Homework 1

Name: Naman Makkar

Student ID:nbm49

- Reasoning and work must be shown to gain partial/full credit
- Please include the cover-page on your homework PDF with your name and student ID. Failure of doing so is considered bad citizenship.

1. (1–4 points) **Fossil fuel generation**: You are an analyst working for an energy company that wants to diversify their portfolio of power assets. Your company wants to build three generating power plants with a total combined capacity of 2500 MW. The company wants you to decide where to build the power plants. To his end, they have given you the following information.

Table 1: Power Plant Characteristics.

	Technology	**Fuel**	**Heat Rate (MMBtu/MWh)**	**Capacity (MW)**	**CO2 Emissions (pounds/MMBtu)**
Plant 1	Gas turbine	Gas	12	500	117
Plant 2	Combined-cycle	Gas	8	1500	117
Plant 3	Steam turbine	Coal	11	500	209

Table 2: Fuel Cost by Location.

	Gas Price (\$/MMBtu)	**Electricity Price (\$/MWh)**	**Coal Price (\$/MMBtu)**
Location A	3	45	4.22
Location B	4.5	55	8
Location C	5	40	5

- **Technology**: Generation technology used by the power plant.
- **Fuel**: Fossil fuel used in the thermodynamic process to generate electricity.
- **Heat Rate**: Average amount of energy used to generate 1 MWh of electricity.
- **Capacity**: Maximum electricity generation that the power plant can provide.
- **CO2 Emissions**: CO2 emitted to the atmosphere per unit of energy burned.
- **Gas/Electricity/Coal Price**: Spot commodity prices at the given location.

Assume that the power plants are running 100% of the time at 100% capacity. Calculate the following for a full calendar year (8760 hours).

(a) (**0.5 pts**) Provide an expression to calculate the yearly revenue (in $) of the power plant.

Solution: The total revenue per year can be calculated as - **Total Energy generated by the power plant in a calendar year (in MWh)** x **Electricity Price (in $/MWh)**. Since there are 8760 hours in a calendar year this can be written as

$$Capacity(MWh) * 8760 * ElectricityPrice(\$/MWh) \tag{1}$$

In [1]:

```python
import numpy as np
import pandas as pd

locations = pd.read_csv('/kaggle/input/energy-systems-datasets/
    datasets/locations.csv')

power_plants = pd.read_csv('/kaggle/input/energy-systems-
    datasets/datasets/power_plants.csv')
#Q1a Revenue calculation for power plants
revenue = power_plants['Power Plant Capacity'] * 8760 *
    locations['Electricity Price (\$/MWh)']
revenue
```

Out[1]:

```
0    197100000
1    722700000
2    175200000
dtype: int64
```

(b) (**0.5 pts**) Provide an expression to calculate the yearly expenses (in $) of the power plant.

Solution: The total expenses for power plants using Gas as fuel can be calculated as - **Total Energy generated by the power plant in a calendar year (in MWh)** x **Heat Rate (MMBtu/MWh)** x **Gas Price($/MMBtu)** which can be written as

$$Capacity(MWh) * 8760 * HeatRate(MMBtu/MWh) * GasPrice(\$/MMBtu) \tag{2}$$

While the total expenses for power plant using Coal as fuel can be calculated as - **Total Energy generated by the power plant in a calendar year (in MWh)** x **Heat Rate (MMBtu/MWh)** x **Coal Price($/MMBtu)** which can be written as -

$$Capacity(MWh) * 8760 * HeatRate(MMBtu/MWh) * CoalPrice(\$/MMBtu) \tag{3}$$

In [2]:

```python
##Q1b Calculation of yearly expenses of power plants
expenses_gas = power_plants['Power Plant Capacity'][:2] * 8760
    * power_plants['Average Heat rate (MMBtu/MWh)'][:2] *
    locations['Gas Price (\$/MMBtu)'][:2]

expenses_gas
```

Out[2]:

```
0    157680000.0
1    473040000.0
dtype: float64
```

In [3]:

```
##Q1b Calculation of yearly expenses of power plants
expenses_coal = power_plants['Power Plant Capacity'][2] * 8760
    * power_plants['Average Heat rate (MMBtu/MWh)'][2] *
    locations['Coal Price (\$/MMBtu)'][2]
expenses_coal
```

Out[3]:

```
240900000.0
```

(c) (**0.5 pts**) Calculate the yearly CO2 emissions (in pounds) of each plant.

Solution:

1. **Calculation for Plant 1** The yearly CO2 emissions for any plant can eb calculated as - **Total Energy generated by the power plant in a calendar year (in MWh)** x **Heat Rate (MMBtu/MWh)** x **CO2 Emissions (pounds/MMBtu)** which can be written as -

$$Capacity(MWh) * 8760 * HeatRate(MMBtu/MWh) * CO2Emissions(pounds/MMBtu) \tag{4}$$

Therefore, for Plant 1, the yearly CO2 generated in pounds would be: **500x8760x12x117 = 6,149,520,000 pounds**

2. **Calculation for Plant 2** Similar to the calculation for Plant 1, the yearly CO2 emissions in pounds for Plant 2 can be calculated as **1500x8760x8x117 = 12,299,040,000 pounds**

3. **Calculation for Plant 3** Similarly, the yearly CO2 emissions for Plant 3 in pounds can be calculated as **500x8760x11x209 = 10,069,620,000 pounds**

In [4]:

```
power_plants['CO2 Emissions (pounds/MMBtu)'] = [117, 117, 209]
co2_emissions = power_plants['Power Plant Capacity'] * 8760 *
    power_plants['Average Heat rate (MMBtu/MWh)'] * power_plants
    ['CO2 Emissions (pounds/MMBtu)']
co2_emissions
```

Out[4]:

```
0     6149520000
1    12299040000
2    10069620000
dtype: int64
```

(d) (**1 pts**) Calculate the yearly profits (in $) that each power plant would generate at each location. (**Note**: Some power plants may not be profitable at certain locations)

Solution: In order to calculate yearly profits we can make use of the equations for yearly revenue and yearly expenses used in Part a and Part b.

1. **Calculation for Plant 1:**

   (a) **Profit generated at Location A:** We can calculate the profit using the following equation:

$$Capacity * 8760 * (ElectricityPrice - (GasPrice * HeatRate)) \tag{5}$$

Which comes out to be **500x8760x(45-36) = $39,420,000**

(b) **Profit generated at Location B:** We can similarly calculate the yearly profit for Plant 1 and Location B as **500x8760x(55 - 54) = $4,380,000**

(c) **Profit generated at Location C:** Similarly yearly profit generated by Plant 1 at Location C is **500x8760x(40 - 60) = - $87,600,000**. Therefore, Plant 1 suffers a loss of $87,600,000 at Location C after a year of operation.

2. **Calculation for Plant 2:**

   (a) **Profit generated at Location A:** This can be calculated similar to Plant 1 **1500x8760x(45-24) = $275,940,000**

   (b) **Profit generated at Location B:** Using the same equations: **1500x8760x(55-36) = $249,660,000**

   (c) **Profit generated at Location C:** We can similarly calculate the profit generated by Plant 2 at Location C as **1500x8760x(40 - 40) = $0**. Plant 2 therefore does not generate any profit at location C.

3. **Calculation for Plant 3:**

   (a) **Profit generated at Location A:** Profit at Location A can be calculated as **500x8760x(45-46.42) = -$6,219,600**. Plant 3 suffers a loss of $6,219,600 at location A.

   (b) **Profit generated at Location B:** Profit generated at Location B by Plant 3 can be calculated as **500x8760x(55-88) = -$144,540,000**. Plant 3 suffers a loss of $144,540,000 at Location B.

   (c) **Profit generated at Location C:** Can be calculated as: **500x8760x(40-55) = -$65,700,000**. Therefore, Plant 3 makes a loss of $65,700,000 at Location C.

In [5]:

```python
## Q1d calculating yearly profit
# Calculation for Plant 1
profit_plant_1_location_A = power_plants['Power Plant Capacity'][0] * 8760 * (locations['Electricity Price ($/MWh)'][0] - (power_plants['Average Heat rate (MMBtu/MWh)'][0] * locations['Gas Price ($/MMBtu)'][0]))
profit_plant_1_location_B = power_plants['Power Plant Capacity'][0] * 8760 * (locations['Electricity Price ($/MWh)'][1] - (power_plants['Average Heat rate (MMBtu/MWh)'][0] * locations['Gas Price ($/MMBtu)'][1]))
profit_plant_1_location_C = power_plants['Power Plant Capacity'][0] * 8760 * (locations['Electricity Price ($/MWh)'][2] - (power_plants['Average Heat rate (MMBtu/MWh)'][0] * locations['Gas Price ($/MMBtu)'][2]))
print(f'Profit generated by Plant 1 at location A is ${profit_plant_1_location_A}')
print(f'Profit generated by Plant 1 at location B is ${profit_plant_1_location_B}')
print(f'Profit generated by Plant 1 at location C is ${profit_plant_1_location_C}')

# Calculation for Plant 2
profit_plant_2_location_A = power_plants['Power Plant Capacity'][1] * 8760 * (locations['Electricity Price ($/MWh)'][0] - (power_plants['Average Heat rate (MMBtu/MWh)'][1] * locations['Gas Price ($/MMBtu)'][0]))
profit_plant_2_location_B = power_plants['Power Plant Capacity'][1] * 8760 * (locations['Electricity Price ($/MWh)'][1] - (power_plants['Average Heat rate (MMBtu/MWh)'][1] * locations['Gas Price ($/MMBtu)'][1]))
profit_plant_2_location_C = power_plants['Power Plant Capacity'][1] * 8760 * (locations['Electricity Price ($/MWh)'][2] - (power_plants['Average Heat rate (MMBtu/MWh)'][1] * locations['Gas Price ($/MMBtu)'][2]))
print(f'Profit generated by Plant 2 at location A is ${profit_plant_2_location_A}')
print(f'Profit generated by Plant 2 at location B is ${profit_plant_2_location_B}')
print(f'Profit generated by Plant 2 at location C is ${profit_plant_2_location_C}')

# Calculation for Plant 3
profit_plant_3_location_A = power_plants['Power Plant Capacity'][2] * 8760 * (locations['Electricity Price ($/MWh)'][0] - (power_plants['Average Heat rate (MMBtu/MWh)'][2] * locations['Coal Price ($/MMBtu)'][0]))
profit_plant_3_location_B = power_plants['Power Plant Capacity'][2] * 8760 * (locations['Electricity Price ($/MWh)'][1] - (power_plants['Average Heat rate (MMBtu/MWh)'][2] * locations['Coal Price ($/MMBtu)'][1]))
profit_plant_3_location_C = power_plants['Power Plant Capacity'][2] * 8760 * (locations['Electricity Price ($/MWh)'][2] - (power_plants['Average Heat rate (MMBtu/MWh)'][2] * locations['Coal Price ($/MMBtu)'][2]))
print(f'Profit generated by Plant 3 at location A is ${profit_plant_3_location_A}')
print(f'Profit generated by Plant 3 at location B is ${profit_plant_3_location_B}')
print(f'Profit generated by Plant 3 at location C is ${profit_plant_3_location_C}')
```

Out[5]:

```
Profit generated by Plant 1 at location A is $39420000.0
Profit generated by Plant 1 at location B is $4380000.0
Profit generated by Plant 1 at location C is $-87600000.0
Profit generated by Plant 2 at location A is $275940000.0
Profit generated by Plant 2 at location B is $249660000.0
Profit generated by Plant 2 at location C is $0.0
Profit generated by Plant 3 at location A is $-6219599.999999977
Profit generated by Plant 3 at location B is $-144540000.0
Profit generated by Plant 3 at location C is $-65700000.0
```

(e) (**1.5 pts**) Provide the optimal placing of power plants, i.e. the generator-location pairs that maximizes revenue. (**Note**: Only one power plant can be built at each location)

Hint: There are two methods to solve this problem. 1) By trying all possible combinations. There are $3!$ *combinations you may want to try. 2) Using optimization (Mixed-integer linear programming). Use 1) if you are not familiar with 2).*

Solution: The maximum revenue can be collected in the following configuration:

1. Plant 1 - Location A
2. Plant 2 - Location B
3. Plant 3 - Location C

This way the energy company would earn a total revenue of **500x8760x45 + 1500x8760x55 + 500x8760x40 = $1,095,000,000**

In [6]:

```python
## Q1e Calculating the best configuration for calculating revenue
revenue_plant_1_location_A = power_plants['Power Plant Capacity'][0] * 8760 * locations['Electricity Price ($/MWh)'][0]
revenue_plant_1_location_B = power_plants['Power Plant Capacity'][0] * 8760 * locations['Electricity Price ($/MWh)'][1]
revenue_plant_1_location_C = power_plants['Power Plant Capacity'][0] * 8760 * locations['Electricity Price ($/MWh)'][2]

revenue_plant_2_location_A = power_plants['Power Plant Capacity'][1] * 8760 * locations['Electricity Price ($/MWh)'][0]
revenue_plant_2_location_B = power_plants['Power Plant Capacity'][1] * 8760 * locations['Electricity Price ($/MWh)'][1]
revenue_plant_2_location_C = power_plants['Power Plant Capacity'][1] * 8760 * locations['Electricity Price ($/MWh)'][2]

revenue_plant_3_location_A = power_plants['Power Plant Capacity'][2] * 8760 * locations['Electricity Price ($/MWh)'][0]
revenue_plant_3_location_B = power_plants['Power Plant Capacity'][2] * 8760 * locations['Electricity Price ($/MWh)'][1]
revenue_plant_3_location_C = power_plants['Power Plant Capacity'][2] * 8760 * locations['Electricity Price ($/MWh)'][2]

print('Total Revenue in configuration Plant 1 - A, Plant 2 - B, Plant 3 - C is $', revenue_plant_1_location_A+revenue_plant_2_location_B+revenue_plant_3_location_C)
print('Total Revenue in configuration Plant 1 - A, Plant 2 - C, Plant 3 - B is $', revenue_plant_1_location_A+revenue_plant_2_location_C+revenue_plant_3_location_B)
print('Total Revenue in configuration Plant 1 - B, Plant 2 - A, Plant 3 - C is $', revenue_plant_1_location_B+revenue_plant_2_location_A+revenue_plant_3_location_C)
print('Total Revenue in configuration Plant 1 - B, Plant 2 - C, Plant 3 - A is $', revenue_plant_1_location_B+revenue_plant_2_location_C+revenue_plant_3_location_A)
print('Total Revenue in configuration Plant 1 - C, Plant 2 - A, Plant 3 - B is $', revenue_plant_1_location_C+revenue_plant_2_location_A+revenue_plant_3_location_B)
print('Total Revenue in configuration Plant 1 - C, Plant 2 - B, Plant 3 - A is $', revenue_plant_1_location_C+revenue_plant_2_location_B+revenue_plant_3_location_A)
```

Out[6]:

```
Total Revenue in configuration Plant 1 - A, Plant 2 - B, Plant 3 - C is $ 1095000000
Total Revenue in configuration Plant 1 - A, Plant 2 - C, Plant 3 - B is $ 963600000
Total Revenue in configuration Plant 1 - B, Plant 2 - A, Plant 3 - C is $ 1007400000
Total Revenue in configuration Plant 1 - B, Plant 2 - C, Plant 3 - A is $ 963600000
Total Revenue in configuration Plant 1 - C, Plant 2 - A, Plant 3 - B is $ 1007400000
Total Revenue in configuration Plant 1 - C, Plant 2 - B, Plant 3 - A is $ 1095000000
```

The company would generate the same amount of revenue in the configuration:

1. Plant 1 - Location C
2. Plant 2 - Location B
3. Plant 3 - Location A

However, the company would generate the most profits in the configuration:

1. Plant 1 - Location B
2. Plant 2 - Location A
3. Plant 3 - Location C

With this configuration the company would generate a profit of **275,940,000 + 4,380,000 - 65,700,000 = $214,620,000**

2. (1–4 points) **Renewable generation**: Your boss changed his mind and now wants you to study the financial viability of a renewable energy project. In the past, your company did some business with Vestas, the leading wind turbine manufacturer in the world. Vestas is willing to give you a good deal on one of their new wind turbine models, the V150-4.5[1], a 4.5 MW wind turbine. The cost of each unit is 7 million US dollars. Similar to problem 1, you have been given the following information

   - **wind_speed.csv**: This file contains one year (2018) of 10-min resolution wind speed time series at the location of interest. The wind speed is measured at the hub height of the turbine.
   - **wind_power_curve.csv**: This is the wind power curve provided by the manufacturer. This curves allows you to simulate the performance of the wind turbine model given the wind speed at any time instance.

   The project consists of the following deliverables.

   (a) (**0.5 pts**) Plot the empirical distribution of wind speeds, i.e. a normalized histogram.

   Solution:

In [7]:

```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

df = pd.read_csv("/kaggle/input/energy-systems-datasets/
    datasets/wind_speed.csv",index_col=0,parse_dates=True)
plt.hist(df.wind_speed_ms, label="wind_speed_ms", histtype="
    step", density=True)
plt.legend()
plt.savefig('/kaggle/working/normalized_histogram.png')
plt.show()
```

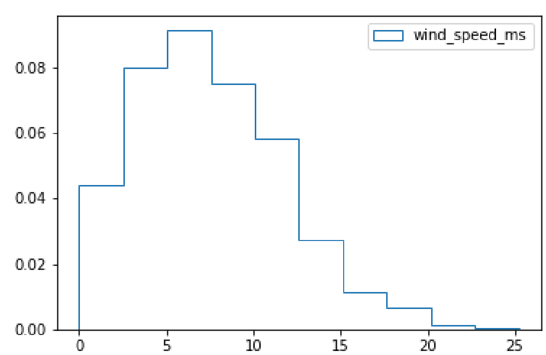

Figure 1: Normalized Histogram plot of Wind Speed in m/s

[1] https://us.vestas.com/en-us/products/4-mw-platform/v150-4-5-mw

(b) (**0.75 pts**) Calculate the wind generation for every time interval available in the dataset, and plot the distribution of wind generation.

Solution:

In [8]:

```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy import interpolate

df2 = pd.read_csv('/kaggle/input/energy-systems-datasets/
    datasets/wind_power_curve.csv')

x = np.array(df2['wind_speed_ms'])
y = np.array(df2['power_output_kw'])
f = interpolate.interp1d(x, y, fill_value='extrapolate')
wind_speeds = np.array(df['wind_speed_ms'])
power = f(wind_speeds)
power[13105] = 0
power
```

Out[8]:

```
array([ 507.79283655,  628.20206917,  475.99147916, ...,
    2096.53014013, 2910.39007685, 3429.19265903])
```

In [9]:

```python
df['power'] = power
fig = df.power.plot()
fig.get_figure().savefig("/kaggle/working/power_generated.png")
plt.ylabel('Power in KW')
```

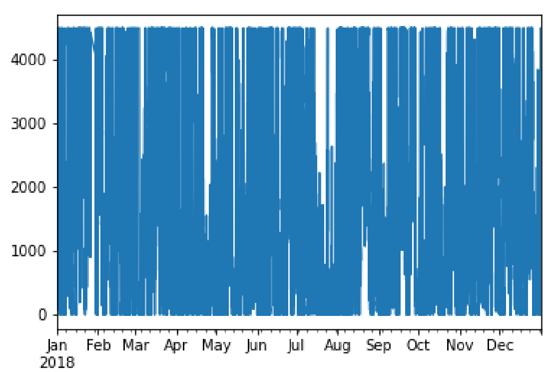

Figure 2: Wind generation plotted

(c) (**0.75 pts**) Calculate the theoretical maximum power that the turbine could harvest for all the time intervals available in the dataset. This can be calculated as follows

$$p_t = \frac{1}{2}\rho A v_t^3 \tag{6}$$

where p_t is the theorical maximum power at time t, in Watts, $\rho := 1.23kg/m^3$ is the air density and is assumed to be constant. A is the area of the rotor and v_t is the wind speed in m/s. To calculate A, you can assume that the diameter of the rotor is 128m.

Solution:

In [10]:
```
p_t_watts = 0.5 * 1.23 * (np.pi * 64 * 64) * np.power(
    wind_speeds, 3)
p_t = p_t_watts * 0.001 # theoretical maximum power in kW
p_t_watts
```

Out[10]:
```
array([1185758.57695139, 1444215.23686529, 1123070.12836216, ...,
       4750020.68156786, 6617996.40361377, 7864830.12213182])
```

(d) (**0.5 pts**) Calculate the efficiency of the wind turbine for every time instance, i.e. the ratio between the solution of Part (b) and Part (c). What is the maximum efficiency achieved by the turbine in the entire time series? What is the average efficiency? How does this compare to the Betz limit? Comment your results.

Solution:

In [11]:
```
efficiency = [] # calculating efficiency of wind turbine
for idx,elem in enumerate(p_t):
    if elem != 0:
        efficiency.append(power[idx] / p_t[idx])
    else:
        efficiency.append(0)
efficiency = np.array(efficiency)
efficiency.max()
```

Out[11]:
```
0.44309899982295775
```

In [12]:
```
efficiency.mean()
```

Out[12]:
```
0.3194899851093223
```

The maximum efficiency achieved by the turbine throughout the time series is **0.44** and the average efficiency is **0.32**. This is much lower than Betz limit of **0.593** which was to be expected since the best wind turbines today can only achieve an efficiency of **0.5**. Additionally, it can be noted that the average power generated in 2018 is **1.86MW** which provides an efficiency of **0.41** for the **4.5MW** wind turbine which is consistent with our analysis.

(e) (**0.75 pts**) Now, assume that on average you can get paid \$40/MWh of electricity you generate. What is the amortization period of the wind turbine? (You can assume the operation and maintenance cost is 0)

Solution: Cost per unit of the wind turbines is \$ 7 million, and each unit is a 4.5MW wind turbine. The amortization period can be calculated as:

In [13]:

```python
investment_per_unit = 7000000
amortization_time_years = investment_per_unit / (4.5 * 8760 *
    40)
print(amortization_time_years)
```

Out[13]:

```
4.439370877727042
```

The amortization period of the wind turbine is approximately **4.44** years.

(f) (**0.75 pts**) Finally, using the average power the turbine generated in 2018, how many turbines would you need to generate as much power as the combined-cycle unit from Problem 1? You can assume that the combined-cycle unit is online 100% of the time and generates electricity at full capacity (1500MW) every single hour of the year.

Solution:

In [14]:

```python
print(power.mean() / 1000) # in MW
```

Out[14]:

```
1.8660316721306367
```

The average power generated by the turbine in 2018 is **1.86MW** while that generated by the combined-cycle unit in Problem 1 is **1500MW**, therefore, we would require **1500/1.86 = 804 wind turbines** in order to generate the same power as the combined cycle unit.