Tutorial-4

① $T(n) = 3T(n/2) + n^2$

Ans⇒ $a=3$, $b=2$ $f(n)=n^2$

$n^{\log_b a} = n^{\log_2 3}$

comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (case 3)

∴ according to master theorem

$T(n) = \Theta(n^2)$

② $T(n) = 4T(n/2) + n^2$

$a=4$, $b=2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (case 2)

∴ acc. to master's theorem $T(n) = \Theta(n^2 \log n)$

③ $T(n) = T(n/2) + 2^n$

$a=1$, $b=2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (Case 3)

∴ Acc. to master theorem $T(n) = \Theta(2^n)$

④ $T(n) = 2^n T(n/2) + n^n$

∴ Master's theorem is not applicable as a function of n.

⑤ $T(n) = 16T(n/4) + n$

$a=16$ $b=4$, $f(n)=n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > f(n)$ (case 1)

$T(n) = \Theta(n^2)$

(6) $T(n) = 2T(n/2) + n \log n$

$a = 2, b = 2, f(n) = n \log n$

$n^{\log_b a} = n^{\log_2 2} = n$

Now $f(n) > n$

$\therefore$ Acc. to master $T(n) = \Theta(n \log n)$

(7) $T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2 \quad f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

$\therefore$ Acc. to master theorem $T(n) = \Theta(n)$

(8) $T(n) = 2T(n/4) + \cancel{\Theta}\, n^{0.51}$

$a = 2, b = 4, f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

$\therefore$ Acc. to master theorem $T(n) = \Theta(n^{0.5})$

(9) $T(n) = 0.5T(n/2) + \frac{1}{n}$

$\therefore$ Master's not applicable as $a < 1$

(10) $T(n) = 16T(n/4) + n!$

$a = 16, b = 4, f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

$\therefore$ Acc. to master $T(n) = \Theta(n!)$

(11) $T(n) = 4T(n/2) + \log n$

$a = 4$, $b = 2$ $\quad f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

$\therefore$ Acc. to master's $T(n) = \Theta(n^2)$

(12) $T(n) = \text{sqrt}(n)T(n/2) + \log n$

$\therefore$ Master's not applicable as a is not const.

(13) $T(n) = 3T(n/2) + n$

$a = 3$, $b = 2$, $f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

$\therefore$ Acc. to master theorem $T(n) = \cancel{\Theta(\log}$

$\Rightarrow \Theta(n^{\log_2 3})$

(14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3$, $b = 3$ $\quad f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

$\therefore$ Acc. to master theorem $T(n) = \Theta(n)$

(15) $T(n) = 4T(n/2) + Cn$

$a = 4$, $b = 2$, $f(n) = C * n$

$n^{\log_b a} = \cancel{\Theta}\, n^{\log_2 4} = n^2$

$n^2 > C * n$

$\therefore$ Acc. to master theorem $T(n) = \Theta(n^2)$

(16) $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

$\therefore$ Acc. to master's theorem $T(n) = \Theta(n\log n)$

(17) $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3,\ f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$\Theta(n) = \Theta(n/2)$

$\therefore$ Acc. to master's theorem.

$T(n) = \Theta(n\log n)$

(18) $T(n) = 6T(n/3) + n^2\log n$

$a = 6,\ b = 3,\ f(n) = n^2\log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2\log n$.

$\therefore$ Acc. to master's theorem $T(n) = \Theta(n^2\log n)$

(19) $T(n) = 4T(n/2) + n/\log n$;

$a = 4,\ b = 2,\ f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$

$\therefore$ Acc. to master's theorem $T(n) = \Theta(n^2)$

(20) $T(n) = 64T(n/8) - n^2 \log n$

Master theorem is not applicable as $f(n)$ is not increasing function.

(21) $T(n) = 7T(n/3) + n^2$

$a = 7$, $b = 3$, $f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

$\therefore$ Acc. to master's, $T(n) = O(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

Master's theorem isn't applicable since regularity condition is isolated in Case 3.