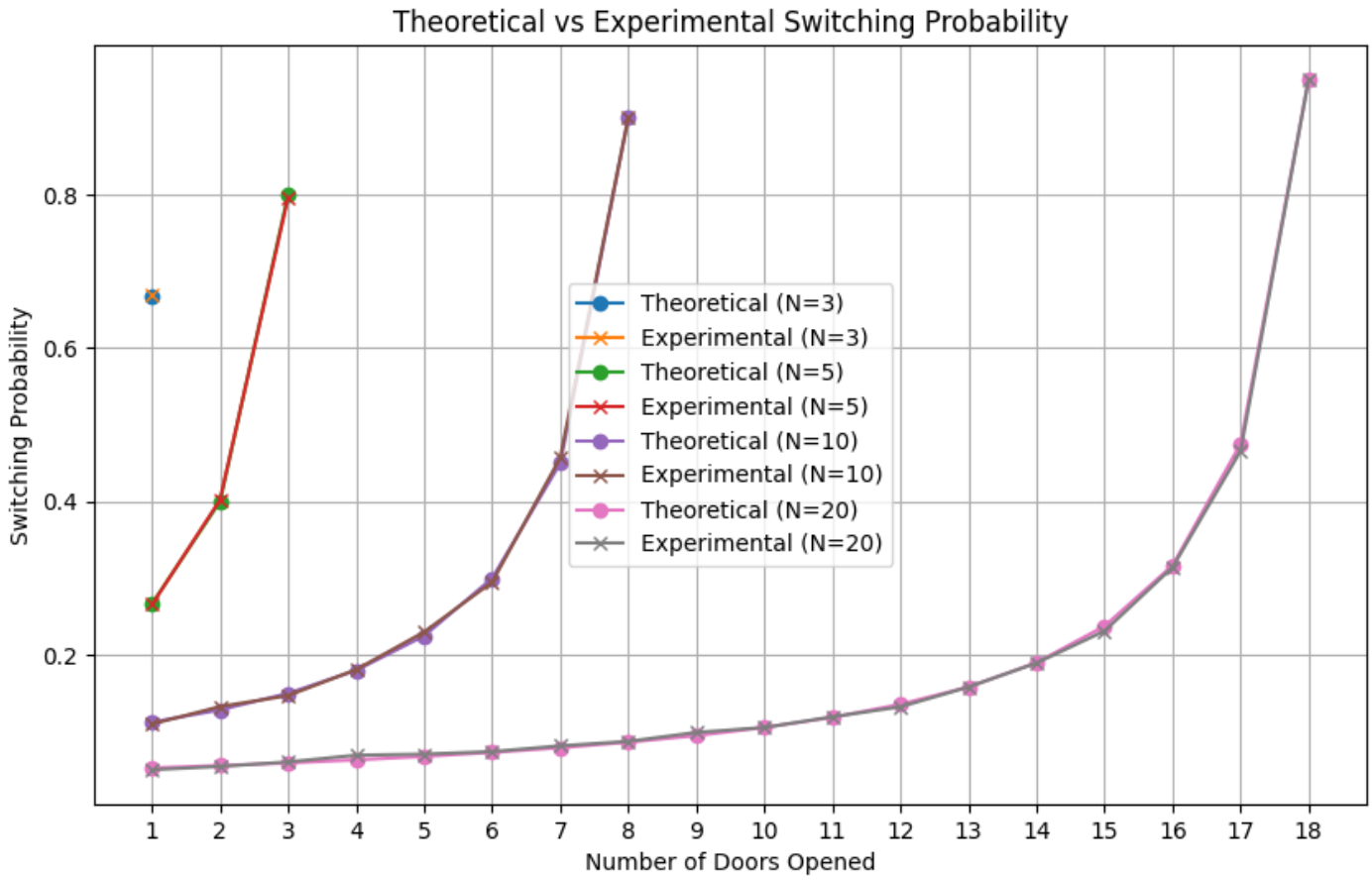

Figure 4: Switching Probability vs. Doors Opened (Simulations=10000)

Example

Let's consider an example with $n = 10$ doors and $k = 5$ doors opened by the host.

Table 1: Probabilities of winning with different numbers of simulations (100, 1000, 10000) and theoretical results

Simulation Type	**Switch Win Probability**	**Stick Win Probability**
100 Simulations	0.88	0.11
1000 Simulations	0.26	0.269
10000 Simulations	0.22	0.063
100000. Simulations	0.236	0.097
1000000 Simulations	0.228	0.098
Theoretical	0.225	0.1

As the number of simulations increases, the experimental results approach the theoretical results.