12.54

EE25BTECH11041 - Naman Kumar

Question:
For a matrix

$$\mathbf{M} = \begin{pmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & x \end{pmatrix} \tag{1}$$

the transpose of the matrix is equal to the inverse of the matrix, i.e., $\mathbf{M}^T = \mathbf{M}^{-1}$.The value of x is given by
Solution:

$$\mathbf{M}^T = \mathbf{M}^{-1} \tag{2}$$

Multiple (2) with $\mathbf{M}$

$$\mathbf{M}\mathbf{M}^T = \mathbf{I} \tag{3}$$

$\mathbf{M}$ is orthogonal matrix

$$\begin{pmatrix} \frac{4}{5} & \frac{-3}{5} \\ \frac{3}{5} & x \end{pmatrix}\begin{pmatrix} \frac{4}{5} & \frac{-3}{5} \\ \frac{3}{5} & x \end{pmatrix}^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} \frac{16}{25} + \frac{9}{25} & \frac{-12}{25} + \frac{3x}{25} \\ \frac{-12}{25} + \frac{3x}{25} & \frac{9}{25} + x^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{5}$$

$$\frac{-12}{25} + \frac{3x}{25} = 0 \tag{6}$$

$$x = \frac{4}{5} \tag{7}$$

$$\frac{9}{25} + x^2 = 1 \tag{8}$$

$$x^2 = \frac{16}{25} \tag{9}$$

$$x = \pm\frac{4}{5} \tag{10}$$

from (7) and (10)

$$x = \frac{4}{5} \tag{11}$$

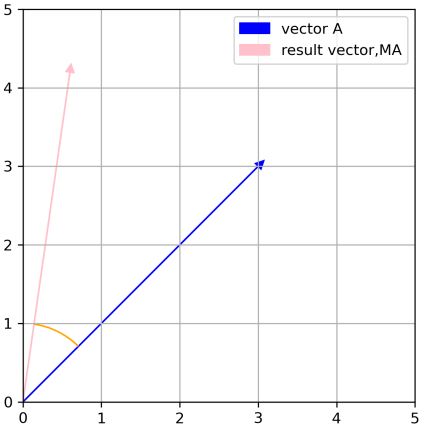

Figure 1

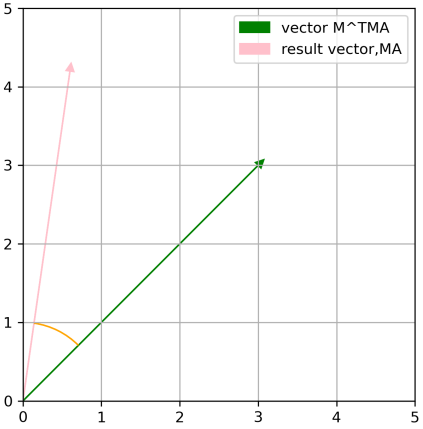

Figure 2