12.54

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Question)

For a matrix

$$\mathbf{M} = \begin{pmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & x \end{pmatrix} \tag{1}$$

the transpose of the matrix is equal to the inverse of the matrix, i.e., $\mathbf{M}^T = \mathbf{M}^{-1}$.The value of x is given by

Solution

$$\mathbf{M}^T = \mathbf{M}^{-1} \tag{2}$$

Multiple (2) with $\mathbf{M}$

$$\mathbf{M}\mathbf{M}^T = \mathbf{I} \tag{3}$$

Solution

M is orthogonal matrix

$$\begin{pmatrix} \frac{4}{5} & \frac{-3}{5} \\ \frac{3}{5} & x \end{pmatrix} \begin{pmatrix} \frac{4}{5} & \frac{-3}{5} \\ \frac{3}{5} & x \end{pmatrix}^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} \frac{16}{25} + \frac{9}{25} & \frac{-12}{25} + \frac{3x}{25} \\ \frac{-12}{25} + \frac{3x}{25} & \frac{9}{25} + x^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{5}$$

Solution

$$\frac{-12}{25} + \frac{3x}{25} = 0 \tag{6}$$

$$x = \frac{4}{5} \tag{7}$$

$$\frac{9}{25} + x^2 = 1 \tag{8}$$

$$x^2 = \frac{16}{25} \tag{9}$$

$$x = \pm\frac{4}{5} \tag{10}$$

Solution

from (7) and (10)

$$x = \frac{4}{5} \tag{11}$$

Figure

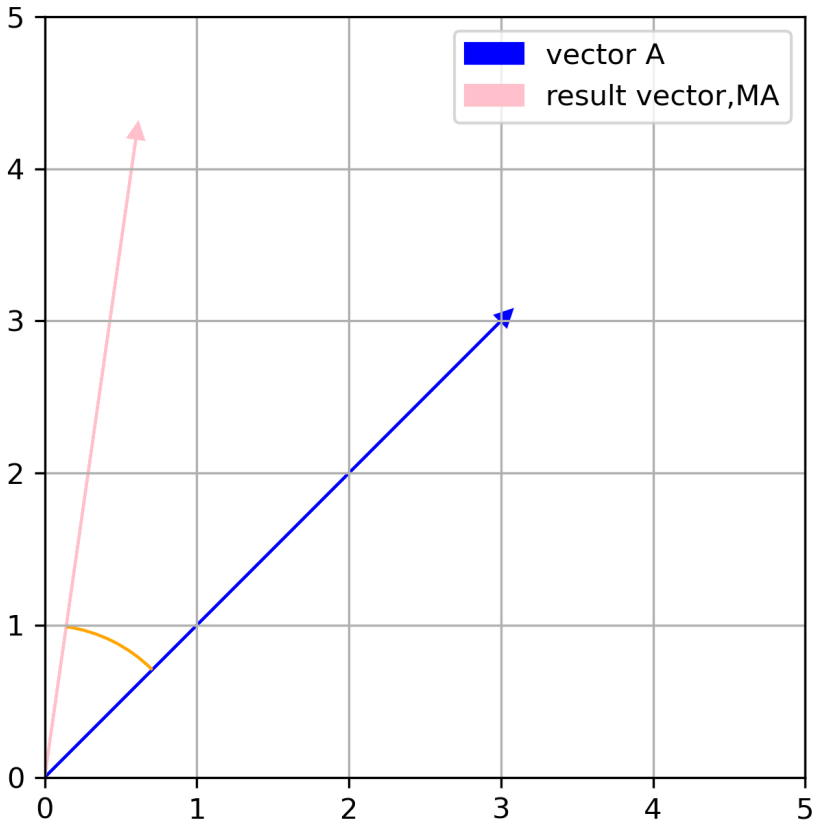

Figure: 1

Figure

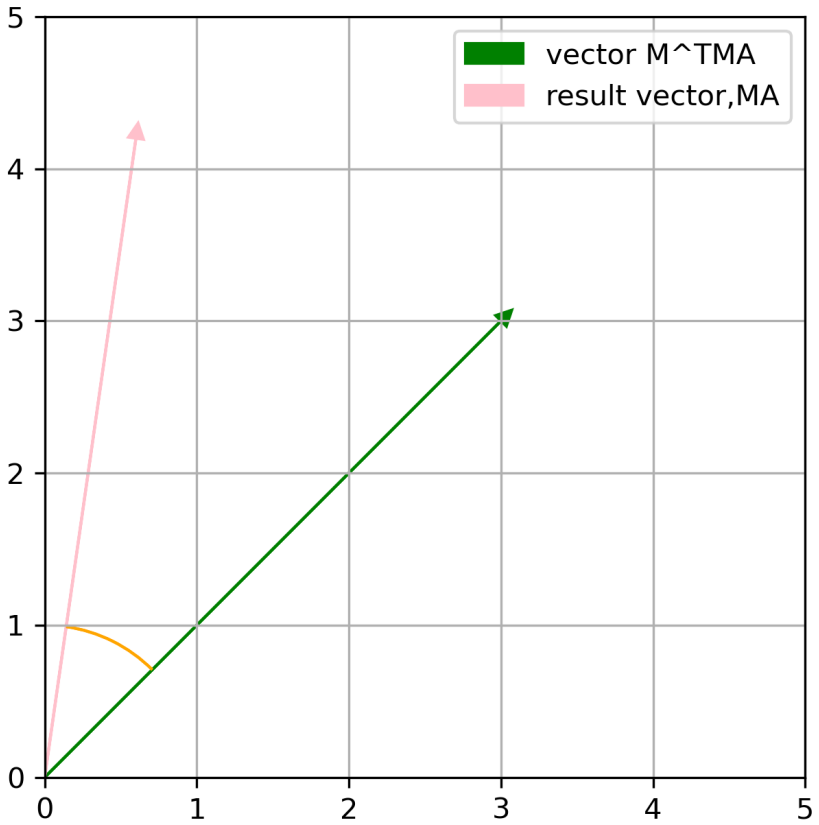

Figure: 2

Direct Python 1

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Arc


x = np.array([3 ,3]).reshape(-1,1)
y= np.array([3,1])
m = np.array([[0.8,-0.6],[0.6,0.8]])
fig, ax = plt.subplots()

ax.arrow(0, 0, 3, 3, head_width=0.1, head_length=0.1, fc='blue',
    ec='blue', label="vector A")
c = m@x
ax.arrow(0, 0, c[0,0], c[1,0], head_width=0.1, head_length=0.1,
    fc='pink', ec='pink',label="result vector,MA")
```

Direct Python 1

```
center = (0, 0)
radius = 1.0
start_angle = 45
end_angle = 81.87

arc = Arc(center, 2 * radius, 2 * radius, theta1=start_angle,
    theta2=end_angle,
          edgecolor='orange', linewidth=1)

ax.add_patch(arc)
ax.add_patch(arc)
```

Direct Python 1

```
ax.set_aspect('equal', adjustable='box')
ax.set_xlim(0, 5)
ax.set_ylim(0, 5)
plt.legend()
plt.grid()
plt.savefig("fig1.png", dpi=300)
plt.show()
```

Direct Python 2

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Arc


x = np.array([3 ,3]).reshape(-1,1)
y= np.array([3,1])
m = np.array([[0.8,-0.6],[0.6,0.8]])
fig, ax = plt.subplots()
```

Direct Python 2

```
ax.arrow(0, 0, 3, 3, head_width=0.1, head_length=0.1, fc='green',
     ec='green', label="vector M^TMA")
c = m@x
ax.arrow(0, 0, c[0,0], c[1,0], head_width=0.1, head_length=0.1,
    fc='pink', ec='pink',label="result vector,MA")
center = (0, 0)
radius = 1.0
start_angle = 45
end_angle = 81.87
```

Direct Python 2

```
arc = Arc(center, 2 * radius, 2 * radius, theta1=start_angle,
    theta2=end_angle,
          edgecolor='orange', linewidth=1)

ax.add_patch(arc)
ax.add_patch(arc)

ax.set_aspect('equal', adjustable='box')
ax.set_xlim(0, 5)
ax.set_ylim(0, 5)
plt.legend()
plt.grid()
plt.savefig("fig2.png", dpi=300)
plt.show()
```